Lesson 31: Solving Functions

Objective: Solve functions using appropriate notation.

Students will be able to:

- Solve an equation to determine a set of inputs that produce a given output.

Prerequisite Knowledge:

- Order of operations.
- Isolating a variable.

Lesson:

In a previous lesson we discussed evaluating functions for a given input, $x = a$. In this lesson, we focus on *solving* functions for a particular output, $y = a$, or $f(x) = a$. There is a fundamental difference between evaluating, $f(a)$, and solving functions, $f(x) = a$. To see this, we will pay close attention to when $a = 0$ and focus on the graphical implications.

When we are evaluating $f(0)$, we are finding the $y-$coordinate associated with when $x = 0$. This corresponds to the $y-$intercept in our graph of f. Alternatively, when we are solving $f(x) = 0$, we are finding *any and all* $x-$coordinates associated with when $y = 0$. This corresponds to the (set of) $x-$intercept(s) in the graph of f.

Whereas evaluating functions requires us to use the standard order of operations, PEMDAS, solving a function usually requires us to use the *reverse* order of operations, SADMEP, in addition to other methods, such as factoring. We do this in order to *isolate* the variable x.

I - Motivating Example(s):

Example: Given $f(x) = x^2 + 3x + 5$, find all x such that $f(x) = 5$.

$f(x) = x^2 + 3x + 5$	Substitute 5 in for $f(x)$
$5 = x^2 + 3x + 5$	Solve for x by factoring
$0 = x^2 + 3x$	Set equal to 0
$0 = x(x + 3)$	Factor
$x = 0$ or $x = -3$	Our solutions

The above answer can be verified by checking. When we input $x = 0$ into the function, we simplify to find that $f(0) = 5$. Similarly, we see that when $x = -3$, $f(-3) = 5$.

II - Demo/Discussion Problems:

1. Given $h(x) = 4x - 1$, find all x such that $h(x) = -3$.

2. Given $g(x) = \dfrac{1}{4x-1}$, find all x such that $g(x) = 0$.

3. Given $k(x) = 2|2x - 3| + 3$, find all x such that $k(x) = 11$.

4. Given $\ell(x) = \sqrt{3 - 5x}$, find all x such that $\ell(x) = 0$

5. Given $m(x) = -\sqrt{25 - x^2}$, find all x such that $m(x) = 0$

6. Given $f(x) = x^2 - 10x + 28$, find all x such that $f(x) = 5$.
   Hint: Use either the Vertex Form or the Quadratic Formula.

III - Practice Problems:

Find $f(0)$ and solve $f(x) = 0$ for each of the given functions.

1. $f(x) = 2x - 1$
2. $f(x) = 3 - \dfrac{2x}{5}$
3. $f(x) = 2x^2 - 6$
4. $f(x) = x^2 - x - 12$
5. $f(x) = \sqrt{x + 4}$
6. $f(x) = \dfrac{1}{2}\sqrt{1 - 2x}$
7. $f(x) = \sqrt{20 - x^2}$
8. $f(x) = \dfrac{3}{4 - x}$
9. $f(x) = \dfrac{3x^2 - 12x}{4 - x^2}$

Find when $f(x) = 2$ for each of the functions above.