New Complexity Results and Algorithms for Minimum Tollbooth Problem

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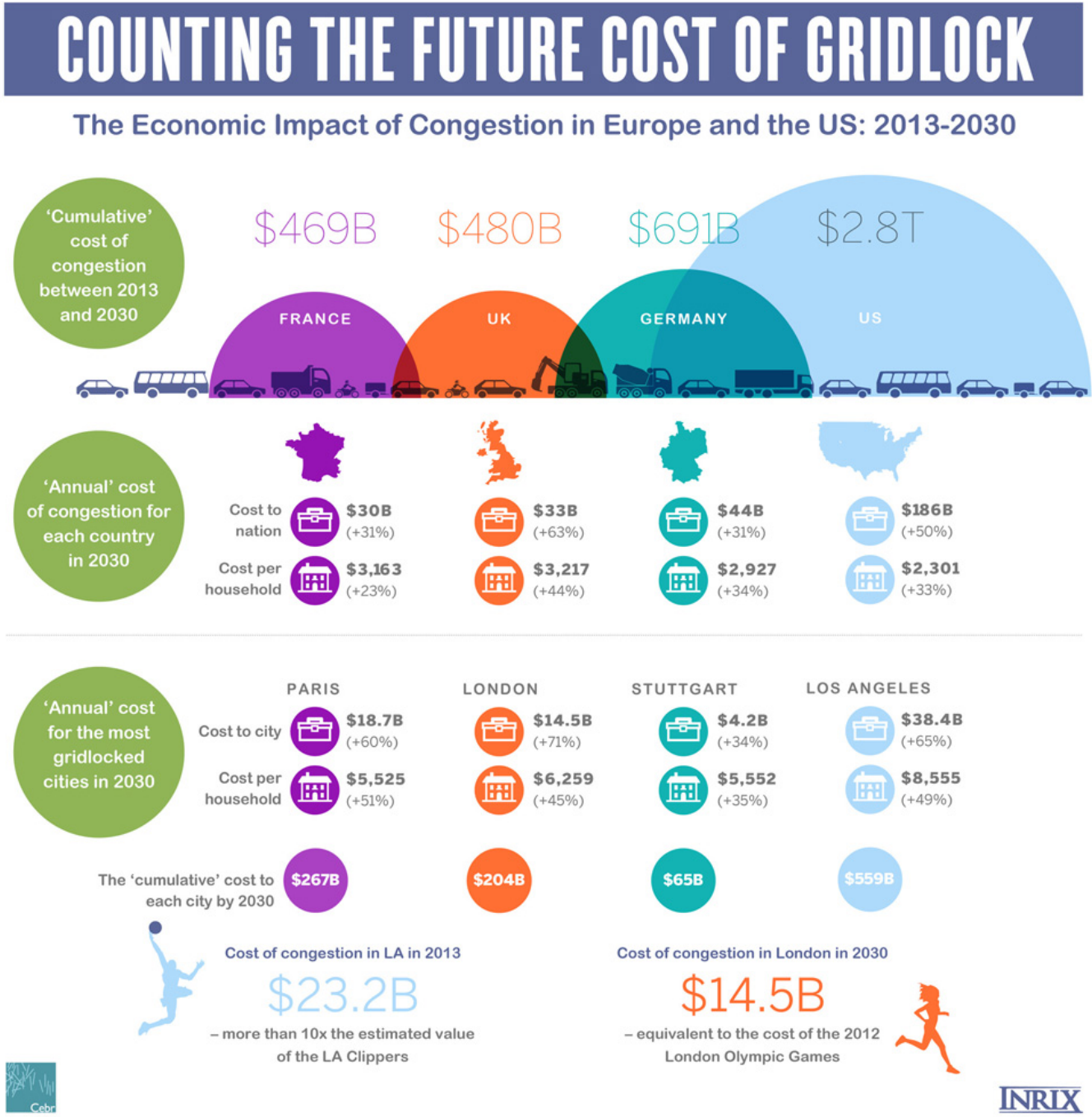
COUNTING THE FUTURE COST OF GRIDLOCK
The Economic Impact of Congestion in Europe and the US: 2013-2030
'Cumulative' cost of congestion between 2013 and 2030
$469B
$480B
$691B
$2.8T
FRANCE
UK
GERMANY
US
'Annual' cost of congestion for each country in 2030
Cost to nation
$30B (+31%)
$33B (+63%)
$44B (+31%)
$186B (+50%)
Cost per household
$3,163 (+23%)
$3,217 (+44%)
$2,927 (+34%)
$2,301 (+33%)
'Annual' cost for the most gridlocked cities in 2030
PARIS
LONDON
STUTTGART
LOS ANGELES
Cost to city
$18.7B (+60%)
$14.5B (+71%)
$4.2B (+34%)
$38.4B (+65%)
Cost per household
$5,525 (+51%)
$6,259 (+45%)
$5,552 (+35%)
$8,555 (+49%)
The 'cumulative' cost to each city by 2030
$267B
$204B
$65B
$559B
Cost of congestion in LA in 2013
$23.2B
– more than 10x the estimated value of the LA Clippers
Cost of congestion in London in 2030
$14.5B
– equivalent to the cost of the 2012 London Olympic Games
Cebr
INRIX

Congestion Gridlock

“ The combined annual cost of gridlock to these (U.S., U.K., France and Germany) countries is expected to soar to $ 293.1 billion by 2030. . . ” –INRIX and the CEBR

How to tackle congestion?

Introduction

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- Infrastructure growth is good.

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- But Not Always: **Selfish users** lead us to **Braess' Paradox**
- Solution: Influence the users by placing appropriate Tolls

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- But Not Always: **Selfish users** lead us to **Braess' Paradox**
- Solution: Influence the users by placing appropriate Tolls
- No Free Lunch: Each toll comes with **large overhead**

Outline

System Model

- Directed graph $G(V, E)$ with single commodity (s, t)
- Flow dependent latency, $\mathcal{L} = \{\ell_e(\cdot) : \forall e \in E\}$
- Traffic network, $\mathcal{G} = \{G, (s, t), \mathcal{L}\}$

Social Optimum (SO)

SO flow o is a flow that minimizes social cost.

Nash Equilibrium (NE)

A flow is said to be in NE iff property (1) and (2) holds:
(1) All used paths cost **L** (2) All unused paths cost $\geq$ **L**

Problem Definition

- **Tolled edge cost**: Under toll θ and flow f, tolled edge cost $c_e = \ell_e(f_e) + \theta_e$
- **Induce flow** f: Under the **tolled edge cost** f is in NE
- **Induced length**: Under NE the **tolled cost** of used paths

Minimum Tollbooth Problem (MINTB)

- Given traffic network $\mathcal{G}$ and an optimal flow **o**
- Find toll with **minimum support** which **induces** o

MINTB: Where do we stand?

