NOTATION AND BASIC EXAMPLES

1) Function

3) Function

5) Not a function

7) Function

9) Not a function

11) Function

13) Function

15) Function

17) Not a function

19) Function

21) Function

23) Function

25) Function

27) Function

29) Not a function

31) Function

33) Function

35) Function

37) Not a function

39) Not a function

41) $f(x) = \frac{2(x+3)}{4}$

43) $f(x) = \sqrt{2x+3}$

45) $f(x) = 2\sqrt{x+3}$

47) $f(x) = \frac{4}{\sqrt{x-13}}$

49) $f(x) = \sqrt{\frac{4}{x} - 13}$

51) $f(x) = 3 - 4x$

- $f(1) = -1$
- $f(-3) = 15$
- $f\left(\frac{3}{2}\right) = -3$
- $f(4x) = 3 - 16x$
- $4f(x) = 12 - 16x$
- $f(-x) = 3 + 4x$
- $f(x-4) = 19 - 4x$
- $f(x) - 4 = -1 - 4x$
- $f\left(x^2\right) = 3 - 4x^2$

53) $f(x) = x^2 - 3x + 2$

- $f(1) = 0$
- $f(-3) = 20$
- $f\left(\frac{3}{2}\right) = -\frac{1}{4}$
- $f(4x) = 16x^2 - 12x + 2$
- $4f(x) = 4x^2 - 12x + 8$
- $f(-x) = x^2 + 3x + 2$
- $f(x-4) = x^2 - 11x + 30$
- $f(x) - 4 = x^2 - 3x - 2$
- $f\left(x^2\right) = x^4 - 3x^2 + 2$

55) $f(x) = \frac{x}{x-1}$

- $f(1) =$ undefined
- $f(-3) = \frac{3}{4}$
- $f\left(\frac{3}{2}\right) = 3$
- $f(4x) = \frac{4x}{4x-1}$
- $4f(x) = \frac{4x}{x-1}$
- $f(-x) = \frac{x}{x+1}$
- $f(x-4) = \frac{x-4}{x-5}$
- $f(x) - 4 = \frac{-3x+4}{x-1}$
- $f\left(x^2\right) = \frac{x^2}{x^2-1}$

57) $f(x) = 0$

- $f(1) = 0$
- $f(-3) = 0$
- $f\left(\frac{3}{2}\right) = 0$
- $f(4x) = 0$
- $4f(x) = 0$
- $f(-x) = 0$
- $f(x - 4) = 0$
- $f(x) - 4 = -4$
- $f\left(x^2\right) = 0$

59) $f(x) = 5 - 2x$

- $f(2) = 1$
- $f(-2) = 9$
- $f(2a) = 5 - 4a$
- $2f(a) = 10 - 4a$
- $f(a + 2) = 1 - 2a$
- $f(a) + f(2) = 6 - 2a$
- $f\left(\frac{2}{a}\right) = \frac{5a-4}{a}$
- $\frac{f(a)}{2} = \frac{5}{2} - a$
- $f(a + h) = 5 - 2a - 2h$

61) $f(x) = 3x^2 + 3x - 2$

- $f(2) = 16$
- $f(-2) = 4$
- $f(2a) = 12a^2 + 6a - 2$
- $2f(a) = 6a^2 + 6a - 4$
- $f(a+2) = 3a^2 + 15a + 16$
- $f(a) + f(2) = 3a^2 + 3a + 14$
- $f\left(\frac{2}{a}\right) = \frac{-2a^2 + 6a + 12}{a^2}$
- $\frac{f(a)}{2} = \frac{3}{2}a^2 + \frac{3}{2}a - 1$
- $f(a + h) = 3a^2 + 6ah + 3h^2 + 3a + 3h - 2$

63) $f(x) = 1$

- $f(2) = 1$
- $f(-2) = 1$
- $f(2a) = 1$
- $2f(a) = 2$
- $f(a + 2) = 1$
- $f(a) + f(2) = 2$
- $f\left(\frac{2}{a}\right) = 1$
- $\frac{f(a)}{2} = \frac{1}{2}$
- $f(a + h) = 1$

65) $f(x) = \dfrac{2}{x}$

- $f(2) = 1$
- $f(-2) = -1$
- $f(2a) = \frac{1}{a}$
- $2f(a) = \frac{4}{a}$
- $f(a + 2) = \frac{2}{a+2}$
- $f(a) + f(2) = \frac{2+a}{a}$
- $f\left(\frac{2}{a}\right) = a$
- $\frac{f(a)}{2} = \frac{1}{a}$
- $f(a + h) = \frac{2}{a+h}$

