Lesson 18: Factor by Grouping

Objective: Factor a tetranomial (four-term) expression by grouping.

Students will be able to:

- Split a tetranomial expression into a pair of binomial expressions that share a common factor.
- Factor out a GCF from each binomial expression, and group them using the distributive property.

Prerequisite Knowledge:

- Multiplication properties of exponents.
- Application of the distributive property.
- Multiplication and division of algebraic expressions.
- Identifying a greatest common factor.

Lesson:

In this lesson, we will introduce another useful factorization strategy, known as *grouping*. Grouping is typically employed when faced with an expression containing four terms. Here, it is important to reinforce the fact that factoring is essentially expansion (multiplication) done in reverse.

The first thing we will always do when factoring is try to factor out a GCF. This GCF is often a monomial (a single term) like in the expression $5xy + 10xz$. Here, the GCF is the monomial $5x$, so we would have $5x(y + 2z)$ as our answer. However, a GCF does not have to be a monomial. It could, in fact, be a binomial and contain two terms, as we will see with grouping.

When attempting to factor by grouping, we will always divide an expression into two parts, or groups: group one will usually contain the first two terms of our expression and group two will contain the last two terms. Then we can identify and factor the GCF out of each group. In doing this, our hope is that what is left over in each group will be the same binomial expression. If the resulting expressions match, we can then factor out this matching expression from both of our designated groups, writing what remains in a new set of parentheses.

The key for grouping to be successful is for the two binomials to match exactly, once the GCF has been factored out of both groups. If there is any difference between the two binomials, we either have to choose two different groups or we cannot factor by grouping.

I - Motivating Example(s):

Example: Find and factor out the GCF of the given expression.

$3ax - 7bx$	Both terms have an x in common, factor it out
$x(3a - 7b)$	Our solution

Now we will work with the same expression, replacing x with $(2a+5b)$. In the same way that we factored out a GCF of x we can factor out a GCF which is a binomial, such as $(2a + 5b)$.

Example: Find and factor out the GCF of the given expression.

$3a(2a + 5b) - 7b(2a + 5b)$	Both terms have $(2a + 5b)$ in common, factor it out
$(2a + 5b)(3a - 7b)$	Our solution

Example: Write the expanded form for the given expression.

$(2a + 3)(5b + 2)$	Distribute $(2a + 3)$ into the second set of parentheses
$5b(2a + 3) + 2(2a + 3)$	Distribute each monomial
$10ab + 15b + 4a + 6$	Our solution

Our solution above has four terms in it. We arrived at this solution by focusing on the two parts, $5b(2a + 3)$ and $2(2a + 3)$. Reversing the process above is the central idea behind grouping.

Example: Factor the given expression by grouping.

$10ab + 15b + 4a + 6$	Split the expression into two groups
$(10ab + 15b) + (4a + 6)$	GCF on the left is $5b$, on the right is 2
$5b(2a + 3) + 2(2a + 3)$	$(2a + 3)$ appears twice! Factor out this GCF.
$(2a + 3)(5b + 2)$	Our solution

II - Demo/Discussion Problems:

Factor each of the given expressions by grouping.

1. $5xy - 8x - 10y + 16$
2. $12ab - 14a - 6b + 7$
3. $6x^2 + 9xy - 14x - 21y$
4. $6x^3 - 15x^2 + 2x - 5$
5. $4a^2 - 21b^3 + 6ab - 14ab^2$
6. $7 + y - 3xy - 21x$
7. $8xy - 12y + 15 - 10$

III - Practice Problems:

Factor each of the given expressions by grouping.

1. $40r^3 - 8r^2 - 25r + 5$
2. $35x^3 - 10x^2 - 56x + 16$
3. $3n^3 - 2n^2 - 9n + 6$
4. $14v^3 + 10v^2 - 7v - 5$
5. $15b^3 + 21b^2 - 35b - 49$
6. $6x^3 - 48x^2 + 5x - 40$
7. $3x^3 + 15x^2 + 2x + 10$
8. $28p^3 + 21p^2 + 20p + 15$
9. $35x^3 - 28x^2 - 20x + 16$
10. $7n^3 + 21n^2 - 5n - 15$
11. $7xy - 49x + 5y - 35$
12. $42r^3 - 49r^2 + 18r - 21$
13. $32xy + 40x^2 + 12y + 15x$
14. $15ab - 6a + 5b^3 - 2b^2$
15. $16xy - 56x + 2y - 7$
16. $3mn - 8m + 15n - 40$
17. $2xy - 8x^2 + 7y^3 - 28y^2x$
18. $5mn + 2m - 25n - 10$
19. $40xy + 35x - 8y^2 - 7y$
20. $8xy + 56x - y - 7$
21. $32uv - 20u + 24v - 15$
22. $4uv + 14u^2 + 12v + 42u$
23. $10xy + 30 + 25x + 12y$
24. $24xy + 25y^2 - 20x - 30y^3$
25. $3uv + 14u - 6u^2 - 7v$
26. $56ab + 14 - 49a - 16b$
27. $16xy - 3x - 6x^2 + 8y$