Lesson 19: Factoring Trinomials with a Leading Coefficient of One

CC attribute: Beginning and Intermediate Algebra by T. Wallace.

Objective: Factor a trinomial with a leading coefficient of one.

Students will be able to:

- Identify two integer values that add to b and multiply to $a \cdot c = c$ in a trinomial expression with ordered coefficients a, b, and c.
- Multiply binomials to verify the accuracy of a factorization.
- Recognize the relationship between factoring and expanding an expression.

Prerequisite Knowledge:

- Identifying a greatest common factor.
- Factor by grouping.
- Application of the distributive property.
- Multiplication and division of algebraic expressions.

Lesson:

I - Motivating Example(s):

Example: Write the expanded form for the given expression.

$$(x+6)(x-4)$$
 Distribute $(x+6)$ through the second set of parentheses.
 $x(x+6)-4(x+6)$ Distribute each monomial through parentheses.
 $x^2+6x-4x-24$ Combine like terms.
 $x^2+2x-24$ Our solution.

Notice that if we reverse the last three steps of the previous example, the process resembles grouping. This is because it is grouping! In the second-to-last line, the GCF of the first two terms is x and the GCF of the last two terms is -4. In this manner, we will factor trinomials by writing them as a polynomial containing four terms, splitting up the middle term, and then factor by grouping. This is demonstrated in the following example, which is the previous one done in reverse.

Example: Factor the given expression.

$$x^2 + 2x - 24$$
 Split middle (linear) term into $+6x - 4x$, since $6 + (-4) = 2$ and $6 \cdot (-4) = -24$. $x^2 + 6x - 4x - 24$ Grouping: GCF on left is x , on right is -4 . $x(x+6) - 4(x+6)$ ($x+6$) appears twice, factor out this GCF. $(x+6)(x-4)$ Our solution.

II - Demo/Discussion Problems:

Factor each of the given trinomial expressions.

1.
$$x^2 + 9x + 18$$

2.
$$x^2 - 4x + 3$$

3.
$$x^2 - 13x + 30$$

4.
$$x^2 + 13x - 30$$

5.
$$5x^2 - 40x - 100$$

6.
$$x^2 - 9xy + 14y^2$$

III - Practice Problems:

Factor each of the given trinomial expressions.

1.
$$p^2 + 17p + 72$$

2.
$$x^2 + x - 72$$

3.
$$n^2 - 9n + 8$$

4.
$$x^2 + x - 30$$

5.
$$x^2 - 9x - 10$$

6.
$$x^2 + 13x + 40$$

7.
$$b^2 + 12b + 32$$

8.
$$b^2 - 17b + 70$$

9.
$$x^2 + 3x - 70$$

10.
$$x^2 + 3x - 18$$

11.
$$n^2 - 8n + 15$$

12.
$$a^2 - 6a - 27$$

13.
$$p^2 + 15p + 54$$

14.
$$p^2 + 7p - 30$$

15.
$$n^2 - 15n + 56$$

16.
$$m^2 - 15mn + 50n^2$$

17.
$$u^2 - 8uv + 15v^2$$

18.
$$m^2 - 3mn - 40n^2$$

19.
$$m^2 + 2mn - 8n^2$$

$$20. \ x^2 + 10xy + 16y^2$$

21.
$$x^2 - 11xy + 18y^2$$

22.
$$u^2 - 9uv + 14v^2$$

23.
$$x^2 + xy - 12y^2$$

$$24. \ x^2 + 14xy + 45y^2$$

25.
$$x^2 + 4xy - 12y^2$$

26.
$$4x^2 + 52x + 168$$

$$27. 5a^2 + 60a + 100$$

28.
$$5n^2 - 45n + 40$$

29.
$$6a^2 + 24a - 192$$

$$30.\ 5v^2 + 20v - 25$$

31.
$$6x^2 + 18xy + 12y^2$$

$$32. 5m^2 + 35mn - 90n^2$$

33.
$$6x^2 + 96xy + 378y^2$$

$$34. 6m^2 - 36mn - 162n^2$$