CHAPTER 1:

INTRODUCTION

Lesson #1: A big picture about Digital Signal Processing

Lesson #2: Analog-to-Digital and Digital-to-Analog conversion

Lesson #3: The concept of frequency in CT & DT signals

Duration: 6 hrs

References: Joyce Van de Vegte, “Fundamentals of Digital Signal Processing,” Chap. 1, 2

John G. Proakis, Dimitris G. Manolakis, “Digital Signal Processing,” Chap. 1

Lecture 1: A big picture about Digital Signal Processing

- **Duration: 2** hr
- **Outline:**

1. Signals

2. Digital Signal Processing (DSP)

3. Why DSP?

4. DSP applications

* * * * * * *

Learning Digital Signal Processing is not something you accomplish; it's a journey you take.

R.G. Lyons, Understanding Digital Signal Processing

Signals

- <u>Function of independent variables</u> such as time, distance, position, temperature
- Convey information
- **Examples:**

1D signal: speech, music, biosensor…

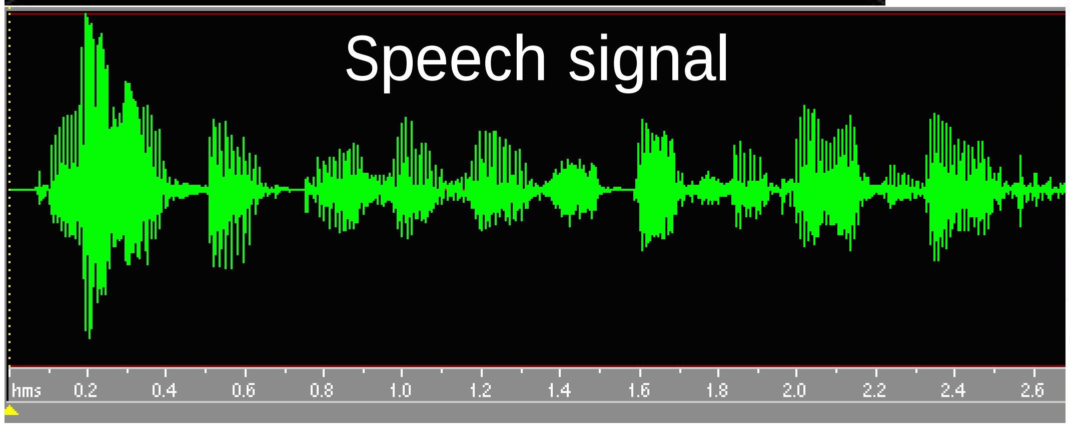
2D signal: image

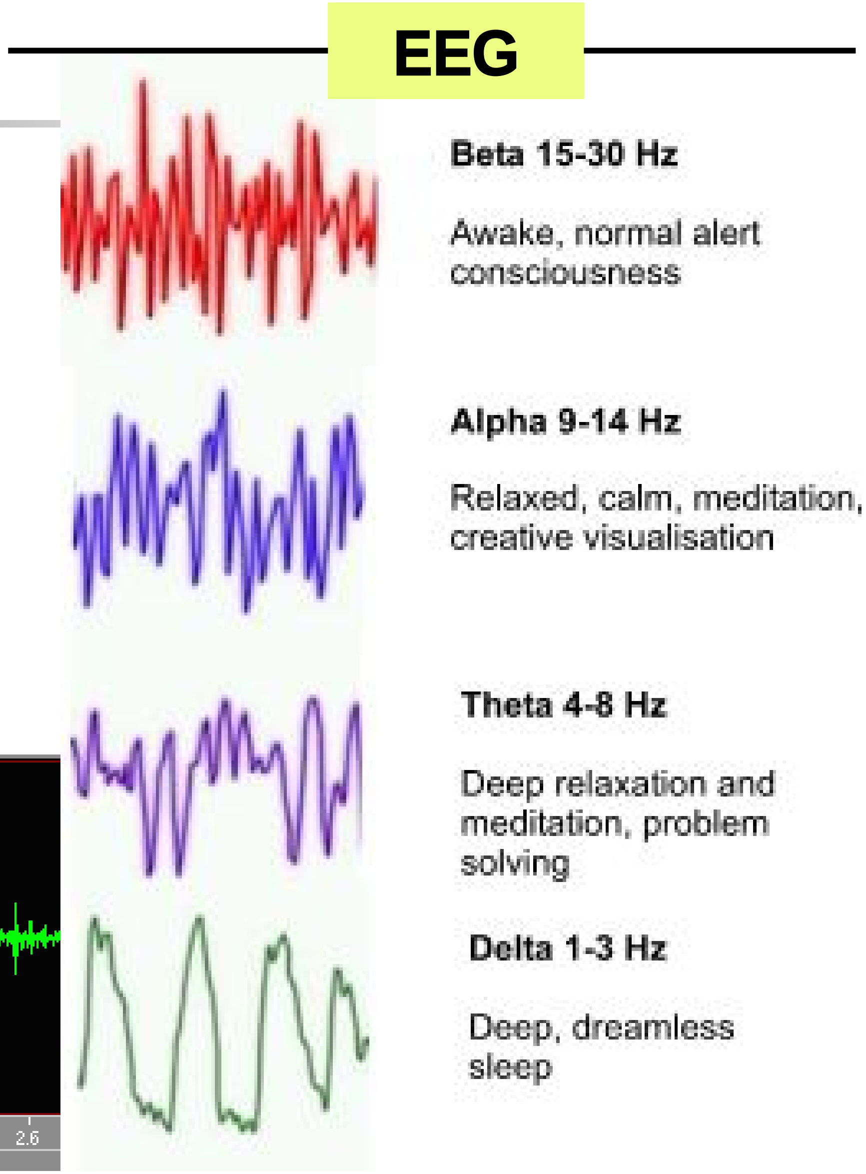
2.5D signal: video (2D image + time)

3D signal: animated

1-D signals


67
ECG
shutterstock


Speech signal
hms 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6


EEG
Beta 15-30 Hz
Awake, normal alert consciousness
Alpha 9-14 Hz
Relaxed, calm, meditation, creative visualisation
Theta 4-8 Hz
Deep relaxation and meditation, problem solving
Delta 1-3 Hz
Deep, dreamless sleep

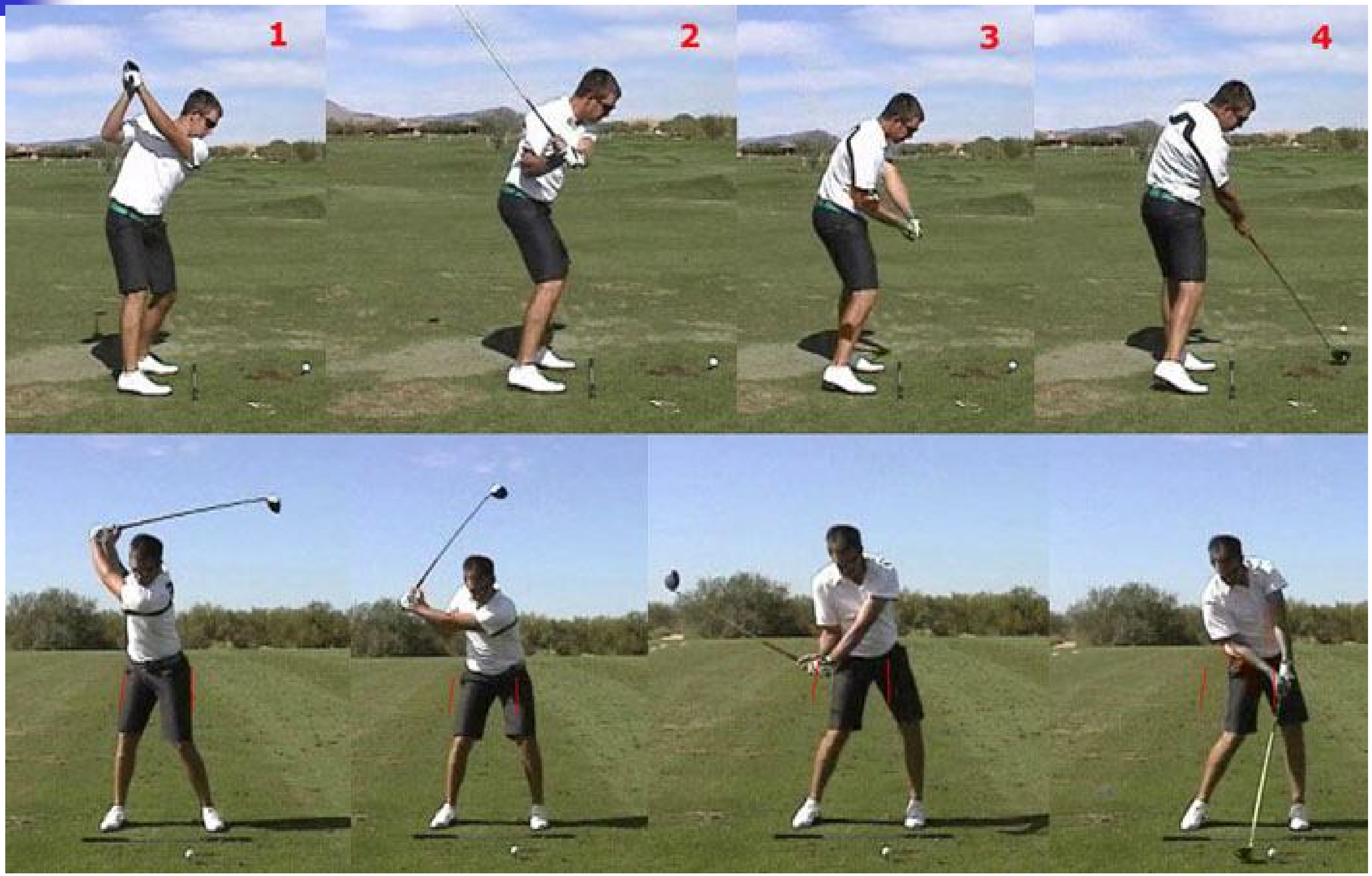
2-D image signals

Binary image

Grey image

Color image

2.5-D video signals



3-D animated signals

Lecture 1: A big picture about Digital Signal Processing

- **Duration: 2** hr
- **Outline:**

  1. Signals

  2. Digital Signal Processing (DSP)

  3. Why DSP?

  4. DSP applications

Discrete-time signal vs. continuous-time signal

- **Continuous-time signal:**

  - define for a continuous duration of time

  - sound, voice...

- **Discrete-time signal:**

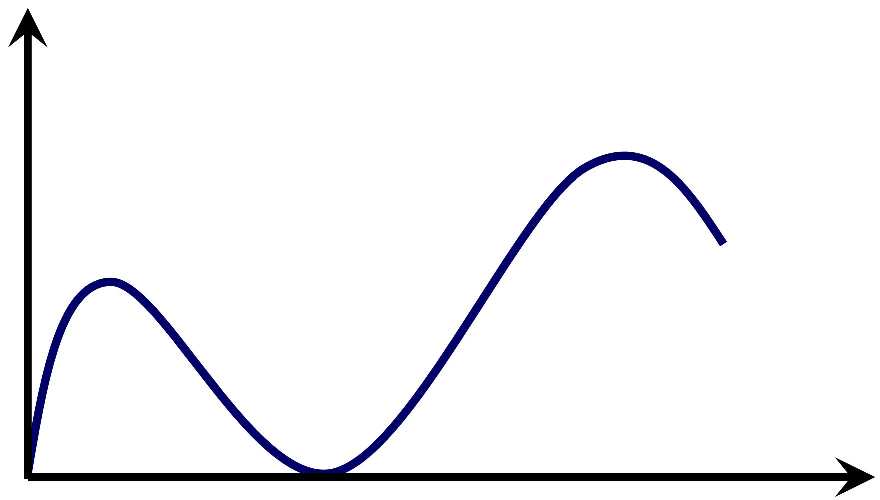
  - define only for discrete points in time (hourly, every second, ...)

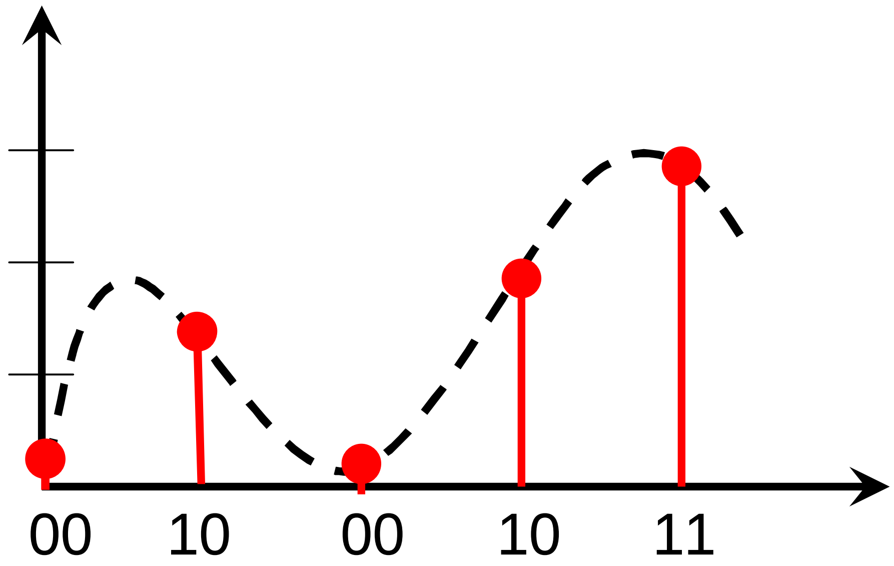
  - an image in computer, a MP3 music file

  - amplitude could be discrete or continuous

  - if the amplitude is also discrete, the signal is digital.

