Lesson 20: Factoring Trinomials with a Leading Coefficient of $a \neq 1$

Objective: Factor a trinomial with a leading coefficient of $a \neq 1$.

Students will be able to:

- Identify two integer values that add to b and multiply to $a \cdot c$ in a trinomial expression with ordered coefficients a, b, and c.
- Multiply binomials to verify the accuracy of a factorization.
- Recognize the relationship between factoring and expanding an expression.

Prerequisite Knowledge:

- Identifying a greatest common factor.
- Factor by grouping.
- Application of the distributive property.
- Multiplication and division of algebraic expressions.

Lesson:

When factoring trinomials we use the *ac*-method to split the middle (or linear) term and then factor by grouping. The *ac*-method gets its name from the general trinomial expression, $ax^2 + bx + c$, where a, b, and c are the leading coefficient, linear coefficient, and constant term, respectively.

The *ac*-method is named as such because we will use the product $a \cdot c$ to help find out what two numbers we will need for grouping later on. In the previous lesson, we always found two numbers whose product was equal to c, since the leading coefficient a was 1 in our expression (so $a \cdot c = 1 \cdot c = c$). Now we will be working with trinomials where $a \neq 1$, so we will need to identify two numbers that multiply to ac and add to b. Aside from this adjustment, the process will be the same as before.

When $a = 1$, we were able to use a shortcut, using the numbers that split the middle coefficient for our factors. As we will see in our examples, this shortcut will not work when $a \neq 1$. Therefore, we must go through all the steps of grouping in order to factor the expression.

I - Motivating Example(s):

Example: Factor the given expression.

$3x^2 + 11x + 6$	Multiply to $a \cdot c$ or $3 \cdot 6 = 18$, add to $b = 11$.
$3x^2 + 9x + 2x + 6$	The numbers are 9 and 2, split the linear term.
$3x(x + 3) + 2(x + 3)$	Factor by grouping.
$(x + 3)(3x + 2)$	Our solution.

Example: Factor the given expression.

$8x^2 - 2x - 15$	Multiply to $a \cdot c$ or $8 \cdot (-15) = -120$, add to $b = -2$.
$8x^2 - 12x + 10x - 15$	The numbers are -12 and 10, split the linear term.
$4x(2x - 3) + 5(2x - 3)$	Factor by grouping.
$(2x - 3)(4x + 5)$	Our solution.

II - Demo/Discussion Problems:

Factor each of the given trinomial expressions.

1. $10x^2 - 27x + 5$
2. $4x^2 - xy - 5y^2$
3. $18x^3 + 33x^2 - 30x$

III - Practice Problems:

Factor each of the given trinomial expressions.

1. $7x^2 - 48x + 36$
2. $7n^2 - 44n + 12$
3. $7b^2 + 15b + 2$
4. $7v^2 - 24v - 16$
5. $5a^2 - 13a - 28$
6. $5n^2 - 7n - 24$
7. $2x^2 - 5x + 2$
8. $3r^2 - 4r - 4$
9. $2x^2 + 19x + 35$
10. $7x^2 + 29x - 30$
11. $2b^2 - b - 3$
12. $5x^2 - 26x + 24$
13. $5x^2 + 13x + 6$
14. $3r^2 + 16r + 21$
15. $3x^2 - 17x + 20$
16. $3u^2 + 13uv - 10v^2$
17. $3x^2 + 17xy + 10y^2$
18. $7x^2 - 2xy - 5y^2$
19. $5x^2 + 28xy - 49y^2$
20. $5u^2 + 31uv - 28v^2$
21. $6x^2 - 39x - 21$
22. $10a^2 - 54a - 36$
23. $21x^2 - 87x - 90$
24. $21n^2 + 45n - 54$
25. $14x^2 - 60x + 16$
26. $4r^2 + r - 3$
27. $6x^2 + 29x + 20$
28. $6p^2 + 11p - 7$
29. $4x^2 - 17x + 4$
30. $4r^2 + 3r - 7$
31. $4x^2 + 9xy + 2y^2$
32. $4m^2 + 6mn + 6n^2$
33. $4m^2 - 9mn - 9n^2$
34. $4x^2 - 6xy + 30y^2$
35. $4x^2 + 13xy + 3y^2$
36. $18u^2 - 3uv - 36v^2$
37. $12x^2 + 62xy + 70y^2$
38. $16x^2 + 60xy + 36y^2$
39. $24x^2 - 52xy + 8y^2$
40. $12x^2 + 50xy + 28y^2$