Chapter 3: Physical-layer transmission techniques

Section 3.1: Performance analysis over AWGN and fading channels

Outline of the lecture notes

Introduction

- We now consider the performance of the digital modulation techniques discussed in the previous chapter when used over AWGN channels and channels with flat-fading.
- There are two performance criteria of interest: the probability of error, defined relative to either symbol or bit errors, and the outage probability, defined as the probability that the instantaneous signal-to-noise ratio falls below a given threshold.
- Wireless channels may also exhibit frequency selective fading and Doppler shift. Frequency-selective fading gives rise to intersymbol interference (ISI), which causes an irreducible error floor in the received signal.

Signal-to-Noise power ratio and bit/symbol energy

- In this section we define the signal-to-noise power ratio (SNR) and its relation to energy-per-bit (E_b) and energy-per-symbol (E_s).
- We then examine the error probability on AWGN channels for different modulation techniques as parameterized by these energy metrics. Our analysis uses the signal space concepts of previous section.
- In an AWGN channel, the modulated signal $s(t) = \text{Re}\left[u(t)e^{j2\pi f_c t}\right]$ has receiver noise $n(t)$ added to it prior to reception. The noise $n(t)$ is a white Gaussian random process with zero-mean and power spectral density $N_0/2$.
- The received signal is thus $r(t) = s(t) + n(t)$.

Signal-to-Noise power ratio and bit/symbol energy (cont.)

- We define the received signal-to-noise power ratio (SNR) as the ratio of the received signal power P_r to the power of the noise within the bandwidth of the transmitted signal $s(t)$.
- The received power P_r is determined by the transmitted power and the path loss and multipath fading.
- The noise power is determined by the bandwidth of the transmitted signal and the spectral properties of $n(t)$. Specifically, if the bandwidth of the complex envelope $u(t)$ of $s(t)$ is B then the bandwidth of the transmitted signal $s(t)$ is $2B$.

Signal-to-Noise ratio and bit/symbol energy (cont.)

- Since the noise $n(t)$ has uniform power spectral density $N_0/2$, the total noise power within the bandwidth $2B$ is $P_n = N_0/2 \times 2B = N_0 B$. So, the received SNR is given by

$$SNR = \frac{P_r}{N_0 B}. \tag{1}$$

- In systems with interference, we often use the received signal-to-interference-plus-noise power ratio (SINR) in place of SNR for calculating error probability. If the interference statistics approximate those of Gaussian noise then this is a reasonable approximation.
- The received SINR is given by

$$SNR = \frac{P_r}{N_0 B + P_I}. \tag{2}$$

where P_I is the average power of the interference.

Signal-to-Noise ratio and bit/symbol energy (cont.)

- The SNR is often expressed in terms of the signal energy per bit E_b or per symbol E_s as

$$SNR = \frac{P_r}{N_0 B} = \frac{E_s}{N_0 B T_s} = \frac{E_b}{N_0 B T_b}. \tag{3}$$

  where T_s and T_b are the symbol and bit durations, respectively. For binary modulation (e.g., BPSK), $T_s = T_b$ and $E_s = E_b$.
- For data shaping pulses with $T_s = 1/B$ (e.g., raised cosine pulses with $\beta = 1$), one will have SNR $= E_s/N_0$ for multilevel signaling and SNR $= E_b/N_0$ for binary signaling. For general pulses, $T_s = k/B$ for some constant k, we have $k \times$ SNR $= E_s/N_0$.
- The quantities $\gamma_s = E_s/N_0$ and $\gamma_b = E_b/N_0$ are sometimes called the SNR per symbol and the SNR per bit, respectively.