Analog signal vs. digital signal



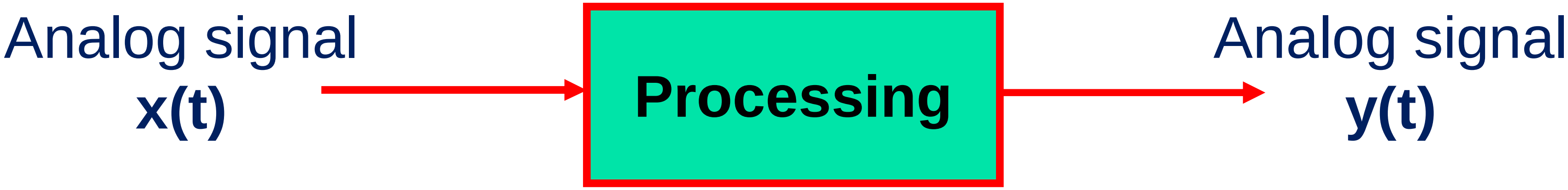


Analog signal | **Digital signal**

What is Digital Signal Processing?

- Represent a signal by a **sequence of numbers** (called a "**discrete-time signal**" or "**digital signal").**
- Modify this sequence of numbers by a **computing process** to **change** or **extract information** from the original signal
- The "computing process" is a system that **converts** one digital signal into another— it is a "discrete-time system" or "digital system".
- **Transforms** are tools using in computing process

Signal processing systems



Analog signal processing

Digital signal processing

Digital Signal Processing implementation

Performed by:

- Special-purpose (custom) chips: **a**pplication-**s**pecific **i**ntegrated **c**ircuits (**ASIC**)
- **F**ield-**p**rogrammable **g**ate **a**rrays (**FPGA**)
- General-purpose microprocessors or microcontrollers (***μ*P / *μ*C**)
- General-purpose **d**igital **s**ignal **p**rocessors (**DSP processors**)
- **DSP processors** with application-specific hardware (**HW**) accelerators

Digital Signal Processing implementation

	ASIC	FPGA	μP/μC	DSP processor	DSP processors with HW accelerators
Flexibility	None	Limited	High	High	Medium
Design time	Long	Medium	Short	Short	Short
Power consumption	Low	Low–medium	Medium–high	Low–medium	Low–medium
Performance	High	High	Low–medium	Medium–high	High
Development cost	High	Medium	Low	Low	Low
Production cost	Low	Low–medium	Medium–high	Low–medium	Medium

Digital Signal Processing implementation

- Use basic operations of addition, multiplication and delay
- Combine these operations to accomplish processing: a discrete-time input signal → another discrete-time output signal

An example of main step: “DT signal processing”

- From a discrete-time input signal:

  { 1 2 4 -9 5 3 }

- Create a discrete-time output signal:

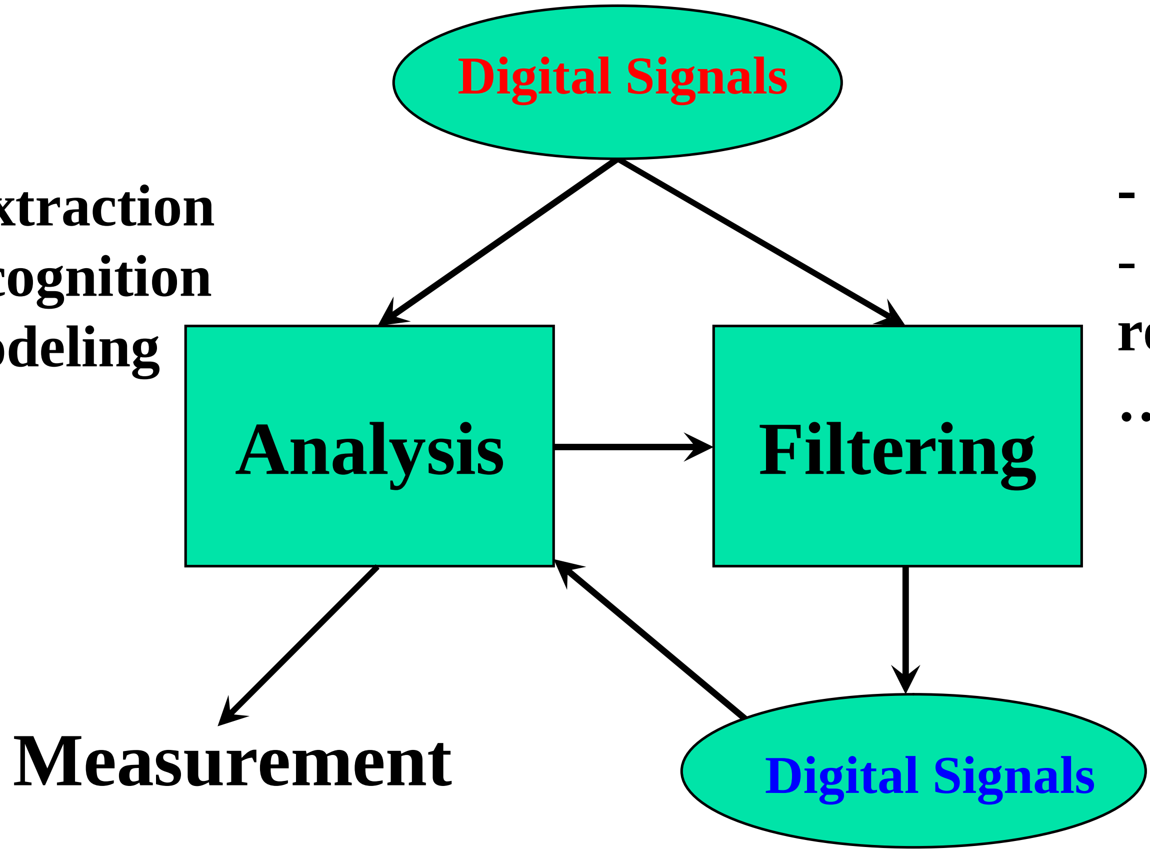
  { 1/3 1 7/3 -1 0 -1/3 8/3 1 }

What is the relation between input and output signal?

Two main categories of DSP

- feature extraction
- signal recognition
- signal modeling

....



- noise removal
- interference removal

......

Lecture 1: A big picture about Digital Signal Processing

- **Duration: 2** hr
- **Outline:**

  1. Signals
  2. Digital Signal Processing (DSP)
  3. **Why DSP?**
  4. DSP applications

Advantages of Digital Signal Processing

- Flexible: re-programming ability
- More reliable
- Smaller, lighter → less power
- Easy to use, to develop and test (by using the assistant tools)
- Suitable to sophisticated applications
- Suitable to remote-control applications

Limitations of Digital Signal Processing

- A/D and D/A needed → aliasing error and quantization error
- Not suitable to high-frequency signal
- Require high technology

Mixed Signal Processing

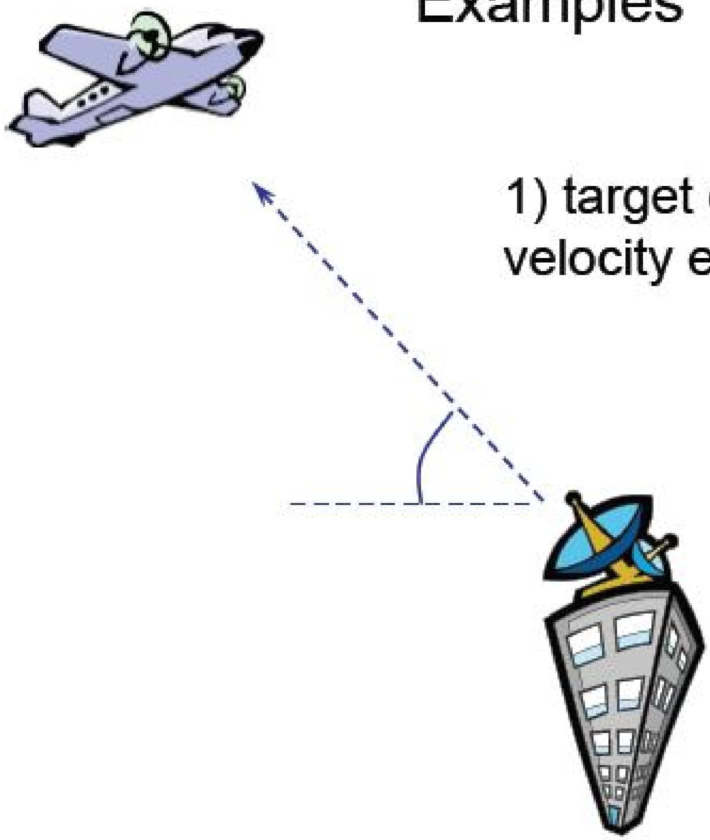
Lecture 1: A big picture about Digital Signal Processing

- **Duration: 2** hr
- **Outline:**

1. Signals
2. Digital Signal Processing (DSP)
3. Why DSP?
4. **DSP applications**

Radar

Examples



1) target detection – position and velocity estimation

2) tracking

Biomedical

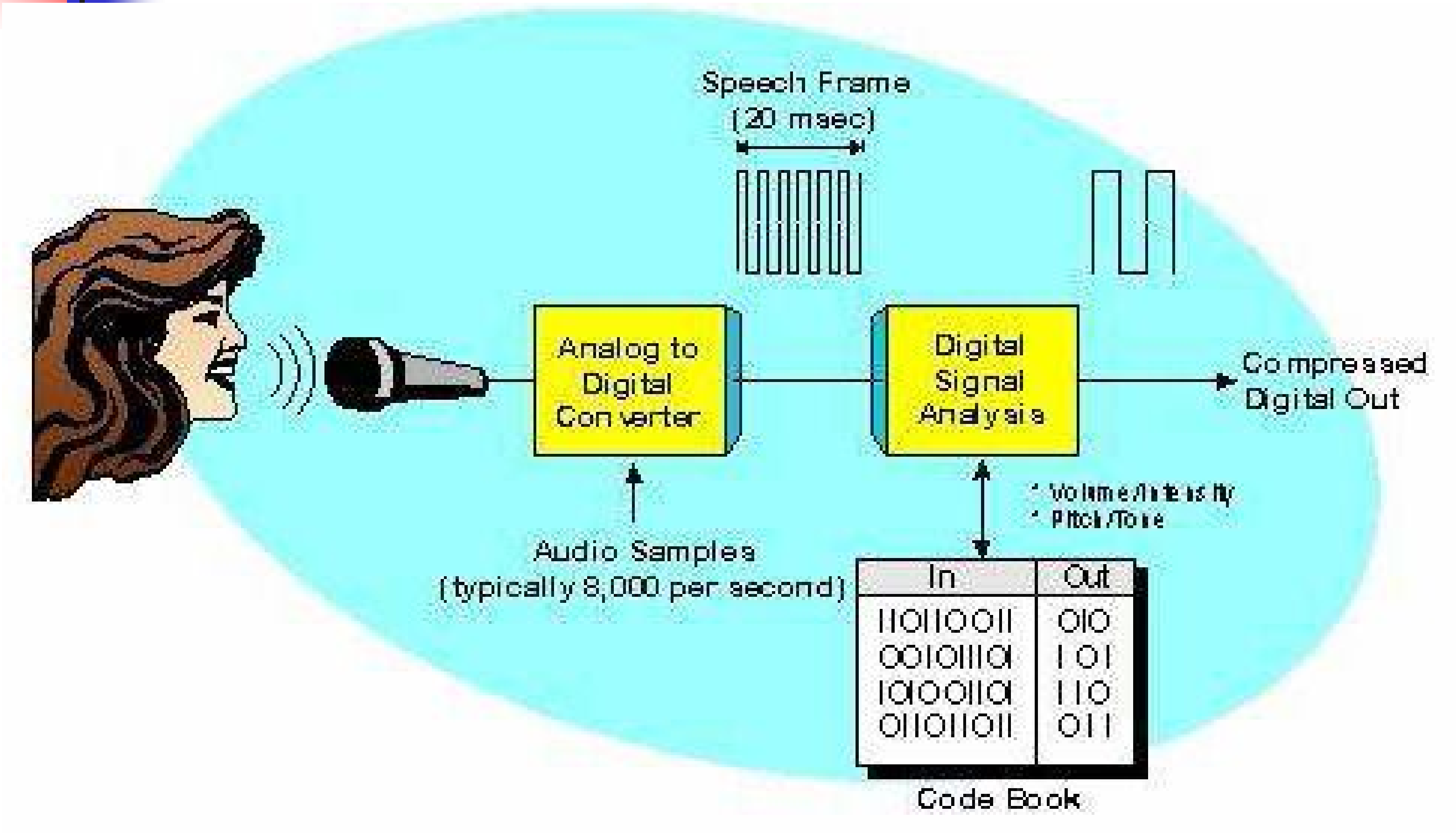
- Analysis of biomedical signals, diagnosis, patient monitoring, preventive health care