- Formulated as Mixed integer linear program (**MILP**). *Hearn et al., 1998*
- Heuristics developed: **Genetic Algorithm, LP relaxation**. *Hardwood et al., '08; Bai et al., '09*
- In Multi-commodity networks it is **NP-hard**. *Bai et al.,'08*
- Design toll for inducing **general flow**. *Harks et al.,'08*

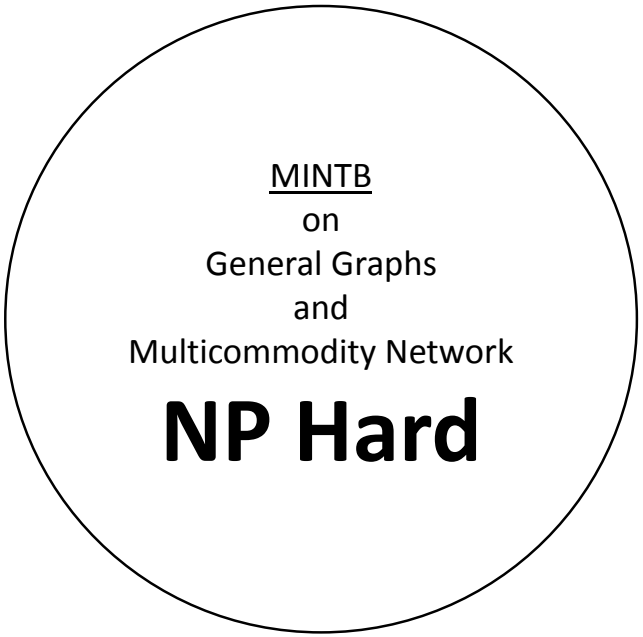


Figure: Complexity Diagram

Contributions

- **First APX-hardness:**
  Single commodity networks with **linear latencies**.
- **MINTB with used edges only**:
  Is *MINTB* efficiently solvable? **No still NP hard.**
- **First Exact Poly-time Algorithm**:
  Algorithm for **Series-Parallel** graphs.

First APX hardness result

Theorem

*For instances with **linear latencies** and **single commodity**, it is **NP-hard to approximate** the solution of MINTB by a factor of less than* **1.1377**.

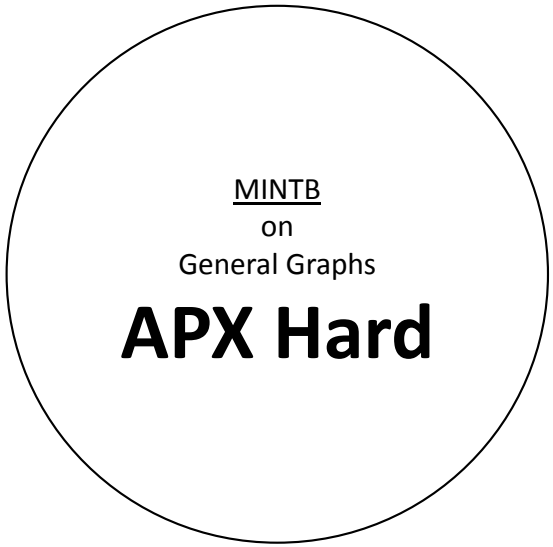


Figure: Complexity Diagram

MINTB Reduction: Vertex Cover

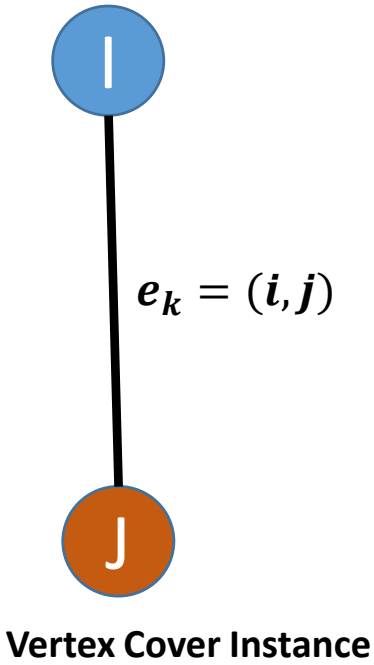
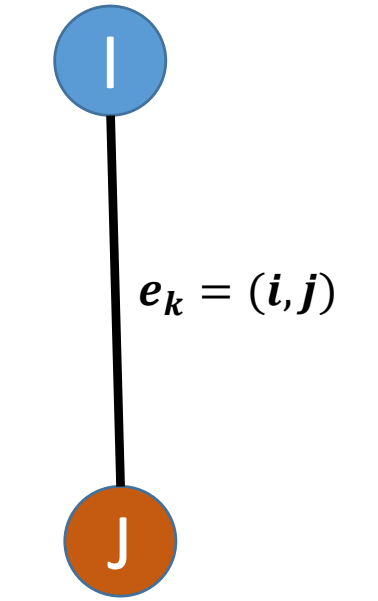
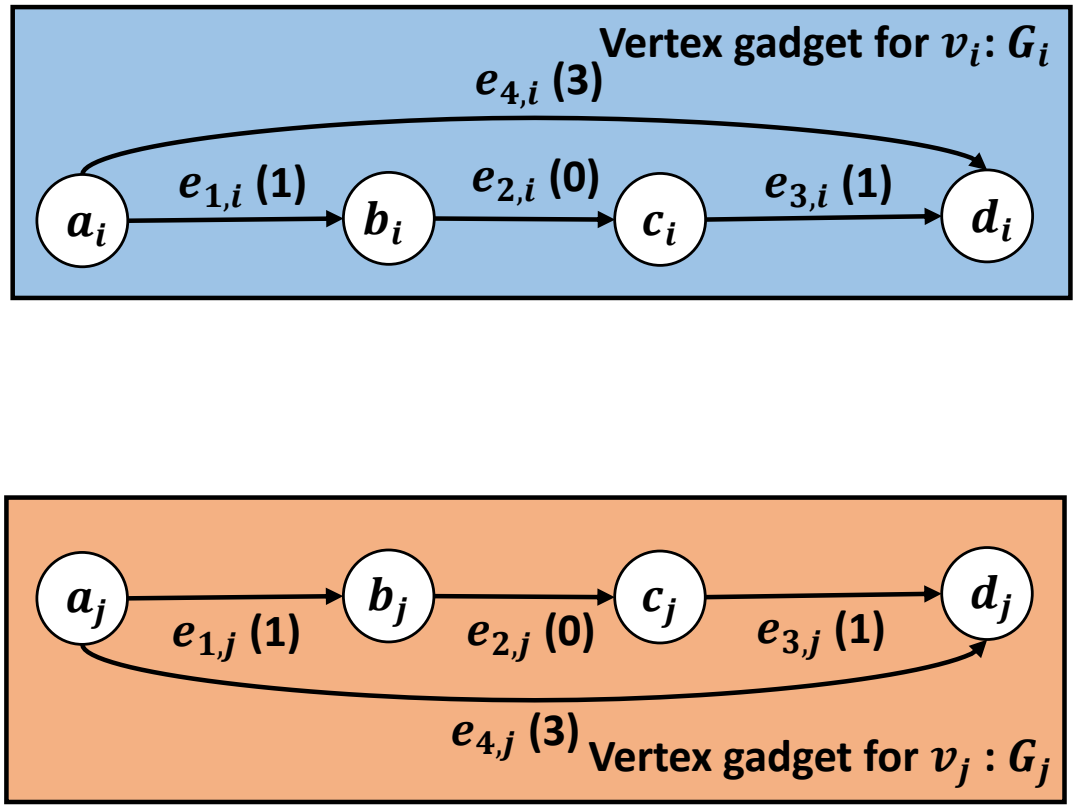


Figure: MINTB Reduction

MINTB Reduction: Vertex Cover



Vertex Cover Instance



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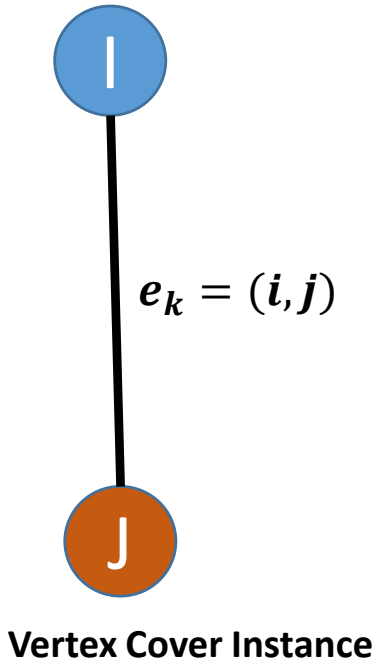

