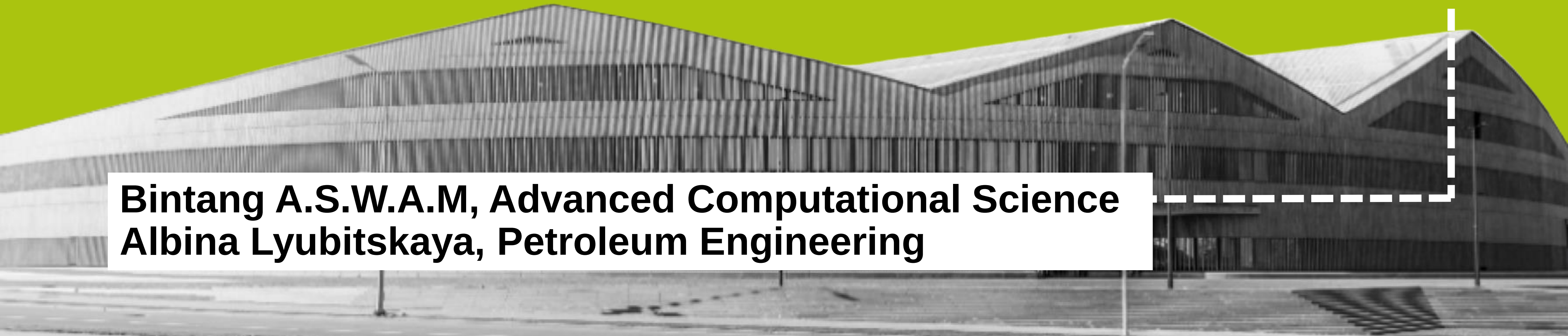
High Performance Pressure-Driven Pipe Flow



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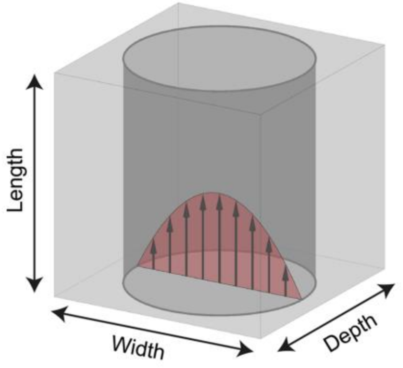
Application of studying the pressure-driven flow in oil and gas fields investigations

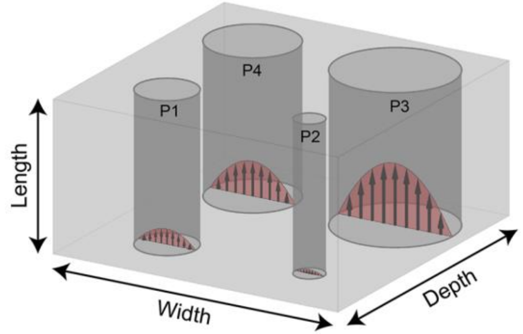
The flow of fluids through porous media such as groundwater flow or oil migration is a key process in geological sciences. Flow is controlled by the permeability of the rock; thus, an accurate determination and prediction of flow value is of crucial importance.

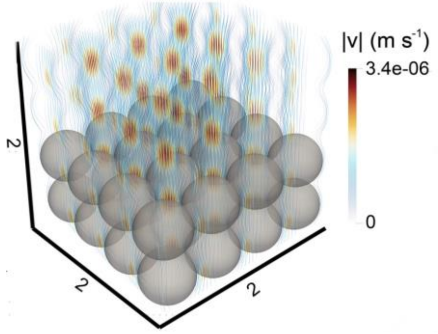
Computing permeabilities and understanding the microstructures and flow patterns in 3D pore structures are achieved with numerical modeling. In recent years, the flow of non-Newtonian fluids through porous media has gained additional importance due to the use of nanofluids for enhanced oil recovery.

