Chapter 3: Physical-layer transmission techniques

Section 3.5: **Diversity techniques**

Introduction

- As observed in Section 3.2, Rayleigh fading induces a **very large power penalty** on the performance of modulation over wireless channels.
- One of the most powerful techniques to mitigate the effects of fading is to use diversity-combining of **independently** fading signal paths.
- Diversity-combining exploits the fact that independent signal paths have a **low probability** of experiencing deep fades **simultaneously**.
- These independent paths are combined in some ways such that the fading of the resultant signal is reduced.
- Diversity techniques that mitigate the effect of multipath fading are called **microdiversity**.
- Diversity to mitigate the effects of shadowing from buildings and objects is called **macrodiversity**. Macrodiversity is generally implemented by combining signals received by several base stations or access points.

Space diversity

- There are many ways of achieving independent fading paths in a wireless system.
- One method is to use multiple transmit or receive antennas, also called an antenna array, where the elements of the array are separated in distance. This type of diversity is referred to as **space diversity**.
- Note that with receiver space diversity, independent fading paths are generated without an increase in transmit signal power or bandwidth.
- Coherent combining of the diversity signals leads to an increase in SNR at the receiver over the SNR that would be obtained with just a single receive antenna.
- Space diversity also requires that the separation between antennas is large enough so that the fading amplitudes corresponding to each antenna are approximately **independent**.

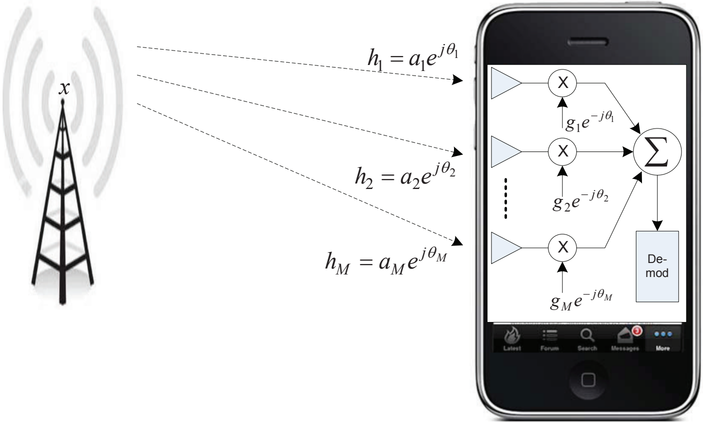
Frequency diversity

- Frequency diversity is achieved by transmitting the same narrowband signal at **different** carrier frequencies.
- This technique requires additional transmit power to send the signal over multiple frequency bands.
- Spread spectrum techniques are sometimes described as providing frequency diversity since the channel gain varies across the bandwidth of the transmitted signal.
- However, this is not equivalent to sending the **same** information signal over indepedently fading paths.

Time diversity

- Time diversity is achieved by transmitting the same signal at different times.
- Time diversity does not require increased transmit power, but it does decrease the data rate since data is repeated in the diversity time slots rather than sending new data in these time slots.
- Time diversity can also be achieved through coding and interleaving.
- Time diversity cannot be used for stationary wireless applications, since fading gains are highly correlated over time.

Maximal Ratio Combining (MRC)



Maximal Ratio Combining (cont.)

- In receiver diversity the independent fading paths associated with multiple receive antennas are combined to obtain a resultant signal that is then passed through a standard demodulator.
- Under the use of M receive antennas over flat-fading (single channel-tap, i.e., $L = 1$) channels, the received signals are

$$y_i = h_i x + n_i, \quad i = 1, ..., M \tag{1}$$

where $h_i = h_{i,R} + jh_{i,I} = a_i e^{j\theta_i}$ and $n_i \sim \mathcal{CN}(0, N_0)$.

