Lesson 22: Square Roots

Objective: Simplify and evaluate expressions involving square roots.

Students will be able to:

- Simplify square roots.

Prerequisite Knowledge:

- Prime factorization of a number.
- Perfect square numbers.
- Find square roots of square numbers.

Lesson:
I - Motivating Example(s):

Example:

$\sqrt{0} = 0$	$\sqrt{121} = 11$
$\sqrt{1} = 1$	$\sqrt{625} = 25$
$\sqrt{4} = 2$	$\sqrt{-81} =$ undefined

The final example of $\sqrt{-81}$ is currently considered to be undefined, since the square root of a negative number does not equal a real number. This is because if we square a positive or a negative number, the answer will be positive, not to mention that $0^2 = 0$. Thus we can only take square roots of nonnegative numbers (positive numbers or zero). In a future lesson, we will define a method we can use to work with and evaluate negative square roots. For now we will simply say they are undefined.

Not all numbers have a "nice" (or *rational*) square root. For example, if we found $\sqrt{8}$ on our calculator, the answer would be 2.828427124746190097..., and even this number is a rounded approximation of the square root. To be as accurate as possible, we will never use the calculator to find decimal approximations of square roots. Instead we will express roots in simplest radical form. We will do this using a property known as the product rule of radicals (in this case, square roots).

$$\textbf{Product Rule of Square Roots}: \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

More generally,

$$\textbf{Product Rule of Radicals}: \sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

We can use the product rule of square roots to simplify an expression such as $\sqrt{180} = \sqrt{36 \cdot 5}$ by splitting it into two roots, $\sqrt{36} \cdot \sqrt{5}$, and simplifying the first root, $6\sqrt{5}$. The trick in this process is being able to recognize that an expression like $\sqrt{180}$ may be rewritten as $\sqrt{36 \cdot 5}$, since $180 = 36 \cdot 5$. In the case of $\sqrt{8}$, we may write $\sqrt{8} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$.

There are several ways of applying the product rule of square roots. The most common and, with a bit of practice, fastest method is to find perfect squares that divide nicely into the radicand (the number under the radical). This is demonstrated in the next example.

Example: Completely simplify the given radical.

$\sqrt{75}$	75 is divisible by 25, a perfect square
$\sqrt{25 \cdot 3}$	Split into factors
$\sqrt{25} \cdot \sqrt{3}$	Product rule, take the square root of 25
$5\sqrt{3}$	Our solution

II - Demo/Discussion Problems:

Completely simplify the given radicals.

1. $5\sqrt{63}$
2. $\sqrt{72}$
3. $-5\sqrt{72x^3y^4}$
4. $-5\sqrt{18x^4y^6z^{10}}$

III - Practice Problems:

Completely simplify each of the following square roots completely.

1. $\sqrt{245}$
2. $\sqrt{125}$
3. $\sqrt{36}$
4. $\sqrt{196}$
5. $\sqrt{12}$
6. $\sqrt{72}$
7. $3\sqrt{12}$
8. $5\sqrt{32}$
9. $6\sqrt{128}$
10. $7\sqrt{128}$
11. $-8\sqrt{392}$
12. $-7\sqrt{63}$
13. $\sqrt{192n}$
14. $\sqrt{343b}$
15. $\sqrt{196v^2}$
16. $\sqrt{100n^3}$
17. $\sqrt{252x^2}$
18. $\sqrt{200a^3}$
19. $-\sqrt{100k^4}$
20. $-4\sqrt{175p^4}$
21. $-7\sqrt{64x^4}$
22. $-2\sqrt{128n}$
23. $-5\sqrt{36m}$
24. $8\sqrt{112p^2}$
25. $\sqrt{45x^2y^2}$
26. $\sqrt{72a^3b^4}$
27. $\sqrt{16x^3y^3}$
28. $\sqrt{512a^4b^2}$
29. $\sqrt{320x^4y^4}$
30. $\sqrt{512m^4n^3}$
31. $6\sqrt{80xy^2}$
32. $8\sqrt{98mn}$
33. $5\sqrt{245x^2y^3}$
34. $2\sqrt{72x^2y^2}$
35. $-2\sqrt{180u^3v}$
36. $-5\sqrt{96x^4y^3}$
37. $-8\sqrt{180x^4y^2z^4}$
38. $6\sqrt{50a^4bc^2}$
39. $2\sqrt{80hj^4k}$
40. $-\sqrt{32xy^2z^3}$
41. $-4\sqrt{54mnp^2}$
42. $-8\sqrt{32m^2p^4q}$