Lesson 6: Linear Inequalities

Objective: Solve, graph, and give interval notation for the solution to a linear inequality. Create a sign diagram to identify those intervals where a linear expression is positive or negative.

Students will be able to:

- Solve a linear inequality by isolating the variable.
- Recognize the need to change the direction of an inequality when multiplying or dividing by a negative value.
- Graph a linear inequality on a one-dimensional axis.
- Express solutions using interval notation.

Prerequisite Knowledge:

- Apply the distributive property.
- Verify the accuracy of a solution to an inequality by checking.

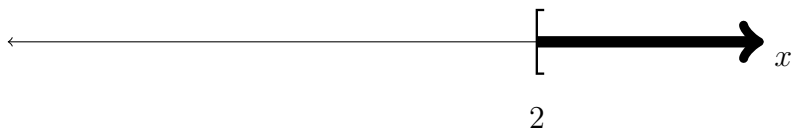
Lesson:
I - Motivating Example(s):

Solve the linear inequality $4x - 3 \geq 5$.

$$\begin{array}{rl} 4x - 3 \geq 5 & \\ \underline{+\mathbf{3} \quad +\mathbf{3}} & \text{Add 3 to both sides} \\ 4x \geq 8 & \\ \overline{\mathbf{4}} \quad \overline{\mathbf{4}} & \text{Divide both sides by 4} \\ x \geq 2 & \text{Our solution} \end{array}$$

Our solution can be expressed as follows.

1. Verbally: "The set of all values of x that are greater than or equal to (at least) 2".
2. Inequality: $\{x | x \geq 2\}$
3. Interval: $[2, \infty)$
4. Real-number Line (Graphically):



Note: A closed (shaded) circle at $x = 2$ is also acceptable in place of a bracket.

Check:

Test Location	Test Value	Unsimplified	Simplified	Result
Shaded region	$x = 3$	$4(3) - 3 \geq 5$	$9 \geq 5$	True
Boundary value	$x = 2$	$4(2) - 3 \geq 5$	$5 \geq 5$	True
Unshaded region	$x = 0$	$4(0) - 3 \geq 5$	$-3 \geq 5$	False

II - Demo/Discussion Problems:

Solve the linear inequality $-1 - 2(x - 3) \leq 5x - 9$.

III - Practice Problems:

Draw a graph for each inequality below and provide interval notation.

1) $n > -5$
2) $n > 4$
3) $-2 \geq k$
4) $1 \geq k$
5) $5 \geq x$
6) $-5 < x$

Solve each inequality, graph each solution, and provide interval notation.

7) $\frac{x}{11} \geq 10$
8) $-2 \leq \frac{n}{13}$
9) $2 + r < 3$
10) $\frac{m}{5} \leq -\frac{6}{5}$
11) $8 + \frac{n}{3} \geq 6$
12) $11 > 8 + \frac{x}{2}$
13) $2 > \frac{a - 2}{5}$
14) $\frac{v - 9}{-4} \leq 2$
15) $\frac{6 + x}{12} \leq -1$

16) $-47 \geq 8 - 5x$
17) $-2(3 + k) < -44$
18) $-7n - 10 \geq 60$
19) $18 < -2(-8 + p)$
20) $5 \geq \frac{x}{5} + 1$
21) $24 \geq -6(m - 6)$
22) $-8(n - 5) \geq 0$
23) $-r - 5(r - 6) < -18$
24) $-60 \geq -4(-6x - 3)$
25) $24 + 4b < 4(1 + 6b)$
26) $-8(2 - 2n) \geq -16 + n$
27) $-5v - 5 < -5(4v + 1)$
28) $-36 + 6x > -8(x + 2) + 4x$
29) $4 + 2(a + 5) < -2(-a - 4)$
30) $3(n + 3) + 7(8 - 8n) < 5n + 5 + 2$
31) $-(k - 2) > -k - 20$
32) $-(4 - 5p) + 3 \geq -2(8 - 5p)$