Examples:

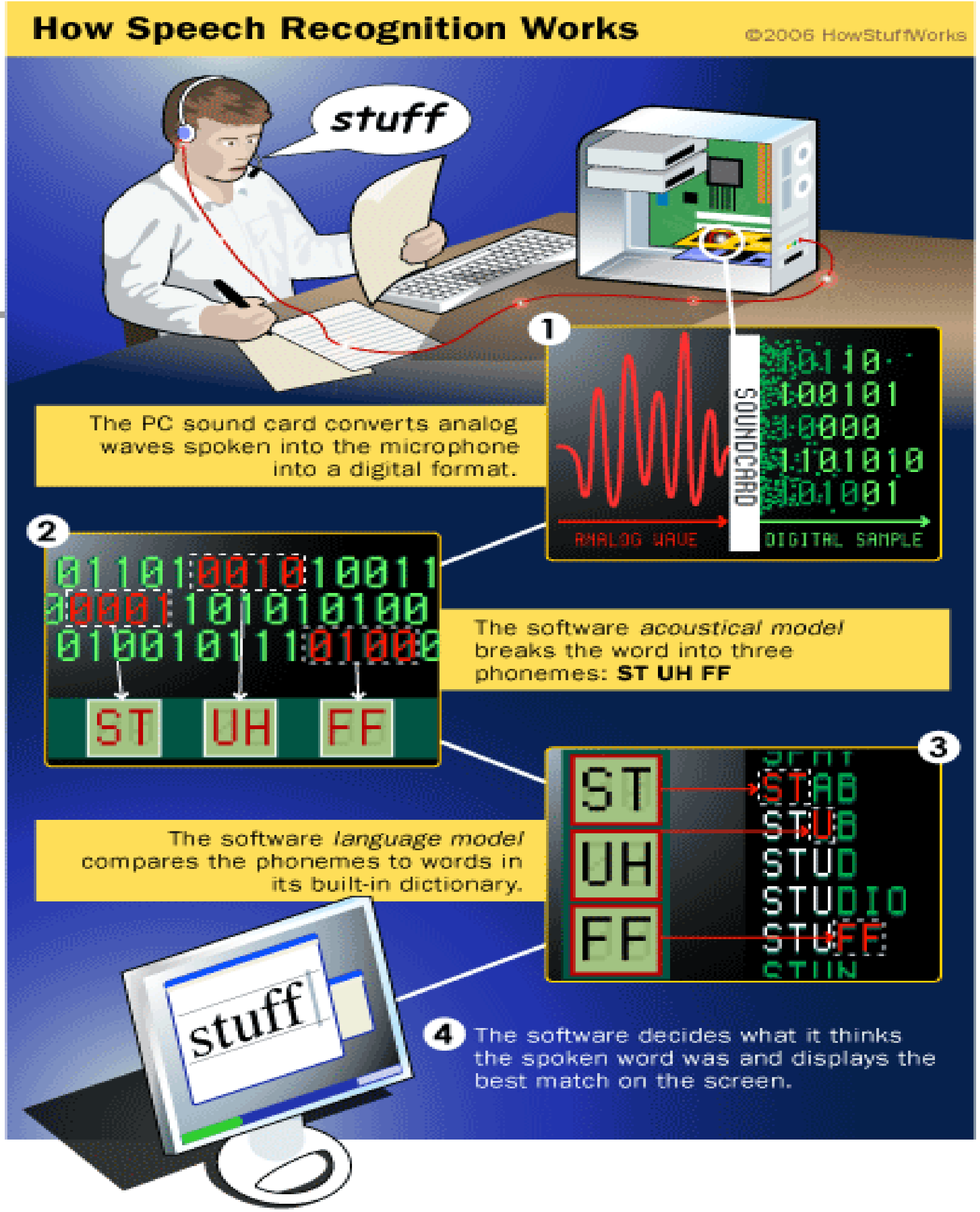
1) electrocardiogram (ECG) signal – provides doctor with information about the condition of the patient's heart

2) electroencephalogram (EEG) signal – provides Information about the activity of the brain

Speech compression



Speech recognition



Communication

Digital telephony: transmission of information in digital form via telephone lines, modern technology, mobile phone

Image processing

Image enhancement: processing an image to be more suitable than the original image for a specific application

It makes all the difference whether one sees darkness through the light or brightness through the shadows

David Lindsay

Image processing

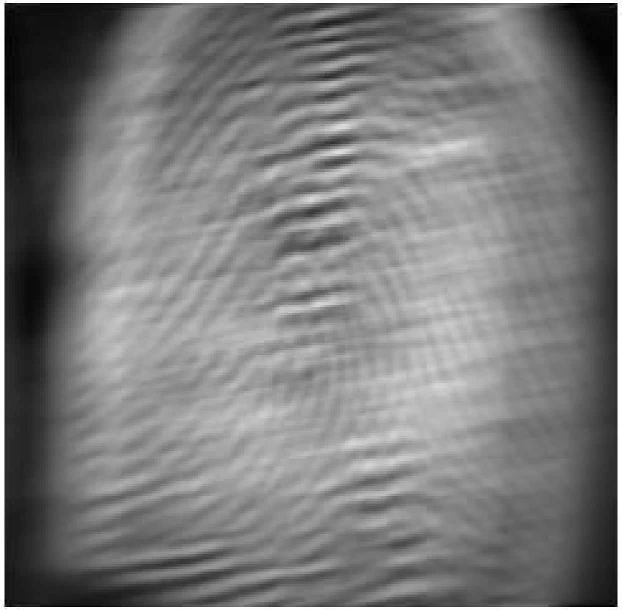
Image compression: reducing the redundancy in the image data

UW Campus (bmp) 180 kb

UW Campus (jpg) 13 kb

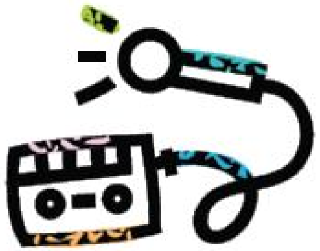
Image processing

Image restoration: reconstruct a degraded image using a priori knowledge of the degradation phenomenon



