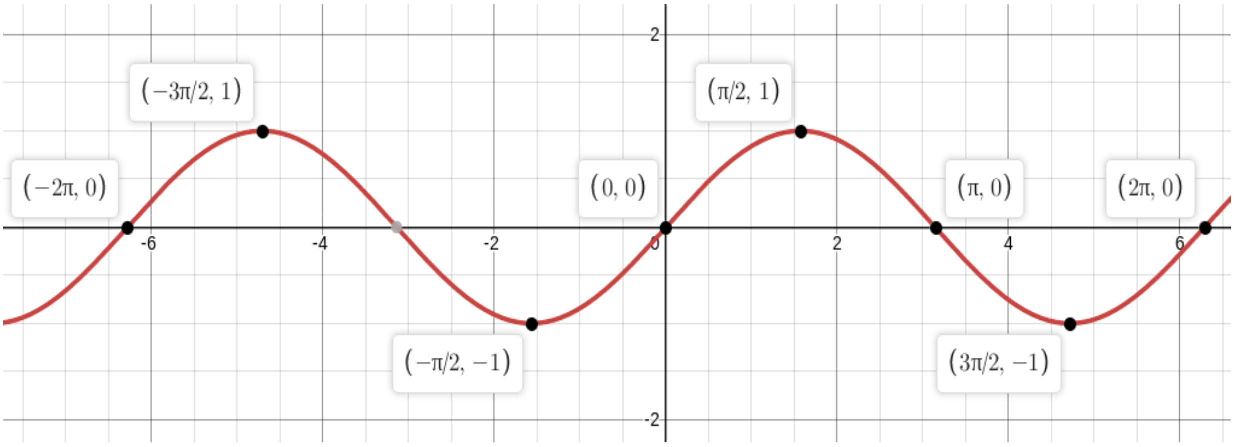
# Common Mathematical Subjects

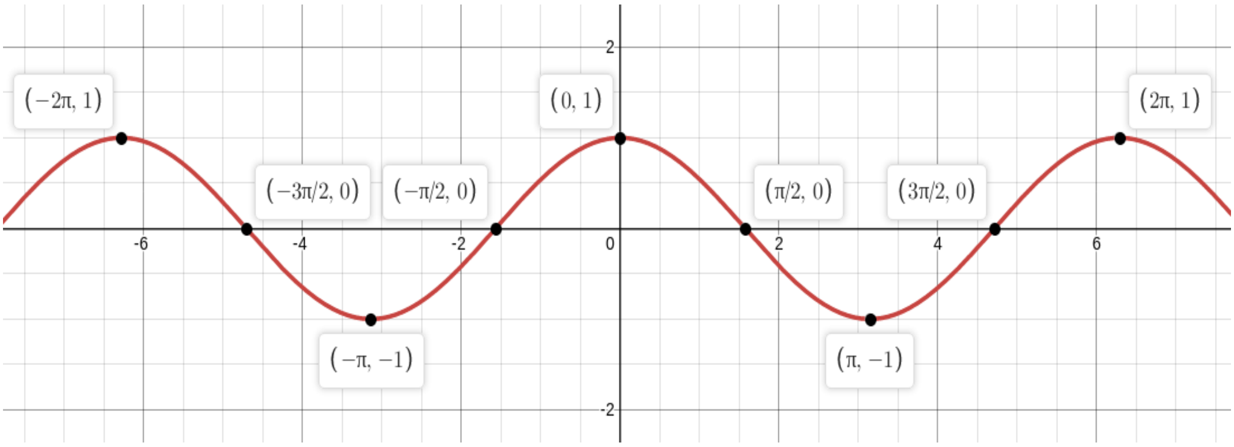
## Trigonometry

### Trigonometric Graphs

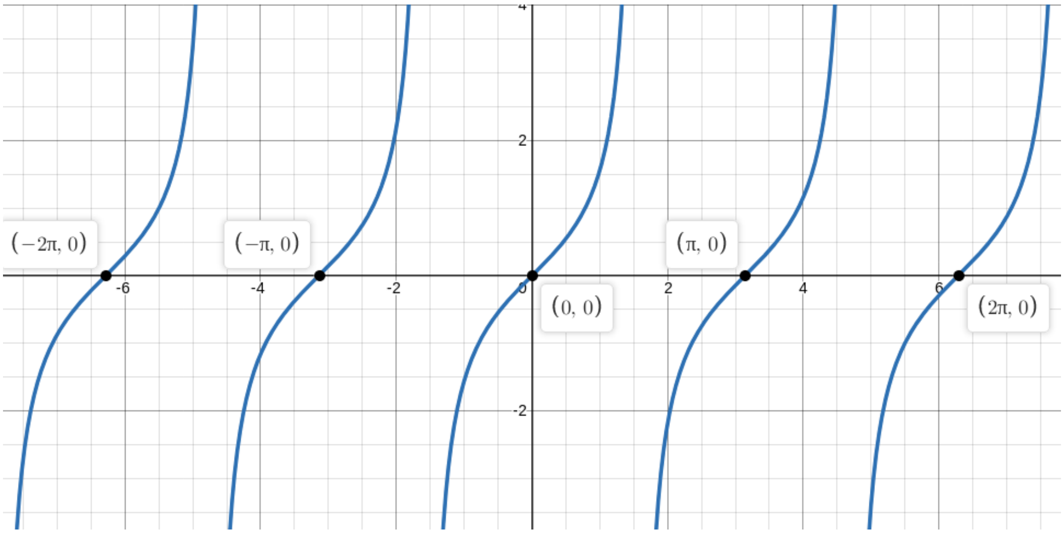
**Sinus Graph**



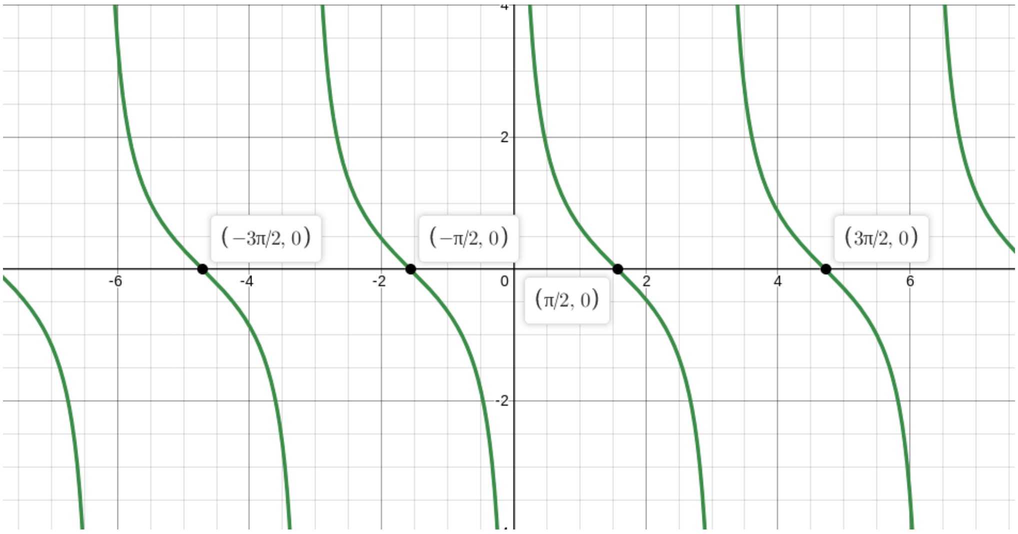

**Cosinus Graph**



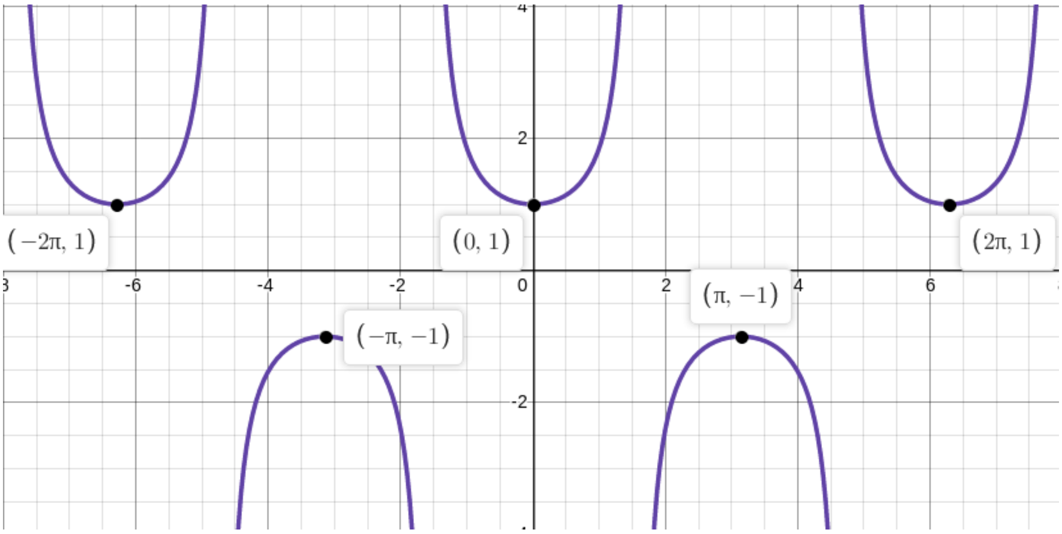

**Tangent Graph**



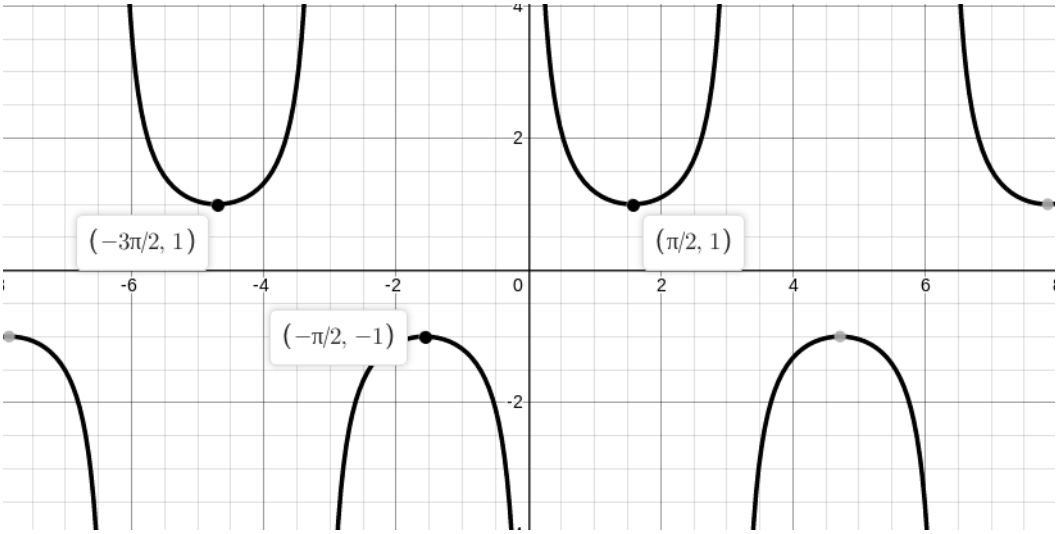

## Cotangent Graph




## Secant Graph




## Cosecant Graph




## Trigonometric Values

| Trigonometry Ratio Table | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Angles (In Degrees)** | $0^\circ$ | $30^\circ$ | $45^\circ$ | $60^\circ$ | $90^\circ$ | $180^\circ$ | $270^\circ$ | $360^\circ$ |
| **Angles (In Radians)** | $0$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |
| $\sin$ | $0$ | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | $1$ | $0$ | $-1$ | $0$ |
| $\cos$ | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | $0$ | $-1$ | $0$ | $1$ |
| $\tan$ | $0$ | $\frac{1}{\sqrt{3}}$ | $1$ | $\sqrt{3}$ | Not Defined | $0$ | Not Defined | $1$ |
| $\cot$ | Not Defined | $\sqrt{3}$ | $1$ | $\frac{1}{\sqrt{3}}$ | $0$ | Not Defined | $0$ | Not Defined |
| $\csc$ | Not Defined | $2$ | $\sqrt{2}$ | $\frac{2}{\sqrt{3}}$ | $1$ | Not Defined | $-1$ | Not Defined |
| $\sec$ | $1$ | $\frac{2}{\sqrt{3}}$ | $\sqrt{2}$ | $2$ | Not Defined | $-1$ | Not Defined | $1$ |