Newtonian flow through a single vertical tube

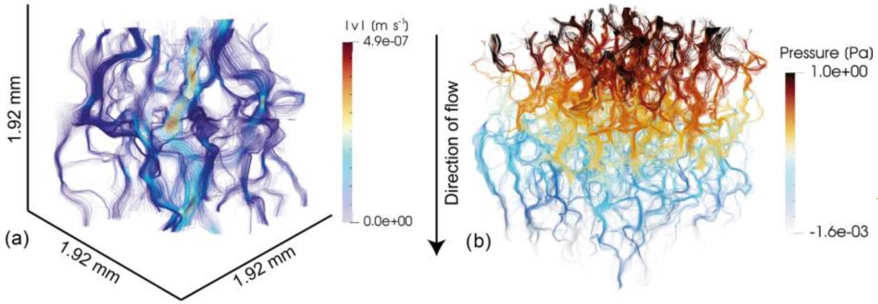


Newtonian flow through multiple vertical tubes





Newtonian flow through simple cubic (SC) sphere pack



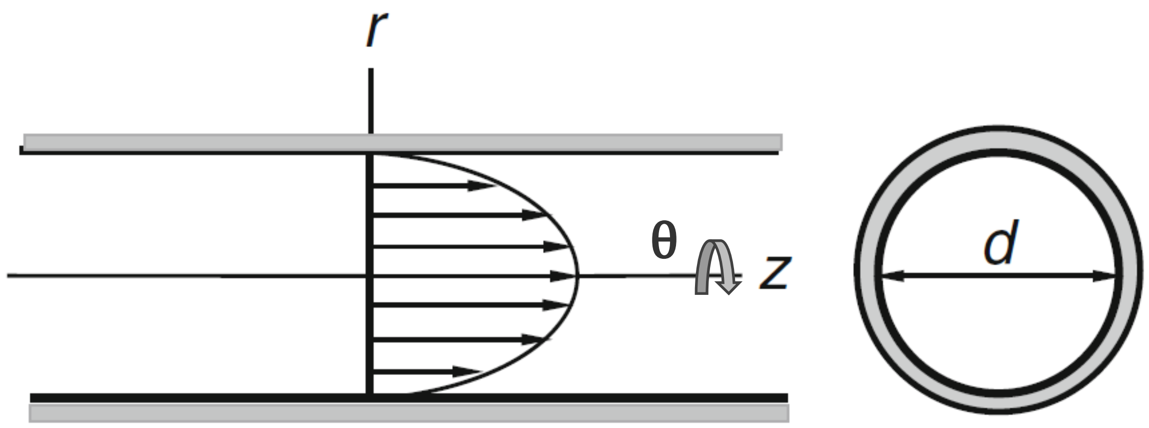
Newtonian fluid flow through the Fontainebleau sandstone sample. Streamlines colored using computed fluid velocities are shown in panel (a) and streamlines colored using fluid pressures are shown in panel (b).

Analytical derivation of Hagen-Poiseuille Flow

Relevant physical assumptions:

1. Steady -state flow
2. The velocity profile of u_r, u_θ could be neglected
3. Fully developed-based drifting flow will be physically considered

Three-dimensional Hagen–Poiseuille flow is the steady flow of an incompressible fluid through a straight pipe of circular cross-section with rotational symmetry (Schlichting, 1960).



a – radius of the pipe, $a = \frac{1}{2}d$

z - define the axis of the pipe

r - radial coordinate measured from the axis outwards

θ – tangential direction

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Analytical derivation of Hagen-Poiseuille Flow

Hagen-Poiseuille flow is a steady unidirectional axisymmetric flow in a circular cylinder. Thus, we look for the velocity field in cylindrical coordinates in the following form:

$$u_r = 0, u_\theta = 0, u_z = u(r)$$

1.Continuity Equation

incompressible flow

$$\nabla \cdot u \equiv 0 \rightarrow div(u) \equiv 0, \quad u = \begin{Bmatrix} u_r \\ u_\theta \\ u_z \end{Bmatrix}$$

In z-direction:

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial r}(ru_r)}_{=0} + \underbrace{\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}}_{=0} + \frac{\partial u_z}{\partial z} = 0$$

$$\boxed{\frac{\partial u_z}{\partial z} = 0}$$

u - velocity

z - define the axis of the pipe

r - radial coordinate measured from the axis outwards

θ - the radial coordinate measured from the axis outwards

2. Momentum Equation

(the Navier–Stokes equation)

$$\rho(\frac{\partial u}{\partial t} + (u \cdot \nabla u) = -P + g + \mu \nabla^2 u$$

In z-direction:

$$\rho\left(\frac{\partial u_z}{\partial t} + \frac{\partial u_z}{\partial r}\frac{dr}{dt} + \frac{\partial u_z}{\partial z}\frac{dz}{dt}\right) = -\frac{\partial P}{\partial z} + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{\partial^2 u_z}{\partial z}\right) + g_z$$

$$\underbrace{\rho\left(\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z}\right)}_{=0} = -\frac{\partial P}{\partial z} + g_z + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \underbrace{\frac{\partial^2 u_z}{\partial z^2}}_{=0}\right) = 0$$

$$\boxed{0 = -\frac{\partial P}{\partial z} + g_z + \frac{\mu}{\rho}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)\right)}$$