Music

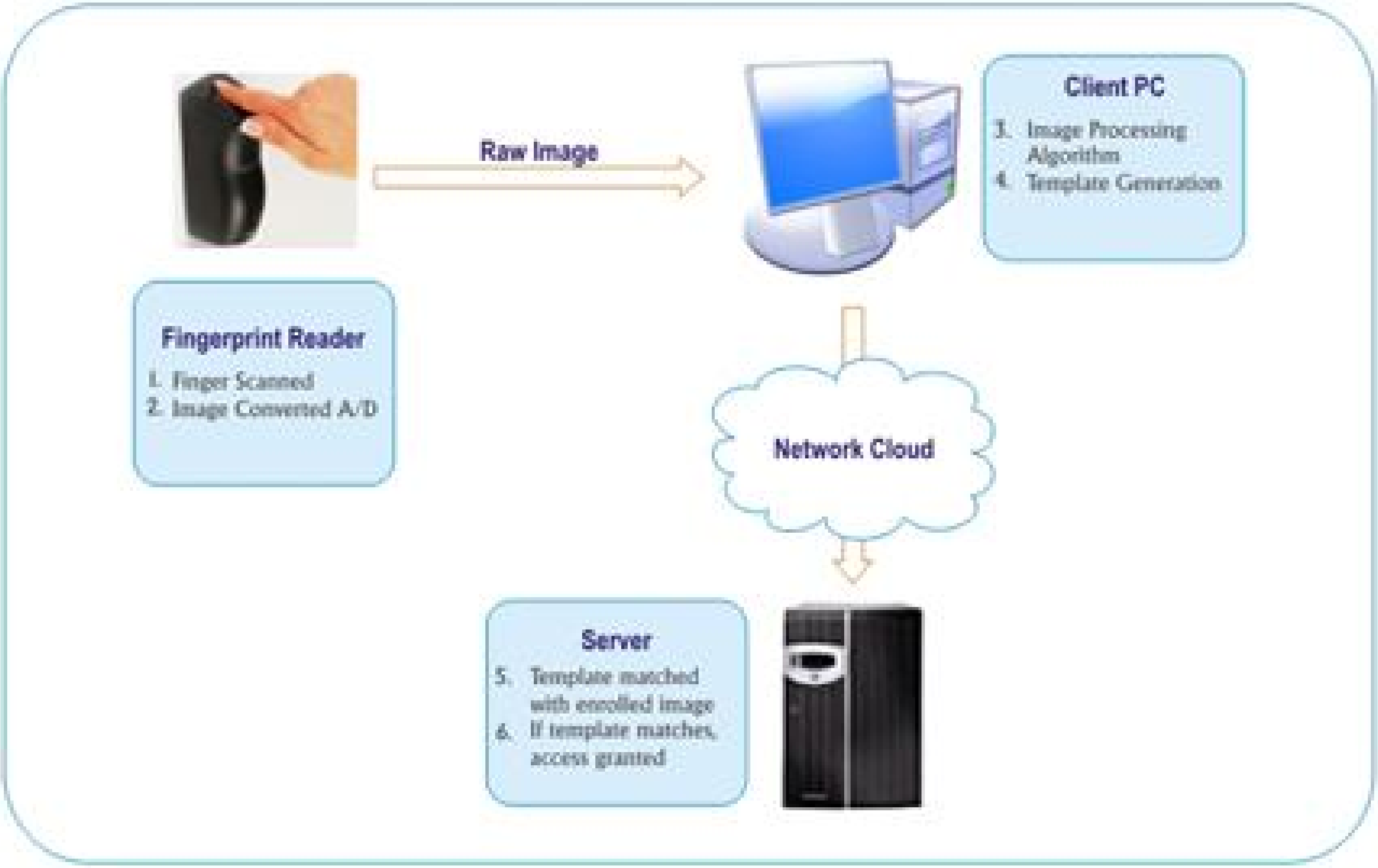
Recording, encoding, storing



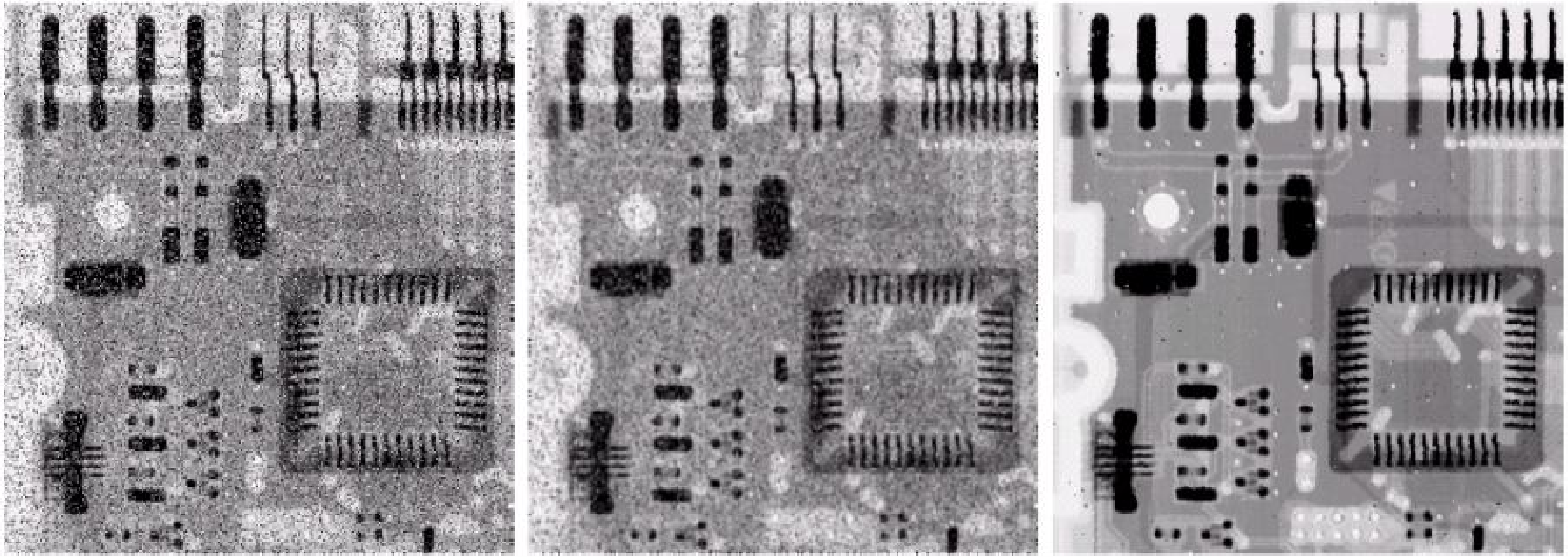
Playback

Manipulation/mixing

Finger print recognition



Noise removal



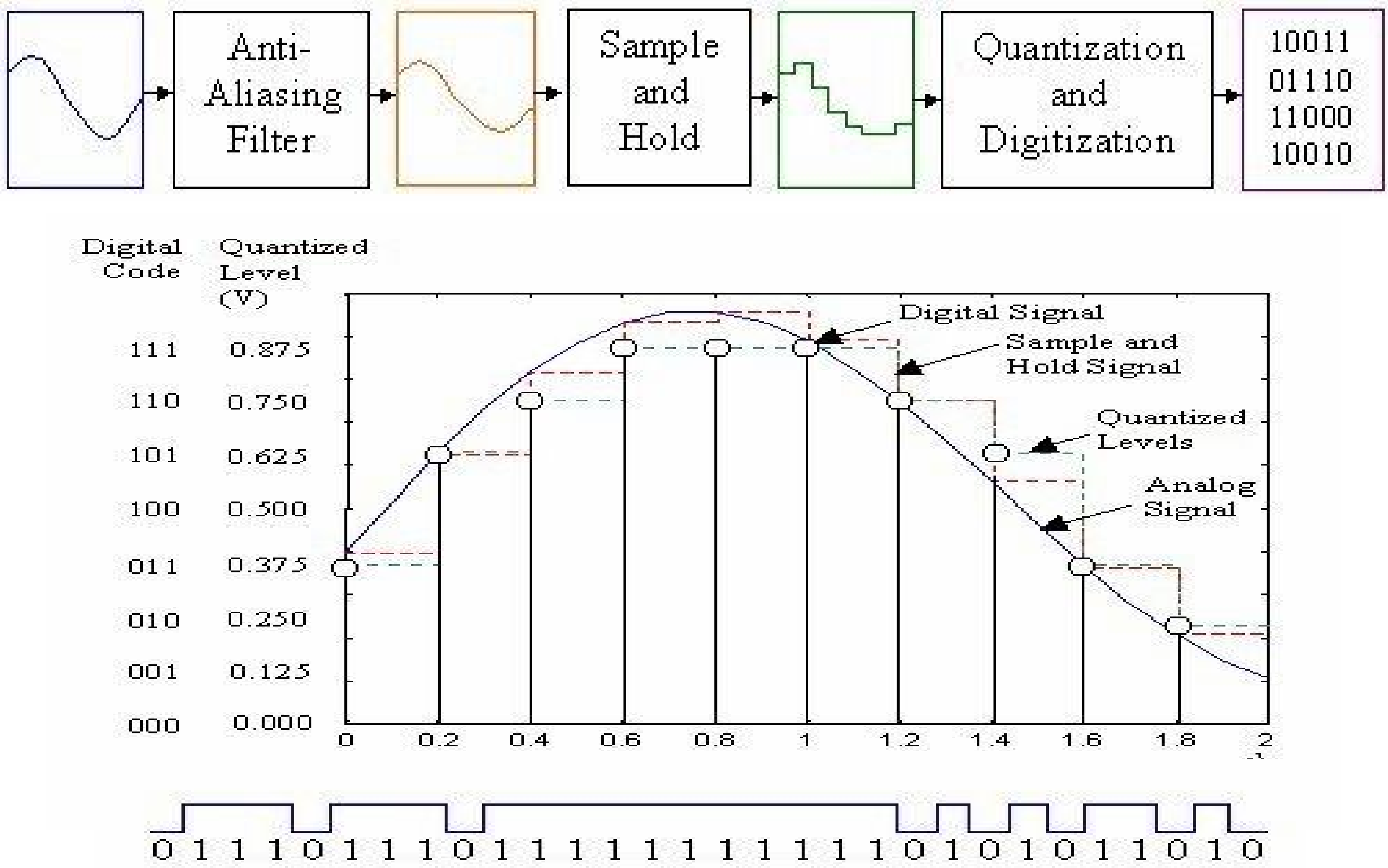
Lecture 2: Analog-to-Digital and Digital-to-Analog conversion

- **Duration: 2** hr
- **Outline:**

1. A/D conversion

2. D/A conversion

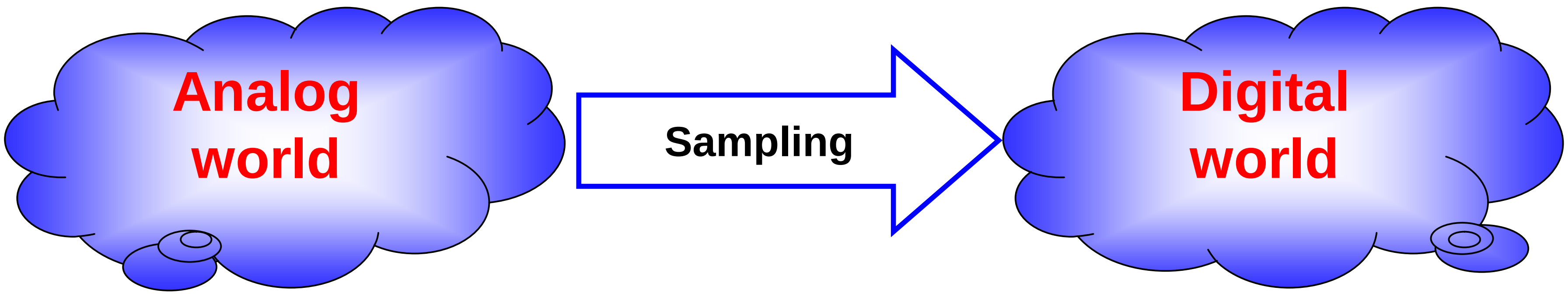
ADC



Anti-Aliasing Filter
Sample and Hold
Quantization and Digitization
10011
01110
11000
10010
Digital Code
Quantized Level (V)
111 0.875
110 0.750
101 0.625
100 0.500
011 0.375
010 0.250
001 0.125
000 0.000
Digital Signal
Sample and Hold Signal
Quantized Levels
Analog Signal
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
0 1 1 1 0 1 1 1 0 1 1 1 1 1 1 1 1 1 1 1 0 1 0 1 0 1 1 0 1 0

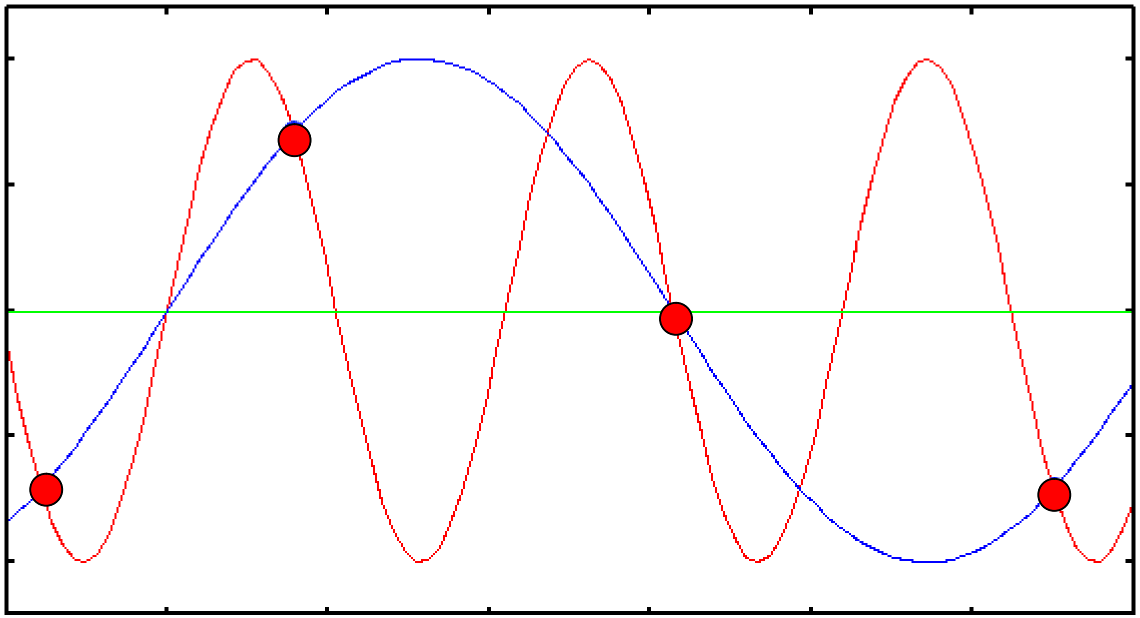
Sampling

- Continuous-time signal → discrete-time signal



Sampling

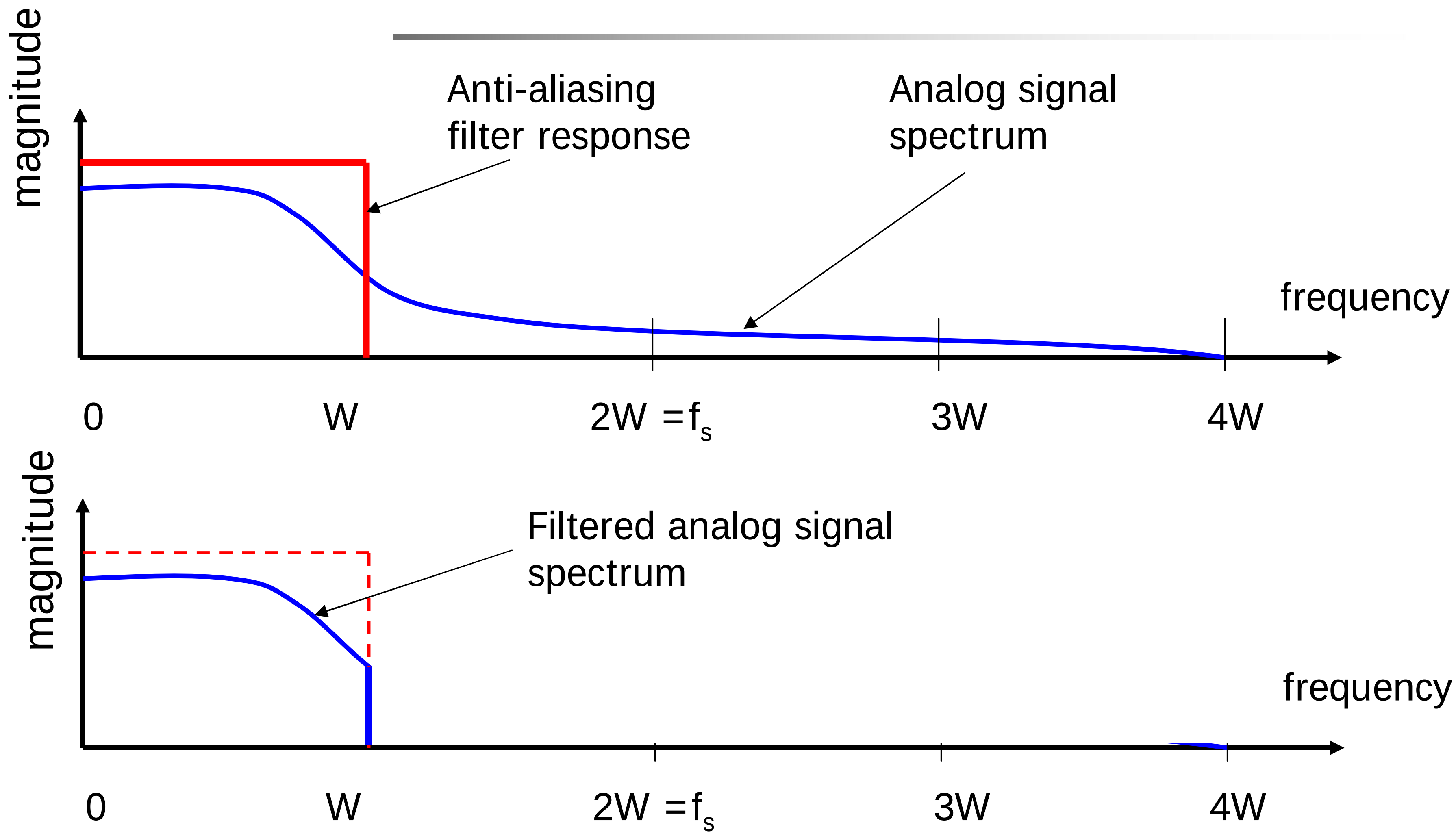
- Taking samples at intervals and don’t know what happens in between → can’t distinguish higher and lower frequencies: ***aliasing***
- How to avoid aliasing?



