Lesson 53: Polynomial Inequalities

Objective: Solve a polynomial inequality by constructing a sign diagram.

Students will be able to:

- Identify the solution for a polynomial inequality using interval notation.

Prerequisite Knowledge:

- Sign diagrams.
- Factoring.
- Evaluating a function at a given value.
- Interval notation.

Lesson:
I - Motivating Example(s):

Example: Solve the polynomial inequality

$$x^4 + 6x^2 - 15x \leq x^4 + 2x^3 - 7x^2.$$

Just as with quadratic inequalities, we begin by setting one side equal to zero. This gives us

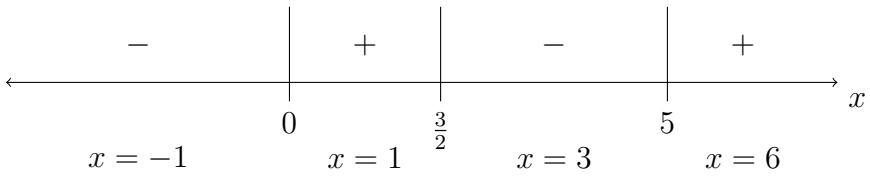
$$2x^3 - 13x^2 + 15x \geq 0.$$

In order to construct a sign diagram, we must find a factorization and identify the roots of the left-hand side of our inequality.

$$2x^3 - 13x^2 + 15x = 2x\left(x - \frac{3}{2}\right)(x - 5)$$

So the dividers in our diagram will be the roots $x = 0, \frac{3}{2}$, and 5. Below is a chart for testing the intervals in our sign diagram, as well as the end result.

Interval	Test Value	Signs	Result
$(-\infty, 0)$	$x = -1$	$(-)(-)(-)$	$-$
$(0, \frac{3}{2})$	$x = 1$	$(+)(-)(-)$	$+$
$(\frac{3}{2}, 5)$	$x = 3$	$(+)(+)(-)$	$-$
$(5, \infty)$	$x = 6$	$(+)(+)(+)$	$+$



So, using our diagram as an aide, we see that the solution to the inequality

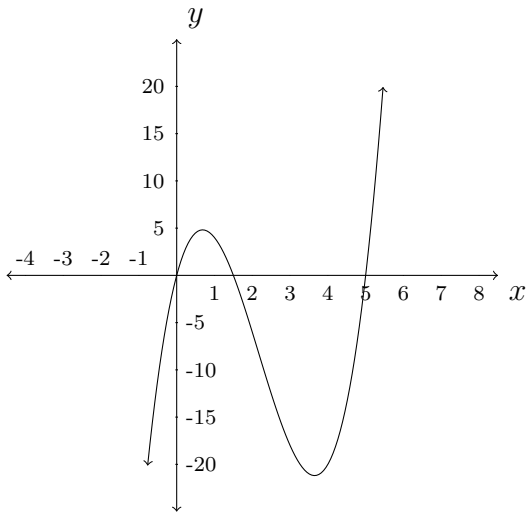
$$2x^3 - 13x^2 + 15x \geq 0,$$

as well as our original inequality

$$x^4 + 6x^2 - 15x \leq x^4 + 2x^3 - 7x^2,$$

will be

$$\left[0, \frac{3}{2}\right] \cup [5, \infty).$$



Since our given inequality was inclusive ($\leq$ or $\geq$), we include the corresponding endpoints in our answer.

We can verify that our answer is correct by comparing it to the graph of the function

$$f(x) = 2x^3 - 13x^2 + 15x,$$

which lies above (or on) the $x-$axis over the intervals in our answer.

II - Demo/Discussion Problems:

Solve each polynomial inequality below, expressing your answers using interval notation. Use Desmos to help confirm that each answer is correct.

1. $x^3 < 4x^2$

2. $x^3 - 7x^2 \leq 12x - 84$

3. $(x-1)^2 \geq 4$

4. $2x^4 > 5x^2 + 3$

III - Practice Problems:

Solve each polynomial inequality below, expressing your answers using interval notation. Use Desmos to help confirm that each answer is correct.

1. $x^4 + x^2 \geq 6$
2. $x^4 - 9x^2 \leq 4x - 12$
3. $4x^3 \geq 3x + 1$
4. $x^4 \leq 16 + 4x - x^3$
5. $3x^2 + 2x < x^4$
6. $\dfrac{x^3 + 2x^2}{2} < x + 2$
7. $\dfrac{x^3 + 20x}{8} \geq x^2 + 2$
8. $19x^2 + 20 > 2x^3 + 49x$