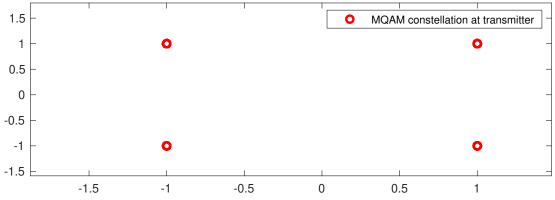
[Digital Modulation] Lab 5: Plot the QAM constellation with noise

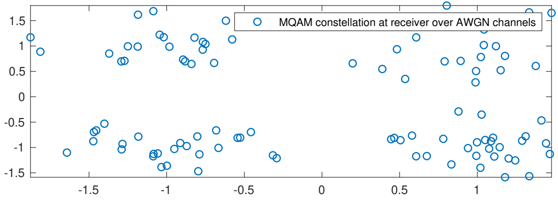
Lab 5. Based on Lab 4, write a MatLAB script to illustrate the M-QAM Digital modulation performance with the plots of MQAM constellation before (at TX) and after (at RX) transmitting via the AWGN channel, where

- Modulation level of M-ary quadrature amplitude modulation (MQAM): $M = 4$,
- The number of MQAM complex symbols per one transmission burst/frame $N = 100(1e2)$,
- The SNR level: SNR$= 10$[dB],

[Digital Modulation] Lab 5: Plot the QAM constellation with noise

Lab 5. Expected plots of MQAM constellation





[Digital Modulation] Lab 5: Plot the QAM constellation with noise

Lab 5. Suggested steps:

- Generate the binary bit sequences at transmitter with length of product of $N*$number of bits per one MQAM complex symbol
- At the TX: Write a function MQAM-modulator to convert the generated bit sequence into M-QAM transmitted sequence with length of N complex symbols (e.g. $(1-1*j;-1+1*j;...)$)
- Generate the AWGN noise sequence with length of N complex symbols, knowing that SNR$=10$[dB] and add to the transmitted signal.
- Plot the MQAM constellation before and after transmitting.

Bonus: Extend for the case $M=16$.

Signal-to-Noise ratio and bit/symbol energy (cont.)

- For performance specification, we are interested in the bit error probability P_b as a function of γ_b.
- However, for M-array signaling (e.g., MPAM and MPSK), the bit error probability depends on both the symbol error probability and the mapping of bits to symbols. Thus, we typically compute the symbol error probability P_s as a function of γ_s based on the signal space concepts of previous section and then obtain P_b as a function of γ_b using an exact or approximate conversion.
- These assumptions for M-array signaling lead to the approximations:

$$\gamma_b \approx \frac{\gamma_s}{\log_2 M} \quad \text{and} \quad P_b \approx \frac{P_s}{\log_2 M}. \tag{4}$$

Error probability for BPSK and QPSK

- Consider BPSK modulation with coherent detection and perfect recovery of the carrier frequency and phase. With binary modulation each symbol corresponds to one bit, so the symbol and bit error rates are the same.
- The transmitted signal is $s_0(t) = Ag(t)cos(2\pi f_c t)$ to send a 0 bit and $s_1(t) = -Ag(t)cos(2\pi f_c t)$ to send a 1 bit. Note that for binary modulation where $M = 2$, there is only one way to make an error and d_{min} is the distance between the two signal constellation points, so the probability of error is also the bound:

$$P_b = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) \quad (5)$$

where $Q(z) = \frac{1}{2}\text{erfc}\left(\frac{z}{\sqrt{2}}\right)$. Remind that:

$$\text{erf}z = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt, \quad (6)$$

and the complementary error function is $\text{erfc}z = 1 - \text{erf}z$.

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Error probability for BPSK and QPSK (cont.)

- Moreover, we have $d_{min} = \| \mathbf{s}_0 - \mathbf{s}_1 \| = \| A - (-A) \| = 2A$. The energy-per-bit can be determined by

$$E_b = \int_0^{T_b} s_0^2(t)dt = \int_0^{T_b} s_1^2(t)dt = \int_0^{T_b} A^2 g^2(t) \cos^2(2\pi f_c t)dt = A^2. \tag{7}$$

- Thus, the signal constellation for BPSK in terms of energy-per-bit is given by $\mathbf{s}_0 = \sqrt{E_b}$ and $\mathbf{s}_1 = -\sqrt{E_b}$. This yields the minimum distance $d_{min} = 2A = 2\sqrt{E_b}$. Substituting this into (5) yields

$$P_b = Q\left(\frac{2\sqrt{E_b}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q\left(\sqrt{2\gamma_b}\right). \tag{8}$$

Error probability for BPSK and QPSK (cont.)