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Analytical derivation of Hagen-Poiseuille Flow

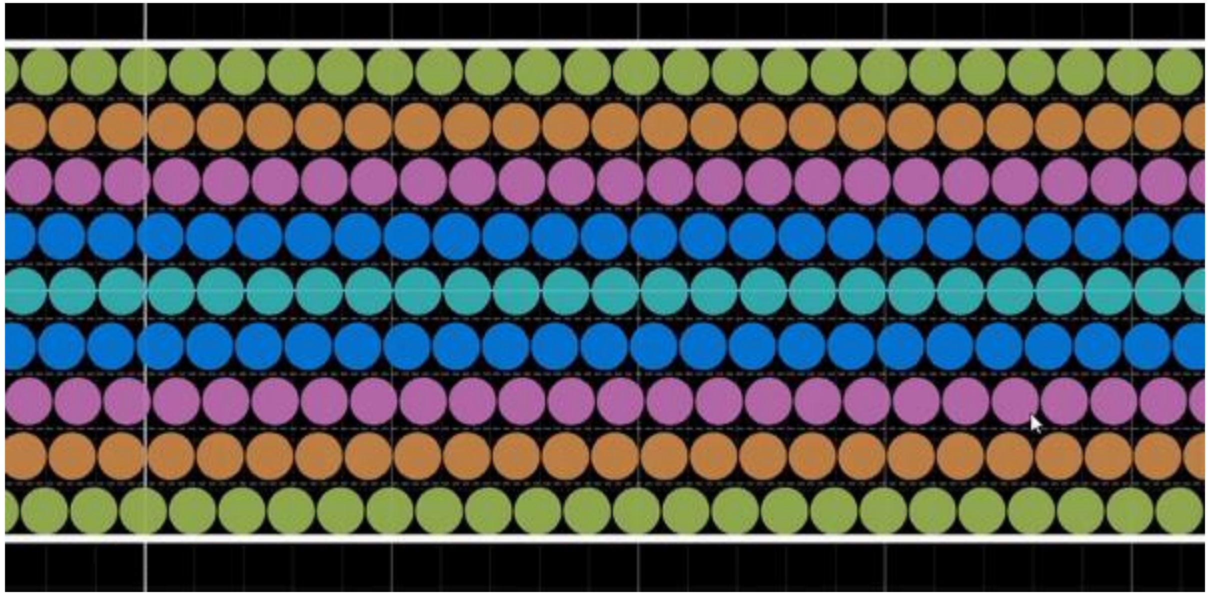
P depends only on z and:

$$\frac{d^2p}{dz^2} = 0 = \text{constant} \quad \longrightarrow \quad \alpha \equiv -\frac{dP}{dz}$$

$$-\frac{\alpha}{\mu} = \frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)$$

Integration of the preceding equation gives:

$$u = -\frac{\alpha r^2}{4\mu} + b\ln r + c$$



Visualization of Hagen-Poiseuille Flow

Since velocity must be bounded in the flow region, the integration constant must be zero. Now the nonslip condition on the cylindrical wall demands that:

$$u = 0\ at\ r = \frac{d}{2} \quad \longrightarrow \quad c = \frac{\alpha}{\mu}\left(\frac{d^2}{16}\right) \quad \text{and} \quad \boxed{u = \frac{\alpha}{4\mu}\left(\frac{d^2}{4} - r^2\right)}$$

- the velocity over the cross-section is distributed in the form of a paraboloid.

The maximum velocity is at r = 0

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Courant-Friedrichs-Lewy Condition

Courant-Friedrichs-Lewy Condition states that, given a space discretization, a time step bigger than some computable quantity should not be taken. Thus the time step must be kept small enough so that information has enough time to propagate through the space discretization.

For any explicit simple linear convection problem, the Courant number must be equal or smaller than 1, otherwise, the numerical viscosity would be negative.