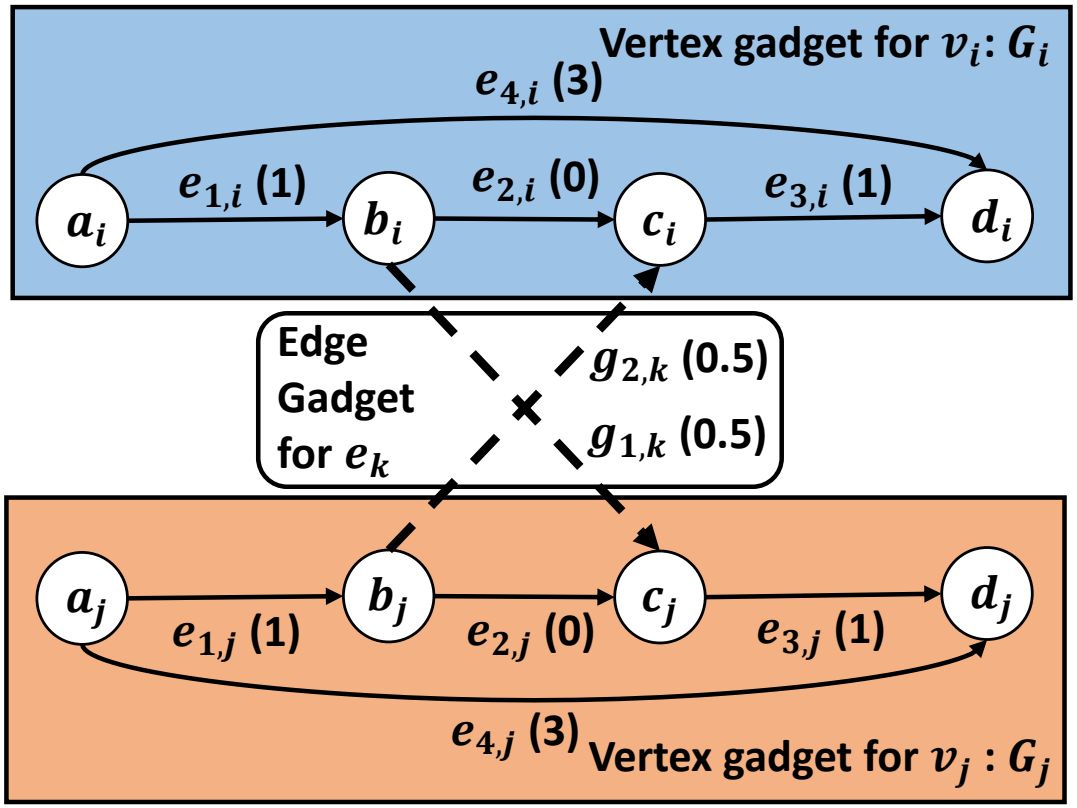

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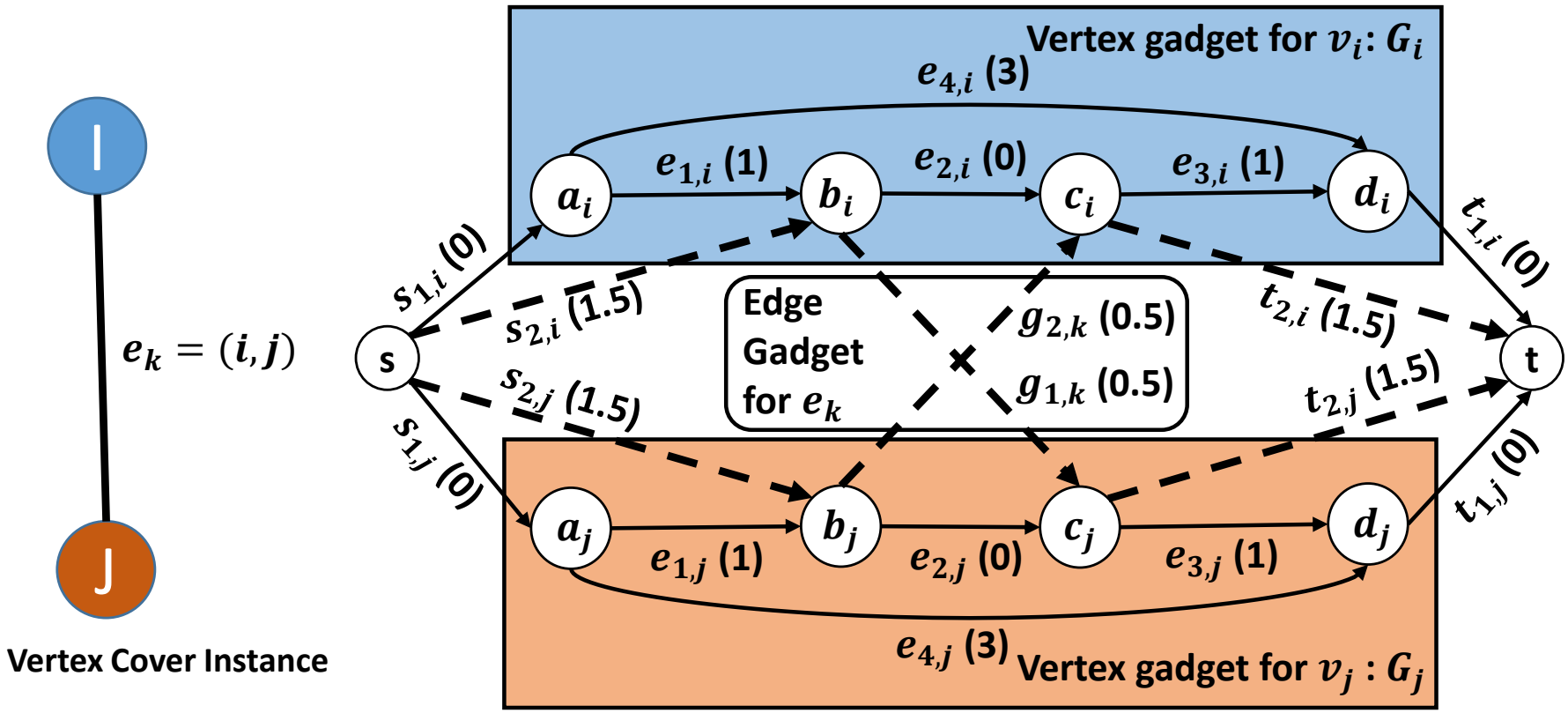