- QPSK modulation consists of BPSK modulation on both the in-phase and quadrature components of the signal. With perfect phase and carrier recovery, the received signal components corresponding to each of these branches are orthogonal and independent. Therefore, the bit error probability on each branch is the same as for BPSK: $P_b = Q\left(\sqrt{2\gamma_b}\right)$.
- The symbol error probability equals the probability that either branch has a bit error:

$$P_s = 1 - \left[1 - Q\left(\sqrt{2\gamma_b}\right)\right]^2. \tag{9}$$

Q: Explain the intuition of this formula?

Error probability for BPSK and QPSK (cont.)

Q: Explain the intuition of this formula?

This approach is based on the idea that a symbol error occurs if there is an error in either the I or Q component, as QPSK transmits two bits per symbol, where:

- P_b is the bit error probability for BPSK. It represents the probability of an error in a single bit.
- $1 - P_b$ is the probability of no error in a single bit. This term represents the probability that both the I and Q branches are error-free.
- $(1 - P_b)^2$ is the probability that both the I and Q branches are error-free for two consecutive bits.
- The symbol error probability (P_s) for QPSK is the probability that at least one of the I or Q branches has an error:

$$P_s = 1 - \left[1 - Q\left(\sqrt{2\gamma_b}\right)\right]^2.$$

Error probability for BPSK and QPSK (cont.): Example

- **On-class Exercise**: Find the bit error probability P_b and symbol error probability P_s of QPSK assuming $\gamma_b = 7$ dB?

Error probability for BPSK and QPSK (cont.): Example

- **On-class Exercise**: Find the bit error probability P_b and symbol error probability P_s of QPSK assuming $\gamma_b = 7$ dB.
- *Solution*:
  - We have $\gamma_b = 10^{7/10} = 5.012$,
  - then $P_b = Q\left(\sqrt{2\gamma_b}\right) = 7.726 \times 10^{-4}$
  - and $P_s = 1 - \left[1 - Q\left(\sqrt{2\gamma_b}\right)\right]^2 = 1.545 \times 10^{-3}$.

Digital Modulation] Lab 6: Recover the M-QAM modulated signals

Lab 6. Based on Lab 5, write a MatLAB function, entitled as MQAM-demodulator to demodulate the noisy received signals after transmitting via the AWGN channels and compare with the original binary sequences.

- Modulation level of M-ary quadrature amplitude modulation (MQAM): $M = 4$,
- The number of MQAM complex symbols per one transmission burst/frame $N = 100(1e2)$,
- The SNR level: SNR$= 10$[dB]

Extend for the case $M = 16$.

Lab 6. Hints:

- At the RX: Write a function MQAM-demodulator to convert the M-QAM received sequence into the binary sequence.
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Approximate symbol and bit error probabilities for typical modulations

Many of the approximations or exact values for P_s derived above for coherent modulation are in the following form:

$$P_s(\gamma_s) \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right) \tag{10}$$

where α_M and β_M depend on the type of approximation and the modulation type. In the below table, we summarize the specific values of α_M and β_M for common P_s expressions for PSK, QAM, and FSK modulations based on the derivations in the prior sections.

