Chapter 3: Physical-layer transmission techniques

Section 3.4: **Orthogonal Frequency Division Multiplexing (OFDM) Fundamentals**

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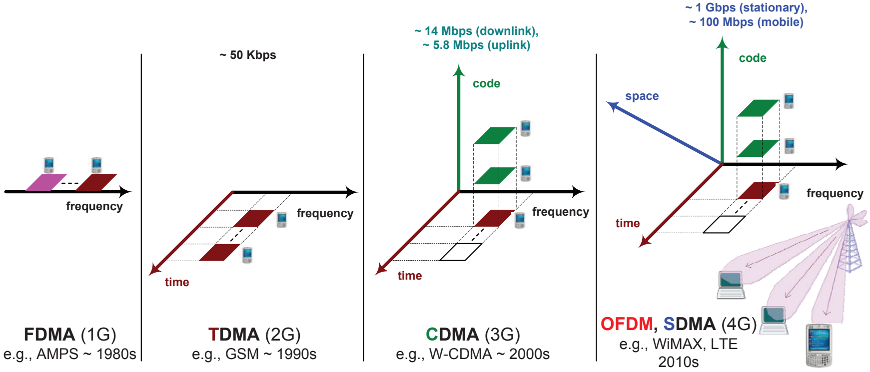
1. Introduction
   - Development of mobile communications systems - Revisit
   - Mobile broadband technology evolution - Revisit

2. Fundamentals of OFDM transmissions
   - Functional blocks in OFDM transmitter and receiver
   - Transmitted OFDM signals
   - Time-variant multipath channels
   - Received OFDM signals
   - MQAM symbol detection in the frequency domain

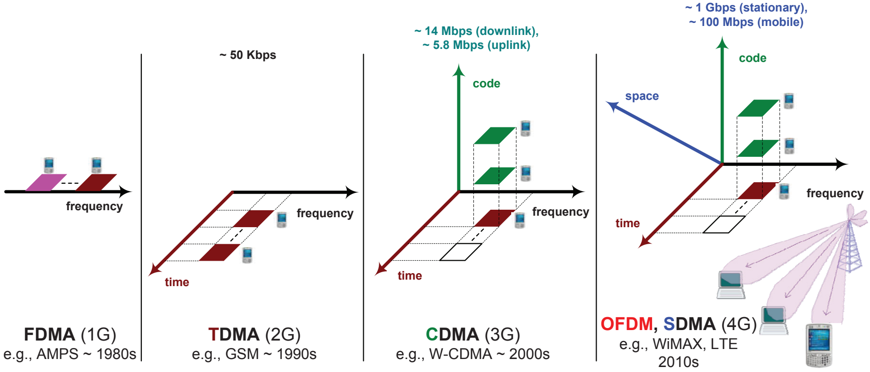
3. Possible research problems in OFDM transmissions
   - Carrier frequency offset (CFO) estimation
   - Doubly selective channel estimation

Development of mobile communications systems - Revisit



Orthogonal multiple access is still exploited in 5G!

Development of mobile communications systems - Revisit



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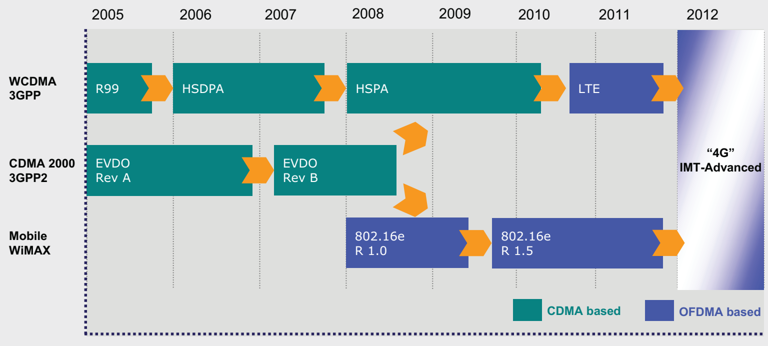
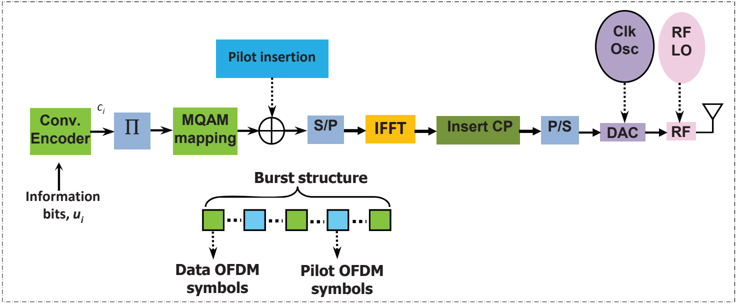
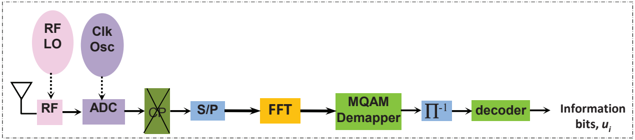
Mobile broadband technology evolution - Revisit