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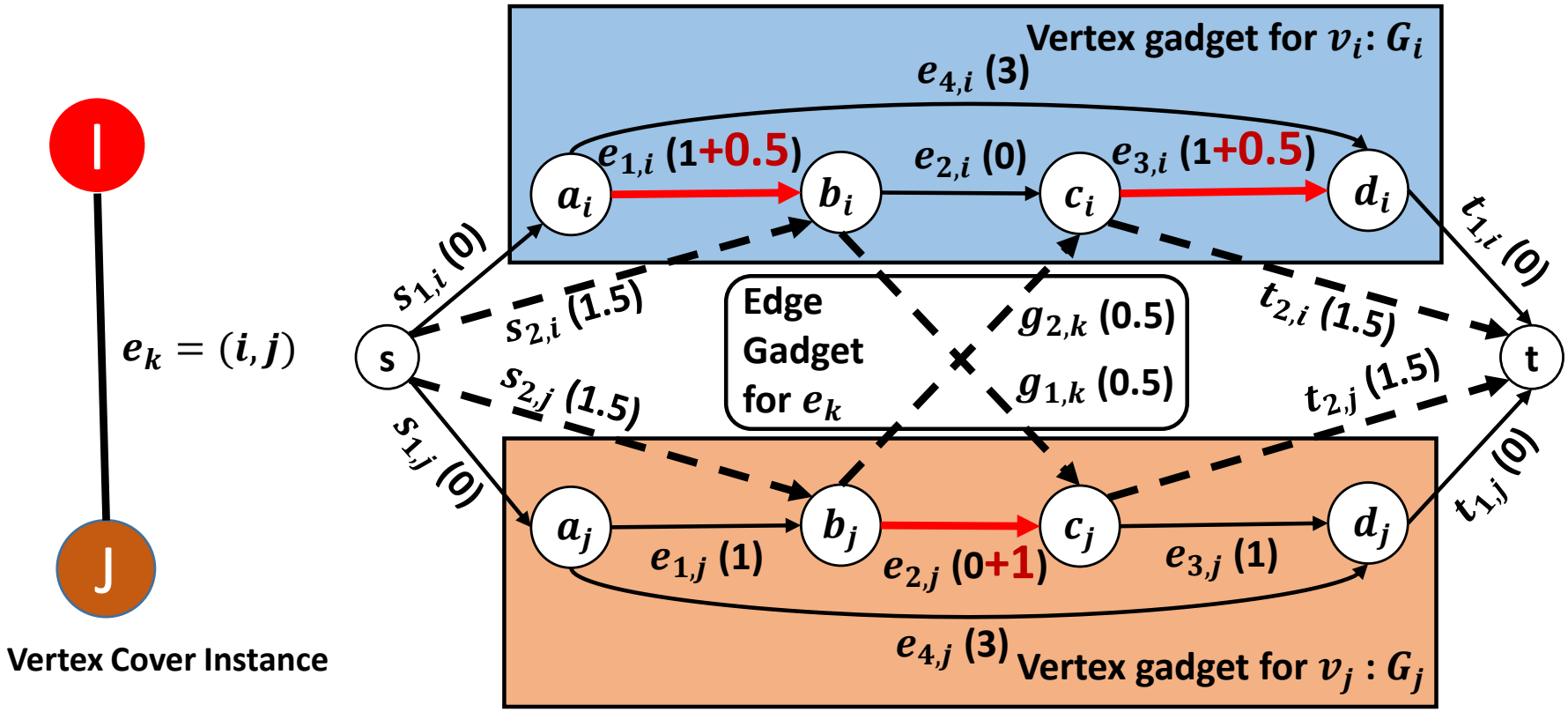


Figure: MINTB Reduction

MINTB Reduction: Vertex Cover

- Given Vertex Cover instance G_{vc} with n_{vc} vertices
- Create a Single Commodity network $\mathcal{G}$

Lemma 1

$\exists$ A Vertex Cover of size $\mathbf{x}$ in G_{vc} $\iff$
$\exists$ Opt-inducing toll with support $\mathbf{n_{vc}} + \mathbf{x}$ in $\mathcal{G}$.

Theorem

It is NP hard to approximate Minimum Vertex Cover to within a factor of **1.3606**. *Dinur et al 2005*

Are Unused Edges the Troublemaker?

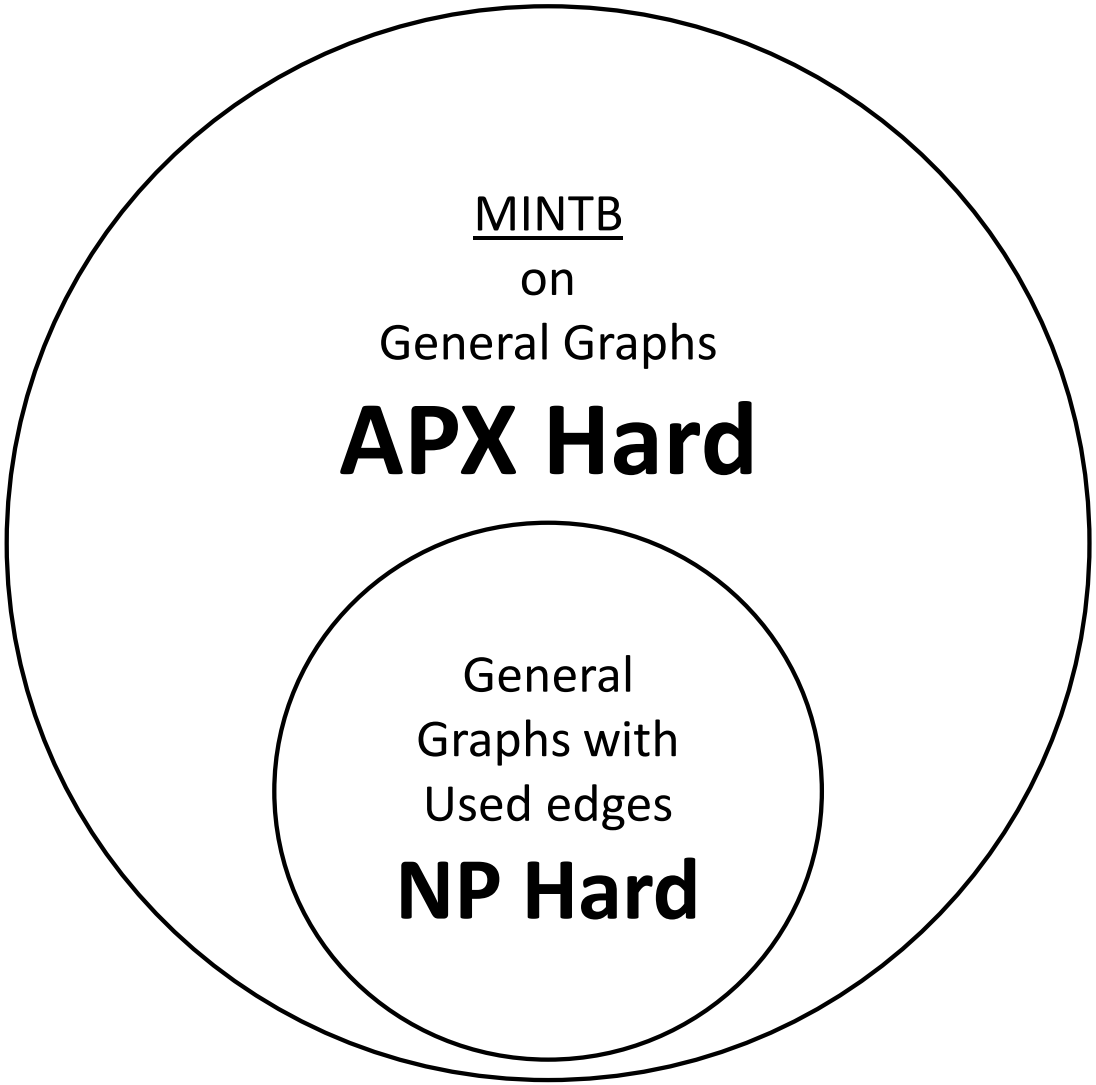
- **Motivation:** Less overhead present in edge removal.
- **Model:** Unused edges in social optimum flow are removed.