- Weight each branch with $g_i e^{-j\theta_i}$: Co-phasing. If not co-phasing, then what happens ?
- Combine signals from these M receive antennas, one have

$$y = \sum_{i=1}^{M} g_i e^{-j\theta_i} y_i = \left(\sum_{i=1}^{M} g_i a_i \right) x + \sum_{i=1}^{M} g_i e^{-j\theta_i} n_i. \tag{2}$$

Maximal Ratio Combining (cont.)

- After combining the signals, the resultant SNR is

$$\text{SNR} = \frac{\left(\sum_{i=1}^{M} g_i a_i\right)^2}{N_0 \sum_{i=1}^{M} g_i^2}, \tag{3}$$

- One needs to find $\{g_i\}_{i=1}^{M}$ to maximize SNR ?.
- The solution to the simple optimization problem can be obtained by taking partial derivatives of (3) or using the Swartz inequality. In particular, the solution is

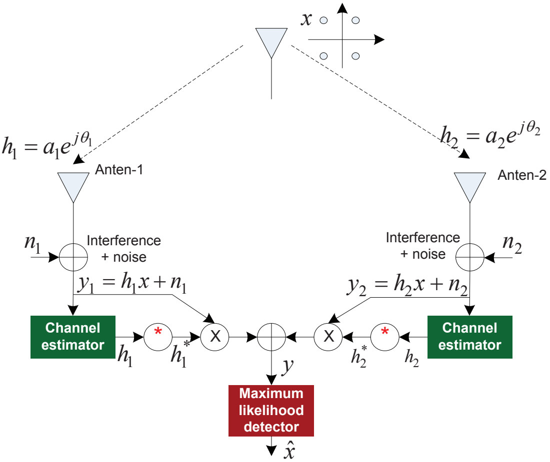
$$\boxed{g_r = a_i/\sqrt{N_0}} \tag{4}$$

and the resultant combined SNR γ_Σ is

$$\boxed{\gamma_\Sigma = \frac{\sum_{i=1}^{M} a_i^2}{N_0} = \sum_{i=1}^{M} \gamma_i}. \tag{5}$$

- The γ_Σ increases linearly with the number of diversity branches M.

Maximal Ratio Combining: An example of 2 Rx-antennas



MRC: Probability of error in symbol detection

- The detection performance of a diversity system, whether it uses space diversity or another form of diversity, in terms of probability of symbol error P_e for demodulation in AWGN with SNR γ_Σ can be determined by

$$\overline{P}_e = \int_0^\infty P_e(\gamma) p_{\gamma_\Sigma}(\gamma) d\gamma. \tag{6}$$

- We can obtain a simple upper bound on the average probability of error by applying the Chernoff bound $Q(x) \leq e^{-x^2/2}$ to the Q function.
- Recall that for static channel gains with MRC, we can approximate the probability of error as

$$P_e = \alpha_M Q\left(\sqrt{\beta_M \gamma_\Sigma}\right) \leq \alpha_M e^{-\beta_M \gamma_\Sigma/2} = \alpha_M e^{-\beta_M(\gamma_1+\ldots+\gamma_M)/2}. \tag{7}$$

MRC: Probability of error in symbol detection (cont.)

- Integrating over the chi-squared distribution for γ_Σ yields

$$\overline{P}_e \leq \alpha_M \prod_{i=1}^{M} \frac{1}{1+\beta_M \overline{\gamma}_i/2}. \tag{8}$$

- In the limit of high SNR and assuming that the γ_i's are identically distributed with $\overline{\gamma}_i = \overline{\gamma}$, one will have

