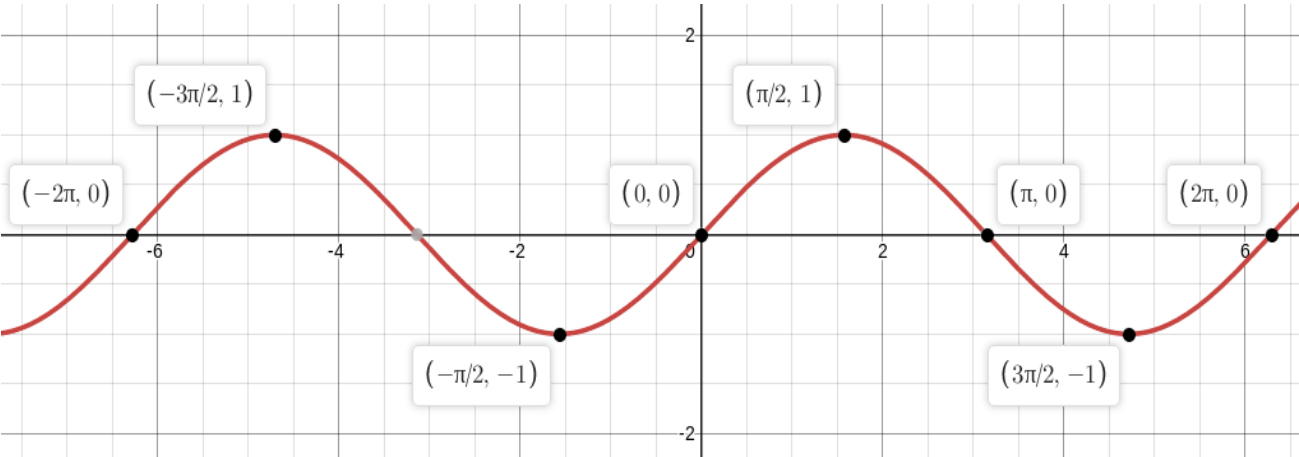# Common Mathematical Subjects
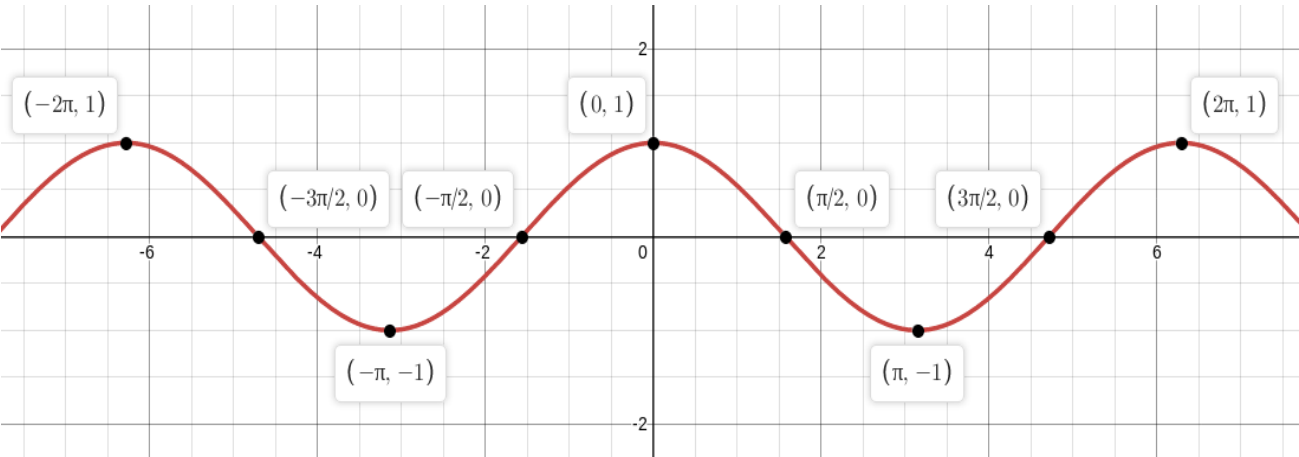
Trigonometry

