Lesson 16: Introduction to Quadratics

Objective: Recognize a quadratic equation and graphically.

Students will be able to:

- Determine if an equation is linear or quadratic.
- Determine if the corresponding graph of a quadratic is concave up or down.
- Identify the $y-$intercept for the graph of a quadratic.
- Recognize the vertex form of a quadratic.

Prerequisite Knowledge:

- Application of the distributive property.
- Order of operations and combining like terms.

Lesson:

A quadratic equation is an equation of the form

$$y = ax^2 + bx + c,$$

where the *coefficients* of a,b, and c are real numbers and $a \neq 0$. This form is most commonly referred to as the *standard form* of a quadratic. We call a the *leading coefficient*, ax^2 the *leading term* (also known as the *quadratic term*), bx the *linear term* and c the *constant term*. The quadratic term ax^2, must have a nonzero coefficient in order for the equation to be a quadratic (otherwise f would be linear, in slope-intercept form). The most fundamental quadratic equation is $y = x^2$ and its graph, like all quadratics, is known as a *parabola*.

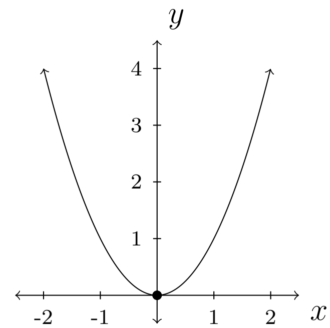
The most useful form for graphing a quadratic equation is the *vertex form*. A quadratic equation is said to be in vertex form if it is represented as

$$y = a(x - h)^2 + k,$$

where h and k are real numbers. The vertex form, unlike the standard form, allows us to immediately identify the vertex of the resulting parabola, which will be the point (h, k).

I - Motivating Example(s):

Example: $y = x^2$

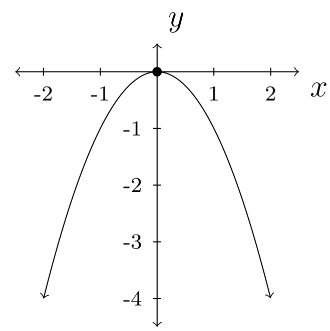


From the standard form, since $a > 0$, the graph opens upwards and is said to be *concave up*.

As a result, there is a minimum point, known as the *vertex*, located at the origin, $(0, 0)$.

Notice the symmetry over the y-axis.

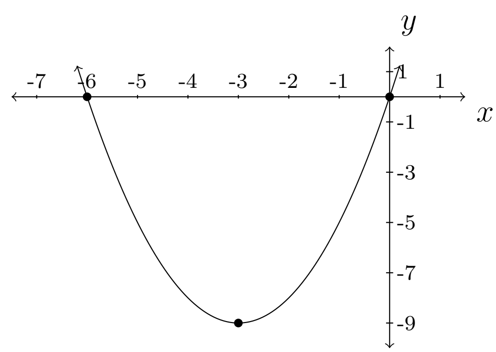
Example: $y = -x^2$



Since $a = -1$, the graph opens downward or we say that it is *concave down*.

Every parabola with a negative leading coefficient ($a < 0$) will be concave down with a maximum value at its vertex.

Example: $y = (x + 3)^2 - 9$



The vertex is at $(-3, -9)$ and the graph can be realized as the graph of $y = x^2$ shifted left 3 units and down 9 units from the origin.

Since our graph is concave up there will be two x-intercepts as the curve opens upward from below the x-axis.

II - Demo/Discussion Problems:

Describe the following equations as either linear or quadratic. If quadratic, identify the $y-$intercept and determine whether the corresponding graph is concave up or down.

1. $y = x^2 - 4x + 2x^2 - 3x + 5$
2. $y = -2(x - 3)(x + 5) - 6$
3. $y = -2x^2 - 4x + 2x^2 - 7x + 7$

Identify the vertex, $y-$intercept, and concavity of each of the following quadratic equations. Find the standard form for each quadratic.

4. $y = -3(x-1)^2 + 2$
5. $y = -\frac{1}{2}(x+2)^2$
6. $y = x^2 + 3$

III - Practice Problems:

Simplifying each of the following equations and classify each as linear, quadratic, or neither. If the equation is a quadratic, identify its $y-$intercept and concavity.

1. $y = x^2 + 9$
2. $y = 5 - 2x + x^2$
3. $y = x + 6 - 3x$
4. $y = 5x + x^2 - 3x - 3x^2$
5. $y = -5x + 3 + 2x - 3x^2 + 6$
6. $y = 3x^2 + -x + x - 3x^2 + 6$
7. $y = (x-1)(x+2) + 3$
8. $y = (x-5)(2x+3) - 2(x-3)$
9. $y = (x-4)(x+4) - (x+1)^2$
10. $y = (2x-4)(x-1) - 2(x+3)^2 + 3x^2$

Identify the vertex and concavity of each of the following quadratics.

11. $y = (x-3)^2 + 4$
12. $y = (x-2)^2 + 5$
13. $y = 6(x+3)^2 + 4$
14. $y = -2(x-3)^2 + 4$
15. $y = -2(x-1)^2 - 7$
16. $y = -(x+1)^2$
17. $y = 7x^2 + 4$
18. $y = -\frac{1}{2}(x-8)^2 + 5$
19. $y = x^2 + 4$
20. $y = 5x^2 + 23$