Theorem

*For instances with **linear latencies**, it is **NP-hard** to solve MINTB even if **all edges are used** by the optimal solution.*

Reduction follows from *PARTITION* problem.

Complexity Diagram



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Series Parallel Graphs

Series Parallel Graphs (*SP*)

An *SP* graph is created by starting from a *directed edge* and *inductively connecting* two graphs in *series or in parallel*.

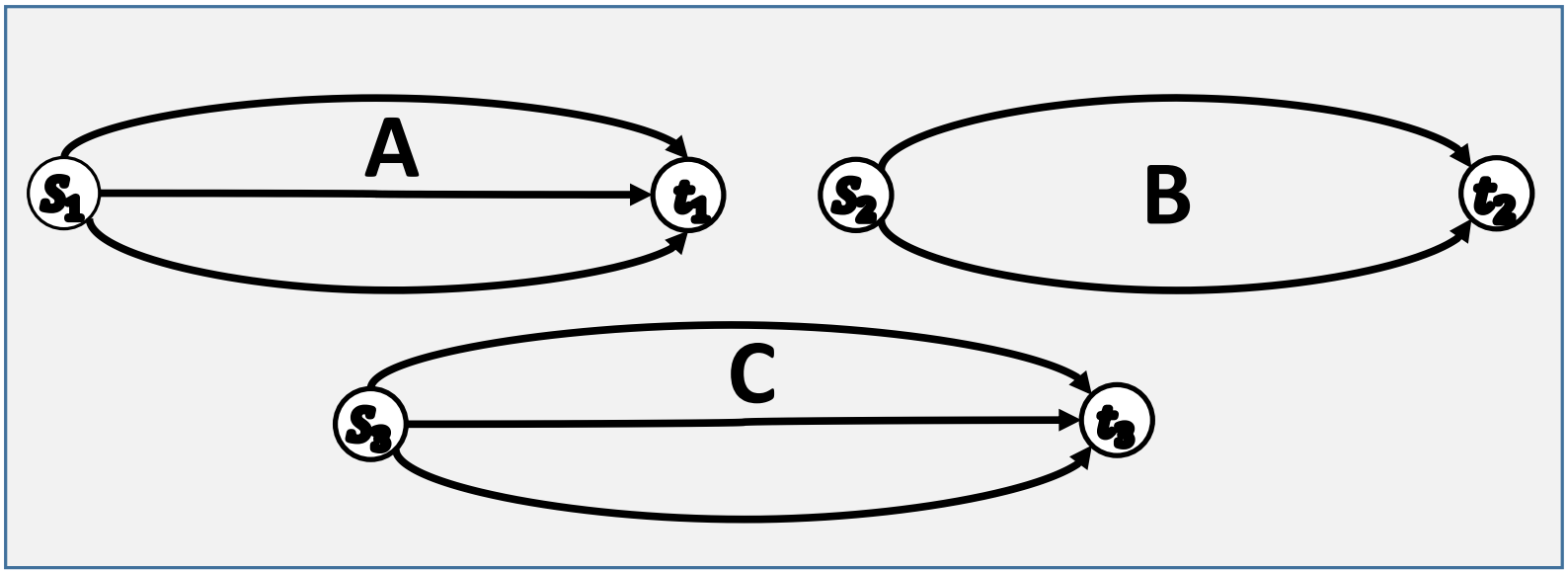

Figure: *SP* Graph

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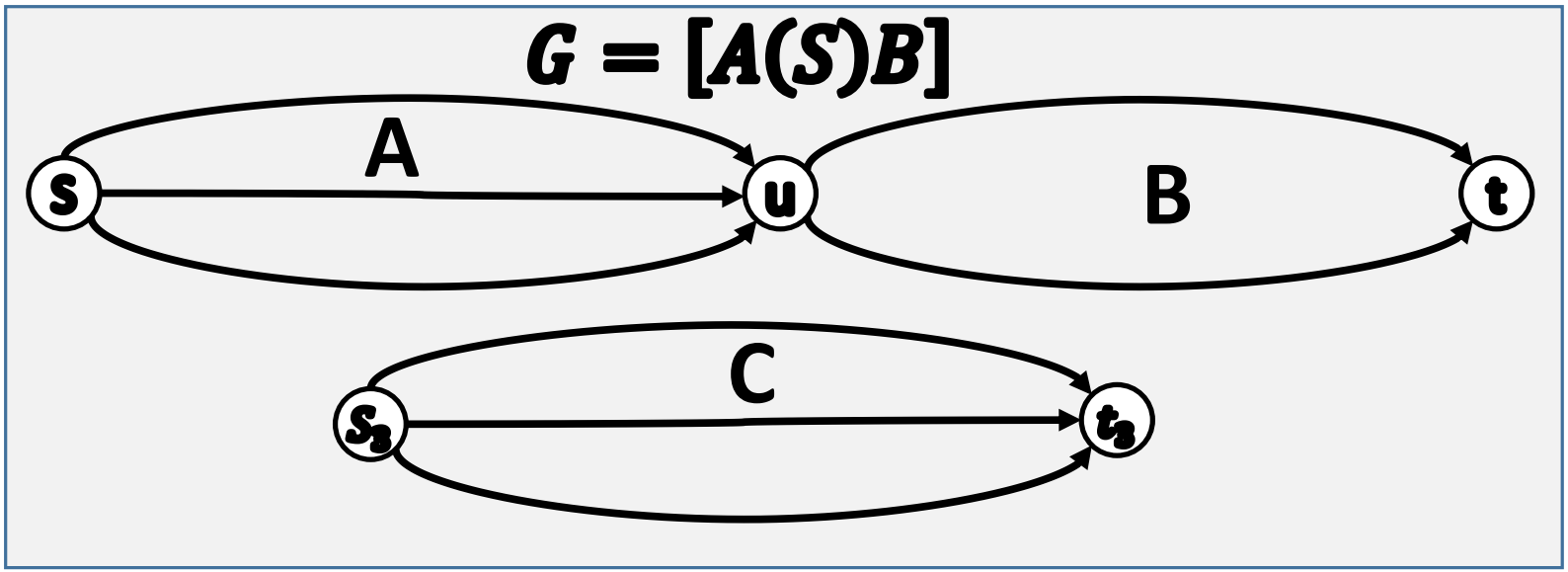