Trigonometric Graphs
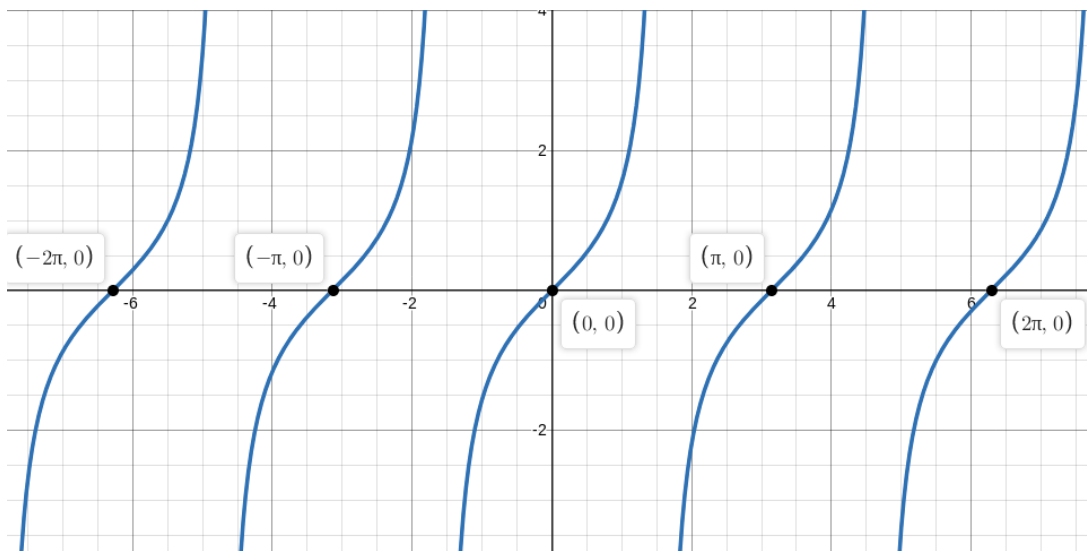
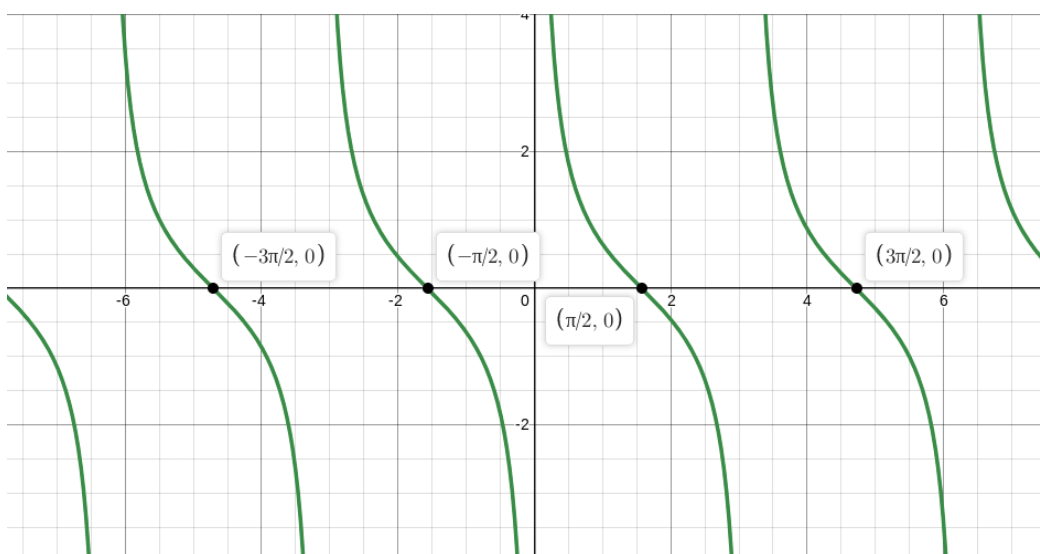**Sinus Graph**

