Lesson 59: Holes

Objective: Identify the precise location of one or more holes in the graph of a rational function.

Students will be able to:

- Identify removable discontinuities and their corresponding holes in the graph of a rational function.

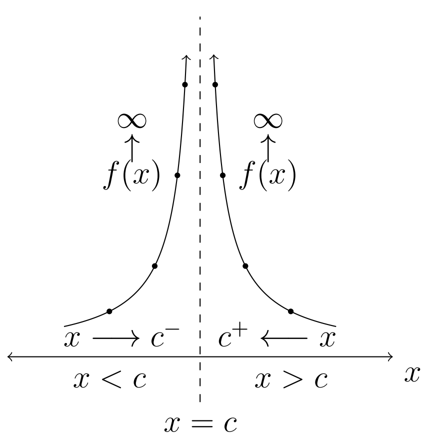
Prerequisite Knowledge:

- Evaluating a function.
- Factoring.
- The Rational Root Theorem.
- Polynomial and /or Synthetic Division.
- Multiplicative identity /inverse.

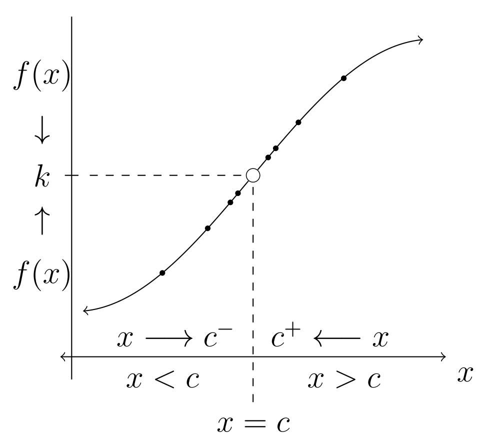
Lesson:

While vertical asymptotes correspond to infinite discontinuities, a hole corresponds to a *removable discontinuity*, since the removal of a single point along a continuous curve creates the hole.

Suppose that the rational function $f(x)$ has a discontinuity at $x = c$, i.e., c is not in the domain of f. If $x = c$ is a vertical asymptote of the graph of f, in the last lesson we saw that as $x \to c$, $f(x) \to \pm\infty$. If $x = c$ represents a hole in the graph of f, however, we will see that as $x \to c$, $f(x) \to k$, for some real number k. This is the fundamental difference between infinite and removable discontinuities.



Infinite Discontinuity



Removable Discontinuity

In the case of the graph of the left, recall that we have the following statements.

$$\text{As } x \to c^+,\ f(x) \to \infty. \qquad \text{As } x \to c^-,\ f(x) \to \infty.$$

Similarly, in the case of the graph on the right, we employ the same idea, using k^+ and k^- in order to identify whether or not the graph of f approaches k from *above* if $f(x) > k$ and *below* if $f(x) < k$.

$$\text{As } x \to c^+,\ f(x) \to k^+. \qquad \text{As } x \to c^-,\ f(x) \to k^-.$$

In virtually all cases, however, it will be sufficient enough to simply state that as $x \to c,\ f(x) \to k$, since further analysis will often prove difficult.

We now state the requirement for a hole, which, as with vertical asymptotes, depends on both the rational function f and its simplified expression.

Let $f(x)$ be a rational function and let $g(x)$ represent the simplified expression for f. If $x = c$ is not in the domain of f, but *is* in the domain of g, then the graph of f will have a hole at $(c, g(c))$.

II - Demo/Discussion Problems:

Identify the coordinates of any holes in the graph of each of the following rational functions. Use Desmos to verify your answers.

1. $f(x) = \dfrac{-2x+4}{4x-8}$
2. $g(x) = \dfrac{x^2+25}{x^2-10x+25}$
3. $h(x) = \dfrac{x^2-9x+20}{x^2-3x-10}$

III - Practice Problems:

Identify the coordinates of any holes in the graph of each of the following rational functions. Use Desmos to verify your answers.

1. $a(x) = \dfrac{2x+6}{x+3}$
2. $g(x) = \dfrac{x^3-16x}{x^2-4x}$
3. $h(x) = \dfrac{x^2-4x-9}{x+2}$
4. $j(x) = \dfrac{18x^3-4x-1}{x^2-4}$
5. $k(x) = \dfrac{x^2-17x+72}{x-9}$
6. $p(x) = \dfrac{x^2+6x+8}{2x+4}$
7. $q(x) = \dfrac{x^5}{x(x-5)}$
8. $r(x) = \dfrac{x^2-11x+30}{x^2-36}$
9. $t(x) = \dfrac{3x^2-12x+12}{x^2+2x-8}$
10. $v(x) = \dfrac{x^2+2x-8}{3x^2-12x+12}$