Lesson 41: Functions Containing Absolute Values

Objective: Graph a variety of functions that contain an absolute value.

Students will be able to:

- Create a table of values for a function containing an absolute value.
- Identify the intercepts of a function containing an absolute value.
- Graph a function that contains an absolute value.
- Solve equations containing an absolute value graphically.

Prerequisite Knowledge:

- Creating a table of values for a function.
- Definition of an absolute value.

Lesson:

The most basic of functions containing an absolute value is $\ell(x) = |x|$. Later, when we cover transformations of functions, we will see more general forms of such functions. In particular, if we consider the function

$$f(x) = a|x-h| + k,$$

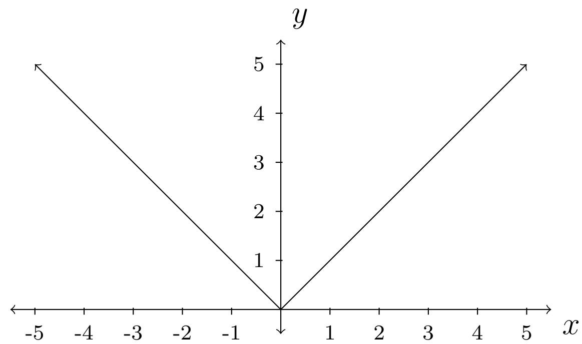
we can make some simple observations about the graph of f. For example, if $a > 0$, the graph of f will point upwards. In this case, the graph of f will have a *minimum* at $y = k$, corresponding to the point (h, k). Alternatively, if $a < 0$, the graph of f will point downwards, and the graph will achieve its *maximum* value at the point (h, k). The magnitude of the coefficient a (i.e. its absolute value) will also determine whether the graph of f is wide or narrow.

I - Motivating Example(s):

Example:

Function Type:	Absolute Value
Representative:	$\ell(x) = \lvert x \rvert$

x	$\ell(x)$
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3



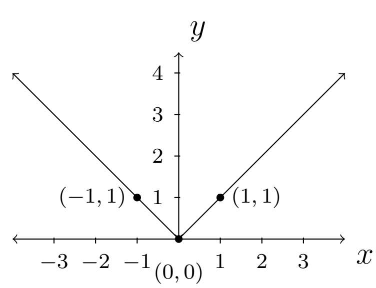
Graph of $\ell(x) = \lvert x \rvert$

$y-$intercept:	$(0,0)$	Domain:	$(-\infty, \infty)$
$x-$intercept(s):	$(0,0)$	Range:	$[0, \infty)$

Notes: The domain of an absolute value function of the form $f(x) = a|x-h|+k$ will remain the same as above. If $a > 0$, the range of f will be $[k, \infty)$. If $a < 0$, the range of f will be $(-\infty, k]$.

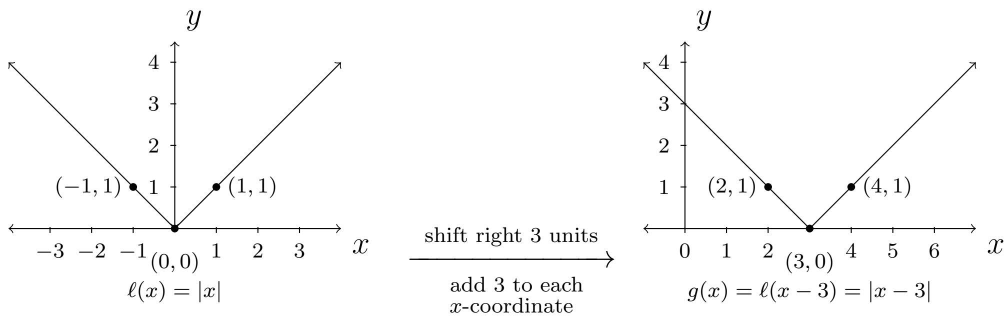
Example: Use the graph of $\ell(x) = |x|$ to graph the function $g(x) = |x-3|$.

We begin by graphing $\ell(x) = |x|$ and labeling three reference points: $(-1,1)$, $(0,0)$ and $(1,1)$.



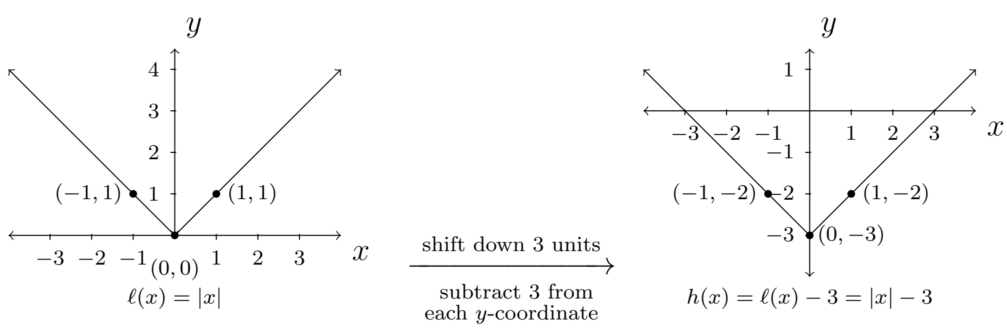
$\ell(x) = |x|$

Since $g(x) = |x-3| = \ell(x-3)$, we will add 3 to each of the $x-$coordinates of the points on the graph of $y = \ell(x)$ to obtain the graph of $y = g(x)$. This shifts the graph of $y = \ell(x)$ to the *right* by 3 units and moves the points $(-1,1)$ to $(2,1)$, $(0,0)$ to $(3,0)$ and $(1,1)$ to $(4,1)$. Connecting these points in the classic '$\vee$' fashion produces the graph of $y = g(x)$.



Example: Use the graph of $\ell(x) = |x|$ to graph the function $h(x) = |x| - 3$.

Since $h(x) = |x| - 3 = f(x) - 3$, we will subtract 3 from each of the $y-$coordinates of the points on the graph of $y = \ell(x)$ to obtain the graph of $y = h(x)$. This shifts the graph of $y = \ell(x)$ *down* by 3 units and moves the points $(-1,1)$ to $(-1,-2)$, $(0,0)$ to $(0,-3)$ and $(1,1)$ to $(1,-2)$. Connecting these points with the '$\vee$' shape produces our graph of $y = h(x)$.



II - Demo/Discussion Problems:

Graph each of the following functions. In each case, make a table of values. Then graph $\ell(x) = |x|$ and f on Desmos and compare the two graphs. Find the intercepts, domain, and range of f.

1. $f(x) = |x+3|$
2. $f(x) = 2|x+3|$
3. $f(x) = -2|x-3|$
4. $f(x) = \frac{1}{2}|3x+1|$
5. $f(x) = \frac{1}{2}|3x+1| - 5$
6. $f(x) = 4 - 2|3x+1|$

III - Practice Problems:

Graph each of the following functions. Make a table of values if necessary. Find the intercepts, domain, and range of the function.

1. $f(x) = -|x| - 7$
2. $g(x) = 4|x| + 2$
3. $h(x) = 5|x-3| + 6$
4. $k(x) = -2|x+1| + 10$
5. $m(x) = |x-3|$
6. $n(x) = -4|x-1| + 1$