Figure: *SP* Graph and Parse Tree

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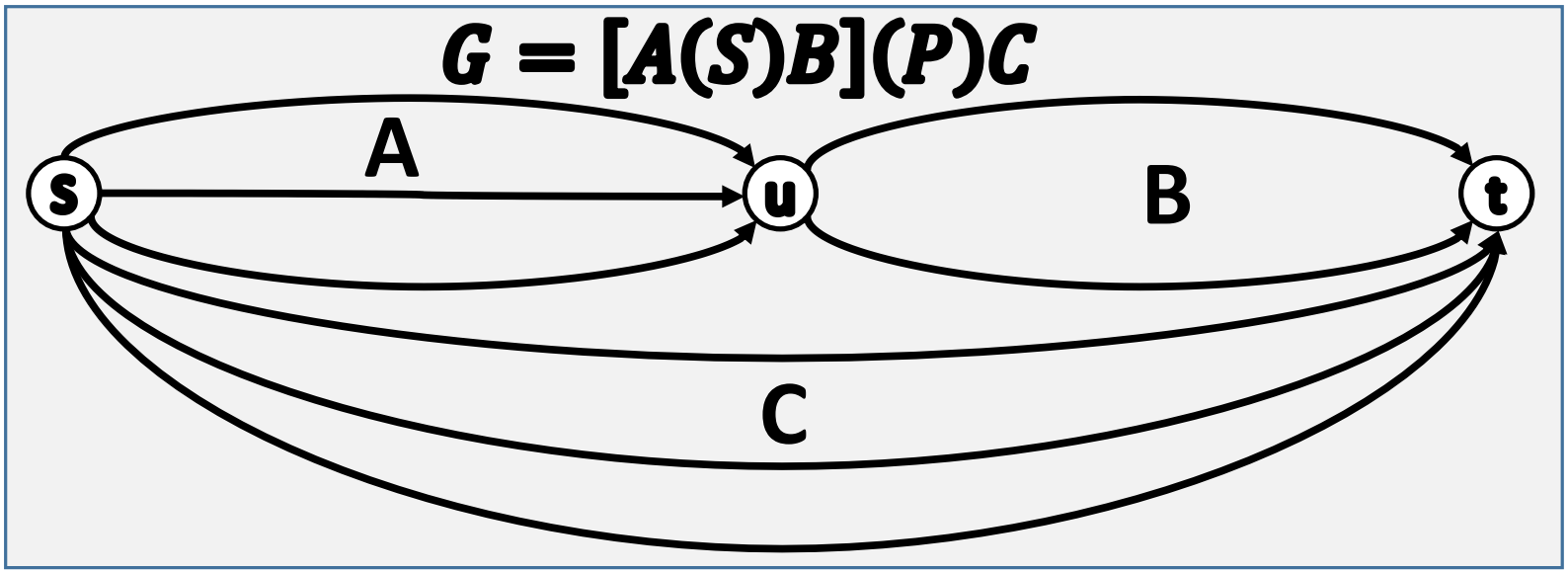

Figure: *SP* Graph

Algorithm for Parallel Link (PL) Graph

Algorithm 1

Input: A parallel link network
Output: Induce L with min support

1. Sort edges
2. Append length $\ell_{end} = \infty$

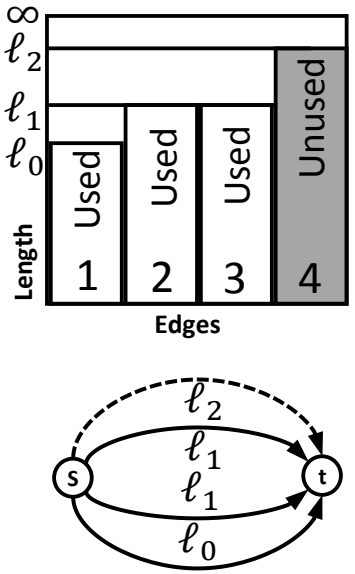

Figure: Example

Algorithm for Parallel Link (PL) Graph

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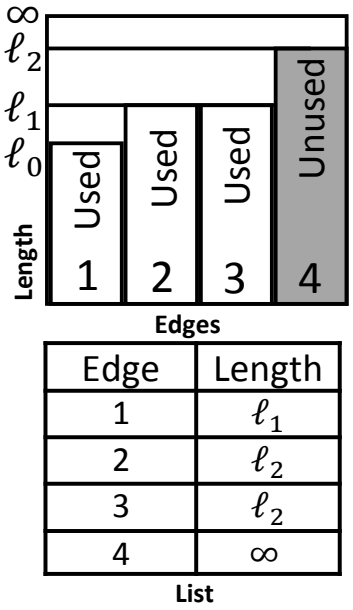

Figure: Example: $\ell_{max} = \ell_1$, $i_0 = 1$

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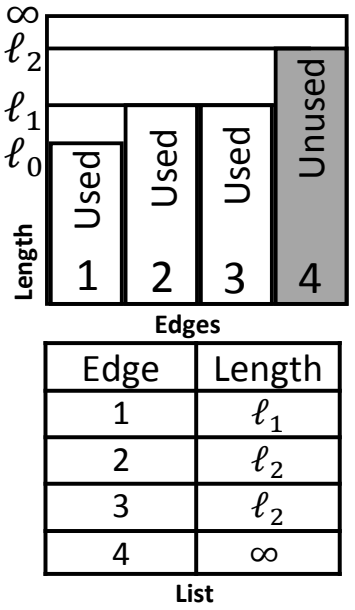
4. Create list $\{(\eta, \ell)\}$
   - Toll on edges $1, \ldots, \eta$
   - Maximally induce ℓ



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   - Maximally induce ℓ
5. Place toll to induce L

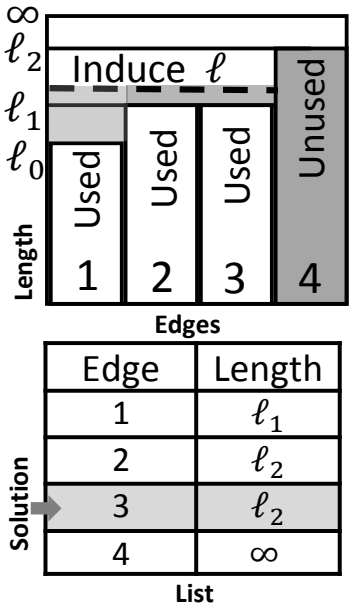

Figure: Example: Placing tolls

Series Parallel in P: Extension to SP

Induce length L

- Series comb. $\mathbf{G_1(S)G_2}$: Induce $\mathbf{L_1}(\leq L)$ in $\mathbf{G_1}$ and $\mathbf{L - L_1}$ in $\mathbf{G_2}$.
- Parallel comb. $\mathbf{G_1(P)G_2}$: Induce $\mathbf{L}$ in $\mathbf{G_1}$ and $\mathbf{G_2}$.

Monotonicity Lemma

- In a SP graph G with maximum used path length $\ell_{\mathbf{max}}$,
  We can induce length $\mathbf{L} \iff \mathbf{L} \geq \ell_{\mathbf{max}}$
- $\mathbf{L}$ is **induced optimally** with support $\mathbf{T}$
  $\Rightarrow \ell \leq \mathbf{L}$ can be **induced optimally** with support $\mathbf{t} \leq \mathbf{T}$

Series Parallel in *P*: Extension to *SP*

MakeList: Bottom-up List creation: Dynamic Programming

1) **Start** from PL 2) **Combine** list in series/parallel 3) **Recur** till root