The explicit scheme is **conditionally stable** under the following CFL-type condition:

$$\boxed{\Delta t \leq \frac{1}{2v}\Delta x^2} \longrightarrow \boxed{\frac{vdt}{dx^2} \leq \frac{1}{2},}$$

v – kinematic viscosity, t – time-step length

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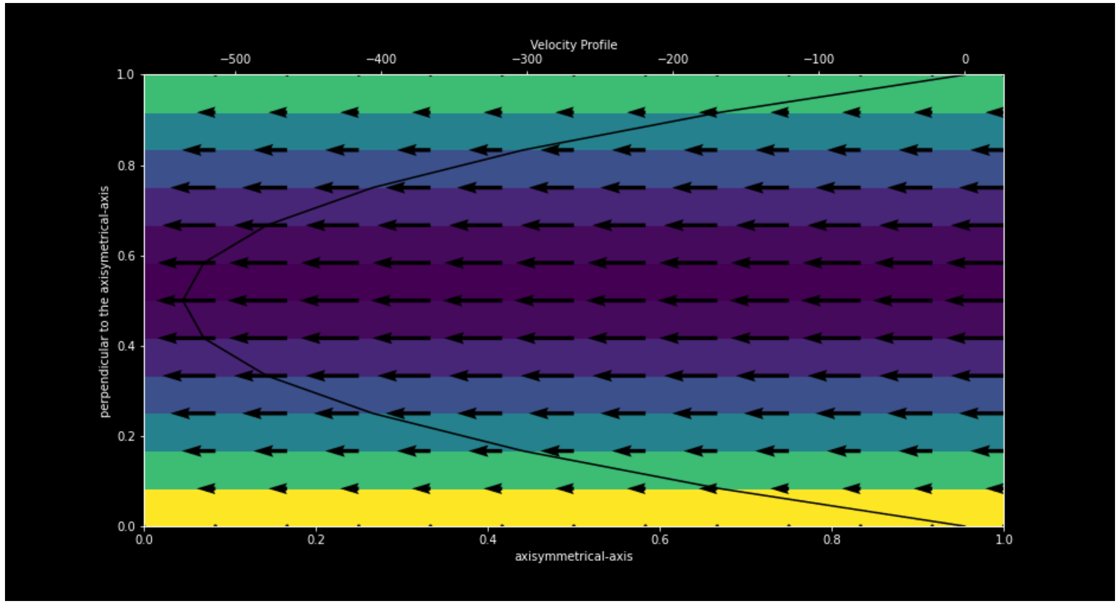
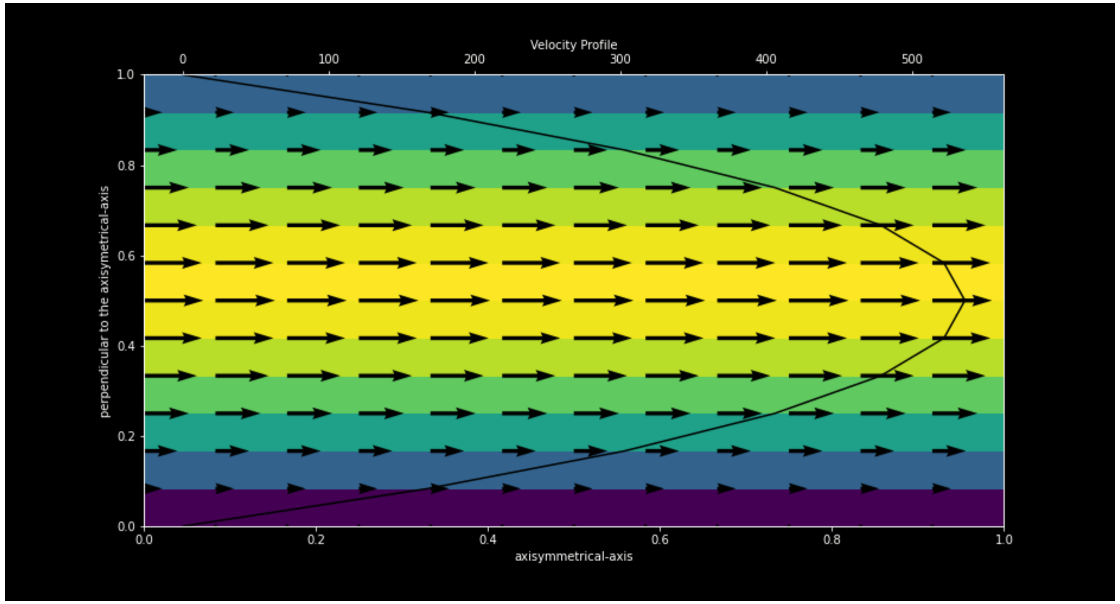
Pressure gradient





Adverse parabolic flow, >0

Favorable parabolic flow, <0

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