Lesson 7: Compound Inequalities

Objective: Solve, graph, and give interval notation to the solution of a compound inequality.

Students will be able to:

- Understand the distinctions between "OR" and "AND" inequalities.
- Recognize a double inequality as an "AND" inequality.
- Represent the solution to a compound inequality using the three notations (algebraically, graphically, and using interval notation).

Prerequisite Knowledge:

- Solving linear inequalities.
- Graphing on a number line.
- Interval notation, including unions and intersections.

Lesson:

Several inequalities can be combined together to form what are called compound inequalities. There are three types of compound inequalities which we will investigate in this lesson.

- The first type of a compound inequality is an "OR" inequality. A solution for this type of inequality will produce a true statement for either one inequality OR the other inequality OR both. Solutions to OR inequalities will often (but not always) consist of a union of two intervals, denoted by a $\cup$.
- The second type of compound inequality is an "AND" inequality. These inequalities require *both* statements to be true for a given solution. Solutions to AND inequalities equal the intersection (or overlap) of two intervals. Intersections are denoted by a $\cap$, but can be always be simplified.
- The third type of compound inequality is a special type of AND inequality. When a variable (or expression containing the variable) is between two numbers, we can write this as a single mathematical sentence with three parts, such as $5 < x \leq 8$, to show x is greater than 5 AND x is less than or equal to 8. Since this sentence contains two inequalities (both always pointing in the same direction), we refer to it as a *double inequality*.

I - Motivating Example(s):

Example: Solve the inequality below, graph the solution, and provide the interval notation of your solution.

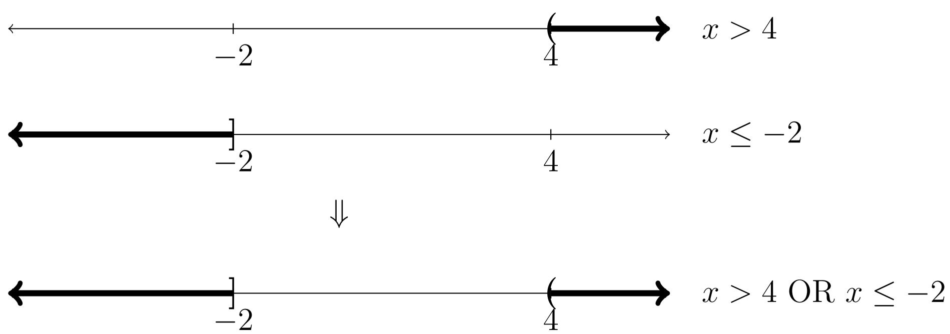
$$2x - 5 > 3 \text{ OR } 4 - x \geq 6$$

Here, we isolate the variable for each inequality. In the first case, adding 5 and dividing by 2 produces $x > 4$.

For the second inequality, we subtract 4 and multiply (or divide) by a -1, which will change the direction of our inequality. Here, we get $x \leq -2$.

$$x > 4 \text{ OR } x \leq -2$$

To express our answer graphically, we will sketch three separate intervals: one for the first inequality, one for the second inequality, and one for the union of the two, which will be our answer.



Since our graphical answer includes two pieces, we use a union in our interval notation.

$$(-\infty, -2] \cup (4, \infty)$$

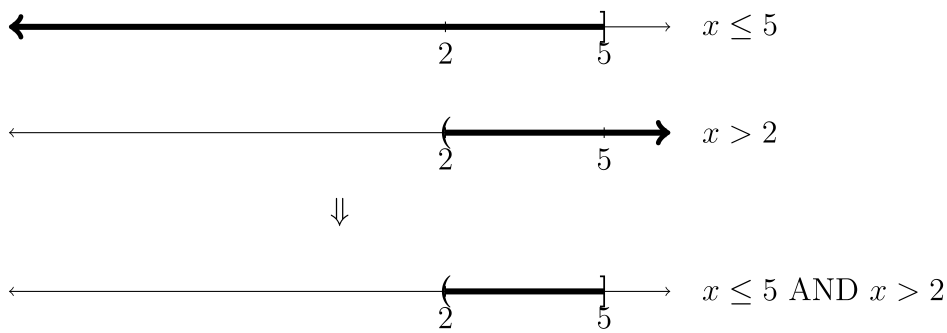
Example: Solve the inequality below, graph the solution, and provide the interval notation of your solution.