Cosinus Graph

Tangent Graph

Cotangent Graph

Secant Graph

Cosecant Graph

$(-3\pi/2, 1)$

$(\pi/2, 1)$

$(-\pi/2, -1)$
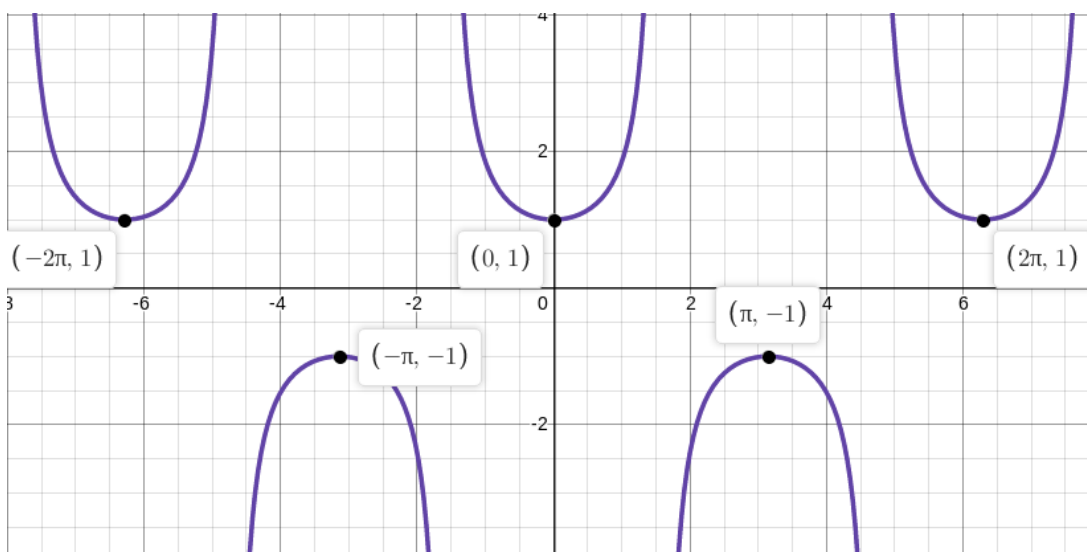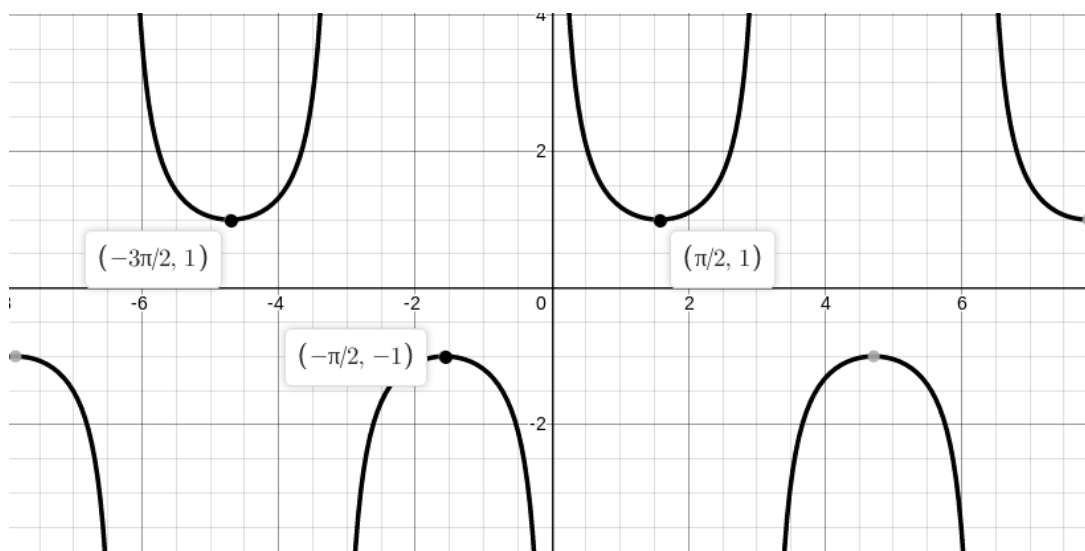
Trigonometric Values