Modulation	$P_s(\gamma_s)$	$P_b(\gamma_b)$
BFSK:		$P_b = Q\left(\sqrt{\gamma_b}\right)$
BPSK:		$P_b = Q\left(\sqrt{2\gamma_b}\right)$
QPSK,4QAM:	$P_s \approx 2\,Q\left(\sqrt{\gamma_s}\right)$	$P_b \approx Q\left(\sqrt{2\gamma_b}\right)$
MPAM:	$P_s \approx \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\overline{\gamma}_s}{M^2-1}}\right)$	$P_b \approx \frac{2(M-1)}{M\log_2 M} Q\left(\sqrt{\frac{6\overline{\gamma}_b \log_2 M}{(M^2-1)}}\right)$
MPSK:	$P_s \approx 2Q\left(\sqrt{2\gamma_s}\sin(\pi/M)\right)$	$P_b \approx \frac{2}{\log_2 M} Q\left(\sqrt{2\gamma_b \log_2 M}\sin(\pi/M)\right)$
Rectangular MQAM:	$P_s \approx \frac{4(\sqrt{M}-1)}{\sqrt{M}} Q\left(\sqrt{\frac{3\overline{\gamma}_s}{M-1}}\right)$	$P_b \approx \frac{4(\sqrt{M}-1)}{\sqrt{M}\log_2 M} Q\left(\sqrt{\frac{3\overline{\gamma}_b \log_2 M}{(M-1)}}\right)$
Nonrectangular MQAM:	$P_s \approx 4Q\left(\sqrt{\frac{3\overline{\gamma}_s}{M-1}}\right)$	$P_b \approx \frac{4}{\log_2 M} Q\left(\sqrt{\frac{3\overline{\gamma}_b \log_2 M}{(M-1)}}\right)$

BER performance over AWGN channels: Lab 7

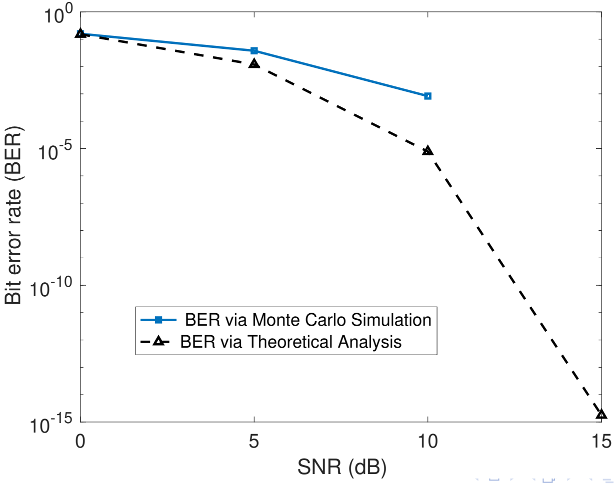
Lab 7 Based on Lab 6, plot Fig. 1 with the MQAM constellation before and after transmitting the AWGN channels and Fig.2 to compare the theoretical BER (bit error rate) performance of the AWGN channels with the BER (BERsim) obtained by Monte Carlo simulations when SNR varies from 0 to 30 dB. Note that the BERsim is computed as the average bit error rate after a number of iteration.

- Modulation level of M-ary quadrature amplitude modulation (MQAM): $M = 4$,
- The number of MQAM complex symbols per one transmission burst/frame $N = 100(1e2)$,
- The SNR level: SNR$= [0:5:30]$[dB],
- The number of interation: num_ iter $= 100$.

Provide comments when the number of iteration and number of complex symbols increase.

BER performance over AWGN channels: Lab 7

Lab 7. Expected plots of BER performance via theoretical analysis and simulation



Introduction

- In AWGN the probability of symbol error depends on the received SNR γ_s. In a fading channel, the received signal power varies randomly over distance or time due to shadowing and/or multipath fading. Thus, in fading γ_s is a random variables with distribution $p_{\gamma_s}(\gamma)$, and therefore $P_s(\gamma_s)$ is also random.
- The performance metric when γ_s is random depends on the rate of change of the fading. There are two main performance criteria that can be used to characterize the random variable P_s:
  - The outage probability, P_{out}, defined as the probability that γ_s falls below a given value corresponding to the maximum allowable P_s.
  - The average error probability, $\overline{P_s}$, averaged over the distribution of γ_s.

