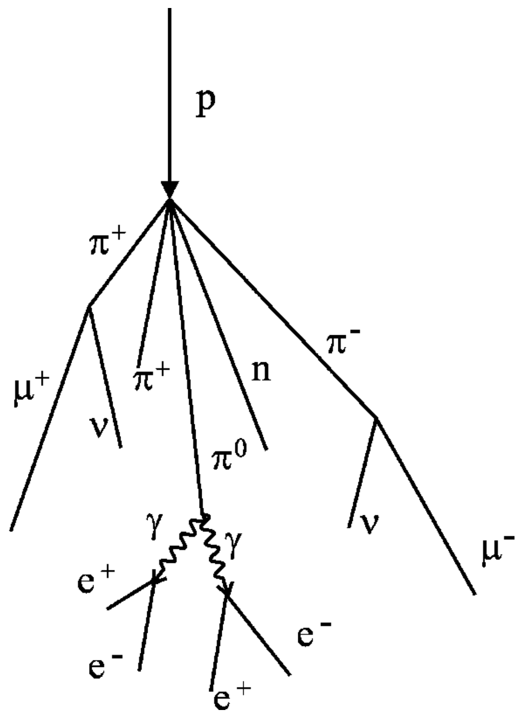
Muon Detection and Time Dilation

The muon (denoted μ) is one of nature's fundamental building blocks of matter and acts in many ways as if it were an unstable heavy electron. It was discovered in 1937 by C.D. Anderson and S.H. Neddermeyer when they exposed a cloud chamber to cosmic rays. In 1941 F. Rasetti demonstrated that it had a finite lifetime, which we now know to be approximately 2.197 μs. It is a negatively charged spin-$1/2$ particle, has a positively charged antimatter pair (the antimuon), has a mass of $105.7\ \mathrm{MeV/c^2}$, is classified as a lepton, and is unusually penetrative of ordinary matter. In the rest of this document we will broaden the use of the term *muon* to refer to both muons and antimuons, which differ only in their charge and cannot be distinguished by the apparatus that we are going to use in this experiment.

What are these cosmic rays that produce muons? Essentially, they are high energy particles produced in other parts of the universe that bombard the top of the earth's atmosphere. Approximately 98% of these high energy particles are protons or heavier nuclei and 2% are electrons. Of the protons and nuclei, about 87% are protons, 12% helium nuclei, and the rest are still heavier nuclei that are the end products of stellar nucleosynthesis.

The primary cosmic rays collide with the nuclei of air molecules and produce a shower of particles that include protons p, neutrons n, pions π^+, π^-, and π^0 (both charged and neutral), neutrinos ν, photons γ, electrons e^-, and positrons e^+. These secondary particles then undergo electromagnetic and nuclear interactions with atmospheric nuclei to produce yet additional particles in a cascade process. Figure 1 indicates the general idea. Not all of the particles produced in the cascade in the upper atmosphere survive down to sea-level due to their interaction with atmospheric nuclei and their own spontaneous decay.

Of particular interest are charged pions produced in the cascade. Some of these will interact via the strong force with air molecule nuclei, producing additional particles, but others will spontaneously decay via the weak force into a muon plus a neutrino or antineutrino. Muons do not interact with matter via the strong force but only through the weak and electromagnetic forces. They travel a relatively long distance while losing kinetic energy and decay by the weak force into an electron plus a neutrino and antineutrino. The flux of sea-level muons is approximately one per minute per square centimeter with a mean kinetic energy of about 4 GeV. While we can detect muons with our scintillator, neutrinos require a much more extensive, deep, and massive apparatus, such as the Super-Kamiokande.



π^+, π^-: charged pions

π^0: uncharged pion

n: neutron

ν: neutrino and antineutrino

γ: photon

e^-: electron

e^+: positron

μ^-, μ^+: muon and antimuon

Figure 1. Cosmic ray cascade induced by a cosmic ray proton p striking an air molecule nucleus.