Trigonometry Ratio Table

Angles (In Degrees)	0°	30°	45°	60°	90°	180°	270°	360°
Angles (In Radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined	1
cot	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not Defined	0	Not Defined
csc	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not Defined	-1	Not Defined
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined	-1	Not Defined	1

Identities

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$1 + \tan^2(\theta) = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

Addition Formulas

$$\cos(A \pm B) = \cos(A).\cos(B) \mp \sin(A).\sin(B)$$

$$\sin(A \pm B) = \sin(A).\cos(B) \pm \cos(A).\sin(B)$$

$$\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$$

Double Angle Formulas

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\cot(2\theta) = \frac{\cot^2(\theta) - 1}{2\cot(\theta)}$$

Half-Angle Formulas

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}} = \frac{\sin(\theta)}{1 + \cos(\theta)} = \frac{1 - \cos(\theta)}{\sin(\theta)}$$

Hyperbolic Function

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth(x) = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\text{sech(x)}\frac{1}{\cosh(x)}$$

$$\text{cosech(x)} = \frac{1}{\sinh(x)}$$

The Law Of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos(\theta)$$

Eular Formulas

$$e^{ix} = \cos(x) + i\sin(x)$$

$$e^{-ix} = \cos(x) - i\sin(x)$$

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$$

$$\tan(x) = \frac{1}{(i)}\frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}}$$

Other Equalities

$$\sin(mx).\sin(nx) = \frac{1}{2}\left[\cos((m-n)x) - cos((m+n)x)\right]$$

$$\sin(mx).\cos(nx) = \frac{1}{2}\left[sin((m-n)x) + \sin((m+n)x)\right]$$

$$\cos(mx).\cos(nx) = \frac{1}{2}\left[\cos((m-n)x) + cos((m+n)x)\right]$$

Limit

Existance

Limit exists $\iff$ left-hand and right-hand limits exists and $\lim\limits_{x \to a^-} f(x) = \lim\limits_{x \to a^+} f(x)$.

Properties

Let,

$$\lim_{x \to c} f(x) = L$$

$$and$$

$$\lim_{x \to c} g(x) = M$$

Then,

1. Summation Rule: $\lim\limits_{x \to c}(f(x) + g(x)) = L + M$

2. Difference Rule: $\lim\limits_{x \to c}(f(x) + g(x)) = L + M$

3. Product Rule: $\lim\limits_{x\to c}(f(x).\,g(x)) = L.\,M$

4. Constant Multiplication Rule: $\lim\limits_{x\to c} k.\,f(x) = k.\,L$

5. Quotient Rule: $\lim\limits_{x\to c}(\frac{f(x)}{g(x)}) = \frac{L}{M}$

6. Power Rule: $\lim\limits_{x\to c} f(x)^{\frac{r}{s}} = L^{\frac{r}{s}}$

Limits of Common Functions

$$\lim_{\theta\to 0}\frac{\sin(\theta)}{\theta} = 1$$

$$\lim_{x\to 0}\frac{ln(1+c.\,x)}{x} = c$$

$$\lim_{x\to 0}\frac{\cos(x)-1}{x} = 0$$

$$\lim_{x\to 0}\frac{\sin(x)}{x} = 1$$

Logarithms

1. $\log MN) = \log(M) + \log(N)$

2. $\log(\frac{M}{N}) = \log(M) - \log(N)$

3. $\log(M^k) = k\log(M) \qquad ((\log(M^k) \neq (\log(M))^k)$

4. $\log_b(M) = \frac{\log(M)}{\log(b)}$

Differentiation

Differentiation Properties

Let u and v are functions, and c is constant.

