Lesson 26: Solve by Extracting Square Roots

Objective: Solve quadratic equations using the method of extracting square roots.

Students will be able to:

- Solve quadratic equations in vertex form as an alternative to factoring or when factoring fails.
- Approximate an irrational root to a quadratic equation for the purposes of graphing.

Prerequisite Knowledge:

- Simplifying radicals.
- Solving by isolating a quadratic term.
- Complex numbers.

Lesson:

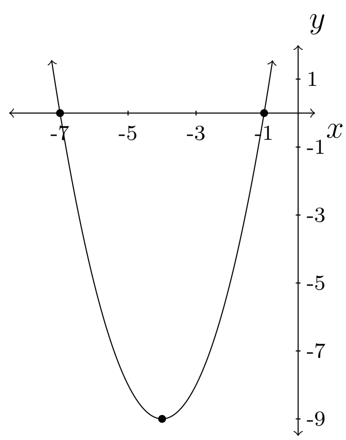
We now introduce a new technique for solving quadratic equations, known as *extracting square roots*. This method will only be employed once we have identified the vertex form for a given quadratic, $y = a(x-h)^2 + k$. The general steps for extracting square roots are shown in our first example, and the requirement of the vertex form will be essential.

I - Motivating Example(s):

Example: Determine the zeros of the quadratic equation $y = ax^2 + bx + c$, where $a \neq 0$. First obtain the vertex form: $h = -\dfrac{b}{2a}$, set $x = h$ to find k.

$$
\begin{aligned}
a(x-h)^2 + \cancel{k} &= 0 && \text{Vertex form} \\
\underline{-\cancel{k}} \quad & \underline{-k} && \text{Subtract } k \text{ from both sides} \\
\frac{\cancel{a}(x-h)^2}{\cancel{a}} &= \frac{-k}{a} && \text{Divide both sides by } a \\
(x-h)^2 &= -\frac{k}{a} && \\
\sqrt{(x-h)^2} &= \pm\sqrt{-\frac{k}{a}} && \text{Take square root of both sides} \\
& && \quad \text{to extract radicand, } x-h \\
x - \cancel{h} &= \pm\sqrt{-\frac{k}{a}} && \\
\underline{+\cancel{h}} \quad & \quad \underline{+h} && \text{Add } h \text{ to both sides} \\
x &= h \pm \sqrt{-\frac{k}{a}} && \text{Our solution}
\end{aligned}
$$

Example: Use the method of extracting square roots to find the zeros of the equation $y = (x+4)^2 - 9$.



$$
\begin{array}{rl}
0 = (x+4)^2 - 9 & \text{Set equal to zero} \\
\underline{\mathbf{+9}} \qquad\qquad \underline{\mathbf{+9}} & \text{Isolate the square} \\
9 = (x+4)^2 & \text{Introduce a} \\
\pm\sqrt{9} = \sqrt{(x+4)^2} & \quad \text{square root; include } \pm \\
\pm 3 = x + 4 & \text{Solve for } x \\
\underline{\mathbf{-4}} \quad \underline{\mathbf{-4}} & \\
x = \pm 3 - 4 & \text{Two solutions} \\
x = 3 - 4 \ \Rightarrow x = -1 & \text{First solution} \\
x = -3 - 4 \Rightarrow x = -7 & \text{Second solution}
\end{array}
$$

Our zeros are $x = -7$ and -1. The corresponding $x-$intercepts are at $(-7, 0)$ and $(-1, 0)$.

Example: Use the method of extracting square roots to find the zeros of the equation $y = -2(x+3)^2 + 48$.

We show our solution below, this time omitting each step in the overall simplification.

$$
\begin{array}{rcl}
-2(x+3)^2 + 48 & = & 0 \\
-2(x+3)^2 & = & -48 \\
(x+3)^2 & = & 24 \\
\sqrt{(x+3)^2} & = & \pm\sqrt{24} \\
x + 3 & = & \pm\sqrt{4}\sqrt{6} \\
x & = & -3 \pm 2\sqrt{6}
\end{array}
$$

Note that we can approximate our two roots, by realizing that

$$2 = \sqrt{4} < \sqrt{6} < \sqrt{9} = 3.$$

Since $\sqrt{6} \approx 2.4$, we can say that our two roots are $x \approx -3 \pm 4.8$. This reduces to $x \approx 1.8$ and $x \approx -7.8$. We may conclude that our $x-$intercepts are approximately located at $(1.8, 0)$ and $(-7.8, 0)$.

II - Demo/Discussion Problems:

Solve each of the following equations for all possible x. Classify each solution as either real or imaginary. If your answer includes a square root, find a decimal approximation for your answer(s).

1. $y = -3(x-1)^2 + 12$
2. $y = (x+5)^2 - 8$
3. $y = \frac{1}{8}(x-6)^2 - 5$
4. $y = -1(x+3)^2 - 4$
5. $y = -\frac{1}{4}(x+3)^2 + 27$

III - Practice Problems:

Solve each of the following equations for all possible x. Classify each solution as either real or imaginary. If your answer includes a square root, find a decimal approximation for your answer(s).

1. $y = 2(x-4)^2 - 200$
2. $y = -2(x-7)^2 + 50$
3. $y = (x-4)^2 - 98$
4. $y = (x-12)^2 - 5$
5. $y = x^2 + 18$
6. $y = (x-16)^2$
7. $y = -3(x-3)^2 + 30$
8. $y = -4(x-1)^2 + 20$
9. $y = -4(x+6)^2 + 8$
10. $y = \frac{1}{20}(x-1)^2 - 15$
11. $y = (x+2)^2 + 12$
12. $y = 9(x-11)^2 - 81$