Lesson 14: Finding Domain and Range from a Graph

Objective: Find the domain and range of a function from its graph.

Students will be able to:

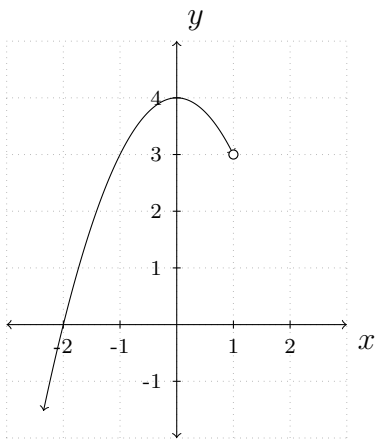
- Find domain graphically.
- Find range graphically.

Prerequisite Knowledge:

- Interval notation
- The definition of a function.
- Graphing a function on the coordinate plane.

Lesson:
I - Motivating Example(s):

Example: Find the domain and range of the function f whose graph is given below.

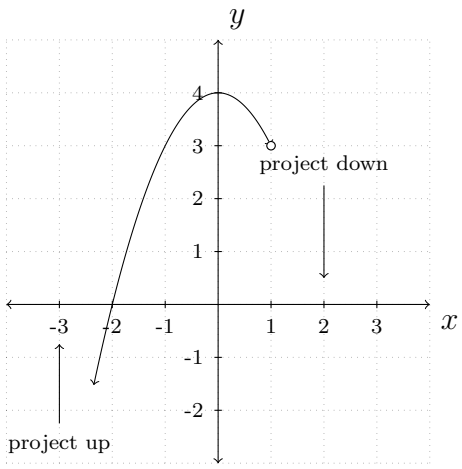


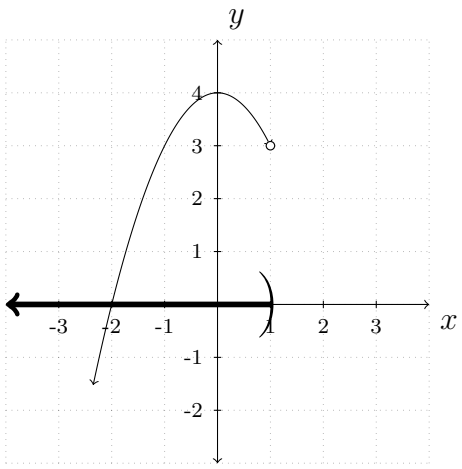
The graph of f

To determine the domain and range of f, we need to determine which x and y-values respectively occur as coordinates of points on the given graph.

To find the domain, it will be helpful to imagine collapsing the curve onto the x-axis and determining the portion of the x-axis that gets covered. This is often described as **projecting** the curve onto the x-axis.

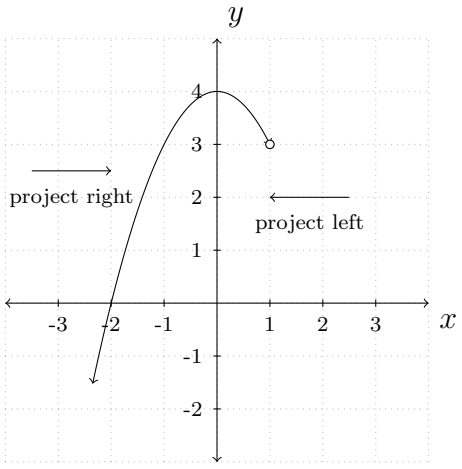
Before we project, we need to pay attention to two subtle notations on the graph: the arrowhead on the lower left corner of the graph indicates that the graph continues to curve downwards to the left forever; and the open circle at $(1, 3)$ indicates that the point $(1, 3)$ is *not* on the graph, but all the points on the curve leading up to $(1, 3)$ are on the graph.

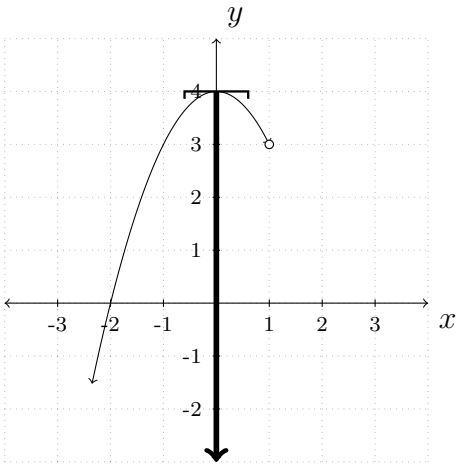




We see from the figure that if we project the graph of f to the x-axis, we get all real numbers less than 1. Using interval notation, we write the domain of f as $(-\infty, 1)$.

