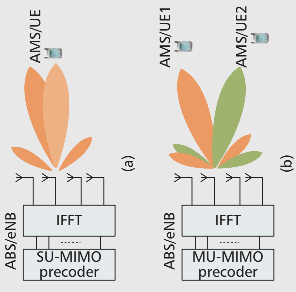
Chapter 3: Physical-layer transmission techniques

Section 3.6: Space Division Multiple Access (SDMA)

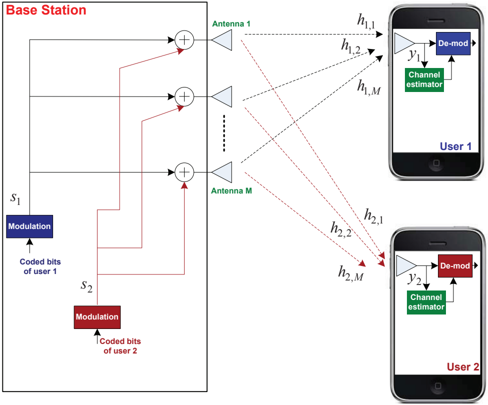
Department of Electronics & Telecommunications Engineering
Danang University of Technology, University of Danang

SDMA with OFDM

- The integration of multi-antenna and OFDM techniques has provided remarkable diversity and capacity gains in broadband wireless communications.
- In multiuser (MU) transmissions, the use of multiantenna array at the base station (BS) enables simultaneous transmission of multiple data streams to multiple users by exploiting spatial separations among users.



A simple example of multiuser (MU) transmission

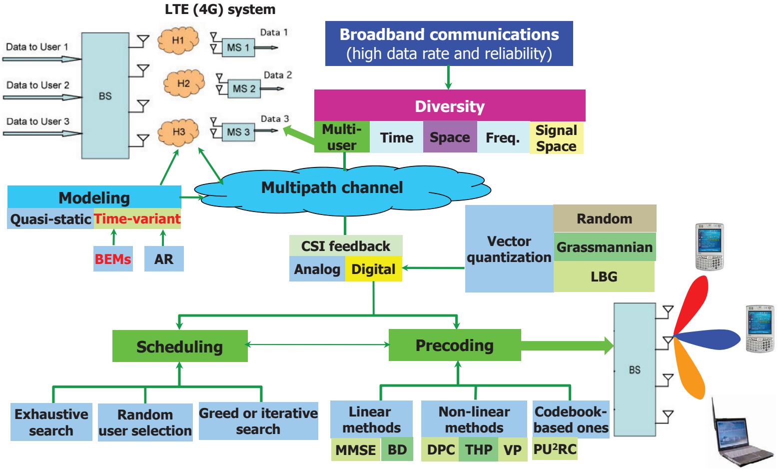


$$y_1 = s_1 \sum_{m=1}^{M} h_{1,m} + s_2 \sum_{m=1}^{M} h_{1,m} + z_1, \text{ and } y_2 = s_2 \sum_{m=1}^{M} h_{2,m} + s_1 \sum_{m=1}^{M} h_{2,m} + z_2$$

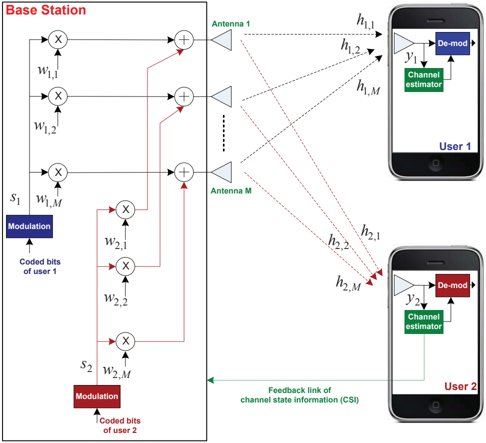
Precoding classification

- In the so-called space division multiple access (SDMA), **multiuser diversity** is the primary factor that increases significantly the system sum-rate (throughput).
- As a result, an appropriate multiuser encoding technique (at the BS) is indispensable to attain the considerable sum-rate gain in SDMA.
- It is well-known that dirty paper coding (DPC) is an **optimal** multiuser encoding strategy that achieves the capacity limit of MU broadcast (BC) channels but at the cost of **extremely high** computation burden as the number of users is large.
- Recent studies have introduced several **suboptimal** multiuser encoding techniques with **lower complexity** (relative to DPC) that can be categorized into:
    - **nonlinear** precoding such as: vector perturbation, Tomlinson Harashima techniques
    - **linear** precoding such as: minimum mean squared error (MMSE), zero-forcing.