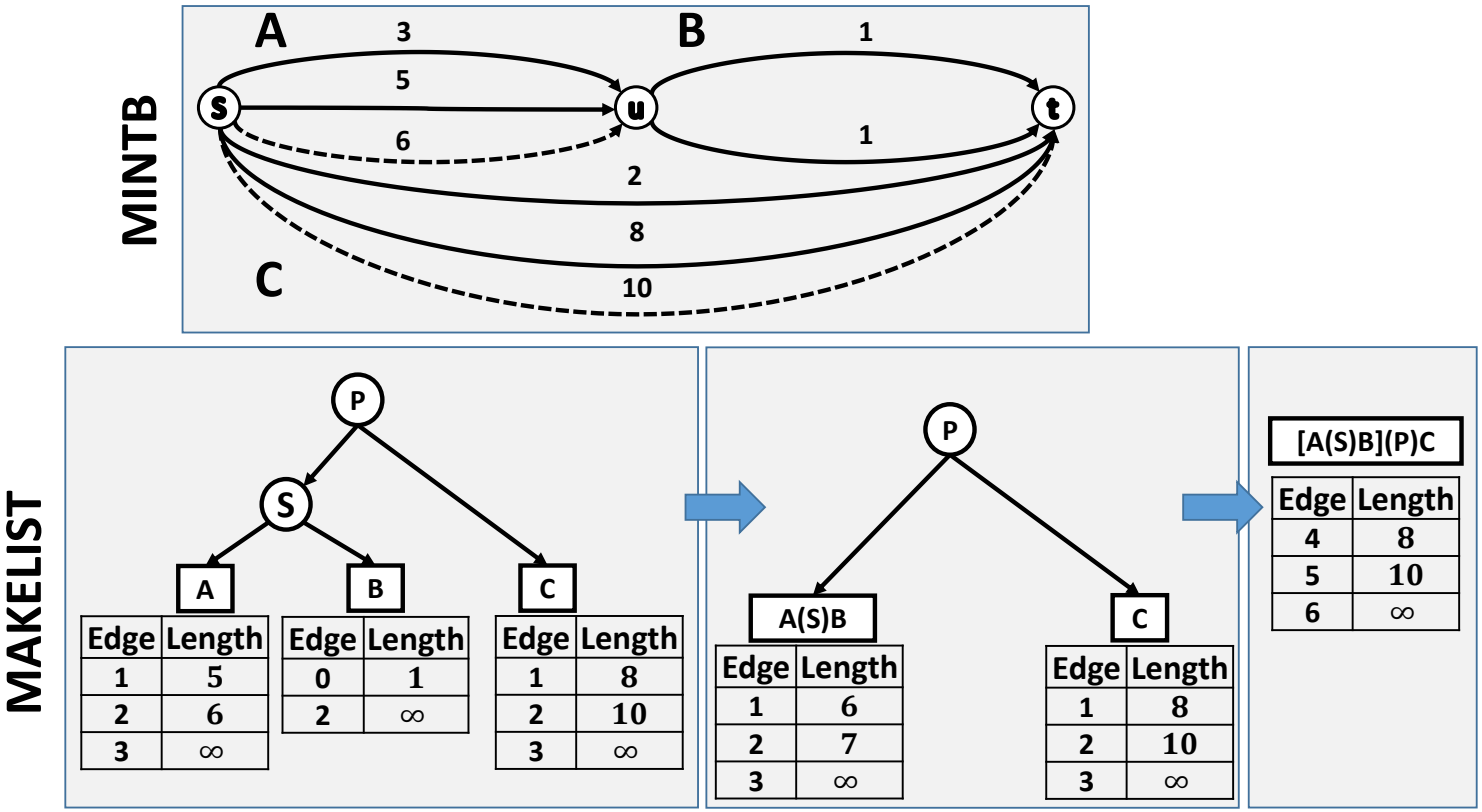

Figure: Step1: Example Run of MINTB Algorithm

Series Parallel in *P*: Extension to *SP*

Placetoll: Top-down Induction of length: Traceback

1) **Start** from root. 2) **Branch** in series/parallel. 3) **Recur** till PL.

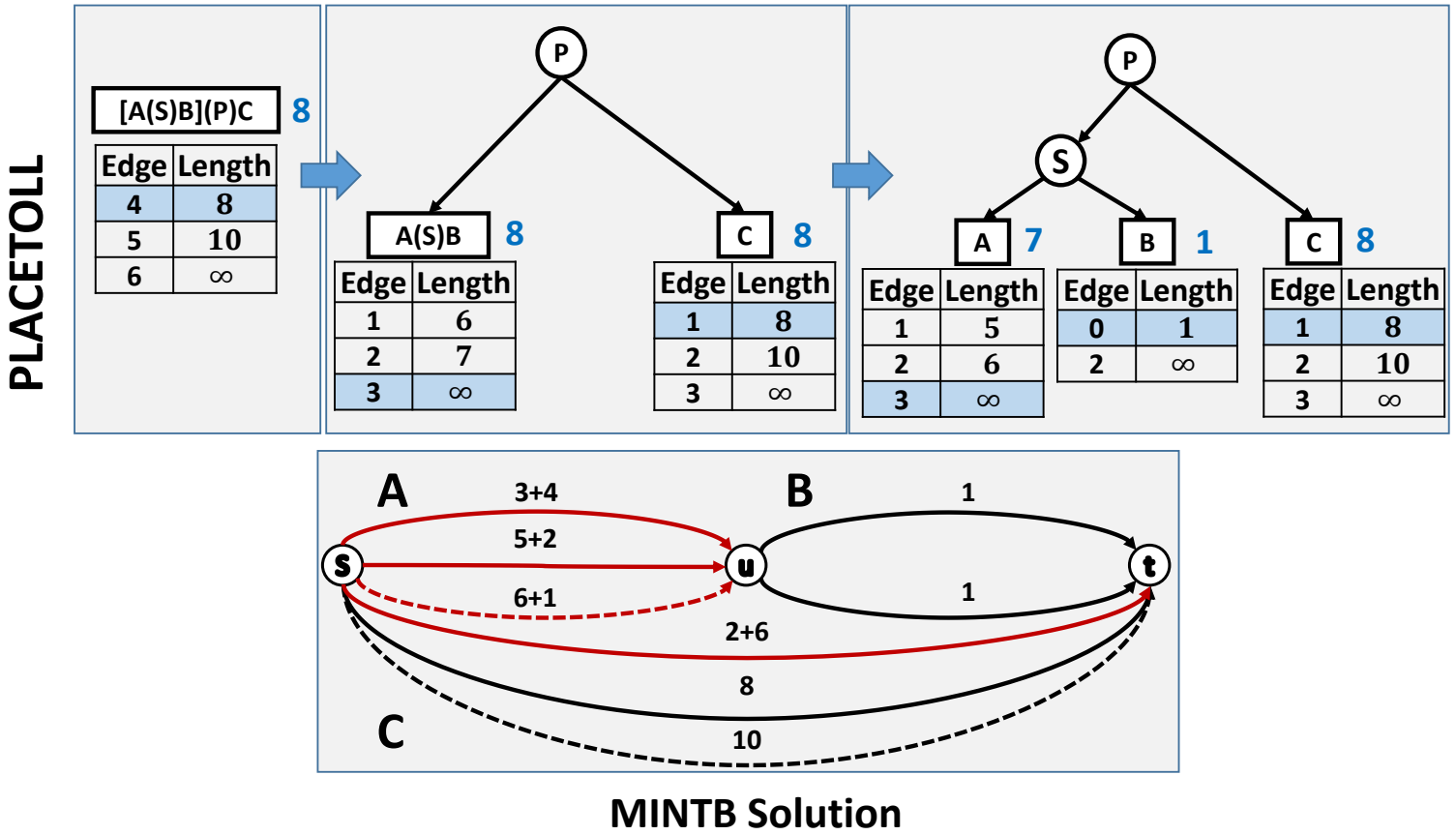

Figure: Step2: Example Run of MINTB Algorithm

Series Parallel in P: Extension to SP

Theorem

The MINTB on a SP graph with $|E| = m$, is solved optimally in time $\mathcal{O}\left(m^3\right)$.

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Presence of Braess Structure:

- The **Monotonicity Lemma** breaks.
- Edges to induce length $3 = 3$.
- Edges to induce length $4 = 2$.