To determine the range of f, we use a similar method, projecting the curve onto the y-axis as follows.

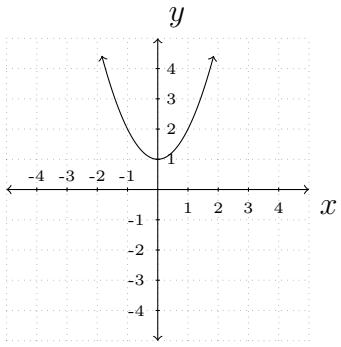


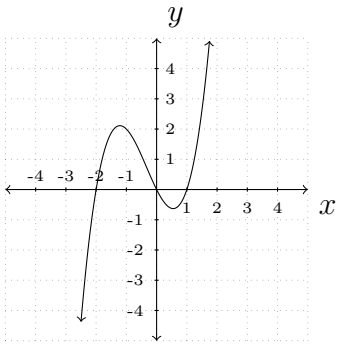


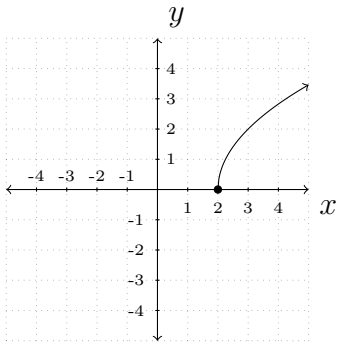
Note that even though there is an open circle at $(1, 3)$, we still include the y value of 3 in our range, since the point $(-1, 3)$ is on the graph of f. We also include $y = 4$ in our answer, since the point $(0, 4)$ is also on our graph. Consequently, the range of f is all real numbers less than or equal to 4, or $(-\infty, 4]$.

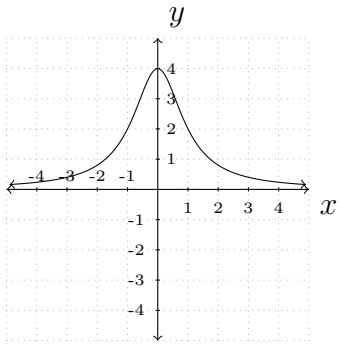
II - Demo/Discussion Problems:

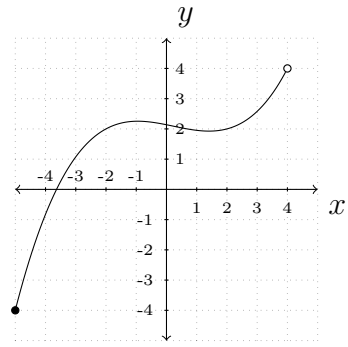
For each of the following graphs, identify the corresponding domain and range. Express your answers using interval notation.

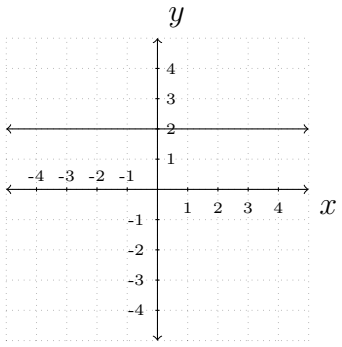
1. 

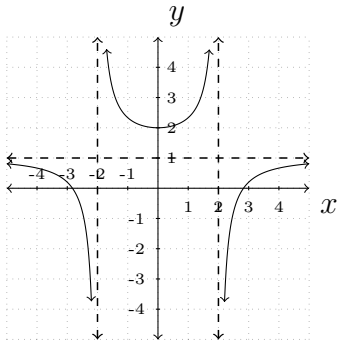
2. 

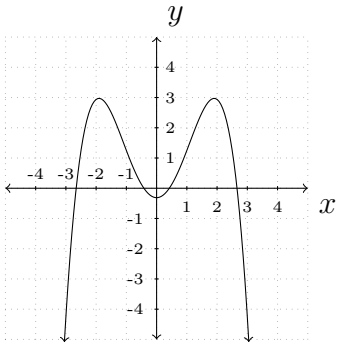
3. 

