Lesson 27: The Discriminant

Objective: Use the discriminant to determine the number of real solutions to a quadratic equation.

Students will be able to:

- Find, simplify, and interpret the discriminant of a quadratic equation in standard form.

Prerequisite Knowledge:

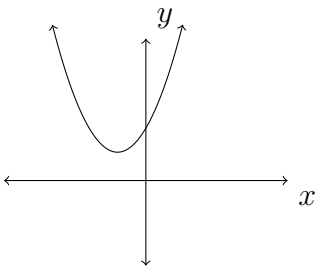
- Identifying coefficients of a quadratic in standard form.
- Order of operations.

Lesson:

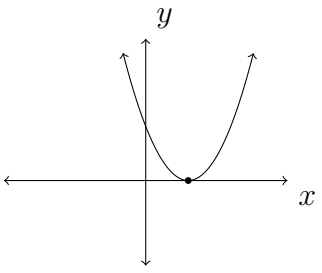
The *discriminant* of a quadratic expression $ax^2 + bx + c$ is defined as the real number $D = b^2 - 4ac$. In the next lesson, we will see that the discriminant is one piece of the larger quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

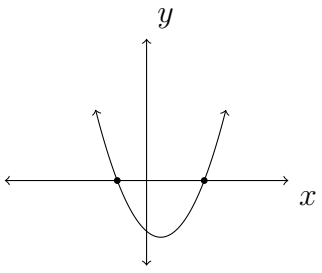
which is used for identifying the roots of the equation $y = ax^2+bx+c$. Since the discriminant appears underneath of a square root in the quadratic formula, whether it is positive, negative, or zero will determine the number of real roots of a quadratic, and consequently the number of $x-$intercepts on its corresponding parabola.



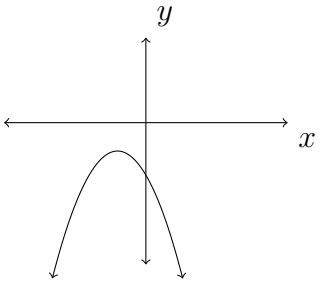
Negative Discriminant
$b^2 - 4ac < 0$
No Real Solutions

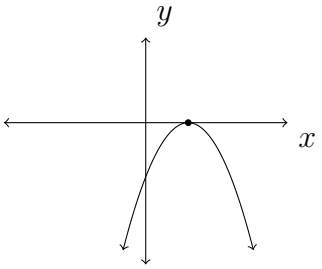


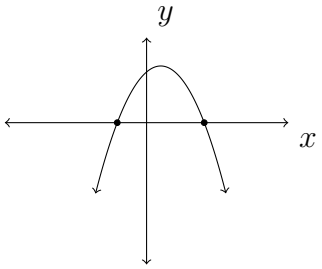
Zero Discriminant
$b^2 - 4ac = 0$
One Real Solution



Positive Discriminant
$b^2 - 4ac > 0$
Two Real Solutions







I - Motivating Example(s):

Example: Determine the number of real roots for the quadratic equation below.

$$y = 3x^2 - 5x + 2$$

$$\begin{aligned} D &= b^2 - 4ac \\ &= (-5)^2 - 4(3)(2) \\ &= 25 - 24 \\ &= 1 \end{aligned}$$

Since $D > 0$, the given equation has two real roots.

II - Demo/Discussion Problems:

Determine the number of real roots for each of the quadratic equations below.

1. $y = -4x^2 - 15$
2. $y = 3x^2 - 12x - 15$
3. $y = 2x^2 + 4x + 2$
4. $y = 10x^2 + 31x + 24$
5. $y = x^2 - 4x + 13$

III - Practice Problems:

Determine the number of real roots for each of the quadratic equations below.

1. $y = x^2 + 6$
2. $y = x^2 + 2x - 1$
3. $y = -3x^2 - 12x - 5$
4. $y = 3x^2 + 12x - 1$
5. $y = -5x^2 - 40x$
6. $y = x^2 - 8x + 15$
7. $y = x^2 + 4x - 2$
8. $y = x^2 + 16x - 2$
9. $y = 4x^2 + 10x$
10. $y = 5x^2 - 4x + 1$
11. $y = -x^2 + 3x - 9$
12. $y = x^2 + 6x + 9$