Lesson 43: Polynomials Introduction

Objective: Identify key features of and classify a polynomial by degree and number of nonzero terms.

Students will be able to:

- Identify a polynomial from its definition, and arrange a polynomial in descending-power order.
- Identify the degree, set of coefficients, leading coefficient, leading and constant term of a polynomial.
- Classify a polynomial by both its degree and number of nonzero terms.

Prerequisite Knowledge:

- Order of operations.

Lesson:

A *polynomial* in terms of a variable x is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_2 x^2 + a_1 x + a_0,$$

where each *coefficient*, a_i, is a real number ($a_n \neq 0$) and the exponent, or *degree* of the polynomial, n, is a nonnegative integer.
Examples of polynomials include: $f(x) = x^2 + 5$, $f(x) = x$ and $f(x) = -3x^7 + 4x^3 - 5x$. For our general polynomial above, the:

degree	is	n
set of coefficients	is	$\{a_n, a_{n-1}, \ldots, a_1, a_0\}$
leading coefficient	is	a_n
leading term	is	$a_n x^n$
constant term	is	$a_0 x^0 = a_0$.

We will categorize polynomials based upon their degree, as well as the number of terms, after all necessary simplification.

Polynomials Classification by Degree

Degree	Type	Example
0	Constant	-1
1	Linear	$2x + \sqrt{5}$
2	Quadratic	$5x^2 - 32x + 2$
3	Cubic	$-0.5x^3$
4	Quartic	$-3x^4 + 2x^2 + 3x + 1$
5	Quintic	$-2x^5$
6 or more	n^{th} Degree	$-2x^7 + 52x^6 + 12$

One point of note in the table above is the appearance of both rational and irrational coefficients $\left(-0.5 \text{ and } \sqrt{5}\right)$. The appearance of such coefficients is permissible in polynomials, since our coefficients a_i are only required to be real numbers. A coefficient containing the imaginary number $i = \sqrt{-1}$, on the other hand, is not permitted.

Polynomial Classification by Number of Nonzero Terms

Number of Terms	Name	Example
1	Monomial	$4x^5$
2	Binomial	$2x^3 + 1$
3	Trinomial	$-23x^{18} + 4x^2 + 3x$
4	Tetranomial	$-23x^{18} + 4x^2 + 3x + 1$
5 or more	Polynomial	$-2x^4 + x^3 + 15x^2 - 41x + 12$

I - Motivating Example(s):

Example: Identify the degree, set of coefficients, leading coefficient, leading term and constant term for the polynomial

$$f(x) = -19x^5 + 4x^4 - 6x + 21.$$

Classify the polynomial by both degree and number of nonzero terms.

- The degree of this polynomial is $n = 5$, since five is the greatest exponent.
- The leading term, which is the term that contains the greatest exponent (degree), is $a_n x^n = -19x^5$.
- The leading coefficient is the real number being multiplied by x^n in the leading term, namely $a_n = -19$.
- The constant term is $a_0 = 21$, which also represents the $y-$intercept for the graph of the given polynomial.
- The complete set of coefficients for the given polynomial is

$$\{a_5 = -19,\ a_4 = 4,\ a_3 = 0,\ a_2 = 0,\ a_1 = -6,\ a_0 = 21\}.$$

- The polynomial is a quintic tetranomial, as it is degree 5 and consists of 4 nonzeroterms.

II - Demo/Discussion Problems:

Identify the degree, set of coefficients, leading coefficient, leading term and constant term for each of the polynomials listed. Classify each polynomial by both degree and number of nonzero terms. If it is not already provided, write the polynomial in descending-power order.

1. $f(x) = 1$
2. $g(x) = x^3 - x^2$
3. $h(x) = x^2 - x^3$
4. $k(x) = \sqrt{2}x^4 + \pi x^2 - e$
5. $\ell(x) = 21x^4 + 12x^2 - 3x^2 - 9x^2 - 22x^4$
6. $m(x) = 3(x + 1)(x - 1) + 2x + 4x^3 + 3$

III - Practice Problems:

Identify the degree, set of coefficients, leading coefficient, leading term and constant term for each of the polynomials listed. Classify each polynomial by both degree and number of nonzero terms. If it is not already provided, write the polynomial in descending-power order.

1. $f(x) = -2x^3 - 1$
2. $f(x) = -2x^4 + 4x + 1$
3. $f(x) = 40 - x^3$
4. $f(x) = (x-1)^2$
5. $f(x) = 32x^5 + x^2 + x$
6. $f(x) = 4x^2 - 3x^4$
7. $f(x) = -2x^4 - 4x^2 - 6x - 8$
8. $f(x) = 5x + 3x^2 + x^3 + \sqrt{3}$
9. $f(x) = \frac{1}{2}x^4 - 5x^2 - \frac{1}{2}$
10. $f(x) = 12 - 6x + 3x^2 - 2x^3 - x^6$
11. $f(x) = -3x^4 + 15x^3 + x^2 - 27x^3 - x^2 - 13$