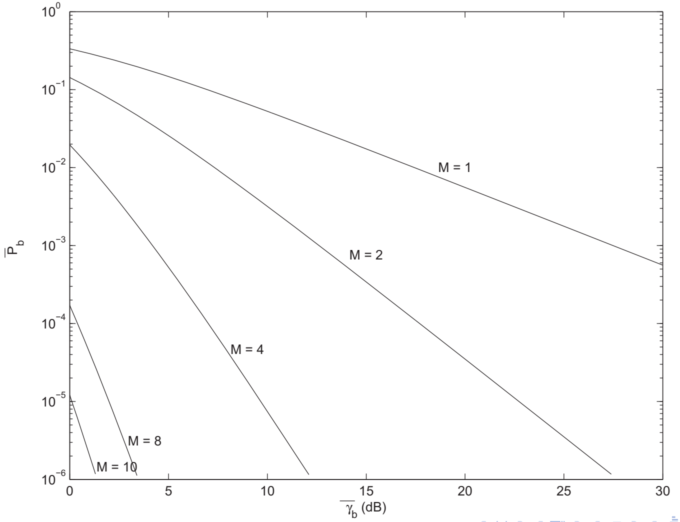
$$\overline{P}_e \approx \alpha_M \left(\frac{\beta_M \overline{\gamma}}{2}\right)^{-M}. \tag{9}$$

- The distribution of the combined SNR $p_{\gamma_\Sigma}(\gamma)$ leads to a decrease in $\overline{P}_e$ due to diversity combining.
- The resultant performance advantage is called the diversity gain.

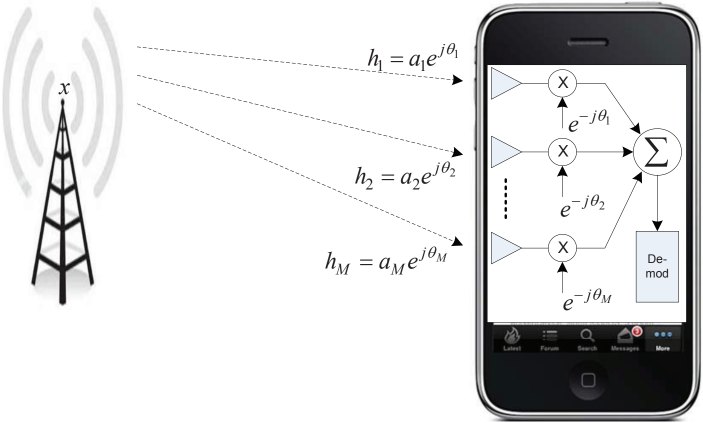
Diversity order

- For some diversity systems, their averaged probability of error can be expressed in the form
$$\boxed{\overline{P}_e = c\overline{\gamma}^{-M}}, \tag{10}$$
where c is a constant depending on the specific modulation and coding, $\overline{\gamma}$ is the averaged received SNR per branch and M is called the **diversity order** of the system.
- The diversity order indicates how the slope of the average probability of error as a function of averaged SNR changes with diversity.
- Recall that a general approximation for average error probability in Rayleigh fading with no diversity is $\overline{P}_e = \alpha_M/(2\beta_M\overline{\gamma})$. This expression has a **diversity order of one**, consistent with **a single receive antenna**.
- The maximum diversity order of a system with M antennas is M, and when the diversity order equals M the system is said to achieve **full diversity order**.

Diversity order: Numerical results of MRC



Equal-Gain Combining (EGC)

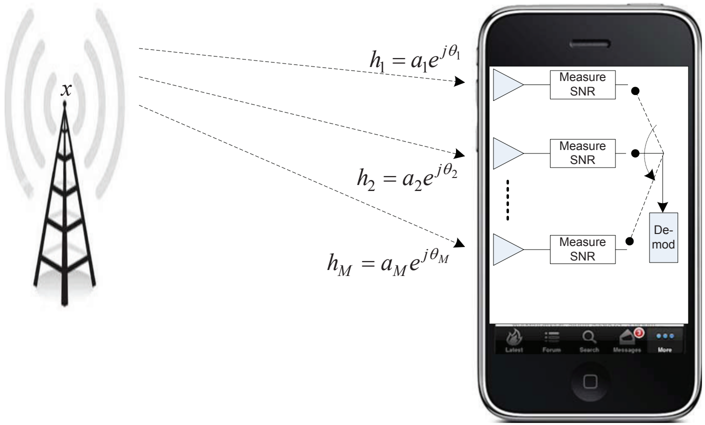


Equal-Gain Combining (cont.)