Outage probability

- The outage probability relative to P_{out} is defined as

$$P_{out} = p\left(\gamma_s < \gamma_0\right) = \int_0^{\gamma_0} p_{\gamma_s}(\gamma)d\gamma. \tag{11}$$

where γ_0 typically specifies the minimum SNR required for acceptable performance.
For example, if we consider digitized voice, $P_b = 10^{-3}$ is an acceptable error rate since it generally cannot be detected by the human ear. Thus, for a BPSK signal in Rayleigh fading, $\gamma_b < 7$ dB would be declared an outage, so we set $\gamma_0 = 7$ dB.

Outage probability

- In Rayleigh fading with $p_{\gamma_s}(\gamma) = \frac{1}{\overline{\gamma}_s} e^{-\gamma_s/\overline{\gamma}_s}$, one will have

$$P_{out} = \int_0^{\gamma_0} \frac{1}{\overline{\gamma}_s} e^{-\gamma_s/\overline{\gamma}_s} d\gamma_s = 1 - e^{-\gamma_0/\overline{\gamma}_s}, \tag{12}$$

where $\overline{\gamma}_s$ is the required average SNR which can be referred as:

$$\overline{\gamma}_s = \frac{\gamma_0}{-\ln(1 - P_{out})}, \tag{13}$$

- For binary signaling, the distribution of γ_b is written as:

$$p_{\gamma_b}(\gamma) = \frac{1}{\overline{\gamma}_b} e^{-\gamma/\overline{\gamma}_b} \tag{14}$$

Outage probability (cont.)

- **Example**: Determine the required $\overline{\gamma}_b$ for BPSK modulation in slow Rayleigh fading such as 95% of the time (or in space) that $P_b(\gamma_b) < 10^{-4}$.

Outage probability (cont.)

- **Example**: Determine the required $\overline{\gamma}_b$ for BPSK modulation in slow Rayleigh fading such as 95% of the time (or in space), $P_b(\gamma_b) < 10^{-4}$.
- *Solution*: For BPSK modulation in AWGN, the target BER is obtained at 8.5 dB (i.e., for $P_b(\gamma_b) = Q(\sqrt{2\gamma_b})$, one have $P_b\left(10^{0.85}\right) = 10^{-4}$). Thus, $\gamma_0 = 8.5$ dB, since we want $P_{out} = p(\gamma_b < \gamma_0) = 0.05$, we have

$$\overline{\gamma}_b = \frac{\gamma_0}{-\ln(1 - P_{out})} = \frac{10^{.85}}{-\ln(1 - 0.05)} = 21.4 \text{ dB}. \qquad (15)$$

BER performance over Rayleigh fading channels: Lab 8

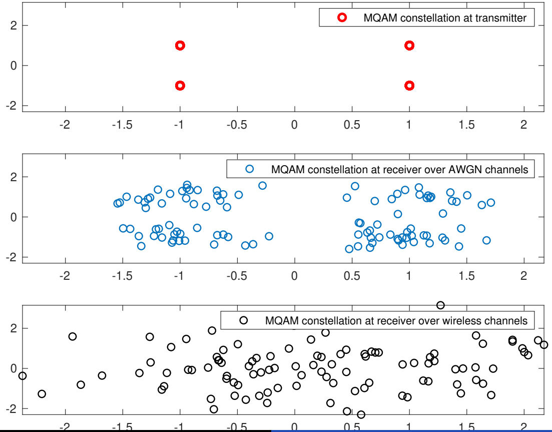
Lab 8 Based on Lab 7, write a MatLAB script to plot Fig. 1 with the MQAM constellation before and after transmitting the AWGN channels and the Rayleigh fading channels and plot Fig.2 to compare BER (bit error rate) performance as a function of SNR, obtained by Monte Carlo simulations, of the AWGN channels and the Rayleigh fading channels with and without knowledge of the channel (without CSI - channel state information).

- Modulation level of M-ary quadrature amplitude modulation (MQAM): $M = 4$,
- The number of MQAM complex symbols per one transmission burst/frame $N = 100(1e2)$,
- The SNR level: SNR$= [0 : 2 : 10]$[dB],
- The number of interation: num_ iter $= 100$.