67) $f(0) = 3$ $f(x) = 0$ for $x = 15/2$
69) $f(0) = -12$ $f(x) = 0$ for $x = -3, 4$
71) $f(0) = 1$ $f(x) = 0$ for $x = 1/2$
73) $f(0) = 0$ $f(x) = 0$ for $x = 0, 4$

Domain and Range

1) $(-\infty, \infty)$

3) $(-\infty, -1) \cup (-1, \infty)$

5) $(-\infty, \infty)$

7) $(-\infty, -6) \cup (-6, 6) \cup (6, \infty)$

9) $(-\infty, 3]$

11) $[-3, \infty)$

13) $[1/3, \infty)$

15) $(-\infty, \infty)$

17) $[1/3, 6) \cup (6, \infty)$

19) $(-\infty, 8) \cup (8, \infty)$

21) $(8, \infty)$

23) $[0, 5) \cup (5, \infty)$

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Combining Functions

1) $f(x) = 3x + 1, \quad g(x) = 4 - x$

- $(f+g)(2) = 9$
- $(f-g)(-1) = -7$
- $(g-f)(1) = -1$
- $(fg)(\frac{1}{2}) = \frac{35}{4}$
- $\left(\frac{f}{g}\right)(0) = \frac{1}{4}$
- $\left(\frac{g}{f}\right)(-2) = -\frac{6}{5}$

3) $f(x) = x^2 - x, \quad g(x) = 12 - x^2$

- $(f+g)(2) = 10$
- $(f-g)(-1) = -9$
- $(g-f)(1) = 11$
- $(fg)(\frac{1}{2}) = -\frac{47}{16}$
- $\left(\frac{f}{g}\right)(0) = 0$
- $\left(\frac{g}{f}\right)(-2) = \frac{4}{3}$

5) $f(x) = \sqrt{x+3}, \quad g(x) = 2x - 1$

- $(f+g)(2) = 3 + \sqrt{5}$
- $(f-g)(-1) = 3 + \sqrt{2}$
- $(g-f)(1) = -1$
- $(fg)(\frac{1}{2}) = 0$
- $\left(\frac{f}{g}\right)(0) = -\sqrt{3}$
- $\left(\frac{g}{f}\right)(-2) = -5$

7) $f(x) = 2x, \quad g(x) = \dfrac{1}{2x+1}$

- $(f+g)(2) = \frac{21}{5}$
- $(f-g)(-1) = -1$
- $(g-f)(1) = -\frac{5}{3}$
- $(fg)(\frac{1}{2}) = \frac{1}{2}$
- $\left(\frac{f}{g}\right)(0) = 0$
- $\left(\frac{g}{f}\right)(-2) = \frac{1}{12}$

9) $f(x) = x^2, \quad g(x) = \dfrac{1}{x^2}$

- $(f+g)(2) = \frac{17}{4}$
- $(f-g)(-1) = 0$
- $(g-f)(1) = 0$
- $(fg)(\frac{1}{2}) = 1$
- $\left(\frac{f}{g}\right)(0) =$ DNE
- $\left(\frac{g}{f}\right)(-2) = \frac{1}{16}$

11) $f(x) = 2x + 1, \quad g(x) = x - 2$

- $(f+g)(x) = 3x - 1$, all reals
- $(f-g)(x) = x + 3$, all reals
- $(fg)(x) = 2x^2 - 3x - 2$, all reals
- $\left(\frac{f}{g}\right)(x) = \frac{2x+1}{x-2}, \ x \neq 2$

13) $f(x) = x^2, \quad g(x) = 3x - 1$

- $(f+g)(x) = x^2 + 3x - 1$, all reals
- $(f-g)(x) = x^2 - 3x + 1$, all reals
- $(fg)(x) = 3x^3 - x^2$, all reals
- $\left(\frac{f}{g}\right)(x) = \frac{x^2}{3x-1}, \ x \neq \frac{1}{3}$

15) $f(x) = x^2 - 4, \quad g(x) = 3x + 6$

- $(f + g)(x) = x^2 + 3x + 2$, all reals
- $(f - g)(x) = x^2 - 3x - 10$, all reals
- $(fg)(x) = 3x^3 + 6x^2 - 12x - 24$, all reals
- $\left(\frac{f}{g}\right)(x) = \frac{x^2 - 4}{3x + 6}$, $x \neq -2$

17) $f(x) = \frac{x}{2}, \quad g(x) = \frac{2}{x}$

- $(f + g)(x) = \frac{x^2 + 4}{2x}$, $x \neq 0$
- $(f - g)(x) = \frac{x^2 - 4}{2x}$, $x \neq 0$
- $(fg)(x) = 1$, $x \neq 0$
- $\left(\frac{f}{g}\right)(x) = \frac{x^2}{4}$, $x \neq 0$

