Subjects and Algorithms Summaries

Limitss

Existance

Limit exists $\iff$ left-hand and right-hand limits exists and $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x)$.

Functions

Monotonic Functions

If function f is increasing ($f'(x) > 0$) or decreasing ($f'(x) < 0$) for an interval $\implies$ f is Monotonic function.

Solving Inverse of a Functions

1)Solve $y = f(x)$ for x

$$x = f^{-1}(y)$$

2)Interchange x and y to obtain

$$y = f^{-1}(x)$$

Continuity

$f(x)$ is continous $\iff$ followings holds:

$$1)\exists f(c) \quad (c \in D(f))$$

$$2)\exists \lim_{x \to c} f(x)$$

$$3) \lim_{x \to 0} f(x) = f(c)$$

(Function is continous $\iff$ there exists $f(c)$ and limit exists at c and limit of f(x) at when x $\to$ c is equalt to $f(c)$).

One-variable Calculus

Vertical Asymptote

If

$$\lim_{x \to a^+} f(x) = \pm\infty \quad \vee \quad \lim_{x \to a^-} f(x) = \pm\infty$$

a is vertical asymptote.

Horizontal Asymptote

If

$$\lim_{x \to \infty} f(x) = b \quad \vee \quad \lim_{x \to -\infty} f(x) = b$$

horizontal asymptote.

Oblique or Slant Asymptote

An oblique or a slant asymptote is an asymptote that is neither vertical or horizontal.

If the degree of the numerator is one more than the degree of the denominator, then the graph of the rational function will have a slant asymptote. Slant asymptote find by long division of the rational function. When denumerator's degree is equal to one, numerator is the asymptote function.

!* A graph can have both a vertical and a slant asymptote, but it CANNOT have both a horizontal and slant asymptote.

L'Hospital's Rule

!This rule is can be used in only one-variable calculus

If $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \vee \frac{\infty}{\infty}$, take derivates of numerator and denumerator seperately ($\frac{f'(x)}{g'(x)}$) until it is $\neq \frac{0}{0}$ or $\frac{\infty}{\infty}$

Sandwich Theorem

Suppose $g(x) \leq f(x) \leq h(x)$ and $\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$. Then,

$$\lim_{x \to c} f(x) = L$$

Tangent Line to the Curve

Let $y = f(x)$ at (x_0, y_0)

1. Calculate slope m: $m = \lim_{h \to 0} \frac{f(x_0+h)-f(x_0)}{h} = f'(x_0)$
2. If limit exists, find tangent line: $y = y_0 + m(x - x_0)$

Instantaneous Speed

$$\lim_{\triangle t \to 0} \frac{\triangle s}{\triangle t} = f'(x_0)$$

Graphing steps for y=f(x)

1. Identify domain of f and symmetries (even/odd)(horizontal/vertical/slant)
2. Find y', y''
3. Find critical points(bounds, $f'(x) = 0$, $\nexists f'(x)$) as (x,y).
4. Find intervals of f that it is increasing ($f'(x) > 0$) or decreasing ($f'(x) < 0$)
5. Find local max and minimums (critical points that change signs $f'(x)$).
6. Calculate all critical points and find absolute min and abosulute max.
7. Find points of inflection ($f''(x) = 0$) and concavity ($f'' < 0$ and $f'' > 0$)
8. Plot key points, intersections then concavities.

Shifting and Strecthing Graph

Let $y = af(b(x + c)) + d$.

a: (+) $\rightarrow$ horizontal stretch or compression, (-) $\rightarrow$ reflection about y-axis

b: (+) $\rightarrow$ vertical strecth or compression, (-) $\rightarrow$ reflection about x-axis

c: horizontal Shifting

d: vertical shift

Rolle's Theorem

Let $f(x) = y$ is continuous at every point of [a,b] and differentiable at every point of (a,b). Then, if $f(a) = f(b)$ then $\exists c$ that $f'(c) = 0$.

Mean-Value Theorem

Mean-Value theorem use Rolle's Theorem to get this equation:

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Integral

Integration by Parts

$$\int (f(x)g'(x))dx = f(x)g(x) - \int (f'(x)g(x))dx$$

Let $u = f(x)$ and $v = g(x)$. Then the statement will be

$$\int (u)dv = uv - \int (v)du$$

*Priority of picking u is shortened by LIATE (L-Logarithms, I-Inverse Trigonometric, A-Algebraic, T-Trigonometric, E-Exponential)

Taking Integral of Powers of Cosinus and Sinus

Let $\int(\sin^m(x).\cos^n(x))dx.$

1. If m-odd $\rightarrow$ replace $\sin^2(x)$ with $\sin^2(x) = 1 - \cos^2(x)$
2. If m-even and n-odd $\rightarrow$ replace $\cos^2(x)$ with $\cos^2(x) = 1 - sin^2(x)$
3. If m-even and n-even $\rightarrow$ replace $\sin^2(x)$ with $\sin^2(x) = \frac{1-\cos(2x)}{2}$ and $\cos^2(x) = \frac{1+\cos(2x)}{2}$