The last decade has witnessed numerous intensive studies in employing orthogonal frequency division multiplexing (OFDM) for the emerging broadband communications systems (e.g., WiFi, WiMAX, LTE) to exploit its high spectral efficiency and robustness against multipath (frequency selective) fading channels.

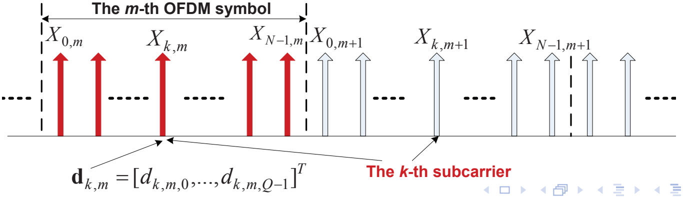
Functional blocks in OFDM transmitter and receiver





Transmitted OFDM signals

- Consider a coded OFDM system using M-ary quadrature amplitude modulation (M-QAM).
- The encoded bit stream is bit-interleaved. Then, the resulting sequence of interleaved bits is organized as a sequence of Q-bit tuples $\{\mathbf{d}_{k,m}\}$ where $Q = \log_2 M$ and $\mathbf{d}_{k,m} = [d_{k,m,0}, \ldots, d_{k,m,Q-1}]^T$.
- The sequence is further mapped to a complex-valued symbol $X_{k,m} \in \mathbb{A}$ where $\mathbb{A}$ is the M-ary modulation signaling alphabet, and m and k denote the indices of OFDM symbol and subcarrier, respectively.



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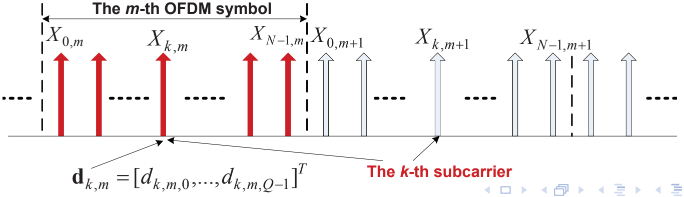
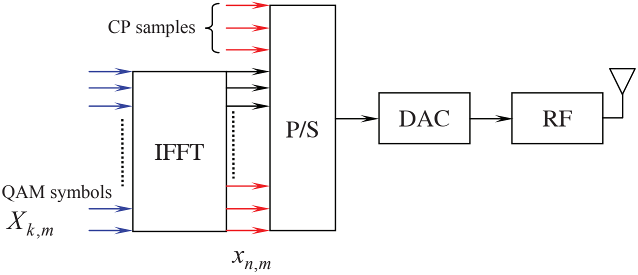
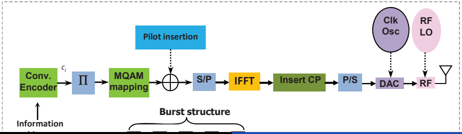


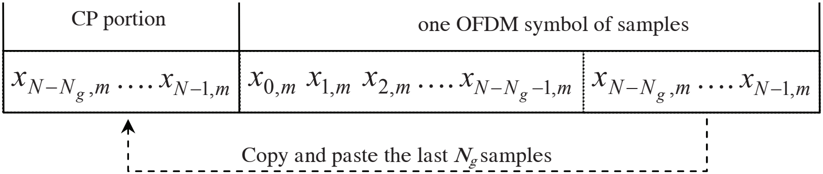
Transmitted OFDM signals (cont.)

Each OFDM symbol consists of N information bearing subcarriers $X_{k,m}$, $k = 0, ..., N-1$, where N is the size of the fast Fourier transform (FFT) and inverse-FFT (IFFT) used in the multicarrier transmission.





Transmitted OFDM signals (cont.)