Nyquist sampling theory

- To guarantee that an analog signal can be perfectly recovered from its sample value
- Theory: a signal with maximum of frequency of *W* Hz must be sampled at least *2W* times per second to make it possible to reconstruct the original signal from the samples
- Nyquist sampling rate: minimum sampling frequency
- Nyquist frequency: half the sampling rate
- Nyquist range: 0 to Nyquist frequency range
- To remove all signal elements above the Nyquist frequency → antialiasing filter

Anti-aliasing filter



Some examples of sampling frequency

- Speech coding/compression ITU G.711, G.729, G.723.1:

  ***fs* = 8 kHz** → T = 1/8000 s = 125μs

- Broadband system ITU-T G.722:

  ***fs* = 16 kHz** → T = 1/16 000 s = 62.5μs

- Audio CDs:

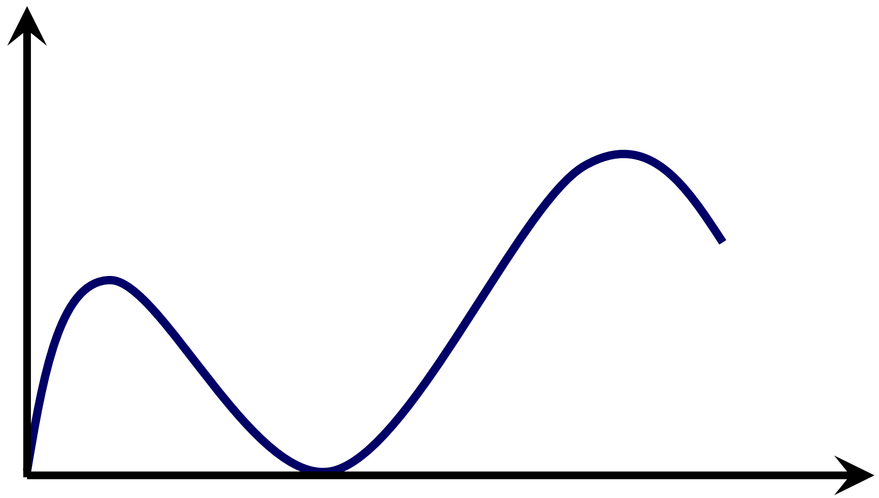
  ***fs* = 44.1 kHz** → T = 1/44100 s = 22.676μs

- Audio hi-fi, e.g., MPEG-2 (moving picture experts group), AAC (advanced audio coding), MP3 (MPEG layer 3):

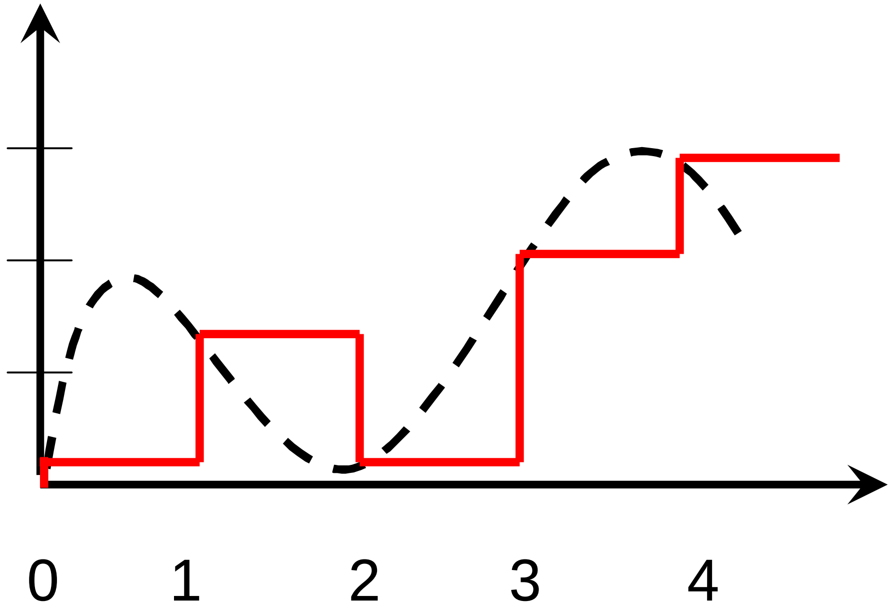
  ***fs* = 48 kHz** → T = 1/48 000 s = 20.833μs

Sampling and Hold

- Sampling interval T_s (sampling period): time between samples
- Sampling frequency f_s (sampling rate): # samples per second



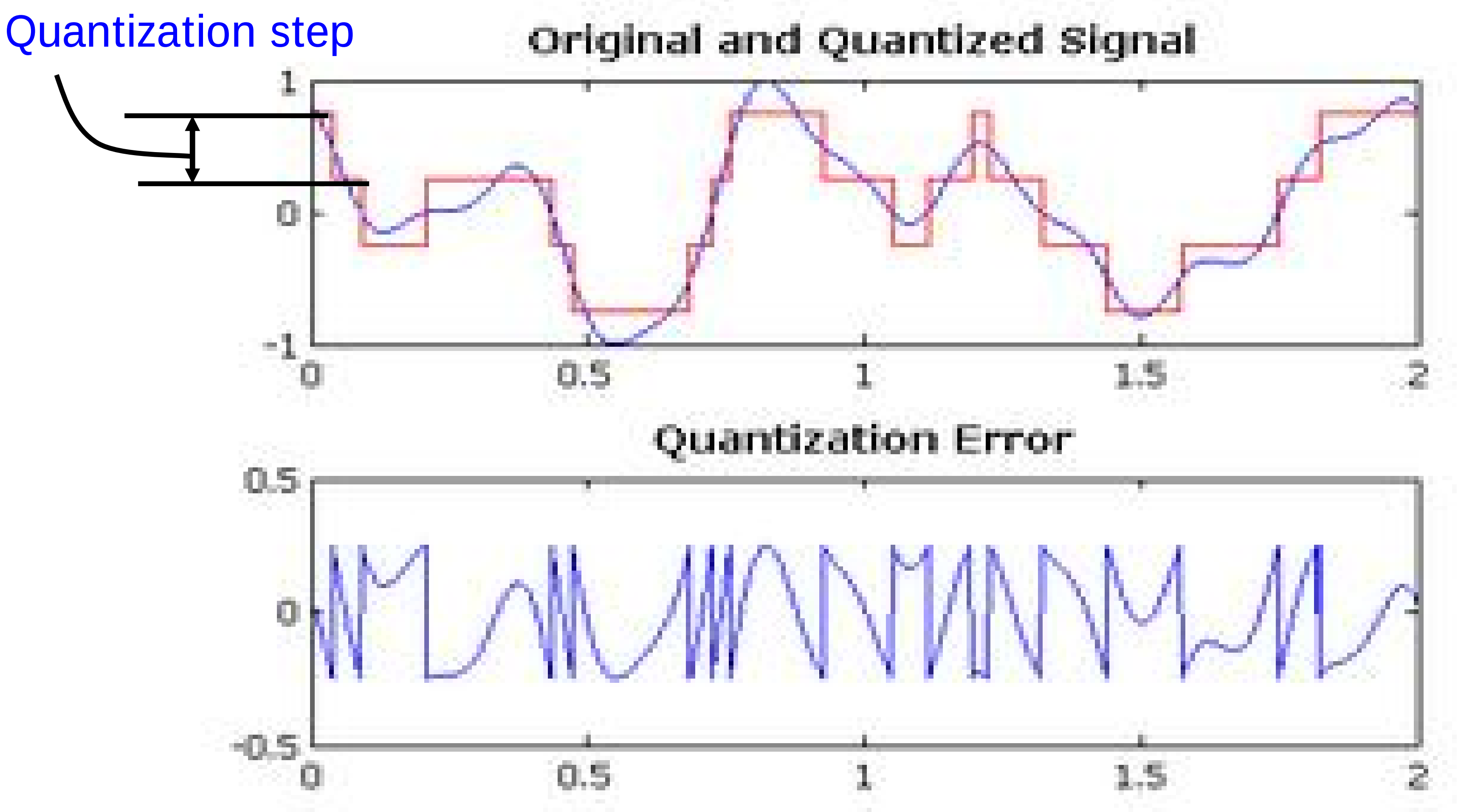
Analog signal



Sample-and-hold signal

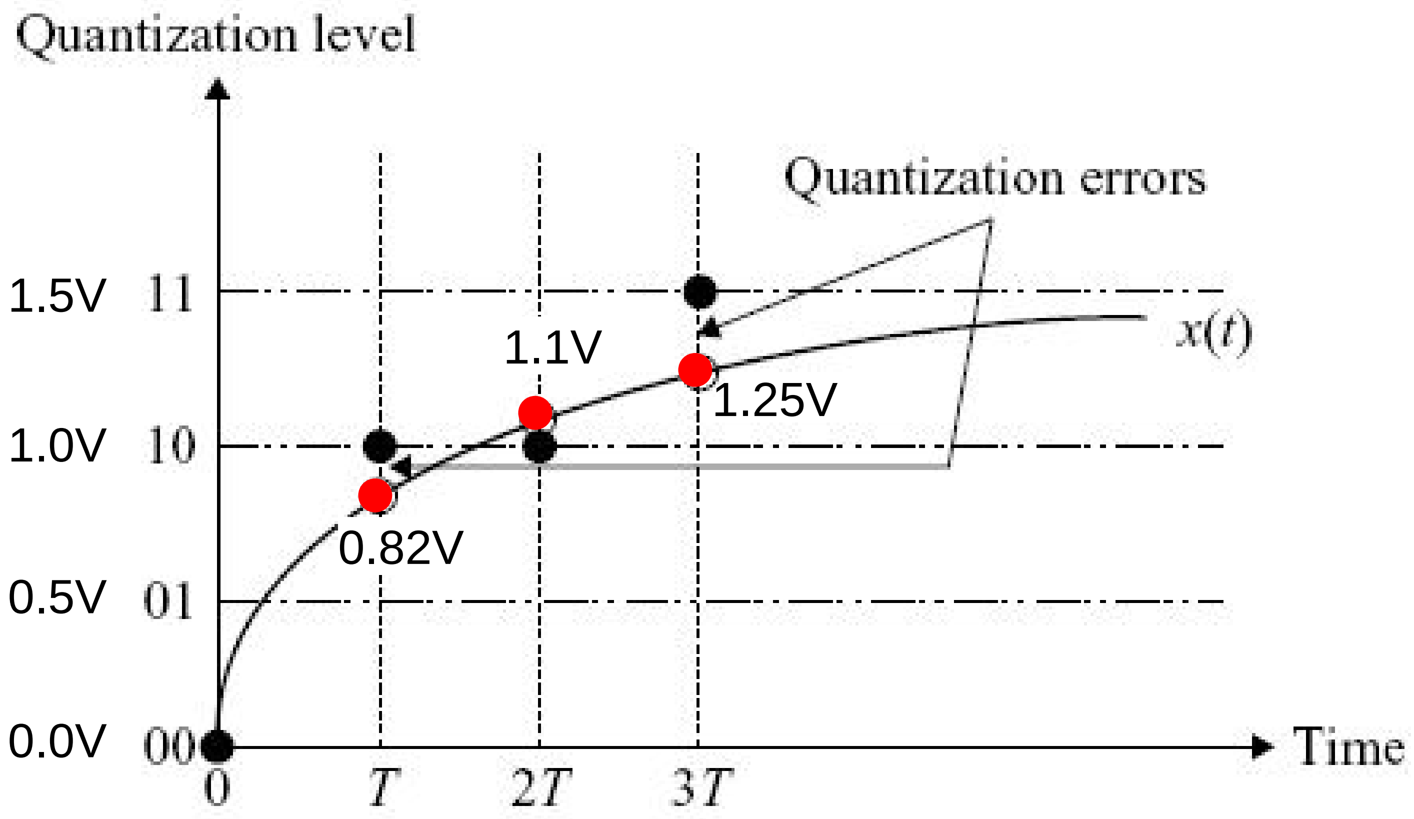
Quantization

- Continuous-amplitude signal → discrete-amplitude signal



Coding

- Quantized sample → N-bit code word



Example of quantization and coding

Analog pressures are recorded, using a pressure transducer, as voltages between 0 and 3V. The signal must be quantized using a 3-bit digital code. Indicate how the analog voltages will be converted to digital values.

Digital code	Quantization Level (V)	Range of analog inputs (V)
000	0.0	0.0-0.1875
001		
010		
011		
100		
101		
110		
111		

Example of quantization and coding

An analog voltage between -5V and 5V must be quantized using 3 bits. Quantize each of the following samples, and record the quantization error for each:

-3.4V; 0V; .625V

Digital code	Quantization Level (V)	Range of analog inputs (V)
100	-5.0	-5.0 → -4.375
101		
110		
111		
000		
001		
010		
011		

Quantization parameters

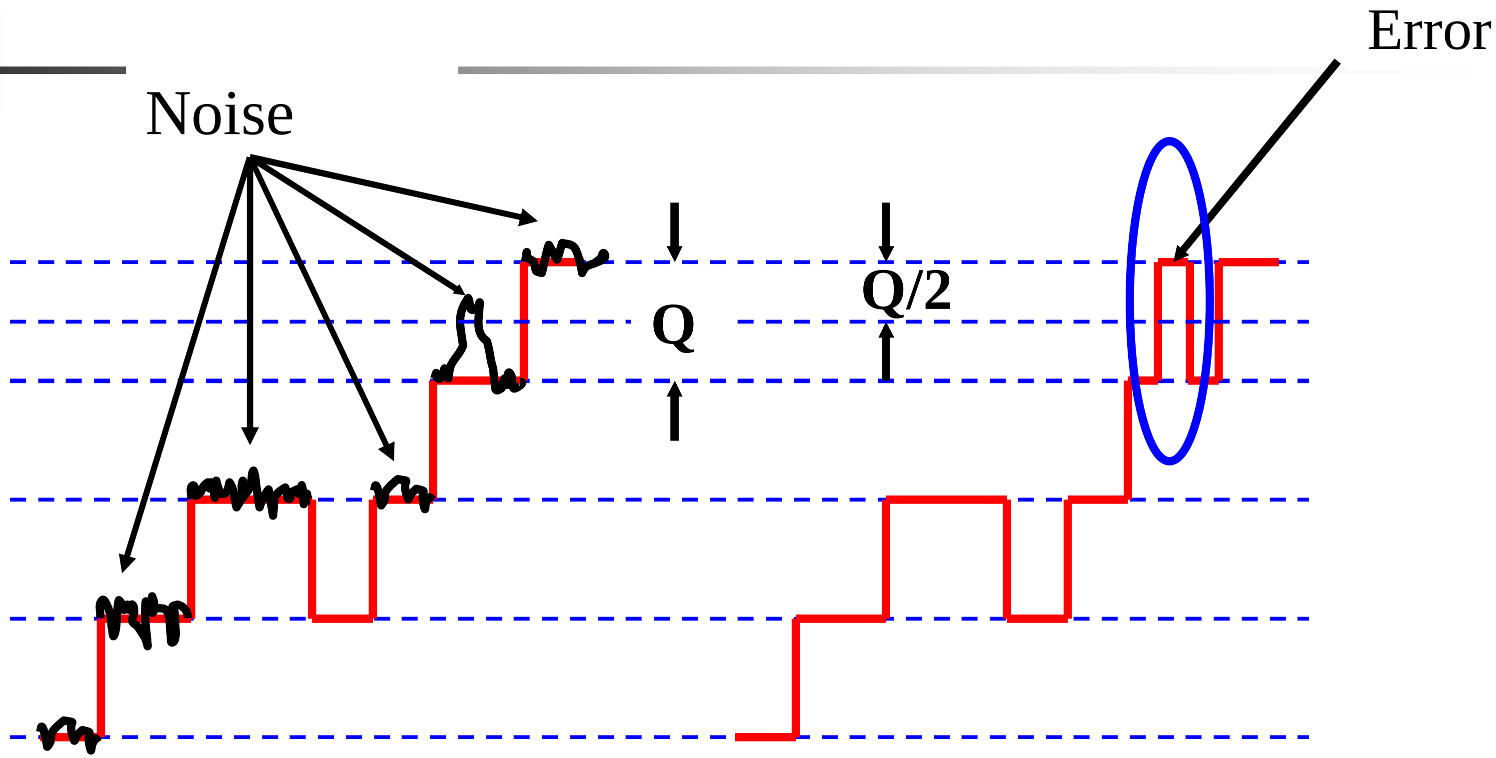
- Number of bits: N
- Full scale analog range: R
- Resolution: the gap between levels $Q = R/2^N$
- Quantization error = *quantized value – actual value*
- Dynamic range: number of levels, in decibel