Vector Calculus

Area of Parallelogram

$$||u \times v|| = |u||v|\sin(\theta)$$

Volume of Parallelpiped (Triple Scalar Product)

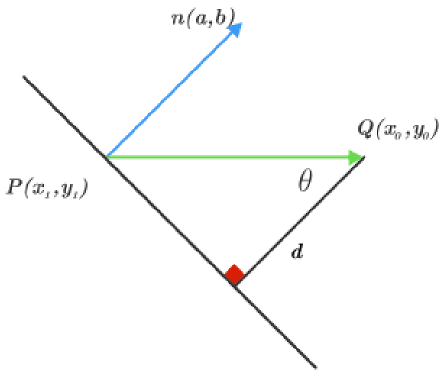
$$(u \times v).w = \begin{vmatrix} u_1 & y_3 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Vector Equation for line

r_0 is initial point, t is parameter (scalar), v is direction vector (usually unit vector)

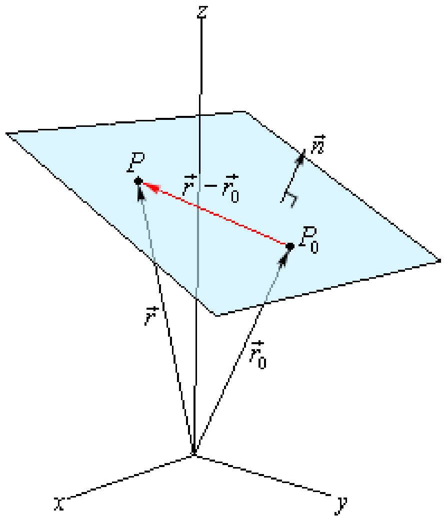
$$r(t) = r_0 + tv$$

Distance from a Point to a line



$$\frac{|\overrightarrow{PQ} \times \overrightarrow{n}|}{||\overrightarrow{n}||} = distance$$

Plane Equation



Normal Vector to a plane: $n = Ai + Bj + Ck$

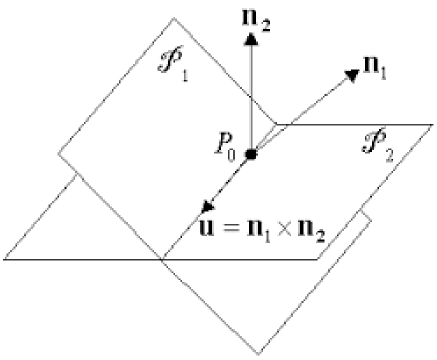
Plane = $n.\overrightarrow{P_0P}$

= $A(x - x_0) + B(y - y_0) + C(z - z_0) = D$

So, the final equation for plane is:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) - D = 0$$

Line of Intersection of Two Planes



Let planes P_1 and P_2 are $P_1 = A_1x + B_1y + C_1z = D_1$ and $P_2 = A_2x + B_2y + C_2z = D_2$

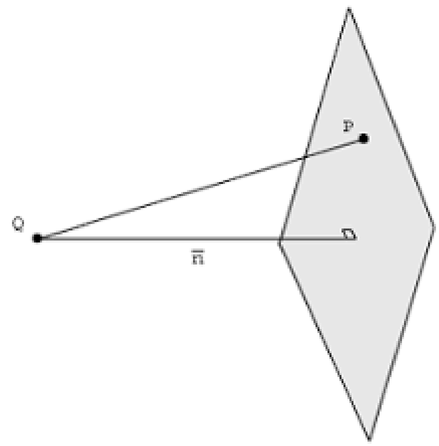
1. Find an intersection point (P_0) of two planes: $P_1 = P_2$
2. Find direction vector ($\overrightarrow{u}$) of intersection line

$$\overrightarrow{u} = \overrightarrow{n_1} \times \overrightarrow{n_2} = \begin{vmatrix} i & j & k \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$$

3. Write new plane equation as follows:

$$A_3(x - x_0) + B_3(y - y_0) + C_3(z - z_0) - D = 0$$

Distance from a Point to a Plane



$$\frac{|\overrightarrow{PQ}.\overrightarrow{n}|}{||\overrightarrow{n}||} = distance$$

Angle between two vectors

$$\cos(\theta) = \frac{\overrightarrow{u}\,.\,\overrightarrow{v}}{||\overrightarrow{u}||.||\overrightarrow{v}||} \implies \theta = \cos^{-1}\left(\frac{\overrightarrow{u}\,.\,\overrightarrow{v}}{||\overrightarrow{u}||.||\overrightarrow{v}||}\right)$$

Angle between two Planes

$$\theta = \cos^{-1}\left(\frac{\overrightarrow{n_1}\,.\,\overrightarrow{n_2}}{||\overrightarrow{n_1}||.||\overrightarrow{n_2}||}\right)$$