Multiuser transmission techniques



An example of linear precoding



$$y_1 = s_1 \sum_{m=1}^{M} w_{1,m} h_{1,m} + s_2 \sum_{m=1}^{M} w_{2,m} h_{1,m} + z_1, \text{ and } y_2 = s_2 \sum_{m=1}^{M} w_{2,m} h_{2,m} + s_1 \sum_{m=1}^{M} w_{1,m} h_{2,m} + z_2$$

Inter-user interference

- The received signals at user-u can be determined by

$$y_u = s_u \sum_{m=1}^{M} w_{u,m} h_{u,m} + s_{u'} \sum_{m=1}^{M} w_{u',m} h_{u,m} + z_u, \quad u, u' \in \{1, 2\}, \tag{1}$$

where $s_{u'} \sum_{m=1}^{M} w_{u',m} h_{u,m}$ is called as **inter-user interference** that would significantly degrade the performance of the system.

- Precoding design is to find the weighting coefficients $\{w_{u,m}\}_{u=1}^{2}$ that satisfy the following condition

$$\sum_{m=1}^{M} w_{u',m} h_{u,m} = 0 \text{ with } u, u' \in \{1, 2\} \tag{2}$$

to eliminate the inter-user interference $s'_u \sum_{m=1}^{M} w_{u',m} h_{u,m}$.

- The above technique is called as **zero-forcing** (ZF) precoding.
- The problem of finding the weighting coefficients $\{w_{u,m}\}_{u=1}^{2}$ can be easily solved by expressing received signals in a vector form.

Zero forcing (ZF) precoding formulation

- In the presence of two users, the previous equations become

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & \dots & h_{1,M} \\ h_{2,1} & \dots & h_{2,M} \end{bmatrix} \begin{bmatrix} w_{1,1} & w_{2,1} \\ \vdots & \vdots \\ w_{1,M} & w_{2,M} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}.$$

- In the presence of U users, the received signal can be expressed by:

$$\mathbf{y} = \mathbf{HWs} + \mathbf{z}, \tag{3}$$

where $\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_U \end{bmatrix}$, $\mathbf{H} = \begin{bmatrix} h_{1,1} & \dots & h_{1,M} \\ \vdots & \vdots & \vdots \\ h_{U,1} & \dots & h_{U,M} \end{bmatrix}$, $\mathbf{s} = \begin{bmatrix} s_1 \\ \vdots \\ s_U \end{bmatrix}$

$\mathbf{W} = \begin{bmatrix} w_{1,1} & \dots & w_{U,1} \\ \vdots & \dots & \vdots \\ w_{1,M} & \dots & w_{U,M} \end{bmatrix} = [\mathbf{w}_1, \dots, \mathbf{w}_U]$ with

$\mathbf{w}_u = [w_{u,1}, \dots, w_{u,M}]^T$, and $\mathbf{z} = [z_1, \dots, z_U]^T$.

Zero-forcing precoding formulation (cont.)

- To eliminate inter-user interference, precoding matrix $\mathbf{W}$ can be determined by

$$\mathbf{W} = \mathbf{H}^H \left(\mathbf{H}\mathbf{H}^H\right)^{-1} \triangleq \mathbf{H}^{\dagger} \tag{4}$$

so that

$$\mathbf{y} = \mathbf{H}\mathbf{W}\mathbf{s} + \mathbf{z} = \mathbf{s} + \mathbf{z}. \tag{5}$$

- With precoding, the received signal can be written by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}, \tag{6}$$

where $\mathbf{x} = [x_1, \ldots, x_M]^T = \mathbf{W}\mathbf{s}$ are the transmitted signals in a vector form at M antennas in the base station.

- Under the power constraint of P_{max} at the BS, one has

$$\mathbb{E}\left[\sum_{m=1}^{M} |x_m|^2\right] = \mathbb{E}\left[\|\ \mathbf{x}\ \|^2\right] \leq P_{\text{max}}, \tag{7}$$

Power allocation in ZF precoding

- The power constraint (7) is equivalent to

$$\sum_{u=1}^{U} \lambda_u P_u \leq P_{\max}. \tag{8}$$

where $\lambda_u = \left[\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\right]_{u,u}$ and $s_u = \sqrt{P_u}\overline{s_u}$

- After ZF precoding, the received signals at U users are given by

$$\mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_U \end{bmatrix} = \begin{bmatrix} \sqrt{P_1}\overline{s_1} \\ \vdots \\ \sqrt{P_U}\overline{s_U} \end{bmatrix} + \begin{bmatrix} z_1 \\ \vdots \\ z_U \end{bmatrix} \tag{9}$$

- Hence, the resultant sum-rate of the multiuser system is

$$C = \max_{P_u : \sum_{u=1}^{U} \lambda_u P_u \leq P_{\max}} \sum_{u=1}^{U} \log_2 (1 + P_u) \quad \text{(bps/Hz)} \tag{10}$$

Power allocation in ZF precoding (cont.)

