Lesson 15: Fundamental Functions

Objective: Graph and identify the domain, range, and intercepts of any of the ten fundamental functions.

Students will be able to:

- Identify (as well as produce) the graph of a variety of fundamental functions.
- Identify the domain and range of a variety of fundamental functions using a graph.

Prerequisite Knowledge:

- Definitions of domain and range of a function.
- Graph a function by plotting points.

Lesson:

In this lesson, we focus on ten fundamental function types which will be referenced throughout the rest of the course, as well as one example of each. Each type of function represents a "building block" for understanding the concepts of a traditional algebra course.

Students should be able to both identify and sketch a graph of each function, as well as identify its intercepts, domain (both graphically and algebraically), and range (graphically). Each representative form in the table below includes some element of generalization to reinforce understanding.

Function Type	Representative Form	Example
Linear	$mx + b$	$f(x) = 3x - 4$
Quadratic	$ax^2 + bx + c$	$g(x) = x^2$
Square Root	$\sqrt{x-h}$	$k(x) = \sqrt{x}$
Absolute Value	$\lvert x-h \rvert$	$\ell(x) = \lvert x \rvert$
Cubic	$(x-h)^3$	$m(x) = x^3$
Cube Root	$\sqrt[3]{x-h}$	$n(x) = \sqrt[3]{x}$
Reciprocal (Rational)	$\dfrac{1}{x-h}$	$p(x) = \dfrac{1}{x}$

Function Type	Representative Form	Example
Semicircular	$\sqrt{r^2 - x^2},\ r > 0$	$q(x) = \sqrt{9 - x^2}$
Exponential*	$a^x,\ a > 0, a \neq 1$	$r(x) = 2^x$
Logarithmic*	$\log_a(x),\ a > 0, a \neq 1$	$s(x) = \log_2(x)$

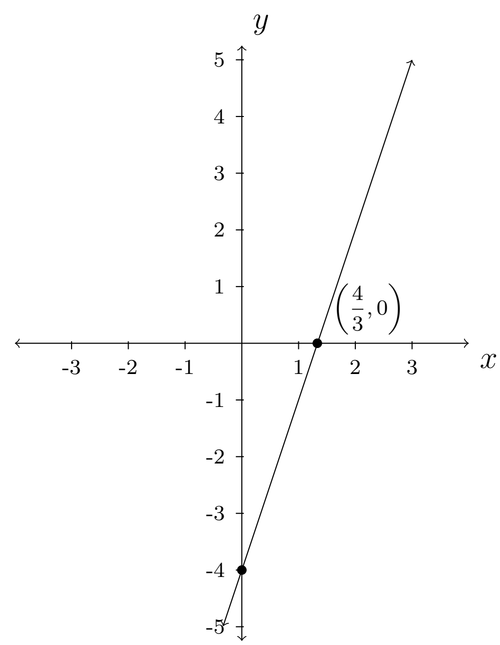
*We have included Exponential and Logarithmic functions for a more complete list. These functions are more formally treated in a Precalculus setting.

I - Motivating Example(s):

Example:

Function Type: Linear ($m \neq 0$)
Representative: $f(x) = 3x - 4$

x	$f(x)$
-3	-13
-2	-10
-1	-7
0	-4
1	-1
$\frac{4}{3}$	0
2	2
3	5



Graph of $f(x) = 3x - 4$

$y-$intercept: $(0, -4)$
$x-$intercept(s): $\left(\frac{4}{3}, 0\right)$

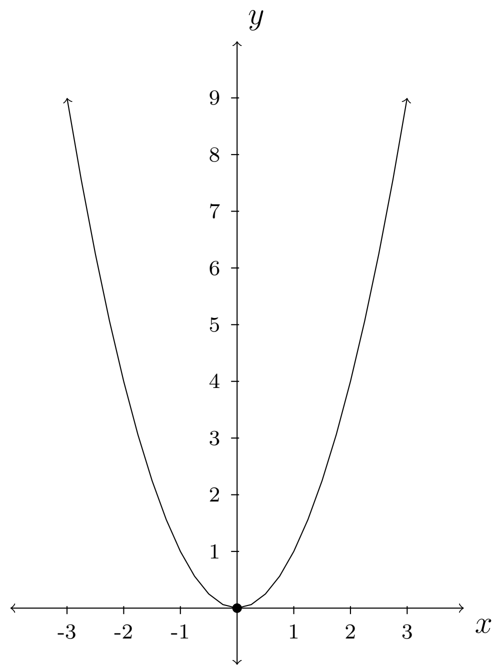
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

Notes: If $m = 0$, then the corresponding graph of $f(x) = b$ is a horizontal line. The domain of f is still $(-\infty, \infty)$, but the range consists of a single value, $\{b\}$.

Example:

Function Type: Quadratic
Representative: $g(x) = x^2$

x	$g(x)$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



Graph of $g(x) = x^2$

$y-$intercept: $(0, 0)$
$x-$intercept(s): $(0, 0)$

Domain: $(-\infty, \infty)$
Range: $[0, \infty)$, or $y \geq 0$

Notes: The domain of any quadratic function is $(-\infty, \infty)$. If $g(x) = a(x - h)^2 + k$, is a quadratic function in vertex form, then if $a > 0$, the corresponding parabola will be concave *up*, and the range of g will be $[k, \infty)$. If $a < 0$, then the corresponding parabola will be concave *down*, and the range of g will be $(-\infty, k]$.

II - Demo/Discussion Problems:

Complete each of the following for the functions listed below.

- Use Desmos to sketch a complete graph of the function.
- Construct a table of points from your graph. Check your table by evaluating the function at the given $x-$coordinates.
- Identify the domain and range of your graph.
- Identify any $x-$ and $y-$intercepts of your graph.

1. $m(x) = x^3$
2. $n(x) = \sqrt[3]{x}$
3. $q(x) = \sqrt{9 - x^2}$
4. $r(x) = 2^x$

III - Practice Problems:

Complete each of the following for the functions listed below.

- Use Desmos to sketch a complete graph of the function.
- Construct a table of points from your graph. Check your table by evaluating the function at the given $x-$coordinates.
- Identify the domain and range of your graph.
- Identify any $x-$ and $y-$intercepts of your graph.

1. $k(x) = \sqrt{x}$
2. $\ell(x) = |x|$
3. $p(x) = \dfrac{1}{x}$
4. $s(x) = \log_2(x)$