Velocity, Speed and Acceleration

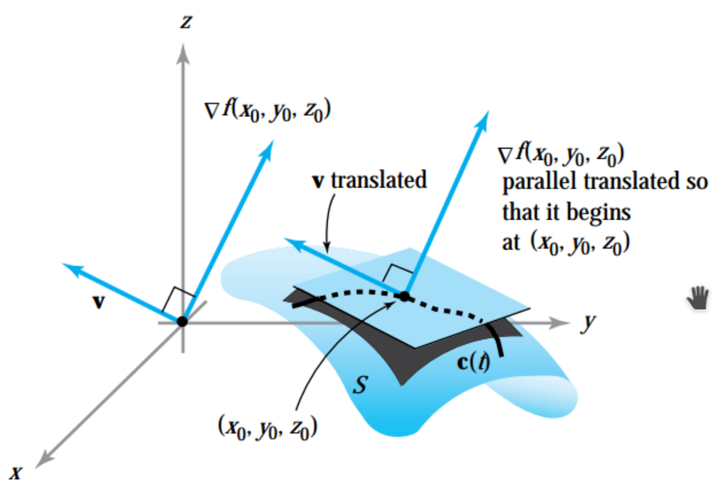
Let $c(t) = (x(t), y(t), z(t))$ is path.

$$c'(t) = v \rightarrow \text{velocity}$$

$$|c'(t)| = |v| \rightarrow \text{speed}$$

$$a = c''(t) = v' \rightarrow \text{acceleration}$$

Gradient of f

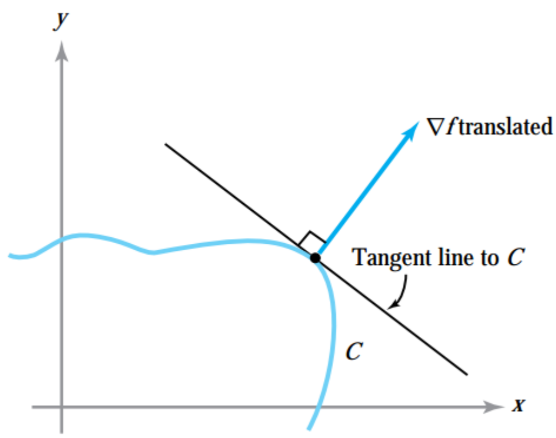


$$\triangledown f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

*$\triangledown f \neq 0 \implies \triangledown f$ gives direction of f is increasing fastest.

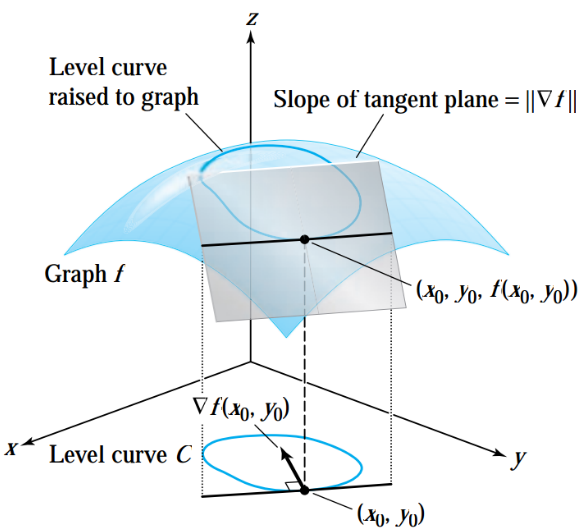
*$\triangledown f$ is normal to Level Surfaces.

Tangent Line



$$l(t) = c(t0) + (t - t0).\, c'(t0)$$

Tangent Planes to Level Surfaces



$$\text{Tangent Plane} = \triangledown f(x_0, y_0, z_0).\,(x - x_0, y - y_0, z - z_0) = 0$$

$$\text{Slope of tangent Plane} = || \triangledown f||$$

Taylor Theorem

The main point of the single-variable Taylor theorem is to find approximations of a function near a given point that are accurate to a higher order than the linear approximation.

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n$$

Forms of the Remainder

$$R_n(x) = \frac{f^{n+1}(c)}{(n+1)!}(x-a)^{n+1}$$

$\exists c$ between a and x such that

Arc Length

$$L = \int_a^b |\overrightarrow{v}|dt = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$$

Arc Length

$$s(t) = \int_a^b |v(u)|du$$

Arc Length Parameter

$$s'(t) = |v(t)|$$

Speed on Smooth Curve

Tangent Vector

Tangent Vector $\rightarrow \overrightarrow{v} = r'(t)$

$$\overrightarrow{T}(t) = \frac{v}{|v|} = \frac{r'(t)}{|r'(t)|}$$

Unit Tangent Vector

$$K = \left|\frac{dT}{ds}\right| = \frac{1}{|v|}.\left|\frac{dT}{dt}\right| = \frac{|v \times a|}{|v|^3}$$

Curvature

$$P = \frac{1}{K}$$

Radius of Curvature

$$N(t) = \frac{1}{K}.\frac{dT}{ds} = \frac{T'(t)}{|T'(t)|}$$

$$B = T \times N$$

Unit Binormal Vector