

Lesson 44: Sign Diagrams for Polynomials



Objective: Construct a sign diagram for a given polynomial expression.

Students will be able to:

- Evaluate a factored polynomial expression at specified test values in order to determine its sign.

Prerequisite Knowledge:

- Factoring.
- Identifying roots of a factored polynomial expression.
- Evaluating functions.
- Order of operations.

Lesson:

If a polynomial function or expression is completely factored, it will be beneficial to us to construct a sign diagram for the polynomial, in order to answer questions about its graph and confirm any other findings. Therefore, we devote this lesson to the construction of a sign diagram for a factored polynomial. Note that expanded polynomials first require us to find a complete factorization prior to constructing a sign diagram.

Recall that the roots of a quadratic expression represent the dividers in its corresponding sign diagram. This carries over directly to a polynomial expression.

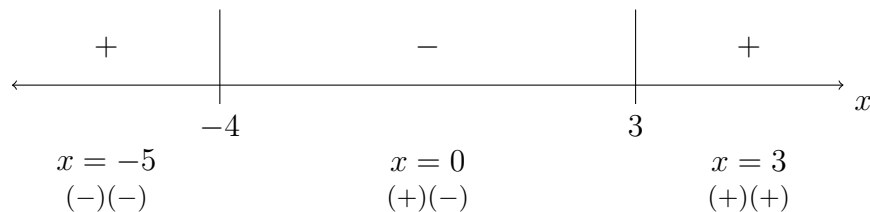
I - Motivating Example(s):

Example: Construct a sign diagram for the polynomial function $f(x) = 2x^2 + 3x - 20$.

Although our first example is not factored, we can apply the *ac*-method to quickly factor our function.

$$\begin{aligned}f(x) &= 2x^2 + 3x - 20 \\&= 2x^2 + 8x - 5x - 20 \\&= 2x(x + 4) - 5(x + 4) \\&= (x + 4)(2x - 5)\end{aligned}$$

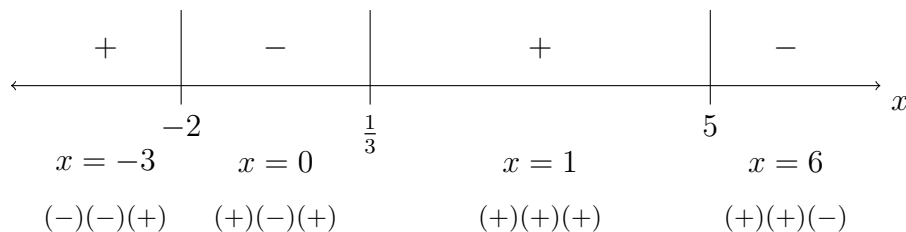
This gives us two roots, $x = -4$ and $x = \frac{5}{2}$, which serve as the dividers in our accompanying diagram. For our three test values, we will use $x = -5, 0$, and 3 .



Example: Construct a sign diagram for the factored polynomial function

$$g(x) = (x + 2)(3x - 1)(5 - x).$$

Our roots are $x = -2, \frac{1}{3}$, and 5. Consequently, the following diagram shows three dividers.



II - Demo/Discussion Problems:

Construct a sign diagram for the factored polynomial functions below. Use [Desmos](#) to graph each function and check the accuracy of your diagram. Identify the interval(s) where the function is positive and where it is negative.

1. $h(x) = (x + 2)^2(3x - 1)(5 - x)$
2. $f(x) = x(x + 1)(x - 2)^2(x^2 + 4)$

III - Practice Problems:

Construct a sign diagram for the factored polynomial functions below. Use [Desmos](#) to graph each function and check the accuracy of your diagram. Identify the interval(s) where the function is positive and where it is negative.

- | | |
|-----------------------------------|--------------------------------------|
| 1. $f(x) = x^3(x - 2)(x + 2)$ | 6. $m(x) = -2(x + 7)^2(1 - 2x)^2$ |
| 2. $g(x) = (x^2 + 1)(1 - x)$ | 7. $f(x) = (x^2 - 1)(x + 4)$ |
| 3. $h(x) = x(x - 3)^2(x + 3)$ | 8. $g(x) = (x^2 - 1)(x^2 - 16)$ |
| 4. $k(x) = (3x - 4)^3$ | 9. $h(x) = -2x^3(3x - 1)(2 - x)$ |
| 5. $\ell(x) = (x^2 + 2)(x^2 + 3)$ | 10. $k(x) = (x^2 - 4x + 1)(x + 2)^2$ |