  Dynamic range = $20\log(R/Q) = 20\log(2^N) = 6.02N$ dB
- Signal-to-noise ratio $SNR = 10\log(\text{signal power}/\text{noise power})$

  Or $SNR = 20\log(\text{signal amplitude}/\text{noise amplitude})$
- Bit rate: the rate at which bits are generated

  $\text{Bit rate} = N.f_s$

Noise removal by quantization

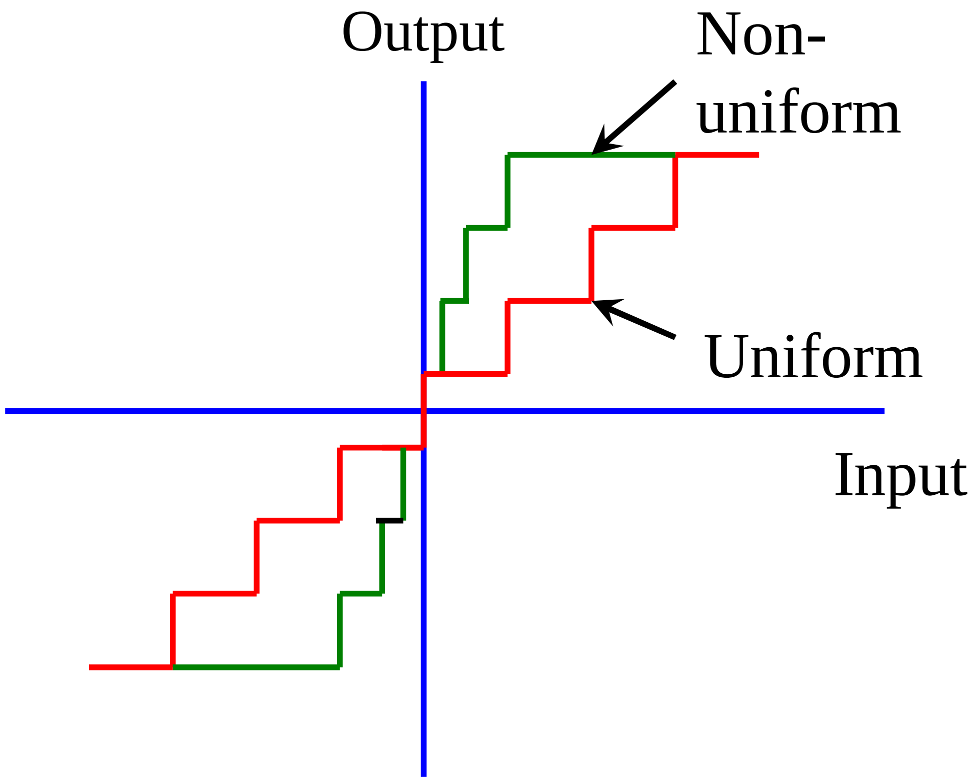


Quantized signal + noise After re-quantization

Non-uniform quantization

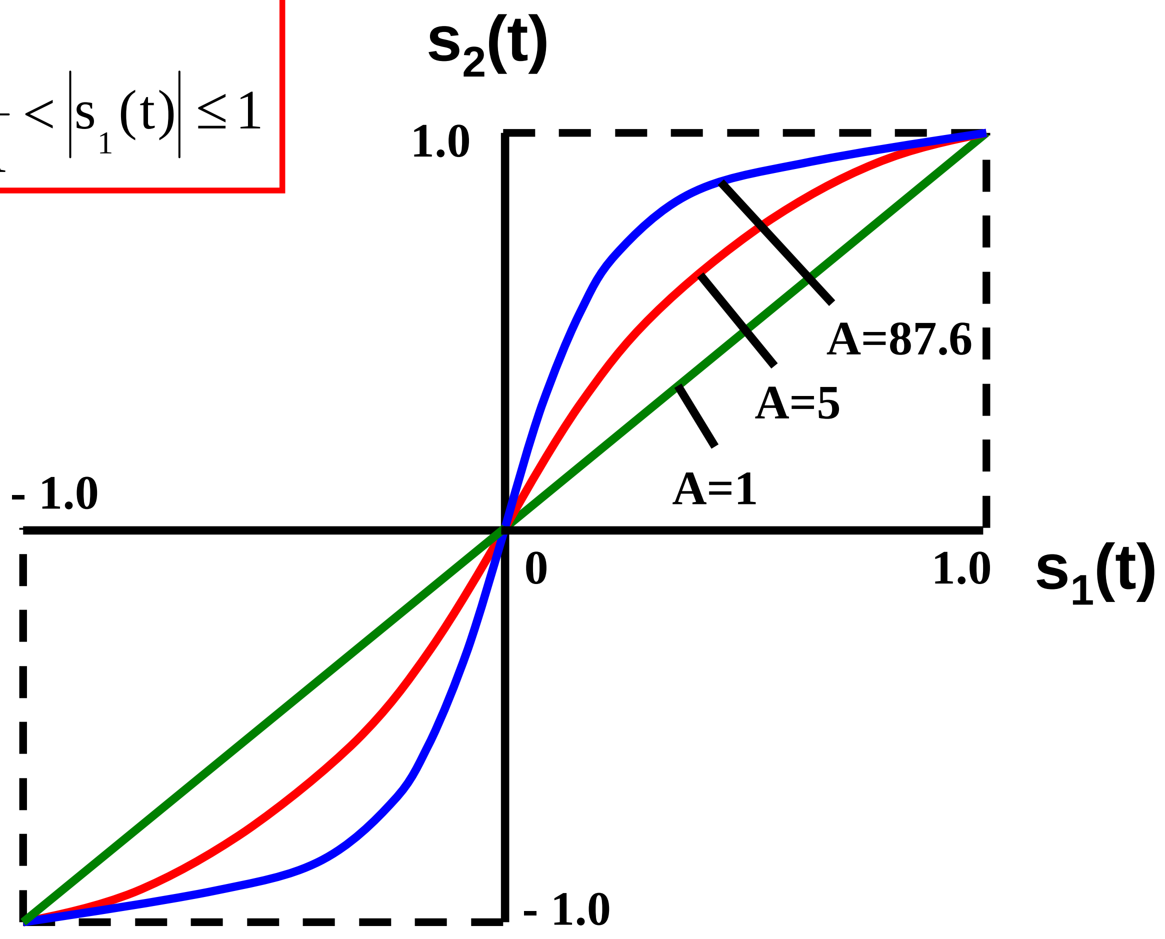
- Quantization with variable quantization step ←→ Q value is variable

- Q value is directly proportional to signal amplitude → SNR is constant
- Most used in speech



A-law compression curve

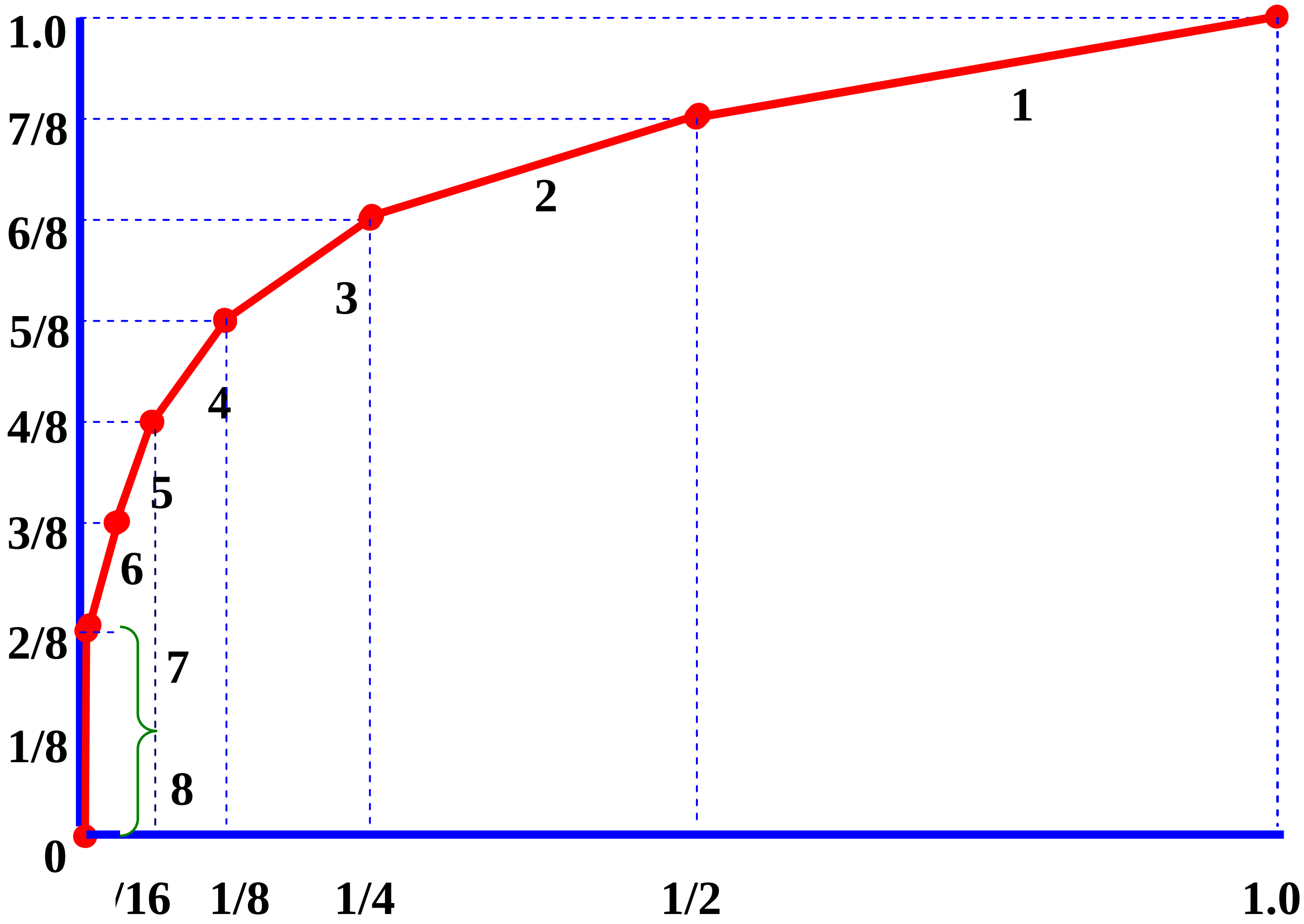
$$
|s_2(t)| = \begin{cases} \dfrac{A|s_1(t)|}{1+\ln A}, & 0 \le |s_1(t)| \le \dfrac{1}{A} \\[2ex] \dfrac{1+\ln(A|s_1(t)|)}{1+\ln A}, & \dfrac{1}{A} < |s_1(t)| \le 1 \end{cases}
$$



ITU G.711 A-law curve

Code-word format:

Sign bit	Part 1 (3bits)	Part 2 (4bits)
0/1	000 → 111	0000 → 1111



ITU G.711 standard

Input range	Step size	Part 1	Part 2	No. code word	Decoding output
0-1 … 30-31	**2**	**000**	**0000** **…** **1111**	**0** **…** **15**	1 … 31
32-33 … 62-63	**2**	**001**	**0000** **…** **1111**	**16** **…** **31**	33 … 63
64-67 … 124-127	**4**	**010**	**0000** **…** **1111**	**32** **…** **47**	66 … 126
128-135 … 248-255	**8**	**011**	**0000** **…** **1111**	**48** **…** **63**	132 … 252
256-271 … 496-511	**16**	**100**	**0000** **…** **1111**	**64** **…** **79**	264 … 504
512-543 … 992-1023	**32**	**101**	**0000** **…** **1111**	**80** **…** **95**	528 … 1008
1024-1087 … 1984-2047	**64**	**110**	**0000** **…** **1111**	**96** **…** **111**	1056 … 2016
2048-2175 … 3968-4095	**128**	**111**	**0000** **…** **1111**	**112** **…** **127**	2112 … 4032

Example of G.711 code word

- A quantized-sample's value is +121

Code word: ?