$$2x + 8 \geq 5x - 7 \text{ AND } 5x - 3 > 3x + 1$$

Solving each inequality separately gives us the following.

$$\begin{aligned} 2x + 8 &\geq 5x - 7 \\ 2x &\geq 5x - 15 \\ -3x &\geq -15 \\ x &\leq 5 \end{aligned} \qquad\qquad \begin{aligned} 5x - 3 &> 3x + 1 \\ 5x &> 3x + 4 \\ 2x &> 4 \\ x &> 2 \end{aligned}$$

Next, we graph the two inequalities separately, and take their intersection (overlap) for our final answer.



Our answer, in interval notation is $(2, 5]$.

II - Demo/Discussion Problems:

Graph each compound inequality on a real number line and provide the corresponding interval notation.

1. $x \le -3$ OR $x < -1$
2. $x \ge -3$ OR $x < -1$
3. $x < -1$ AND $x < -2$
4. $x > 2$ AND $x < -1$

Solve each of the given compound inequalities. Graph each solution on a real number line and provide the corresponding interval notation.

5. $9 + n < 2$ OR $5n > 40$
6. $\dfrac{v}{8} > -1$ AND $v - 2 < 1$
7. $-6 \le -4x + 2 < 2$
8. $-4 < 8 - 3m \le 11$

III - Practice Problems:

Solve each of the given compound inequalities. Graph each solution on a real number line and provide the corresponding interval notation.

1. $\dfrac{n}{3} \le -3$ OR $-5n \le -10$
2. $6m \ge -24$ OR $m - 7 < -12$
3. $x + 7 \ge 12$ OR $9x < -45$
4. $10r > 0$ OR $r - 5 < -12$
5. $x - 6 < -13$ OR $6x \le -60$
6. $-9x < 63$ AND $\dfrac{x}{4} < 1$
7. $-8 + b < -3$ AND $4b < 20$
8. $-6n \le 12$ AND $\dfrac{n}{3} \le 2$
9. $a + 10 \ge 3$ AND $8a \le 48$
10. $-6 + v \ge 0$ AND $2v > 4$
11. $3 + 7r > 59$ OR $-6r - 3 > 33$
12. $-6 - 8x \ge -6$ OR $2 + 10x > 82$

13. $3 \leq 9 + x \leq 7$

14. $0 \geq \frac{x}{9} \geq -1$

15. $11 < 8 + k \leq 12$

16. $-11 \leq n - 9 \leq -5$

17. $-3 < x - 1 < 1$

18. $-2 < 1 - 3x \leq 10$

19. $1 \leq \frac{p}{8} \leq 0$

20. $-22 \leq 2n - 10 \leq -16$

21. $\frac{1}{2} < 5 - \frac{x}{3} \leq 4$

22. $-5b + 10 \leq 30$ AND $7b + 2 \leq -40$

23. $n + 10 \geq 15$ OR $4n - 5 < -1$

24. $3x - 9 < 2x + 10$ AND $5 + 7x \leq 10x - 10$

25. $4n + 8 < 3n - 6$ OR $10n - 8 \geq 9 + 9n$

26. $-8 - 6v \leq 8 - 8v$ AND $7v + 9 \leq 6 + 10v$

27. $5 - 2a \geq 2a + 1$ OR $10a - 10 \geq 9a + 9$

28. $1 + 5k \leq 7k - 3$ OR $k - 10 > 2k + 10$

29. $8 - 10r \leq 8 + 4r$ OR $-6 + 8r < 2 + 8r$

30. $2x + 9 \geq 10x + 1$ AND $3x - 2 < 7x + 2$

31. $-9m + 2 < -10 - 6m$ OR $-m + 5 \geq 10 + 4m$