19) $f(x) = x, \quad g(x) = \sqrt{x + 1}$

- $(f + g)(x) = x + \sqrt{x + 1}$, $x \geq -1$
- $(f - g)(x) = x - \sqrt{x + 1}$, $x \geq -1$
- $(fg)(x) = x\sqrt{x + 1}$, $x \geq -1$
- $\left(\frac{f}{g}\right)(x) = \frac{x}{\sqrt{x+1}}$, $x > -1$

21) 2

27) DNE

23) 0

29) 4

25) 3

31) -2

33) $f(x) = x^2, \quad g(x) = 2x + 1$

- $(g \circ f)(0) = 1$
- $(f \circ g)(-1) = 1$
- $(f \circ f)(2) = 16$
- $(g \circ f)(-3) = 19$
- $(f \circ g)(\frac{1}{2}) = 4$
- $(f \circ f)(-2) = 16$

35) $f(x) = 4 - 3x, \quad g(x) = |x|$

- $(g \circ f)(0) = 4$
- $(f \circ g)(-1) = 1$
- $(f \circ f)(2) = 10$
- $(g \circ f)(-3) = 13$
- $(f \circ g)(\frac{1}{2}) = \frac{5}{2}$
- $(f \circ f)(-2) = -26$

37) $f(x) = 4x + 5, \quad g(x) = \sqrt{x}$

- $(g \circ f)(0) = \sqrt{5}$
- $(f \circ g)(-1) =$DNE
- $(f \circ f)(2) = 57$
- $(g \circ f)(-3) =$DNE
- $(f \circ g)(\frac{1}{2}) = 4\sqrt{\frac{1}{2}} + 5$
- $(f \circ f)(-2) = -7$

39) $f(x) = \dfrac{3}{1-x}, \quad g(x) = \dfrac{4x}{x^2+1}$

- $(g \circ f)(0) = \frac{6}{5}$
- $(f \circ g)(-1) = 1$
- $(f \circ f)(2) = \frac{3}{4}$

- $(g \circ f)(-3) = \frac{48}{25}$
- $(f \circ g)(\frac{1}{2}) = -5$
- $(f \circ f)(-2) =$ DNE

41) $f(x) = 2x + 3, \quad g(x) = x^2 - 9$

- $(g \circ f)(x) = 4x^2 + 12x$
- $(f \circ g)(x) = 2x^2 - 15$
- $(f \circ f)(x) = 4x = 9$

43) $f(x) = x^2 - 4, \quad g(x) = |x|$

- $(g \circ f)(x) = |x^2 - 4|$
- $(f \circ g)(x) = x^2 - 4$
- $(f \circ f)(x) = x^4 - 8x^2 + 12$

45) $f(x) = |x+1|, \quad g(x) = \sqrt{x}$

- $(g \circ f)(x) = \sqrt{|x+1|}$
- $(f \circ g)(x) = |\sqrt{x} + 1|$
- $(f \circ f)(x) = |x+1| + 1$

47) $f(x) = |x|, \quad g(x) = \sqrt{4-x}$

- $(g \circ f)(x) = \sqrt{4 - |x|}$
- $(f \circ g)(x) = \sqrt{4-x}$
- $(f \circ f)(x) = |x|$

49) $f(x) = 3x - 1, \quad g(x) = \dfrac{1}{x+3}$

- $(g \circ f)(x) = \frac{1}{3x+2}$
- $(f \circ g)(x) = \frac{x}{x+3}$
- $(f \circ f)(x) = 9x - 4$

51) $f(x) = \dfrac{x}{2x+1}, \quad g(x) = \dfrac{2x+1}{x}$

- $(g \circ f)(x) = \frac{4x+1}{x}$
- $(f \circ g)(x) = \frac{2x+1}{5x+2}$
- $(f \circ f)(x) = \frac{x}{4x+1}$