- MRC requires knowledge of the time-varying SNR on each branch, which can be very difficult to measure.
- A simpler technique is equal-gain combining, which co-phases the signals on each branch and then combines them with equal weighting, i.e., $g_i = e^{-j\theta_i}$.
- The SNR of the combiner output, assuming the same noise PSD N_0 in each branch, is then given by

$$\gamma_\Sigma = \frac{1}{N_0 M} \left(\sum_{i=1}^{M} |h_i| \right)^2 , \tag{11}$$

Selection combining (SC)



Selection combining (cont.)

- In **selection combining** (SC), the combiner outputs the signal on the branch with the **highest SNR**.
- Since only one branch is used at a time, SC often requires just one receiver that is switched into the active antenna branch.
- A dedicated receiver on each antenna branch may be needed for systems that transmit continuously in order to simultaneously and continuously monitor SNR on each branch.
- Since only one branch output is used, co-phasing of multiple branches is not required.
- As a result, this technique can be used with either coherent or differential modulation.

Selection combining (cont.)

- Under SC implementation, the pdf of γ_Σ is

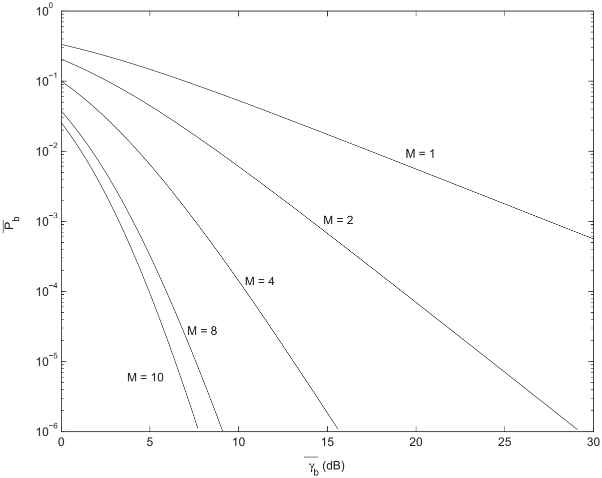
$$p_{\gamma_\Sigma}(\gamma) = \frac{M}{\gamma}\left(1 - e^{-\gamma/\overline{\gamma}}\right)^{M-1} e^{-\gamma/\overline{\gamma}} \qquad (12)$$

- As a result, the averaged output SNR of the combiner in Rayleigh fading is

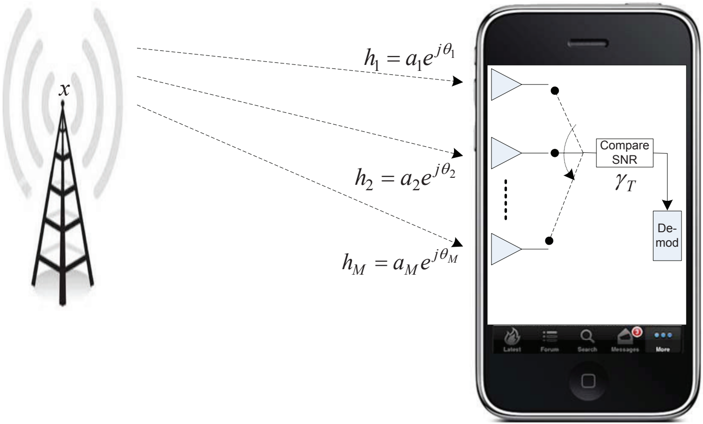
$$\boxed{\overline{\gamma}_\Sigma = \overline{\gamma}\sum_{i=1}^{M}\frac{1}{i}}. \qquad (13)$$

- The average SNR gain increases with M, but not linearly.
- The biggest gain is obtained by going from no diversity $(M = 1)$ to two-branch diversity $(M = 2)$.

