Lesson 30: Finding Domain Algebraically

Objective: Find the domain of a function by algebraic methods.

Students will be able to:

- Determine the appropriate course of action for identifying the domain of a variety of algebraic functions (polynomial, rational, radical, etc.).
- Identify the domain of an arbitrary algebraic function.

Prerequisite Knowledge:

- Solving basic inequalities.
- Interval notation.

Lesson:

When trying to identify the domain of a function that has been described algebraically or whose graph is not known, we will often need to consider what is *not* permissible for the function, then exclude any values of x that will make the function undefined from the interval $(-\infty, \infty)$. What is left will be our domain. With virtually every algebraic function, this amounts to avoiding the following situations.

- Negatives under an even radical $\left(\sqrt{\ }\ ,\ \sqrt[4]{\ }\ ,\ \sqrt[6]{\ }\ ,\ \ldots\right)$
- Zero in a denominator

I - Motivating Example(s):

Example: Find the domain of $f(x) = \frac{1}{3}x^2 - x$.

$f(x) = \dfrac{1}{3}x^2 - x$	No radicals or variables in a denominator
	No values of x need to be excluded
All real numbers or $(-\infty, \infty)$	Our solution

Our next example will be of a *rational function*, which is defined as a ratio of two polynomial functions. We will explore rational functions and their graphs in a later lesson. Since rational functions usually include expressions in a denominator, their domains will often require us to exclude one or more values of x.

Example: Find the domain of the function $f(x) = \dfrac{3x-1}{x^2+x-6}$.

$$\begin{aligned} f(x) &= \frac{3x-1}{x^2+x-6} && \text{Cannot have zero in a denominator} \\ x^2+x-6 &\neq 0 && \text{Solve by factoring} \\ (x+3)(x-2) &\neq 0 && \text{Set each factor not equal to zero} \\ x+3 \neq 0 \text{ and } x-2 &\neq 0 && \text{Solve each inequality} \\ x &\neq -3, 2 && \text{Our solution as an inequality} \\ (-\infty,-3)\cup(-3,2)\cup(2,\infty) & && \text{Our solution using interval notation} \end{aligned}$$

Although one can easily see that $x = \frac{1}{3}$ will make the numerator equal zero, since $x = \frac{1}{3}$ does not coincide with the two values obtained above (either -3 or 2), we should not exclude it from our domain. Whenever we are finding the domain of a rational function, we need not be concerned at all with the numerator, and instead must restrict our domain to exclude any value for x that would make the *denominator* equal to zero.

Example: Find the domain of $f(x) = \sqrt{-2x+3}$.

$$\begin{aligned} f(x) &= \sqrt{-2x+3} && \text{Even radical; cannot have negative underneath} \\ -2x+3 &\geq 0 && \text{Set greater than or equal to zero and solve} \\ -2x &\geq -3 && \text{Remember to switch direction of inequality} \\ x \leq \frac{3}{2} \text{ or } &\left(-\infty, \frac{3}{2}\right] && \text{Our solution as an inequality or an interval} \end{aligned}$$

II - Demo/Discussion Problems:

Find the domain of each of the following functions. Express your answers using interval notation.

1. $f(x) = |3x-2|$
2. $g(x) = (3x-2)^2$
3. $h(x) = \dfrac{1}{3x-2}$
4. $k(x) = \sqrt{3x-2}$
5. $k(x) = \sqrt[3]{3x-2}$
6. $\ell(x) = \sqrt[4]{2-3x}$
7. $m(x) = \dfrac{x-2}{\sqrt{3x-2}}$
8. $n(x) = \dfrac{\sqrt{3x-2}}{x-2}$

III - Practice Problems:

Find the domain of each of the following functions. Express your answers using interval notation.

1. $g(x) - 4x^2$
2. $f(x) = x^4 - 13x^3 + 56x^2 - 19$
3. $g(x) = x^2 - 4$
4. $k(x) = \dfrac{x}{x-8}$
5. $h(x) = \dfrac{x-5}{x+4}$
6. $h(x) = \dfrac{x-2}{x+1}$
7. $k(x) = \dfrac{x-2}{x-2}$
8. $k(x) = \dfrac{3x}{x^2+x-2}$
9. $g(x) = \dfrac{2x}{x^2-9}$
10. $f(x) = \dfrac{2x}{x^2+9}$
11. $h(x) = \dfrac{x+4}{x^2-36}$
12. $f(x) = \sqrt{3-x}$
13. $g(x) = \sqrt{2x+5}$
14. $f(x) = 5\sqrt{x-1}$
15. $h(x) = 9x\sqrt{x+3}$
16. $k(x) = \dfrac{\sqrt{7-x}}{x^2+1}$
17. $f(x) = \sqrt{6x-2}$
18. $g(x) = \dfrac{6}{\sqrt{6x-2}}$
19. $k(x) = \dfrac{4}{\sqrt{x-3}}$
20. $g(x) = \dfrac{x}{\sqrt{x-8}}$
21. $h(x) = \sqrt[3]{6x-2}$
22. $k(x) = \dfrac{6}{4-\sqrt{6x-2}}$
23. $f(x) = \dfrac{\sqrt{6x-2}}{x^2-36}$
24. $g(x) = \dfrac{\sqrt[3]{6x-2}}{x^2+36}$
25. $h(x) = \sqrt{x-7} + \sqrt{9-x}$
26. $h(t) = \dfrac{\sqrt{t}-8}{5-t}$
27. $f(r) = \dfrac{\sqrt{r}}{r-8}$
28. $k(v) = \dfrac{1}{4-\dfrac{1}{v^2}}$
29. $f(y) = \sqrt[3]{\dfrac{y}{y-8}}$
30. $k(w) = \dfrac{w-8}{5-\sqrt{w}}$