53) $f(x) = x^3, \quad g(x) = 2x + 3$

55) $f(x) = \sqrt{x}, \quad g(x) = 2x - 1$

57) $f(x) = \dfrac{2}{x}, \quad g(x) = 5x + 1$

59) $f(x) = \dfrac{x+1}{x-1}, \quad g(x) = |x|$

61) $f(x) = \dfrac{x+1}{3-2x}, \quad g(x) = 2x$

63) $k \circ j \circ f \circ h \circ g$

65) 4 69) -4 73) -3 77) 4

67) 3 71) 0 75) 4 79) 0

Inverse Functions

1) $f^{-1}(x) = \dfrac{x+2}{6}$

3) $f^{-1}(x) = 3x - 10$

5) $f^{-1}(x) = \frac{1}{3}(x-5)^2 + \frac{1}{3},\ x \geq 5$

7) $f^{-1}(x) = \frac{1}{9}(x+4)^2 + 1,\ x \geq -4$

9) $f^{-1}(x) = \frac{1}{3}x^5 + \frac{1}{3}$

11) $f^{-1}(x) = 5 + \sqrt{x+25}$

13) $f^{-1}(x) = 3 - \sqrt{x+4}$

15) $f^{-1}(x) = \dfrac{4x-3}{x}$

17) $f^{-1}(x) = \dfrac{4x+1}{2-3x}$

19) $f^{-1}(x) = \dfrac{-3x-2}{x+3}$

TRANSFORMATIONS

1) $(2, 0)$

3) $(2, -4)$

5) $(2, -9)$

7) $(2, 3)$

9) $(5, -2)$

11) $(2, 13)$

13) $(2, -\frac{3}{2})$

15) $(-1, -7)$

17) $(1, 1)$

Each answer below describes the resulting transformation of the graph of $f(x) = |x|$.

19) Shift down 2 units

21) Shift right 2 units

23) Vertical stretch (or horizontal shrink) by a factor of 2

25) Shift right 2 units

27) (22) and (23) match up; (21) and (25) match up

$|kx| = |k| \cdot |x|$, where $k \in \mathbb{R}$

Each answer below describes the resulting transformation of the graph of $f(x) = \sqrt{9 - x^2}$.

29) Shift down 1/2 units

31) Shift left 4 units

33) Vertical shrink by a factor of 5/3

35) Horizontal stretch by a factor of 3/2

31) Shift right 3 units, vertical stretch by a factor of 4, shift up 6 units

39) $g(x) = -2\sqrt[3]{x + 3} - 1$

43) $g(x) = \sqrt{x - 2} - 3$

45) $g(x) = -\sqrt{x} - 1$

47) $g(x) = \sqrt{-x - 1} + 2$

49) $g(x) = 2\sqrt{x + 3} - 8$

51) $g(x) = \sqrt{2x - 6} + 1$

PIECEWISE-DEFINED AND ABSOLUTE VALUE FUNCTIONS

PIECEWISE-DEFINED FUNCTIONS

1) $f(x) = \begin{cases} x+5 & \text{if} & x \le -3 \\ \sqrt{9-x^2} & \text{if} & -3 < x \le 3 \\ -x+5 & \text{if} & x > 3 \end{cases}$

(a) $f(-4) = 1$ (b) $f(-3) = 2$ (c) $f(3) = 0$

(d) $f(3.1) = 1.9$ (e) $f(-3.01) = 1.99$ (f) $f(2) = \sqrt{5}$

3)   D: $(-\infty, \infty)$ R: $[1, \infty)$ No zeros
5)   D: $(-\infty, \infty)$ R: $[-3, 3]$ $x = 3/2$
7)   D: $(-\infty, \infty)$ R: $(-4, \infty)$ $x = -2, 0$
9)   D: $(-6, -1) \cup (-1, 1) \cup (1, 9)$ R: $(-1, 1) \cup (1, 3)$ $x = 0$

ABSOLUTE VALUE FUNCTIONS

11)   No zeros $y-$int at $(0, 4)$ D: $(-\infty, \infty)$ R: $[4, \infty)$

$$f(x) = \begin{cases} x+4 & \text{if} & x \ge 0 \\ -x+4 & \text{if} & x < 0 \end{cases}$$

13)   Zero at $x = \frac{5}{2}$ $y-$int at $(0, 5)$ D: $(-\infty, \infty)$ R: $[0, \infty)$

$$f(x) = \begin{cases} 2x-5 & \text{if} & x \ge \frac{5}{2} \\ -2x+5 & \text{if} & x < \frac{5}{2} \end{cases}$$

15)   Zero at $x = \frac{5}{2}$ $y-$int at $(0, 5)$ D: $(-\infty, \infty)$ R: $[0, \infty)$

$$f(x) = \begin{cases} 2x-5 & \text{if} & x \ge \frac{5}{2} \\ -2x+5 & \text{if} & x < \frac{5}{2} \end{cases}$$

17)   Zeros at $x = -\frac{16}{3}, -\frac{8}{3}$ $y-$int at $(0, 8)$ D: $(-\infty, \infty)$ R: $[-4, \infty)$

$$f(x) = \begin{cases} 3x+8 & \text{if} & x \ge -4 \\ -3x-16 & \text{if} & x < -4 \end{cases}$$