SC: averaged probability of error in BPSK detection



Threshold Combining (TC)



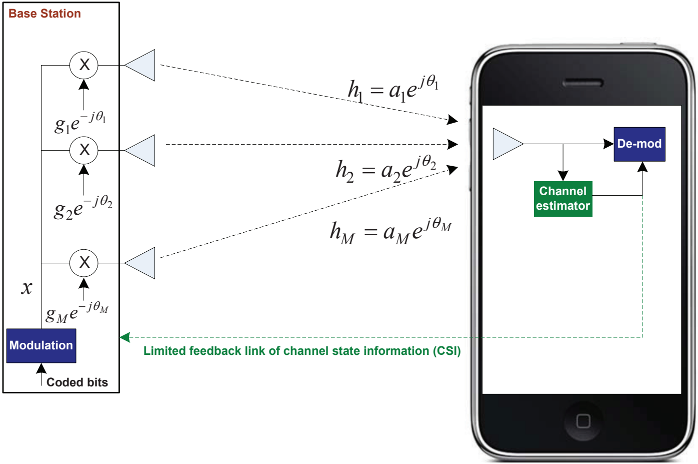
Threshold Combining (cont.)

- SC for wireless systems transmitting continuously may require a dedicated receiver on each branch to continuously monitor branch SNR.
- A simpler type of combining, called **threshold combining**, avoids the need for a dedicated receiver on each branch by **scanning** each of the branches in **sequential order** and outputting the **first** signal with SNR above a given threshold γ_T .
- As in SC, since only one branch output is used at a time, co-phasing is not required.
- Thus, this technique can be used with either coherent or differential (noncoherent) modulation.
- There are several criteria the combiner can use to decide which branch to switch to.
- The simplest criterion is to switch randomly to another branch.

Transmitter Diversity: Introduction

- In transmit diversity, there are multiple transmit antennas with the **transmit power divided among these antennas**.
- Transmit diversity is desirable in cellular systems where more space, power, and processing capability is available on the transmit side rather than the receive side.
- Transmit diversity design depends on whether or not the complex channel gain is **known** at the transmitter or **not**.
- When this gain is known, the system is very similar to receiver diversity.
- However, without this channel knowledge, transmit diversity gain requires a combination of space and time diversity via a novel technique called the Alamouti scheme.

Channel Known at Transmitter: Transmission model



Channel known at transmitter: detailed implementations

- Consider a transmit diversity system with M transmit antennas and one receive antenna.
- Assume the path gain associated with the ith transmit antenna given by $h_i = a_i e^{j\theta_i}$ is known at the transmitter via limited feedback links from mobile terminals.
- This is referred to as having channel side information (CSI) at the transmitter or CSIT.
- Let x denote the transmitted signal with total energy per symbol E_s
- This signal is multiplied by a complex gain $g_i e^{-j\theta_i}$, $0 \leq g_i \leq 1$ and sent through the ith transmit antenna.
- Due to the average total energy constraint E_s, the weights $g_i e^{-j\theta_i}$ must satisfy $\sum_{i=1}^{M} g_i^2 = 1$

Channel known at transmitter (cont.)

- The weighted signals transmitted over all antennas are added via signal superposition at the receive antenna, which leads to a received signal given by

$$y = \sum_{i=1}^{M} g_i a_i x + n, \quad n \sim \mathcal{CN}(0, N_0). \tag{14}$$

- One can obtain the weights g_i that achieve the maximum SNR:

$$\boxed{g_i = \frac{a_i}{\sqrt{\sum_{i=1}^{M} a_i^2}}}, \tag{15}$$

and the resultant SNR is

$$\gamma_\Sigma = \frac{E_s}{N_0} \sum_{i=1}^{M} a_i^2 = \sum_{i=1}^{M} \gamma_i, \tag{16}$$

where $\gamma_i = a_i^2 E_s / N_0$ equal to the branch SNR between the ith transmit antenna and the receive antenna.

