Lesson 54: Rational Function Introduction and Terminology

Objective: Define and Identify key features of rational functions.

Students will be able to:

- Find the y-intercept of a rational function.
- Find the x-intercept(s) of a rational function.
- State the domain of a rational function.

Prerequisite Knowledge:

- Factor polynomials.
- Evaluate functions.
- Write the domain of a function.

Lesson:
I - Motivating Example(s):

Example: $f(x) = \dfrac{(x-2)^2(x+3)}{2(x-6)(1-x)}$

To find the $y-$intercept of the graph of f, we evaluate $f(0)$ below.

$$f(0) = \frac{(0-2)^2(0+3)}{2(0-6)(1-0)} = \frac{(-2)^2(3)}{2(-6)(1)} = \frac{12}{-12} = -1$$

Hence, the $y-$intercept of the graph of f is $(0, -1)$.

To identify the domain of a rational function $f(x) = \frac{p(x)}{q(x)}$, we must eliminate all real numbers x which make the denominator equal to zero. In other words, the domain of f is the set of all x such that $q(x) \neq 0$.

The domain of f is $x \neq 6$ or 1, or, in interval notation, $(-\infty, 1) \cup (1, 6) \cup (6, \infty)$.

To find all possible $x-$intercepts for the graph of $f(x) = \frac{p(x)}{q(x)}$, we set the function equal to zero and solve for all possible x, keeping *only* those values that are also in our domain.

The numerator has two zeros, $x = 2$ and -3, that do not also make the denominator equal to zero. Written as coordinates, the x-intercepts are $(2, 0)$ and $(-3, 0)$.

II - Demo/Discussion Problems:

Find the domain, x-intercept(s), and y-intercept of each function. Write your intercepts as coordinate pairs.

1. $f(x) = \dfrac{x^3 - x^2 - 8x + 12}{-2x^2 + 14x - 12}$

2. $g(x) = \dfrac{(x-2)^2(x+3)}{2(x-6)(1-x)}$

3. $h(x) = \dfrac{3(x+4)(x-2)^2}{(x+3)^2(2x-3)}$

4. $j(x) = \dfrac{-x^2 - 4x + 45}{2x^3 - 5x^2 - 18x + 45}$

III - Practice Problems:

Find the domain, x-intercept(s), and y-intercept of each function. Write your intercepts as coordinate pairs.

1. $f(x) = \dfrac{(x-5)(x-4)}{x+3}$

2. $g(x) = \dfrac{x^2 - 4x}{x^2 - 4}$

3. $h(x) = \dfrac{x^2 + 1}{x^3 + 3x^2 - 10x}$

4. $j(x) = \dfrac{2x^2 - x - 10}{3x^2 + x - 10}$

5. $k(x) = \dfrac{3x^2 + x - 10}{2x^2 - x - 10}$

6. $m(x) = \dfrac{x^3 - 25x}{x^4 - 16x^2}$

7. $n(x) = \dfrac{x(x-3)}{(2x-1)(x+5)}$