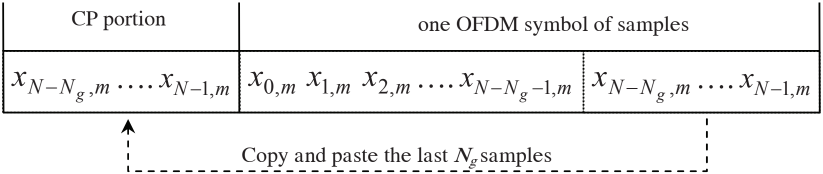


After inverse FFT (IFFT) and cyclic prefix (CP) insertion, the transmitted baseband signal of the mth OFDM symbol can be written as

$$x_{n,m} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_{k,m} \exp\left(\frac{j2\pi kn}{N}\right), \tag{1}$$

where $n \in \{-N_g, ..., 0, ..., N-1\}$, N_g denotes the CP length.

Transmitted OFDM signals (cont.)



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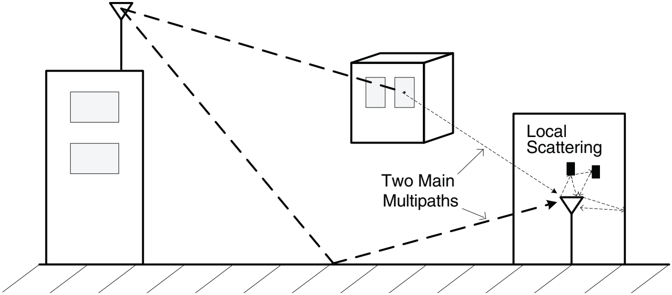
Time-variant multipath channels



$$\boxed{y_m = \sum_l x_{m-l} h_{l,m} + w_b(m/W)} \tag{2}$$

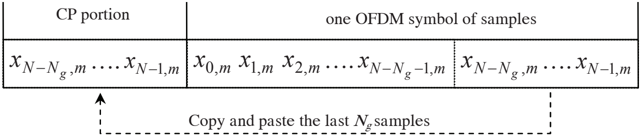
The lth (time-variant) channel tap gain that includes the effect of transmit-receive filters and time-variant channel propagation is denoted by $h_{l,n,m}$ where n and m stand for the indices of the time-domain sample and OFDM symbol, respectively.

Time-variant multipath channels (cont.)

- In the time-variant multipath channels, a number of basis expansion models (BEMs) can be employed for reducing the number of channel parameters while capturing the time-variation of the channels.
- Using BEMs, $h_{l,n,m}$ (**after CP removal**) can be approximated by

$$h_{l,n,m} = \sum_{q=1}^{Q} b_{n+N_g+mN_s,q}\, c_{q,l}, \quad l \in \{0, ..., L-1\}, \tag{3}$$

where $N_s = N + N_g$ denotes the OFDM symbol length after CP insertion, $n = 0, ..., N-1$, $m = 0, ..., M-1$ and M is the number of both data and pilot OFDM symbols in a burst. $b_{n+N_g+mN_s,q}$ stand for the qth basis function values of the used BEM.



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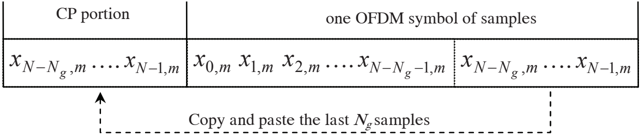

Time-variant multipath channels (cont.)

- It is noted that the use of BEMs offers a significant dimension reduction in the time-variant channel representation, i.e., $Q \ll NM$.
- For instance, in the current LTE system settings, three LTE time slots contain 21 OFDM symbols with 128-FFT (the smallest FFT size used in the LTE settings) and the resulting number of the time-variant channel parameters corresponding to one channel tap gain $h_{l,n,m}$ in (3) will be $NM = 128 \times 21 = 2688$.
- By using the discrete prolate spheroidal (DPS)-BEM as shown in (3), the number of the basis functions Q can vary from 3 (for low user speeds, e.g., about 10km/h) to 5 (for moderate user speeds, e.g., about 100km/h) under the required MSE of the DPS-BEM-based channel approximation below 10^{-10} as shown in Fig. 1.

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Numerical results of BEM-based channel fitting