The Poisson Distribution

The Poisson distribution is used to model experiments in which we count a number n of events that occur *at random* but *with a definite average rate* R. Such an experiment is known as a *counting experiment* or *Poisson process* and assumes that the sample size of potential events is large. If the experiment is repeated many times, then the distribution of number of counted events will be described by the *Poisson distribution*:

$$P_{\mu,t}(n) = \frac{\mu^n}{n!} e^{-\mu}$$

where μ is the number of events counted in a fixed time interval t. As a normalized limiting distribution, $P_{\langle n \rangle}(n)$ represents the relative occurrence of n counts that will be made in a given time interval t. In a *single* experiment, it provides a prediction for the probability of measuring n counts. This ends up being useful for both physical counting processes, such as particle detection and radioactive decay, but also for chi-squared minimization when comparing measured distributions to expected limiting distributions.

Our muon experiment is an example of a Poisson process; muons hit the detector at an average rate and no two muons are correlated, so we cannot accurately predict when the next muon will arrive. Therefore the Poisson distribution, $P_{\langle n \rangle}(n)$, should reasonably describe how many of these *events* (one muon moving through the detector) we count in a given time interval. There are also, it should be noted, several other sources of "events" in our experimental system which you will need to think about in this experiment.

The Apparatus and Detection Process

The active volume of the detector is a plastic (hydrocarbon) scintillator in the shape of a right circular cylinder of 15.0 cm diameter and 12.5 cm height placed at the bottom of a black anodized aluminum alloy tube. A charged particle passing through the scintillator will lose some of its kinetic energy by ionization and atomic excitation of the molecules that make up the plastic. When the molecules relax to their ground state, they emit photons, creating a short (nanosecond timescale) light pulse. This light pulse is detected by a photomultiplier tube (PMT) that converts it to a voltage pulse and triggers readout electronics (see Figure 2).

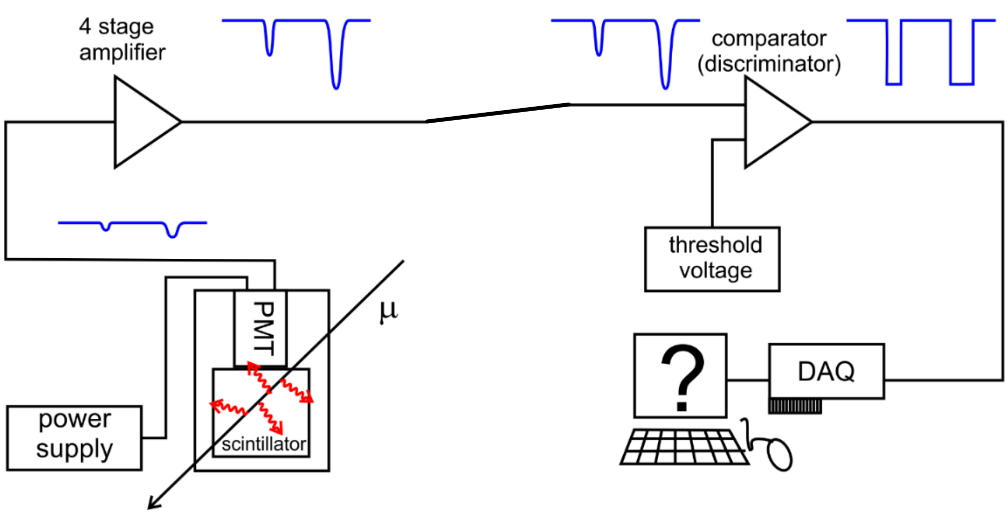


Figure 2. The apparatus schematic: a fraction of the photons produced in a light pulse in the scintillator reach the PMT, whose signal is amplified and compared with a fixed threshold voltage via a comparator. When the pulses exceed the threshold voltage, measurements are made by the DAQ (digital acquisition board) which is directly connected to a computer.

The discriminator compares the signal to a threshold and sets the output signal equal to 0 V when the signal is above this threshold and a fixed nonzero value when the signal is below this threshold, as shown in Figure 3. A derivative threshold is also used to identify and detect the center of a pulse, which should have a derivative near to 0 V/s.