Decoding value: ?

- A quantized-sample's value is -121

Code word: ?

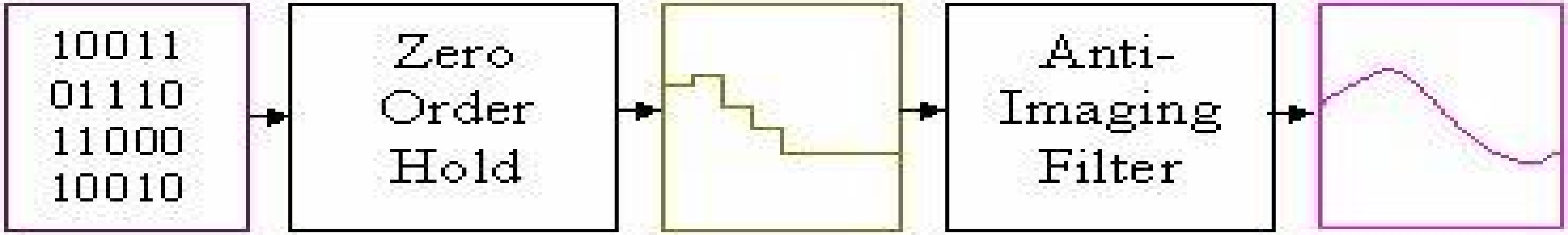
Lecture 2: Analog-to-Digital and Digital-to-Analog conversion

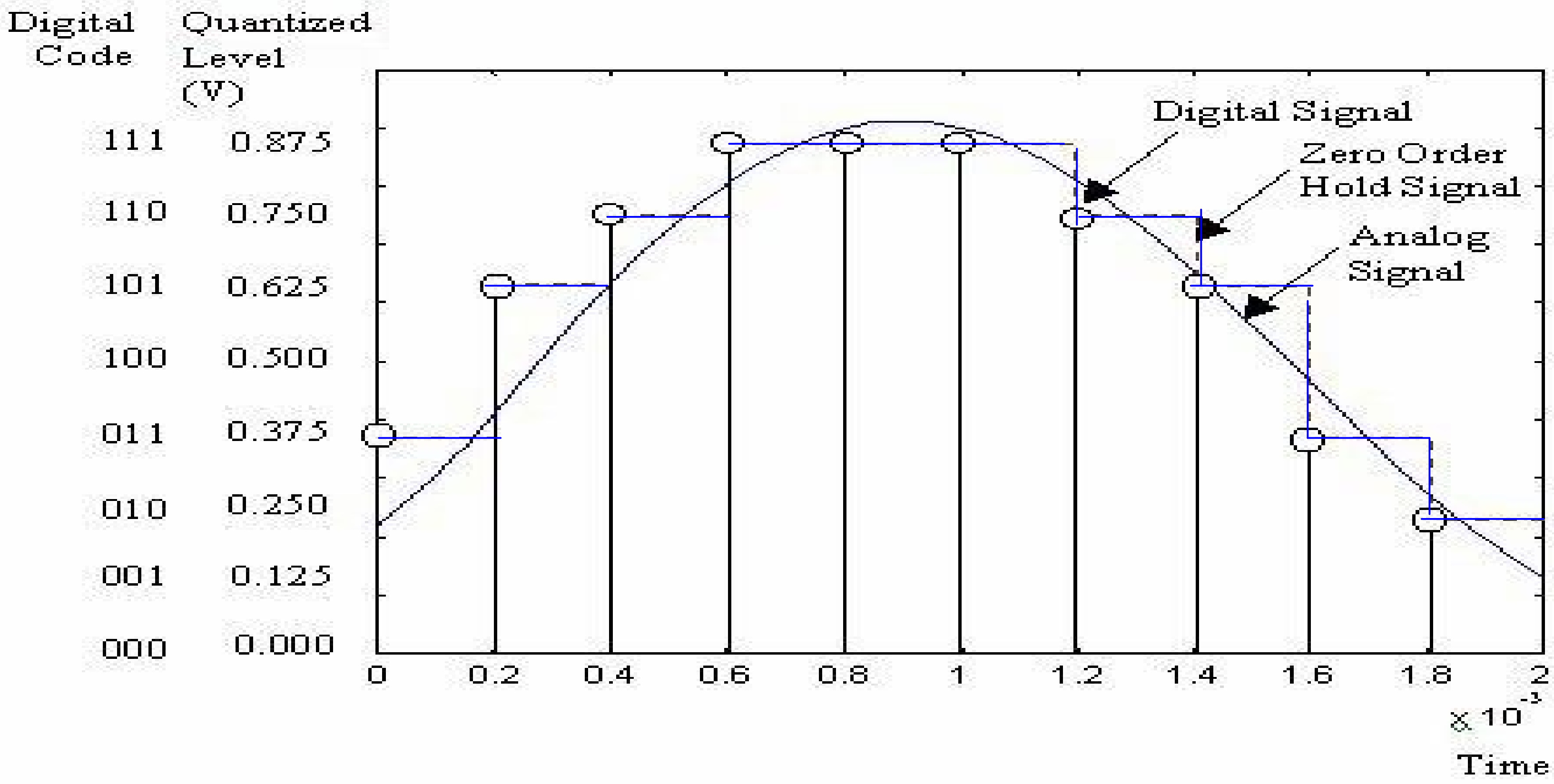
- **Duration: 2** hr
- **Outline:**

  1. A/D conversion

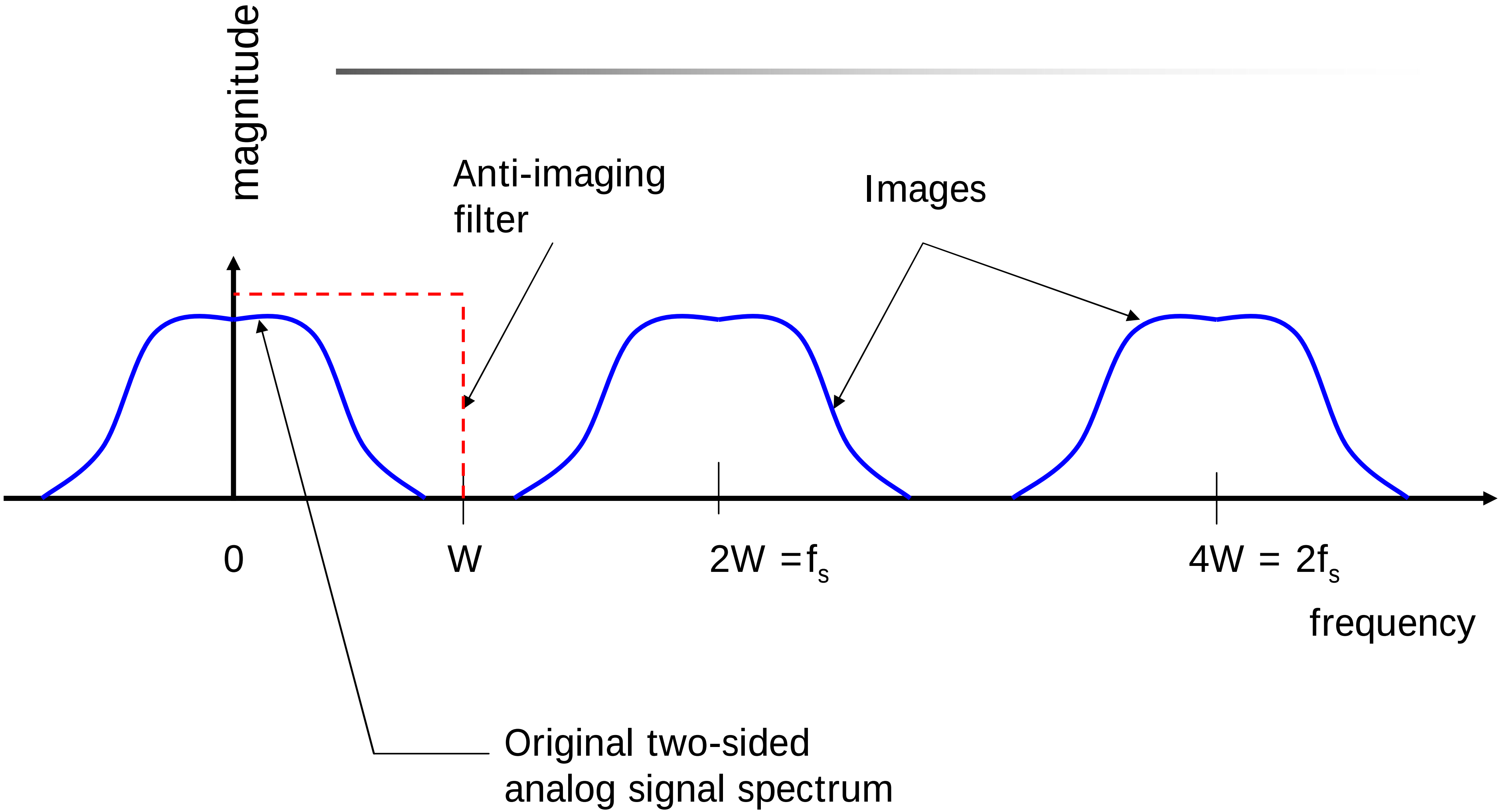
  2. D/A conversion

DAC


10011
01110
11000
10010
Zero Order Hold
Anti-Imaging Filter


Digital Code
Quantized Level (V)
111 0.875
110 0.750
101 0.625
100 0.500
011 0.375
010 0.250
001 0.125
000 0.000
Digital Signal
Zero Order Hold Signal
Analog Signal
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x 10^-3
Time

Anti-imaging filter



Lecture 3
The concept of frequency in CT & DT signals

- **Duration: 2** hrs
- **Outline:**

1. CT sinusoidal signals

2. DT sinusoidal signals

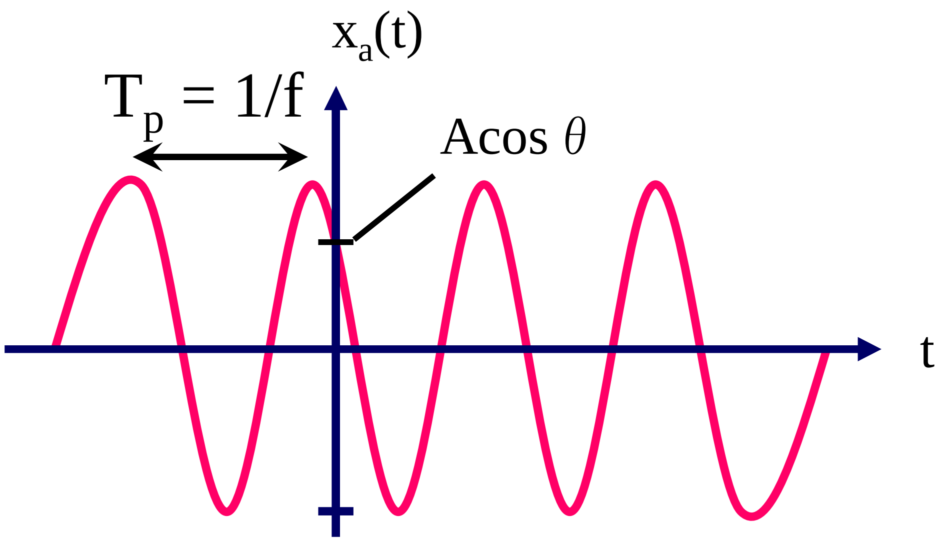
3. Relations among frequency variables

Mathematical description of CT sinusoidal signals

- Functions:

$$\boxed{\begin{aligned} x_a(t) &= A\cos(\omega t + \theta), \quad -\infty < t < +\infty \\ &= A\cos(2\pi f t + \theta), \quad -\infty < t < +\infty \end{aligned}}$$

- Plot:



Properties of CT sinusoidal signals

1. For every fixed value of the frequency f, $x_a(t)$ is periodic: $x_a(t+T_p) = x_a(t)$

T_p = 1/f: fundamental period

2. CT sinusoidal signals with different frequencies are themselves different

3. Increasing the frequency **f** results in an increase in the rate of oscillation of the signal (more periods in a given time interval)

Properties of CT sinusoidal signals (cont)

- For $f = 0 \rightarrow T_p = \infty$
- For $f = \infty \rightarrow T_p = 0$
- Physical frequency: positive
- Mathematical frequency: positive and negative

$$x_a(t) = A\cos(\omega t + \theta) = \frac{A}{2}e^{j(\omega t+\theta)} + \frac{A}{2}e^{-j(\omega t+\theta)}$$