$$(c)' = 0$$

$$(x)' = 1$$

$$(u+v)' = u' + v'$$

$$(cu)' = cu'$$

$$(uv)' = u'v + uv'$$

$$(\frac{u}{v})' = \frac{u'v - uv'}{v^2}$$

$$\left(\frac{c}{u}\right)' = \frac{-cu'}{v^2}$$

Differentiation Table

$$(x^n)' = nx^{n-1}$$

$$(u^n)' = nu^{n-1}u'$$

$$(e^u) = e^u u'$$

$$(a^u)' = a^u \ln(a)u'$$

$$(u^v)' = vu^{v-1}u' + \ln(u)u^v v'$$

$$(log_a u)' = \log_a(e)\frac{u'}{u} = \frac{u'}{u\ln(a)}$$

$$(\ln(u))' = \frac{u'}{u}$$

Derivative of Trigonometric Functions

$$(\sin(u))' = \cos(u)u'$$

$$(\cos(u))' = -\sin(u)u'$$

$$(\tan(u))' = \sec^2(u)u' = \frac{1}{\cos^2(u)}u'$$

$$(\cot(u))' = -\csc^2(u)u' = -\frac{1}{\sin^2(u)}u'$$

$$(\sec(u))' = \sec(u)\tan(u)u'$$

$$(\csc(u))' = -\csc(u)\cot(u)u'$$

Derivative of Inverse Functions

$$(\arcsin(u))' = \frac{u'}{\sqrt{1-u^2}}$$

$$(\arccos(u))' = \frac{-u'}{\sqrt{1-u^2}}$$

$$(\arctan(u))' = \frac{u'}{1+u^2}$$

$$(arccot(u))' = -\frac{u'}{1+u^2}$$

$$(arcsec(u))' = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$(arccsc(u))' = \frac{-u'}{|u|\sqrt{u^2 - 1}}$$

Integral

Table of Integrals

$$\int (x^k)dx = \frac{x^{k+1}}{k+1} + c$$

$$\int (e^x)dx = e^x + c$$

$$\int (a^x)dx = \frac{a^x}{\ln(a)} + c$$

$$\int \frac{1}{x}dx = \ln(|x|) + c$$

Trigonometric Integrals

$$\int \cos^n(x)dx = \frac{1}{n}\sin^{n-1}(x) + c$$

$$\int \sin^n(x)dx = -\frac{1}{n}\cos^{n-1}(x) + c$$

$$\int \tan^n(x)dx = \ln|\sec(x)| + c = \frac{1}{n-1}\tan^{n-1}(x) - \int \tan^{n-2}(x)dx$$

$$\int \cot^n(x)dx = \ln|\sin(x)| + c = \frac{-1}{n-1}\cot^{n-1}(x) - \int \cot^{n-2}(x)dx$$

$$\int \sec^n(x)dx = \ln|\sec(x) + \tan(x)| + c = \frac{1}{n-1}\sec^{n-2}(x)\tan(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)dx$$

$$\int \csc^n(x)dx = \ln|\csc(x) - \tan(x)| + c = \frac{-1}{n-1}\csc^{n-2}(x)\cot(x) + \frac{n-2}{n-1}\int \csc^{n-2}(x)dx$$

$$\int \sec(x)\tan(x)dx = \sec(x) + c$$

$$\int \csc(x)\cot(x)dx = -\csc(x) + c$$

$$\int \sec^2(x)dx = \int \frac{1}{\cos^2(x)}dx = \tan(x) + c$$

$$\int \csc^2(x)dx = \int \frac{1}{\sin^2(x)}dx = -\cot(x) + c$$

$$\int \cosh(x)dx = \sinh(x) + c$$

$$\int \sinh(x)dx = \cosh(x) + c$$

Inverse Trigonometric Integrals

$$\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin(\frac{u}{a}) + c = -\arccos(\frac{u}{a}) + c$$

$$\int \frac{1}{a^2 + u^2} du = \frac{1}{a}\arctan(\frac{u}{a}) + c = -\frac{1}{a}\text{arccot}(\frac{u}{a}) + c$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a}\text{arcsec}(\frac{|u|}{a}) + c = -\frac{1}{a}\text{arccsc}(\frac{|u|}{a}) + c$$