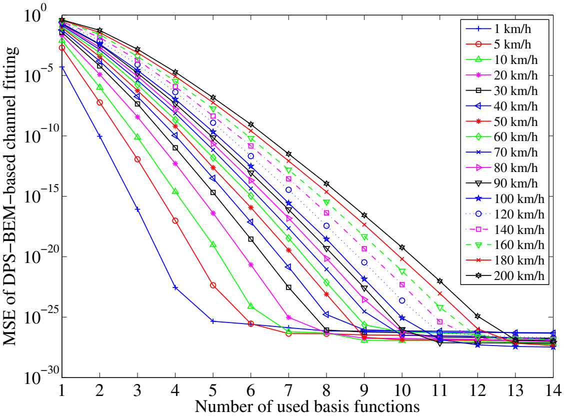


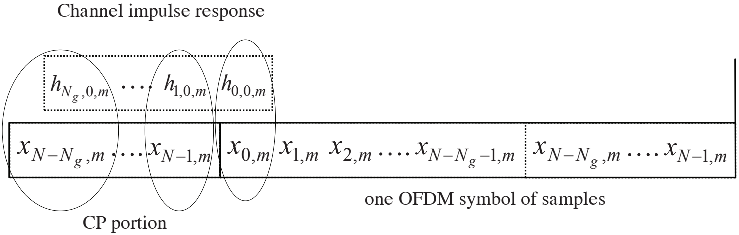
Figure 1: Normalized MSE of DPS-BEM-based approximation (fitting) of time-varying channels generated by Jakes model under different mobile speeds.

Received OFDM signals

Over the above doubly-selective channels, after CP removal, the nth received sample in the mth OFDM symbol can be represented by

$$y_{n,m} = \sum_{l=0}^{L-1} h_{l,n,m} x_{n-l,m} + z_{n,m}, \tag{4}$$

where $n = 0, ..., N-1$ and $z_{n,m}$ is the additive white Gaussian noise (AWGN) with variance N_o.



$$y_{0,m} = h_{N_g,0,m} x_{N-N_g,m} + \cdots + h_{1,0,m} x_{N-1,m} + h_{0,0,m} x_{0,m}$$

Received OFDM signals in the frequency domain

- At the receiver, after performing CP removal and FFT, the received samples in the frequency domain can be determined by

$$Y_{k,m} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y_{n,m} e^{-j2\pi kn/N} = H_{k,m} X_{k,m} + \rho_{k,m} + Z_{k,m}, \quad (5)$$

where $H_{k,m} = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{L-1} h_{l,n,m} e^{-j2\pi kl/N}$, $Z_{k,m} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} z_{n,m} e^{\left(-j\frac{2\pi kn}{N}\right)}$ and $\rho_{k,m} = \frac{1}{N} \sum_{i=0, i\neq k}^{N-1} X_{i,m} \sum_{n=0}^{N-1} \left(\sum_{l=0}^{L-1} h_{l,n,m} e^{-j2\pi il/N} \right) e^{j2\pi n(i-k)/N}$.

- It is noted that the received samples in the **time domain** are:

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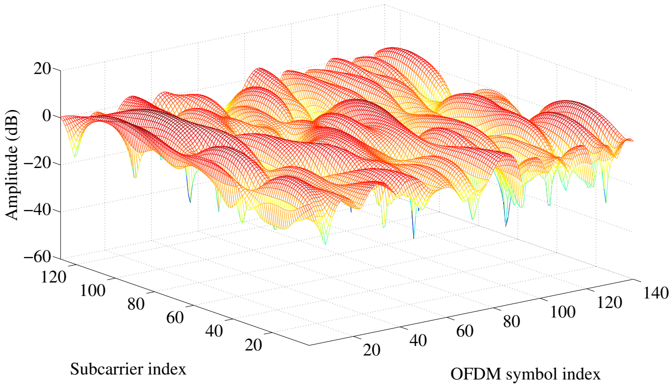
An example of channel frequency response in time- and frequency-domains



Received OFDM signals over **block-fading** channel

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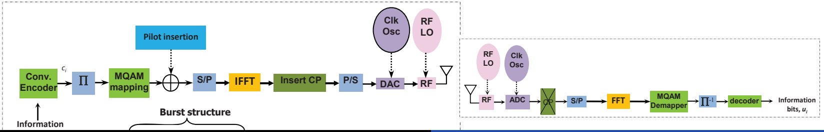

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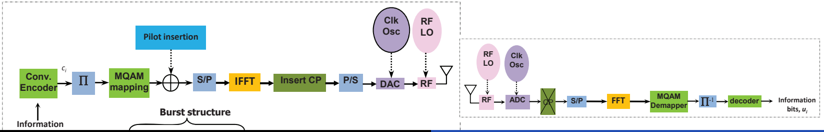

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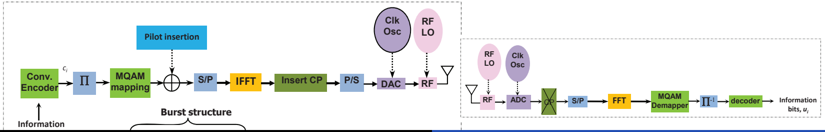