In this experiment, you can vary the PMT voltage, the signal threshold, and the derivative threshold. When handling the equipment, be extremely careful not to touch any components that you do not intend to modify or change connections with, such as the power supplies.

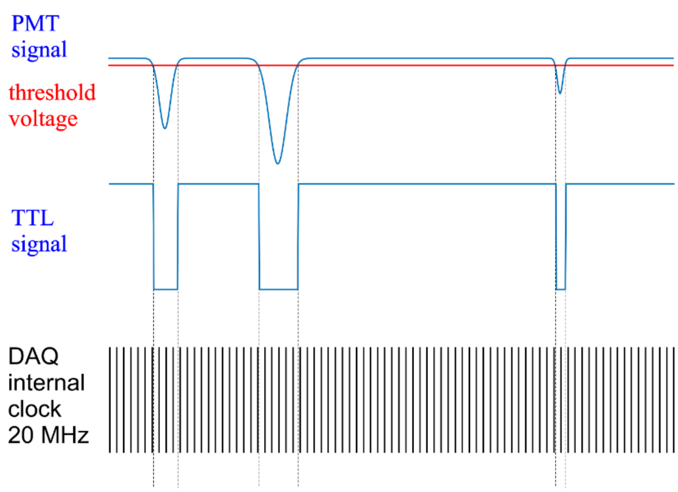


Figure 3. Signal timing diagram; the comparator produces a transistor-transistor logic (TTL) signal in the form of a square pulse when the PMT signal exceeds the threshold voltage. The DAQ internal clock, accessed in LabVIEW, measures the time between changes in the signal corresponding to either a rising trigger. With a rising trigger, the time between the end of one pulse and the beginning of the next pulse is measured.

Questions to address within your lab notebook and report

1) Show that the Poisson distribution is normalized – the total probability of measuring all possible numbers of events in a time interval is 1.
2) Show that the average of the Poisson distribution is μ and that the variance is also μ. What is significant or useful about this finding? How is the Poisson distribution different from the Gaussian distribution?
3) Demonstrate that if a count occurs (say, at $t = 0$), then the probability density describing the "waiting time" for the next count to occur is given by an exponential

decay distribution. (This implies that longer waiting times relative to the average time between counts are increasingly unlikely, while short waiting times are most probable.)

4) Initial experimental exploration (CAREFUL – do not touch the power supplies!)
   (a) Observe the amplified PMT output in an oscilloscope and save and present a sample signal for a detection. Note how the PMT voltage influences your results.
   (b) Observe the "M" signal, which is the derivative of the amplified PMT output while triggering on the amplified PMT output. Properly terminate your split amplified signal at the oscilloscope.
   (c) Observe the TTL signal, triggering on the amplified PMT output. Adjust the signal threshold and note how the TTL output changes. Select a signal threshold that allows for "reasonable" true detections.
   (d) Observe the TTL signal, triggering on the "M" signal. Adjust the derivative threshold and note how the TTL output changes.
   (e) Measure the width of the TTL signal for a few peaks and characterize the width (in time) of this signal. (You may want to use some of your code from the previous experiment here.)
5) Determine what PMT voltage setting and thresholds you will use. Explain how you arrived at this decision. You may consider running the detection software and/or oscilloscope output to sample how effectively the code detects events. Also consider the limits of too-low and too-high thresholds.
6) Begin collecting data – for sufficient statistics, you will likely want to sample for at least 24 hours.
7) **The output data saved from LabVIEW is a sequence of data points that correspond to the time between pulses from the TTL signal (Figure 3).** You can save a quick sample set to begin playing with for automated data processing before the final data set is complete.
   a) How can use this data to check that your results follow Poisson statistics?
   b) Prepare an automated code that will perform this analysis.
8) Once you have the data: Analyze the relevant statistical properties of the data according to the Poisson distribution. When making histograms of your data, include a logarithmic scale to observe low-time behavior, and discuss/explain any differences between the observed and expected distributions. What processes or correlations could explain deviations from the expected distribution?

.