## Identities

$$\cos^2(\theta) + sin^2(\theta) = 1$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

### Addition Formulas

$$\cos(A \pm B) = \cos(A).\cos(B) \mp \sin(A).\sin(B)$$

$$\sin(A \pm B) = \sin(A).\cos(B) \pm \cos(A).\sin(B)$$

$$\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$$

### Double Angle Formulas

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\cot(2\theta) = \frac{\cot^2(\theta) - 1}{2\cot(\theta)}$$

### Half-Angle Formulas

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}} = \frac{\sin(\theta)}{1 + \cos(\theta)} = \frac{1 - \cos(\theta)}{\sin(\theta)}$$

## Hyperbolic Function

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth(x) = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\mathrm{sech(x)}\frac{1}{\cosh(x)}$$

$$\mathrm{cosech(x)} = \frac{1}{\sinh(x)}$$

### The Law Of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos(\theta)$$

### Eular Formulas

$$\mathrm{e}^{ix} = \cos(x) + \mathrm{i}\sin(x)$$

$$\mathrm{e}^{-ix} = \cos(x) - \mathrm{i}\sin(x)$$

$$\sin(x) = \frac{\mathrm{e}^{ix} - \mathrm{e}^{-ix}}{2\mathrm{i}}$$

$$\cos(x) = \frac{\mathrm{e}^{ix} + \mathrm{e}^{-ix}}{2}$$

$$\tan(x) = \frac{1}{(i)}\frac{\mathrm{e}^{ix} - \mathrm{e}^{-ix}}{\mathrm{e}^{ix} + \mathrm{e}^{-ix}}$$

### Other Equalities

$$\sin(mx).\sin(nx) = \frac{1}{2}[\cos((m-n)x) - cos((m+n)x)]$$

$$\sin(mx).\cos(nx) = \frac{1}{2}[sin((m-n)x) + \sin((m+n)x)]$$

$$\cos(mx).\cos(nx) = \frac{1}{2}[\cos((m-n)x) + cos((m+n)x)]$$

# Limit

### Existance

Limit exists $\iff$ left-hand and right-hand limits exists and $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$.

### Properties

Let,

$$\lim_{x\to c} f(x) = L$$

$$and$$

$$\lim_{x \to c} g(x) = M$$

Then,

1. Summation Rule: $\lim_{x \to c}(f(x) + g(x)) = L + M$
2. Difference Rule: $\lim_{x \to c}(f(x) + g(x)) = L + M$
3. Product Rule: $\lim_{x \to c}(f(x). g(x)) = L. M$
4. Constant Multiplication Rule: $\lim_{x \to c} k. f(x) = k. L$
5. Quotient Rule: $\lim_{x \to c}(\frac{f(x)}{g(x)}) = \frac{L}{M}$
6. Power Rule: $\lim_{x \to c} f(x)^{\frac{r}{s}} = L^{\frac{r}{s}}$

### Limits of Common Functions

$$\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1$$

$$\lim_{x \to 0} \frac{ln(1 + c. x)}{x} = c$$

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0$$

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

# Logarithms

1. $\log MN) = \log(M) + \log(N)$
2. $\log(\frac{M}{N}) = \log(M) - \log(N)$
3. $\log(M^k) = k \log(M) \qquad ((\log(M^k) \neq (\log(M))^k)$
4. $\log_b(M) = \frac{\log(M)}{\log(b)}$

# Differentiation

### Differentiation Properties

Let $u$ and $v$ are functions, and $c$ is constant.