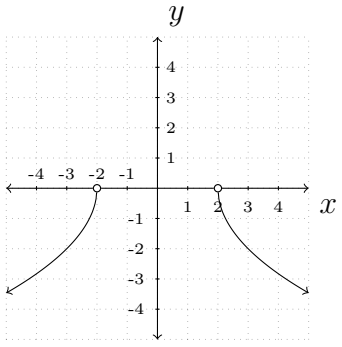
4. 

5. 

6. 

7. 

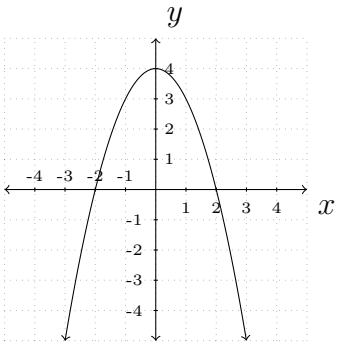
8. 

9. 

III - Practice Problems:

For each of the following graphs, identify the corresponding domain and range. Express your answers using interval notation.

1.



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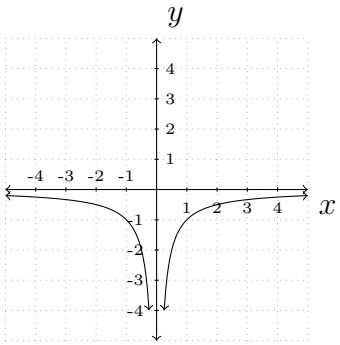

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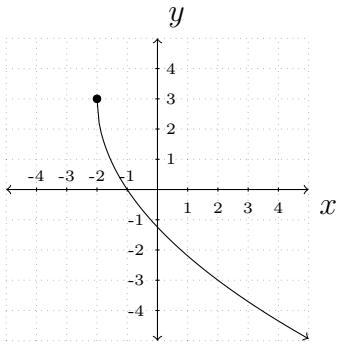

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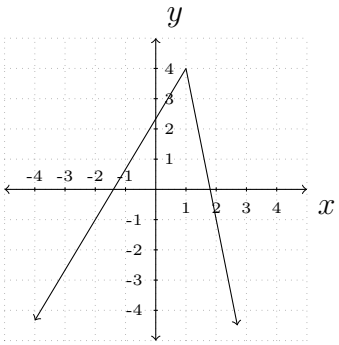

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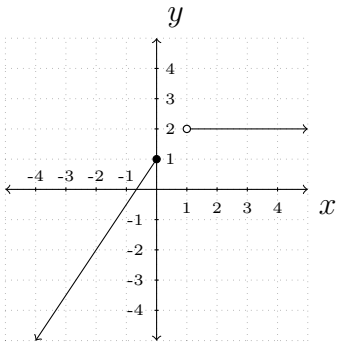

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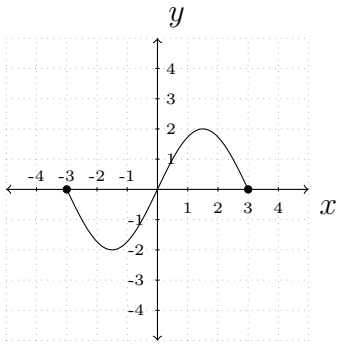

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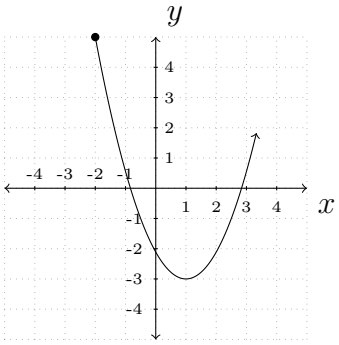

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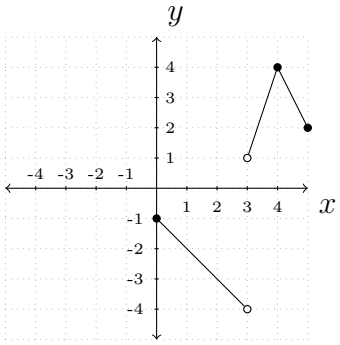

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