- The optimal power allocation $[P_u, u \in \{1, ..., U\}]$ in (10) can be easily determined by the following waterfilling process

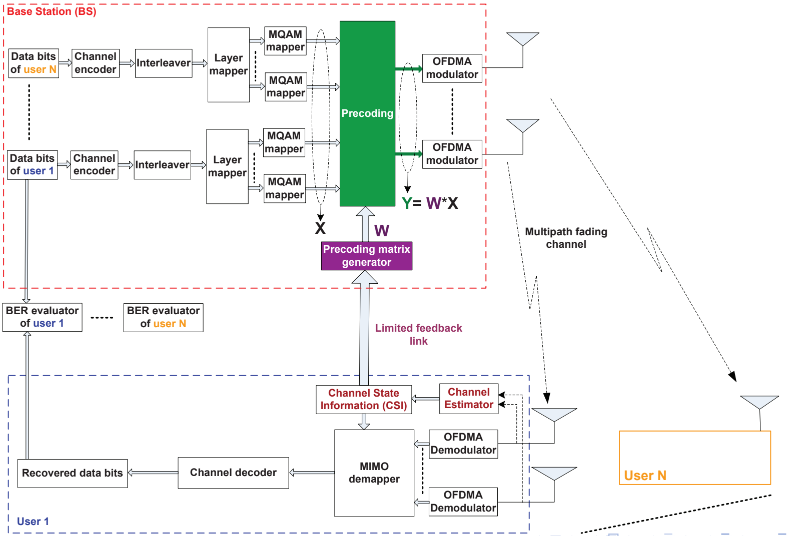
$$P_u = (\mu/\lambda_u - 1)^+ \tag{11}$$

where x^+ denotes $\max(x, 0)$, and the water level μ is chosen to satisfy

$$\sum_{u=1}^{U} (\mu - \lambda_u)^+ = P_{\mathsf{max}}. \tag{12}$$

- Given a set of selected users $\mathbf{\Omega} = \{1, ..., U\}$, the above precoding process attempts to eliminate the inter-user interference and maximize the system sum-rate.
- The problem of how to perform user selection (finding the set $\mathbf{\Omega} = \{1, ..., U\}$) with a reasonable complexity for **maximizing** the system sum-rate will be addressed in the next section.

Precoding in LTE downlink transmissions



Exhaustive selection

- Given a precoding technique, scheduling (user selection) is to find a set of users among all active users to maximize the system sum-rate.
- Obviously, the simple optimal method for user selection is exhaustive search but its complexity is impractically high as the number of users is large.

Greedy selection

Greedy user selection algorithm

1. **Initialization**: $\boldsymbol{\Theta}_0 = \{1, 2, ..., N_U\}$ is the set of all available users' indices
   $\boldsymbol{\Omega}_0 = \{\emptyset\}$ is the set of selected users initially assigned to a null set.
   $\eta = 0$ stands for the number of selected users, initially set to zero.
   $C_0 = 0$ is the system sum-rate of selected users, initially set to zero.
2. **Repetition**: Assuming that selecting user u in the set $\boldsymbol{\Theta}_\eta$ maximizes the resulting sum-rate of the system called C_{max}.
   - $\eta = \eta + 1$
   - If $C_{\text{max}} < C_{\eta-1}$ or $\eta > N_t$ or $\eta > N_u$ go to Step 3 otherwise do:
     - $C_\eta = C_{\text{max}}$
     - $\boldsymbol{\Omega}_\eta = \boldsymbol{\Omega}_{\eta-1} \bigcup \{u\}$ (select one more user)
     - $\boldsymbol{\Theta}_\eta = \boldsymbol{\Theta}_{\eta-1} \backslash \{u\}$ (ignore user-u in later consideration)
   - Go to Step 2.
3. **Stop** the user selection process and compute the ZF weighting vectors based on the composite channel matrix of selected users.