Provide comments when the number of iteration and number of complex symbols increase.

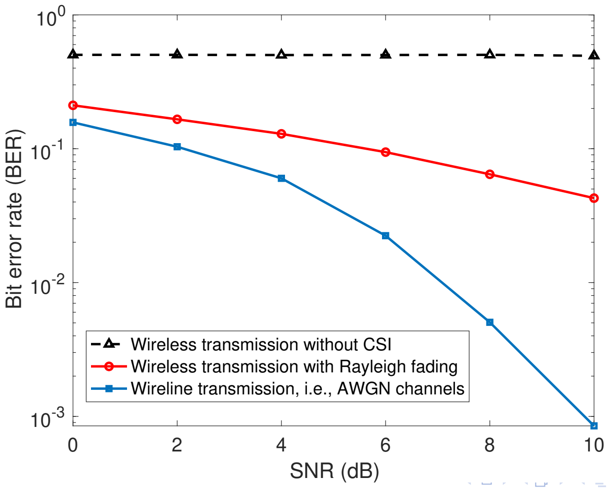
M-QAM performance over AWGN and fading channels: Lab 8

Lab 8. Expected plots of 4-QAM constellation performance via AWGN and fading channels



BER performance over AWGN and fading channels: Lab 8

Lab 8. Expected plots of BER performance pver AWGN and fading channels



BER performance over Rayleigh fading channels: Hints for Lab 8

Lab 8 Hints for Lab 8: We need to generate a random fading channel following Rayleigh distribution.

- Remind: The Rayleigh distribution is related to normal distribution as follows:
- Random variable $R \approx$ Rayleigh (σ) is Rayleigh distributed if $R = \sqrt{X^2 + Y^2}$, where $X \approx N(0, \sigma^2)$ and $Y \approx N(0, \sigma^2)$
- Generation of Rayleigh fading channel can be illustrated in programming as:
  $h = sqrt(1/2) * (randn(1, N) + i * randn(1, N))$
- Write the demod function in case knowing the CSI of the channel.

Average probability of error

- The average probability of error is used as a performance metric when γ_s is roughly constant over a symbol time. Then the averaged probability of error is computed by integrating the error probability in AWGN over the fading distribution:

$$\overline{P}_s = \int_0^{\infty} P_s(\gamma) p_{\gamma_s}(\gamma) d\gamma. \quad (16)$$

where $P_s(\gamma)$ is the probability of symbol error in AWGN channels with SNR γ, which can be approximated by the expressions in the aforementioned table.

- For a given distribution of the fading amplitude r (i.e., Rayleigh, Rician, log-normal, etc.), we compute $p_{\gamma_s}(\gamma)$ by making the change of variable

$$p_{\gamma_s}(\gamma) d\gamma = p(r) dr. \quad (17)$$

Average probability of error (cont.)

- For instance, in Rayleigh fading, the received signal amplitude r has the Rayleigh distribution

$$p(r) = \frac{r}{\sigma^2} e^{-r^2/(2\sigma^2)}, \quad r \geq 0, \tag{18}$$

where the signal power is exponentially distributed with mean $2\sigma^2$.

- The SNR per symbol for a given amplitude r is

$$\gamma = \frac{r^2 T_s}{2\sigma_n^2} = \frac{r^2 T_s}{N_0}. \tag{19}$$

where $\sigma_n^2 = N_0/2$ is the PSD of the noise in the in-phase and quadrature branches.

- Differentiating both sides of this expression yields

$$d\gamma = \frac{r T_s}{\sigma_n^2} dr. \tag{20}$$

Average probability of error (cont.)

