Lesson 1: Solving Linear Equations

Objective: Solve general linear equations with variables on both sides of the equation.

Students will be able to:

- Solve and check the solutions to linear equations.

Prerequisite Knowledge:

- Adding, subtracting, and multiplying fractions.
- Finding a least common multiple (LCM).
- Applying the distributive property.
- Checking solutions to equations.

Lesson:

I - Motivating Example(s):

We wish to solve the equation

$$\frac{2}{3}x - 2 = \frac{3}{2}x + \frac{1}{6}.$$

To do so, we will "clear out" all denominators in the equation by multiplying each term in the equation by a least common multiple of the denominators (LCM). In this case, our LCM is 6.

$$\mathbf{6} \cdot \frac{2}{3}x - \mathbf{6} \cdot 2 = \mathbf{6} \cdot \frac{3}{2}x + \mathbf{6} \cdot \frac{1}{6}$$

Cancel and reduce each term to eliminate all fractions.

$$\begin{aligned} \cancel{\mathbf{6}} \cdot \frac{2}{\cancel{3}}x - \mathbf{6} \cdot 2 &= \cancel{\mathbf{6}} \cdot \frac{3}{\cancel{2}}x + \cancel{\mathbf{6}} \cdot \frac{1}{\cancel{6}} \\ \mathbf{2} \cdot 2x - \mathbf{6} \cdot 2 &= \mathbf{3} \cdot 3x + \mathbf{1} \cdot 1 \\ 4x - 12 &= 9x + 1 \end{aligned}$$

Combine like terms and solve the resulting two-step equation for x.

$$\begin{aligned} 4x - 12 &= 9x + 1 \\ -12 &= 5x + 1 \\ -13 &= 5x \\ x &= -\frac{13}{5} \end{aligned}$$

Check your answer by plugging it back into the **original** equation and simplifying.

$$\begin{aligned} \frac{2}{3} \cdot \left(-\frac{13}{5}\right) - 2 &= \frac{3}{2} \cdot \left(-\frac{13}{5}\right) + \frac{1}{6} \\ -\frac{26}{15} - 2 &= -\frac{39}{10} + \frac{1}{6} \end{aligned}$$

Multiply through by the LCM and simplify.

$$\begin{aligned}
\mathbf{30}\cdot\left(-\frac{26}{15}\right)-\mathbf{30}\cdot 2 &= \mathbf{30}\cdot\left(-\frac{39}{10}\right)+\mathbf{30}\cdot\frac{1}{6}\\
\cancel{\mathbf{30}}\cdot\left(-\frac{26}{\cancel{15}}\right)-\mathbf{30}\cdot 2 &= \cancel{\mathbf{30}}\cdot\left(-\frac{39}{\cancel{10}}\right)+\cancel{\mathbf{30}}\cdot\frac{1}{\cancel{6}}\\
\mathbf{2}\cdot(-26)-\mathbf{30}\cdot 2 &= \mathbf{3}\cdot(-39)+\mathbf{5}\cdot 1\\
-52-60 &= -117+5\\
-112 &= -112\ \checkmark
\end{aligned}$$

Since the resulting equation is true, our solution is correct.

II - Demo/Discussion Problems:

Solve each equation. Check your answer.

1. $\frac{3}{4}x-\frac{7}{2}=\frac{5}{6}$
2. $\frac{3}{2}\left(\frac{5}{9}x+\frac{4}{27}\right)=3$
3. $\frac{3}{4}x-\frac{1}{2}=\frac{1}{3}\left(\frac{3}{4}x+6\right)-\frac{7}{2}$

III - Practice Problems:

Solve each equation.

1) $2-(-3a-8)=1$
2) $2(-3n+8)=-20$
3) $-5(-4+2v)=-50$
4) $2-8(-4+3x)=34$
5) $66=6(6+5x)$
6) $32=2-5(-4n+6)$
7) $0=-8(p-5)$
8) $-55=8+7(k-5)$
9) $-2+2(8x-7)=-16$
10) $-(3-5n)=12$
11) $-21x+12=-6-3x$
12) $-3n-27=-27-3n$
13) $-1-7m=-8m+7$
14) $56p-48=6p+2$
15) $1-12r=29-8r$
16) $4+3x=-12x+4$
17) $20-7b=-12b+30$
18) $-16n+12=39-7n$
19) $-32-24v=34-2v$
20) $17-2x=35-8x$
21) $-2-5(2-4m)=33+5m$
22) $-25-7x=6(2x-1)$
23) $-4n+11=2(1-8n)+3n$
24) $-7(1+b)=-5-5b$
25) $-6v-29=-4v-5(v+1)$
26) $-8(8r-2)=3r+16$
27) $2(4x-4)=-20-4x$
28) $-8n-19=-2(8n-3)+3n$
29) $-a-5(8a-1)=39-7a$
30) $-4+4k=4(8k-8)$
31) $-57=-(-p+1)+2(6+8p)$
32) $16=-5(1-6x)+3(6x+7)$
33) $-2(m-2)+7(m-8)=-67$
34) $7=4(n-7)+5(7n+7)$