$$(c)' = 0$$

$$(x)' = 1$$

$$(u+v)' = u' + v'$$

$$(cu)' = cu'$$

$$(uv)' = u'v + uv'$$

$$(\frac{u}{v})' = \frac{u'v - uv'}{v^2}$$

$$(\frac{c}{u})' = \frac{-cu'}{v^2}$$

## Differentiation Table

$$(x^n)' = nx^{n-1}$$

$$(u^n)' = nu^{n-1}u'$$

$$(e^u) = e^u u'$$

$$(a^u)' = a^u \ln(a) u'$$

$$(u^v)' = vu^{v-1}u' + \ln(u) u^v v'$$

$$(log_a u)' = \log_a(e) \frac{u'}{u} = \frac{u'}{u \ln(a)}$$

$$(\ln(u))' = \frac{u'}{u}$$

## Derivative of Trigonometric Functions

$$(\sin(u))' = \cos(u) u'$$

$$(\cos(u))' = -\sin(u) u'$$

$$(\tan(u))' = \sec^2(u) u' = \frac{1}{\cos^2(u)} u'$$

$$(\cot(u))' = -\csc^2(u) u' = -\frac{1}{\sin^2(u)} u'$$

$$(\sec(u))' = \sec(u) \tan(u) u'$$

$$(\csc(u))' = -\csc(u) \cot(u) u'$$

### Derivative of Inverse Functions

$$(\arcsin(u))' = \frac{u'}{\sqrt{1-u^2}}$$

$$(\arccos(u))' = \frac{-u'}{\sqrt{1-u^2}}$$

$$(\arctan(u))' = \frac{u'}{1+u^2}$$

$$(arccot(u))' = -\frac{u'}{1+u^2}$$

$$(arcsec(u))' = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$(arccsc(u))' = \frac{-u'}{|u|\sqrt{u^2-1}}$$

# Integral

### Table of Integrals

$$\int (x^k)dx = \frac{x^{k+1}}{k+1} + c$$

$$\int (\mathrm{e}^{\mathrm{x}})dx = \mathrm{e}^{\mathrm{x}} + c$$

$$\int (\mathrm{a}^{\mathrm{x}})dx = \frac{\mathrm{a}^{\mathrm{x}}}{\ln(a)} + c$$

$$\int \frac{1}{x}dx = \ln(|x|) + c$$

### Trigonometric Integrals

$$\int \cos^n(x)dx = \frac{1}{n}\sin^{n-1}(x) + c$$

$$\int \sin^n(x)dx = -\frac{1}{n}\cos^{n-1}(x) + c$$

$$\int \tan^n(x)dx = \ln|\sec(x)| + c = \frac{1}{n-1}\tan^{n-1}(x) - \int \tan^{n-2}(x)dx$$

$$\int \cot^n(x)dx = \ln|\sin(x)| + c = \frac{-1}{n-1}\cot^{n-1}(x) - \int \cot^{n-2}(x)dx$$

$$\int \sec^n(x)dx = \ln|\sec(x)+\tan(x)|+c = \frac{1}{n-1}\sec^{n-2}(x)\tan(x)+\frac{n-2}{n-1}\int \sec^{n-2}(x)dx$$

$$\int \csc^n(x)dx = \ln|\csc(x)-\tan(x)|+c = \frac{-1}{n-1}\csc^{n-2}(x)\cot(x)+\frac{n-2}{n-1}\int \csc^{n-2}(x)dx$$

$$\int \sec(x)\tan(x)dx = \sec(x)+c$$

$$\int \csc(x)\cot(x)dx = -\csc(x)+c$$

$$\int \sec^2(x)dx = \int \frac{1}{\cos^2(x)}dx = \tan(x)+c$$

$$\int \csc^2(x)dx = \int \frac{1}{\sin^2(x)}dx = -\cot(x)+c$$

$$\int \cosh(x)dx = \sinh(x)+c$$

$$\int \sinh(x)dx = \cosh(x)+c$$

## Inverse Trigonometric Integrals

$$\int \frac{1}{\sqrt{a^2-u^2}}du = \arcsin(\frac{u}{a})+c = -\arccos(\frac{u}{a})+c$$

$$\int \frac{1}{a^2+u^2}du = \frac{1}{a}\arctan(\frac{u}{a})+c = -\frac{1}{a}\text{arccot}(\frac{\text{u}}{\text{a}})+c$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}du = \frac{1}{a}\text{arcsec}(\frac{|\text{u}|}{\text{a}})+c = -\frac{1}{a}\text{arccsc}(\frac{|\text{u}|}{\text{a}})+c$$