- Substituting (19) and (20) into (18) and then (17) yields

$$p_{\gamma_s}(\gamma) = \frac{\sigma_n^2}{\sigma^2 T_s} e^{-\gamma\sigma_n^2/(\sigma^2 T_s)}. \tag{21}$$

- Since the average SNR per symbol $\overline{\gamma}_s$ is just $\sigma^2 T_s/\sigma_n^2$, one can rewrite (21) as

$$p_{\gamma_s}(\gamma) = \frac{1}{\overline{\gamma}_s} e^{-\gamma/\overline{\gamma}_s}, \tag{22}$$

  which is exponential. For binary signaling, this reduces to

$$p_{\gamma_b}(\gamma) = \frac{1}{\overline{\gamma}_b} e^{-\gamma/\overline{\gamma}_b}. \tag{23}$$

Average probability of error (cont.)

- Integrating the error probability of BPSK in AWGN over the distribution (23) yields the following average probability of error for BPSK in Rayleigh fading:

$$\overline{P}_b = \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma}_b}{1+\overline{\gamma}_b}}\right) \approx \frac{1}{4\overline{\gamma}_b}. \qquad (24)$$

  where the approximation holds for large $\overline{\gamma}_b$.

- If we use the general approximation $P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$ then the average probability of symbol error in Rayleigh fading can be approximated as

$$\begin{aligned} \overline{P}_s &\approx \int_0^\infty \alpha_M Q\left(\sqrt{\beta_M \gamma}\right) \frac{1}{\overline{\gamma}_s} e^{-\gamma/\overline{\gamma}_s} d\gamma_s = \frac{\alpha_m}{2}\left(1 - \sqrt{\frac{.5\beta_M \overline{\gamma}_s}{1+.5\beta_M\overline{\gamma}_s}}\right) \\ &\approx \frac{\alpha_M}{2\beta_M \overline{\gamma}_s}. \end{aligned}$$

Average probability of error: Numerical results of BPSK

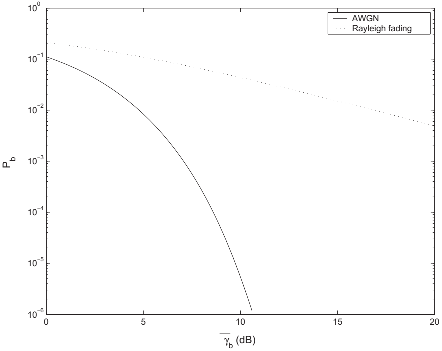


Figure 1: Average probability of bit error P_b for BPSK in Rayleigh Fading and AWGN.

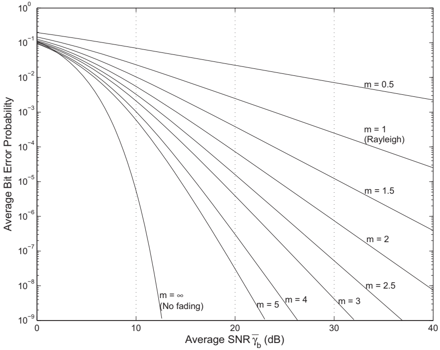


Figure 2: Average P_b for BPSK in Nakagami Fading.

Average probability of error: Numerical results of MQAM

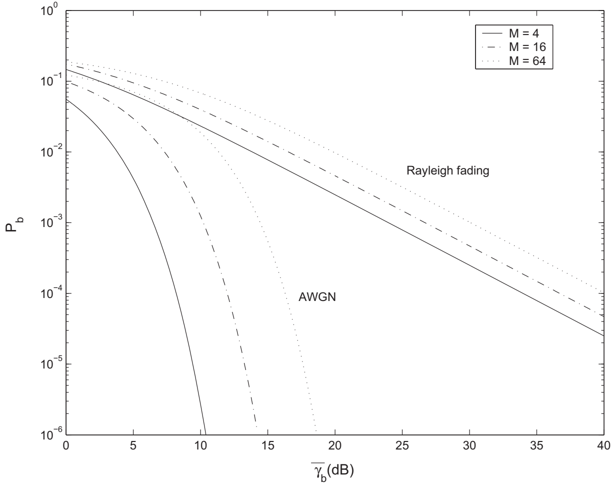


Figure 3: Average P_b for MQAM in Rayleigh Fading and AWGN.