35) $50 = 8(7 + 7r) - (4r + 6)$
36) $-8(6 + 6x) + 4(-3 + 6x) = -12$
37) $-8(n - 7) + 3(3n - 3) = 41$
38) $-76 = 5(1 + 3b) + 3(3b - 3)$
39) $-61 = -5(5r - 4) + 4(3r - 4)$
40) $-6(x - 8) - 4(x - 2) = -4$
41) $-2(8n - 4) = 8(1 - n)$
42) $-4(1 + a) = 2a - 8(5 + 3a)$
43) $-3(-7v + 3) + 8v = 5v - 4(1 - 6v)$
44) $-6(x - 3) + 5 = -2 - 5(x - 5)$
45) $-7(x - 2) = -4 - 6(x - 1)$
46) $-(n + 8) + n = -8n + 2(4n - 4)$
47) $-6(8k + 4) = 8(6k + 3) + 12$
48) $-5(x + 7) = 4(-8x - 2)$
49) $-2(1 - 7p) = 8(p - 7)$
50) $8(-8n + 4) = 4(-7n + 8)$

Solve each equation.

51) $\frac{3}{5}(1 + p) = \frac{21}{20}$
52) $-\frac{1}{2} = \frac{3}{2}k + \frac{3}{2}$
53) $0 = -\frac{5}{4}(x - \frac{6}{5})$
54) $\frac{3}{2}n - \frac{8}{3} = -\frac{29}{12}$
55) $\frac{3}{4} - \frac{5}{4}m = \frac{113}{24}$
56) $\frac{11}{4} + \frac{3}{4}r = \frac{163}{32}$
57) $\frac{635}{72} = -\frac{5}{2}(-\frac{11}{4} + x)$
58) $-\frac{16}{9} = -\frac{4}{3}(\frac{5}{3} + n)$
59) $2b + \frac{9}{5} = -\frac{11}{5}$
60) $\frac{3}{2} - \frac{7}{4}v = -\frac{9}{8}$
61) $\frac{3}{2}(\frac{7}{3}n + 1) = \frac{3}{2}$
62) $\frac{41}{9} = \frac{5}{2}(x + \frac{2}{3}) - \frac{1}{3}x$
63) $-a - \frac{5}{4}(-\frac{8}{3}a + 1) = -\frac{19}{4}$
64) $\frac{1}{3}(-\frac{7}{4}k + 1) - \frac{10}{3}k = -\frac{13}{8}$
65) $\frac{55}{6} = -\frac{5}{2}(\frac{3}{2}p - \frac{5}{3})$
66) $-\frac{1}{2}(\frac{2}{3}x - \frac{3}{4}) - \frac{7}{2}x = -\frac{83}{24}$
67) $\frac{16}{9} = -\frac{4}{3}(-\frac{4}{3}n - \frac{4}{3})$
68) $\frac{2}{3}(m + \frac{9}{4}) - \frac{10}{3} = -\frac{53}{18}$
69) $-\frac{5}{8} = \frac{5}{4}(r - \frac{3}{2})$
70) $\frac{1}{12} = \frac{4}{3}x + \frac{5}{3}(x - \frac{7}{4})$
71) $-\frac{11}{3} + \frac{3}{2}b = \frac{5}{2}(b - \frac{5}{3})$
72) $\frac{7}{6} - \frac{4}{3}n = -\frac{3}{2}n + 2(n + \frac{3}{2})$
73) $-(-\frac{5}{2}x - \frac{3}{2}) = -\frac{3}{2} + x$
74) $-\frac{149}{16} - \frac{11}{3}r = -\frac{7}{4}r - \frac{5}{4}(-\frac{4}{3}r + 1)$
75) $\frac{45}{16} + \frac{3}{2}n = \frac{7}{4}n - \frac{19}{16}$
76) $-\frac{7}{2}(\frac{5}{3}a + \frac{1}{3}) = \frac{11}{4}a + \frac{25}{8}$
77) $\frac{3}{2}(v + \frac{3}{2}) = -\frac{7}{4}v - \frac{19}{6}$
78) $-\frac{8}{3} - \frac{1}{2}x = -\frac{4}{3}x - \frac{2}{3}(-\frac{13}{4}x + 1)$
79) $\frac{47}{9} + \frac{3}{2}x = \frac{5}{3}(\frac{5}{2}x + 1)$
80) $\frac{1}{3}n + \frac{29}{6} = 2(\frac{4}{3}n + \frac{2}{3})$