- The frequency range for CT signal:

$$-\infty < f < +\infty$$

Lecture 3
The concept of frequency in CT & DT signals

- **Duration: 2** hrs
- **Outline:**

1. CT sinusoidal signals

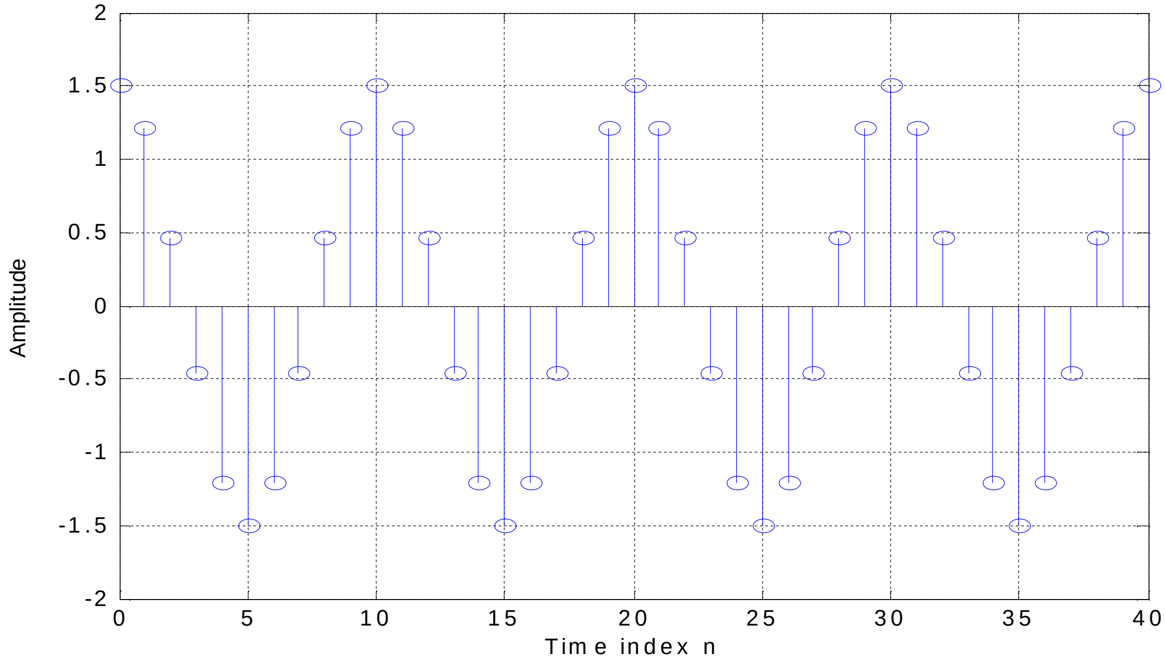
2. DT sinusoidal signals

3. Relations among frequency variables

Mathematical description of DT sinusoidal signals

- Functions: $$x(n) = A\cos(\Omega n + \theta), \quad -\infty < n < +\infty$$
$$= A\cos(2\pi F n + \theta), \quad -\infty < n < +\infty$$

- Plot:



Properties of DT sinusoidal signals

1. A DT sinusoidal signal x(n) is periodic only if its frequency ***F*** is a rational number

$$x(n+N) = x(n) \quad \forall n$$

$$A\cos[2\pi F_0(n+N)+\theta)] = A\cos(2\pi F_0 n+\theta) \quad \forall n$$

$$2\pi F_0 N = 2\pi k$$

$$F_0 = \frac{k}{N}$$

Properties of DT sinusoidal signals

2. DT sinusoidal signals whose frequencies are separated by an integer multiple of 2π are identical

$$x(n) = \cos[(\Omega_0 + 2\pi)n + \theta] = \cos(\Omega_0 n + 2\pi n + \theta) = \cos(\Omega_0 n + \theta)$$

→ All $x_k(n) = A\cos(\Omega_k n + \theta), \quad k = 0, 1, 2, \ldots$

$$\Omega_k = \Omega_0 + 2k\pi, \quad -\pi \le \Omega_0 \le +\pi$$

are identical

Properties of DT sinusoidal signals

3. The highest rate of oscillation in a DT sinusoidal signal is obtained when:

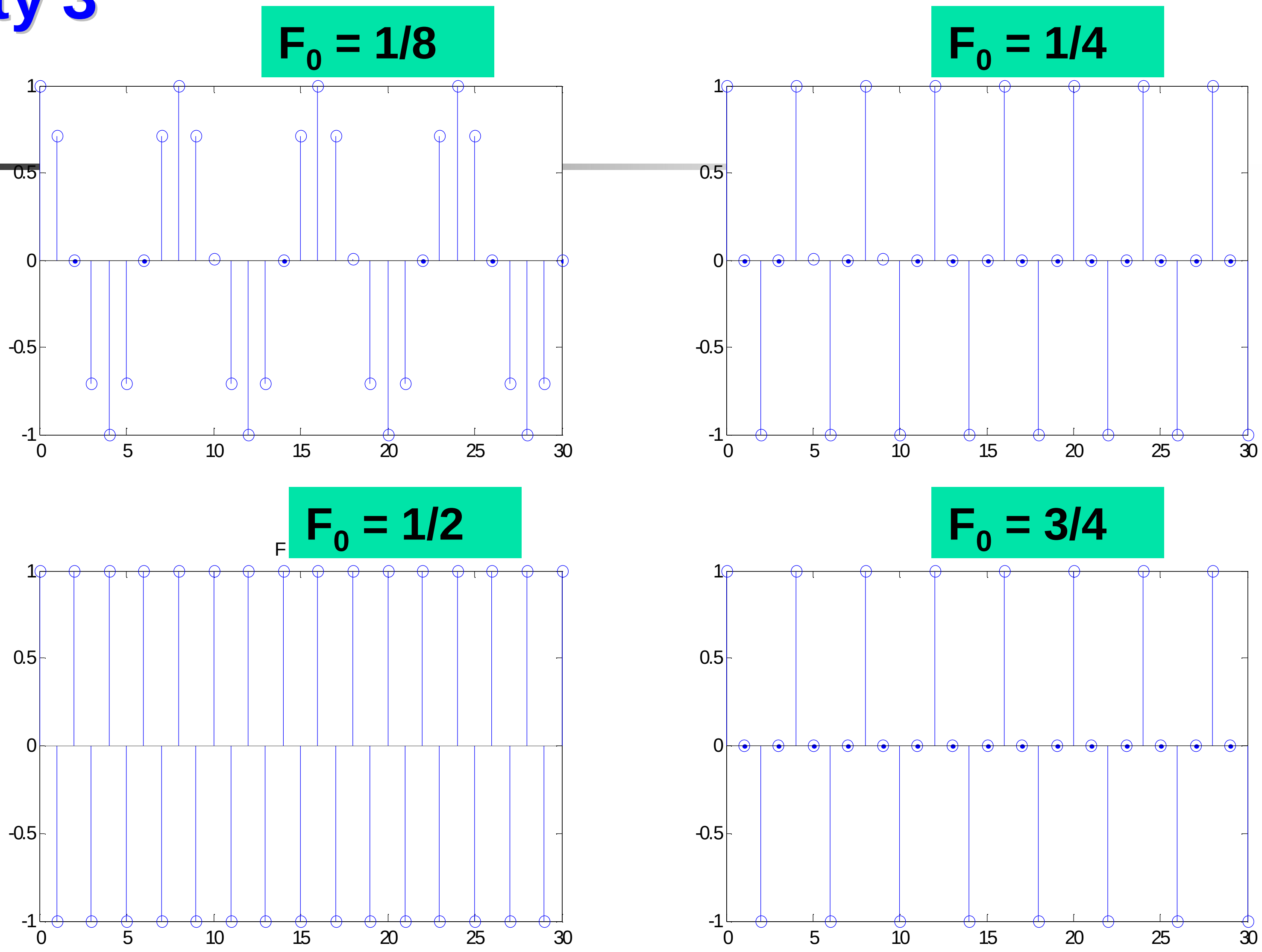
$$\boxed{\Omega = \pi \quad (\text{or } \Omega = -\pi)}$$

or, equivalently,

$$\boxed{F = \frac{1}{2} \quad (\text{or } F = -\frac{1}{2})}$$

Illustration for property 3

$$x(n) = \cos(2\pi F_0 n)$$



$-\pi \le \Omega \le \pi$ or $-1/2 \le F \le 1/2$: fundamental range

Lecture 3
The concept of frequency in CT & DT signals

- **Duration: 2** hrs
- **Outline:**

1. CT sinusoidal signals
2. DT sinusoidal signals
3. **Relations among frequency variables**

Sampling of CT sinusoidal signals

CT signal	Sampling	DT signal
$x_a(t)$		$x_a(nT)$
$A\cos(2\pi f t + \theta)$	$\longrightarrow$	$A\cos(2\pi f nT + \theta)$
		$= A\cos\left(\dfrac{2\pi f n}{f_s} + \theta\right)$

$$\boxed{F = \frac{f}{f_s}}$$

Normalized frequency

Relations among frequency variables

CT signals

$$\omega = 2\pi f$$

$$-\infty < \omega < +\infty$$
$$-\infty < f < +\infty$$

$$-\pi / T \le \omega \le +\pi / T$$
$$-f_s / 2 \le f \le +f_s / 2$$

$$F = \frac{f}{f_s}$$

DT signals

$$\Omega = 2\pi F$$

$$-\pi \le \Omega \le +\pi$$
$$-1/2 \le F \le +1/2$$

Exercise 1

Consider the analog signal

$$x(t) = 3\cos 100\pi t, \quad t[s]$$

a) Determine the minimum sampling rate required to avoid aliasing

b) Suppose that the signal is sampled at the rate $\mathbf{f_s = 200\ Hz}$. What is the DT signal obtained after sampling?

c) Suppose that the signal is sampled at the rate $\mathbf{f_s = 75\ Hz}$. What is the DT signal obtained after sampling?

d) What is the frequency $\mathbf{0 < f < f_s/2}$ of a sinusoidal signal that yields samples identical to those obtained in part (c)?

Solution

$$x(t) = 3\cos 100\pi\, t, \quad t[s]$$

Solution

$$x(t) = 3\cos 100\pi t, \quad t[s]$$

Exercise 2

An analog signal is sampled at its Nyquist rate $\boldsymbol{1/T_s}$, and quantized using $\boldsymbol{L}$ quantization levels. The derived signal is then transmitted on some channels.

(a) Show that the time duration, $\boldsymbol{T}$, of one bit of the transmitted binary encoded signal must satisfy

$$T \leq T_s / (\log_2 L)$$

(b) When is the equality sign valid?

HW – Problem 1 (20%)

1

n	0	1	2	3	4	5	6	7	8
Sample(V)	0.1111	0.6250	0.9555	3.4373	4.0500	2.8755	1.5625	2.7500	4.9676

A set of analog samples, listed in table 1, is digitized using the quantization table 2. Determine the digital codes, the quantized level, and the quantization error for each sample.

2

Digital code	**Quantization Level (V)**	**Range of analog inputs (V)**
000	0.0	0.0 →0.3125
001	0.625	0.3125→0.9375
010	1.250	0.9375→1.5625
011	1.875	1.5625→2.1875
100	2.500	2.1875→2.8125
101	3.125	2.8125→3.4375
110	3.750	3.4375→4.0625
111	4.375	4.0625→5.0

HW – Problem 2 (20%)

Consider that you desire an A/D conversion system, such that the quantization distortion does not exceed ***±1%*** of the full scale range of analog signal.

(a) If the analog signal's maximum frequency is ***4000 Hz***, and sampling takes place at the Nyquist rate, what value of sampling frequency is required?

(b) How many quantization levels of the analog signal are needed?

(c) How many bits per sample are needed for the number of levels found in part (b)?

(d) What is the data rate in bits/s?

HW – Problem 3 (20%)

A 3-bit D/A converter produces a ***0 V*** output for the code ***000*** and a ***5 V*** output for the code ***111***, with other codes distributed evenly between ***0*** and ***5 V.***

Draw the zero order hold output from the converter for the input below:

111 101 011 101 000 001 011 010 100 110

HW – Problem 4 (20%)

Determine whether or not each of the following signals is periodic, specify its fundamental period.

$$a)\, x(t) = 3\cos(5t + \pi/6)$$

$$b)\, x(n) = 3\cos(5n + \pi/6)$$

$$c)\, y(n) = 2\exp[j(n/6) - \pi]$$

$$d)\, h(n) = \cos(n/8)\cos(n\pi/8)$$

$$e)\, w(n) = \cos(n\pi/2) - \sin(n\pi/8) + 3\cos(n\pi/4 + \pi/3)$$

HW – Problem 5 (20%)

Consider the following continuous-time sinusoidal signal

$$x_a(t) = \sin 2\pi f_0 t, \quad -\infty < t < +\infty$$

Its sampled version is described by values every T (s) as the formula below

$$x(n) = x_a(nT) = \sin 2\pi \frac{f_0}{f_s} n, \quad -\infty < n < +\infty$$

where $f_s = 1/T$ is the sampling frequency.

a) Plot the signal $x(n)$, $0 \le n \le 99$ for $f_s = 5$ *kHZ* and $f_0 = 0.5, 2, 3,$ and *4.5 kHz.* Explain the similarities and differences among the various plots.

b) Suppose that $f_0 = 2$ *kHz and* $f_s = 50$ *kHz.*

(1) Plot the signal $x(n)$. What is the frequency F_0 of the signal $x(n)$.

(2) Plot the signal $y(n)$ created by taking the even-numbered samples of $x(n)$. Is this a sinusoidal signal? Why? If so, what is its frequency?