MQAM symbol detection in the frequency domain

- Under maximum likelihood (ML) detection, the transmitted QAM symbols can be recovered in the frequency domain by

$$\widehat{X}_{k,m} = \arg\min_{X_{k,m} \in \mathbb{A}} \left| Y_{k,m} - H_{k,m} X_{k,m} \right|^2 . \quad (7)$$

- Based on the detected QAM symbols $\widehat{X}_{k,m}$, the transmitted data bits can be recovered accordingly by channel decoders.
- It is noted that the ML symbol detection process (7) needs to know channel frequency response (CFR) $H_{k,m}$ before providing a detected version of the transmitted symbol $\widehat{X}_{k,m}$

Possible research problem

In OFDM-based transmissions (e.g., WiFi, WiMAX, LTE systems) over time-variant multipath channels, the problem of estimating channel responses (i.e., $H_{k,m}$) is of importance in research and industry as well.

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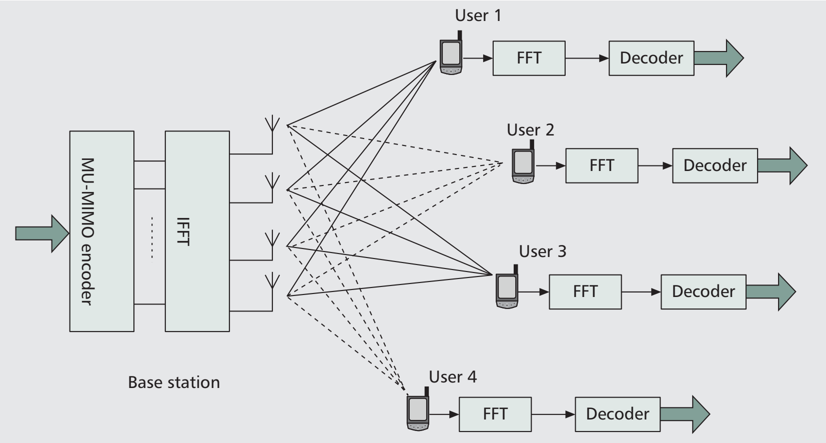
Downlink multiuser MIMO-OFDM transmissions in WiMAX systems (IEEE 802.16e)



Carrier frequency offset (CFO) estimation

- In the presence of CFO, after CP removal, the nth received sample in the mth OFDM symbol can be represented by

$$y_{n,m} = e^{j\frac{2\pi\varepsilon}{N}(n+N_g+mN_s)} \sum_{l=0}^{L-1} h_{l,n,m} x_{n-l,m} + z_{n,m}, \tag{8}$$

where:

  - $n = 0, ..., N-1$
  - Δf and $\varepsilon = \Delta f NT$ denote the absolute and normalized CFOs, respectively.
  - T is the sampling period of the system.
- As observed in (8), the presence of CFO introduces a time-domain phase rotation that will translate into ICI in the frequency domain.
- In addition, the time-variation of the multipath channels also induces ICI in the frequency domain.

Carrier frequency offset (CFO) estimation (cont.)

- Consequently, the presence of both CFO and doubly selective channels would incur a significant ICI power at OFDM receivers, giving rise to a considerable irreducible error floor in the receiver performance.
- For CFO compensation and reliable coherent data detection/decoding, the CFO and CIR estimates are indispensible at OFDM receivers.
- To obtain the estimates, CFO and channel estimates can be obtained either jointly or separately by existing estimation techniques (e.g., RLS, ML, MAP,...).

Doubly selective channel estimation: Introduction

- The last decade has witnessed numerous intensive studies in employing OFDM for broadband communication systems to exploit its high spectral efficiency and robustness against multipath (frequency-selective) fading channels.
- In the current literature, most of these studies have assumed frequency-selective channels to be time-invariant (i.e., quasi-static or block-fading) within a transmission burst.
- This channel assumption can be used in a wireless system with stationary and/or low-speed users.

Doubly selective channel estimation: Introduction (cont.)

- In a wireless network with rapidly moving nodes (e.g., users in cars and trains in 4G-LTE systems), the resultant time-selectivity of the channel impulse response (CIR) introduces a large number of channel parameters (much greater than that of quasi-static/block-fading channels).
- In addition, the time-variation of the channel leads to a loss of subcarrier orthogonality, resulting in inter-carrier interference (ICI) in OFDM receivers.
- Under such a scenario, the assumption of quasi-static fading channels becomes inappropriate.
- As a result, time- and frequency-selective (doubly selective) channels should be considered in the wireless system investigation and analysis.