- Thus, we see that transmit diversity when the channel gains are known at

Channel unknown at transmitter: the Alamouti scheme

- We now consider the same model as in the previous subsection but assume that the transmitter no longer knows the channel gains $h_i = a_i e^{j\theta_i}$, so there is no CSIT.
- In this case it is not obvious how to obtain diversity gain. Consider, for example, a naive strategy whereby for a two-antenna system we divide the transmit energy equally between the two antennas.
- Thus, the transmit signal on antenna i will be $x_i = \sqrt{.5}x$ where x is the transmit signal with energy per symbol E_s.
- Assume two antennas have complex Gaussian channel gains $\{h_i = a_i e^{j\theta_i}\}_{i=1}^{2}$ with zero-mean and unit variant $(N_0 = 1)$.
- The received signal is

$$y = \sqrt{.5}(h_1 + h_2)x + n. \qquad (17)$$

The Alamouti scheme (cont.)

- Note that $h_1 + h_2$ is the sum of two complex Gaussian random variables, and is thus a complex Gaussian as well with mean equal to the sum of means (zero) and variance equal to the sum of variances.
- Thus $\sqrt{.5}(h_1 + h_2)$ is a complex Gaussian random variable with **zero-mean** and **unit-variance** (1), so the received signal has the **same distribution as if we had just used one antenna** with the full energy per symbol.
- In other words, we have obtained **no performance advantage** from the two antennas, since we could not divide our energy intelligently between them or obtain coherent combining through co-phasing.
- Transmit diversity gain can be obtained even in the absence of channel information with an appropriate scheme to exploit the antennas.

The Alamouti scheme (cont.)

- A particularly simple and prevalent scheme for this diversity that combines both space and time diversity was developed by Alamouti.
- Alamouti's scheme is designed for a digital communication system with **two-antenna** transmit diversity.
- The scheme works over **two symbol periods** where it is assumed that the channel gain is constant over this time duration.
- Over the **first symbol period** two different symbols s_1 and s_2 each with energy $E_s/2$ are transmitted simultaneously from antennas 1 and 2, respectively.
- Over the **next symbol period**, symbol $-s_2^*$ is transmitted from antenna 1 and symbol s_1^* is transmitted from antenna 2, each with symbol energy $E_s/2$.

The Alamouti scheme (cont.)

- Assume complex channel gains $\{h_i = a_i e^{j\theta_i}\}_{i=1}^{2}$ between the ith transmit antenna and the receive antenna.
- The received symbol over the **first symbol period** is

$$y_1 = h_1 s_1 + h_2 s_2 + n_1, \tag{18}$$

and the received symbol over the **second symbol period** is

$$y_2 = -h_1 s_2^* + h_2 s_1^* + n_2, \tag{19}$$

where $\{n_i\}_{i=1}^{2}$ is the AWGN noise sample at the receiver associated with the ith symbol transmission. We assume the noise sample has zero-mean and power of N_0.

The Alamouti scheme (cont.)

- The receiver uses these sequentially received symbols to form the vector $\mathbf{y} = [y_1 \ y_2^*]^T$ given by

$$\mathbf{y} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} = \mathbf{H}_A \mathbf{s} + \mathbf{n}, \qquad (20)$$

where $\mathbf{H}_A = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$, $\mathbf{s} = [s_1 \ s_2]^T$ and $\mathbf{n} = [n_1 \ n_2]^T$.