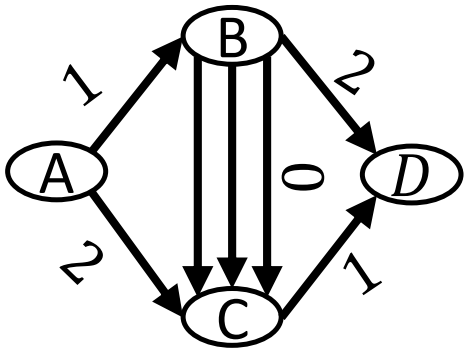


Figure: Counter

Summary

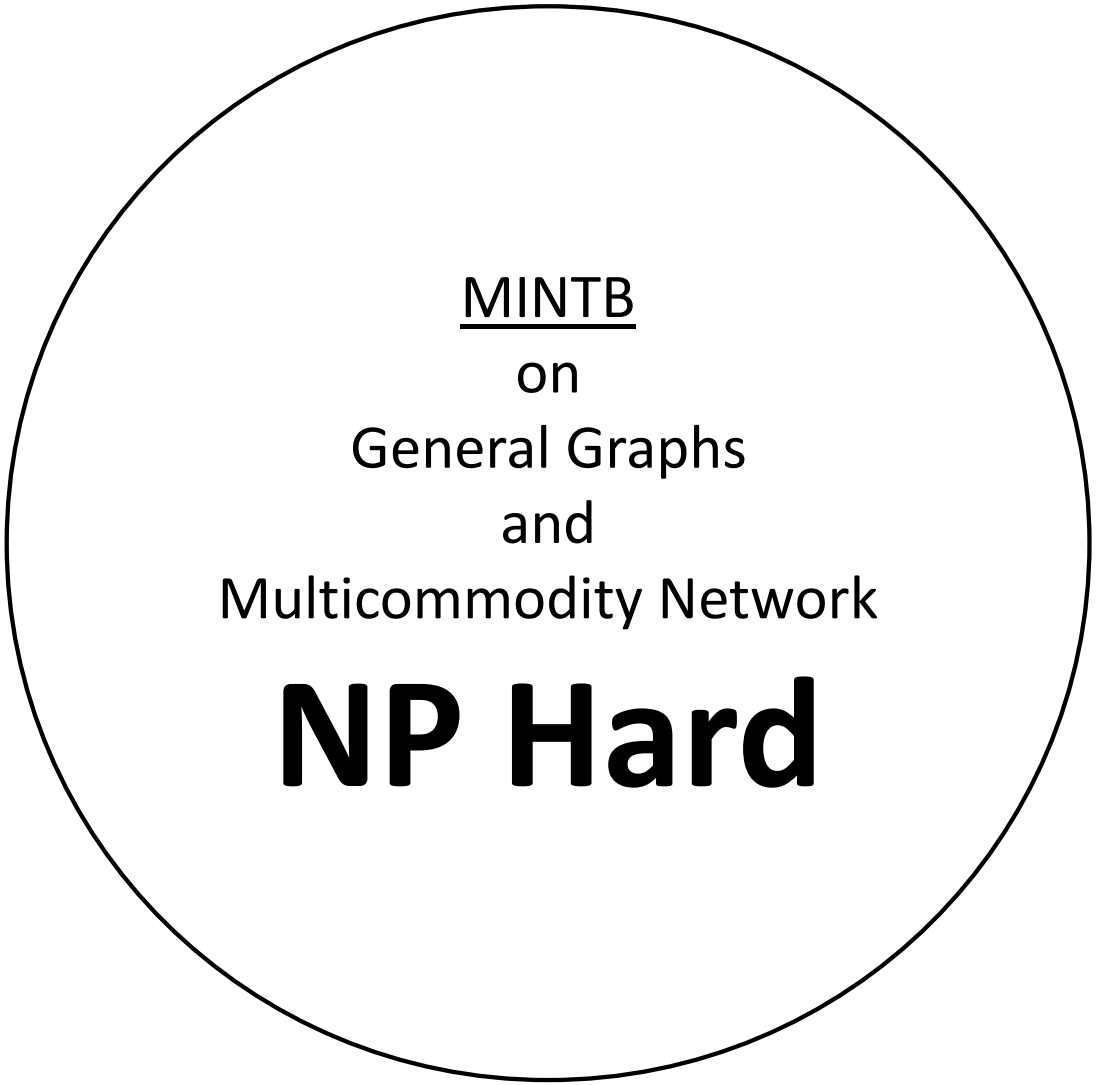


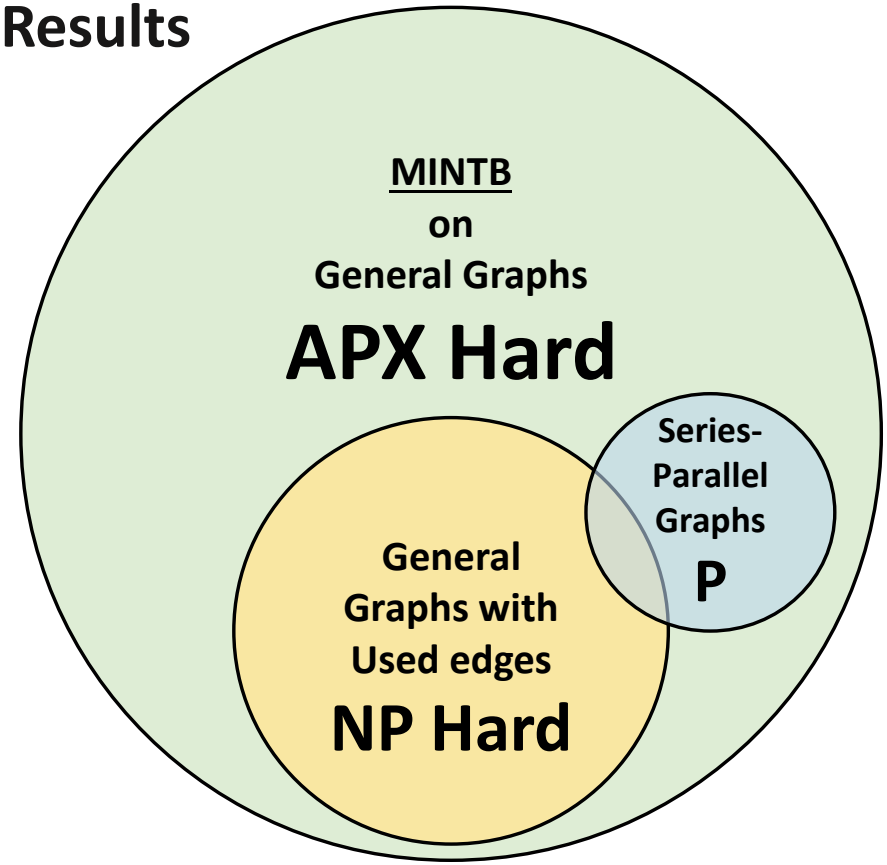
Figure: Complexity Diagram **Prior to the Work**

Summary

- Reduction From Vertex Cover Problem
- APX hardness of 1.1377
- Exploits Unused Edges in an Optimal Flow

- Reduction from Partition Problem
- Exploits Braess Structure

Complexity Results



Algorithm

- Absence of Braess Structure
- Monotonicity Property
- Tree Decomposition

- Dynamic Programming
- Bottom Up List Creation
- Top Down Toll Placement

Future Directions

- Is the APX hardness result **tight**?
  - YES Find matching approximation algorithms.
  - NO Give tighter APX hardness results.
- Design Practical Algorithms:
  - Improved heuristics with **performance guarantee**.
  - Faster algorithms for **large-scale traffic networks**.
- What happens while **Taxing Sub-networks**?

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Questions?