- Let us define the new vector $\mathbf{z} = \mathbf{H}_A^H \mathbf{y}$. The structure of $\mathbf{H}_A$ implies that

$$\mathbf{H}_A^H \mathbf{H}_A = \left(|h_1|^2 + |h_2|^2\right) \mathbf{I}_2 \qquad (21)$$

is diagonal and thus

$$\mathbf{z} = [z_1 \ z_2]^T = \left(|h_1|^2 + |h_2|^2\right) \mathbf{I}_2 \mathbf{s} + \widetilde{\mathbf{n}}, \qquad (22)$$

where $\widetilde{\mathbf{n}} = \mathbf{H}_A^H \mathbf{n}$ is a complex Gaussian noise vector with mean zero and covariance matrix $E\left(\widetilde{\mathbf{n}}\widetilde{\mathbf{n}}^H\right) = \left(|h_1|^2 + |h_2|^2\right) \mathbf{I}_2 N_0$.

The Alamouti scheme (cont.)

- The diagonal nature of $\mathbf{z}$ effectively decouples the two symbol transmissions, so that each component of $\mathbf{z}$ corresponds to one of the transmitted symbols:

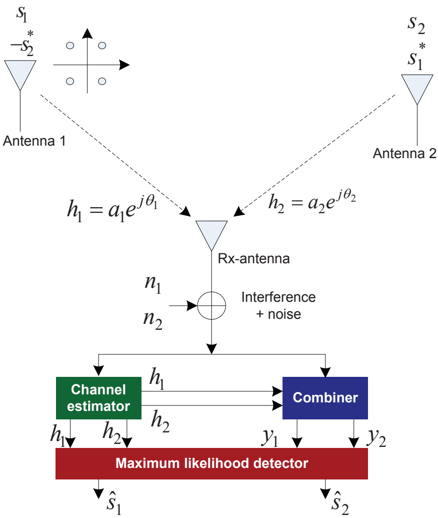
$$\boxed{z_i = \left(|h_1|^2 + |h_2|^2\right) s_i + \widetilde{n}_i}, \quad i = 1, 2. \tag{23}$$

- The received SNR thus corresponds to the SNR for z_i given by

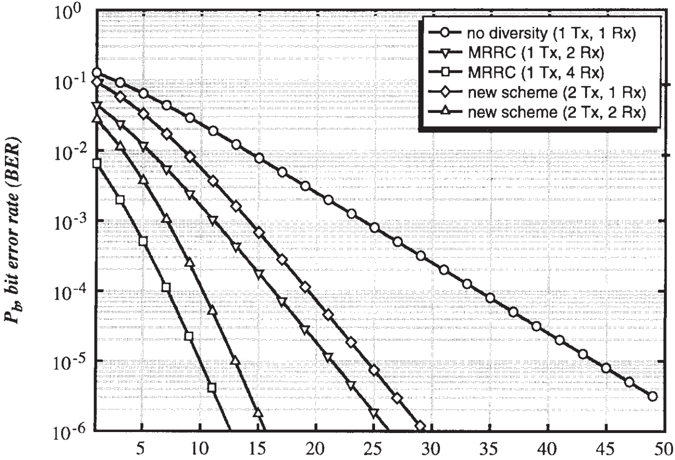
$$\gamma_i = \frac{\left(|h_1|^2 + |h_2|^2\right) E_s}{2N_0}, \tag{24}$$

  where the factor of 2 comes from the fact that s_i is transmitted using half the total symbol energy E_s.
- The received SNR is thus equal to the sum of SNRs on each branch, identical to the case of transmit diversity with MRC assuming that the channel gains are known at the transmitter.
- Thus, the Alamouti scheme achieves a **diversity order of 2**, the maximum possible for a two-antenna transmit system, despite the fact that channel knowledge is not available at the transmitter.

The Alamouti scheme: An example of 2 Tx-antennas



The Alamouti scheme: BER results of BPSK



Possible problems to be considered in theses

- In Alamouti's scheme, wireless channels are assumed to be flat- and block-fading.
- Doubly selective channels can be considered in Alamouti's scheme by using OFDM and BEMs.
- The study results can be employed in LTE downlink transmissions with mobile terminals.