A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 2EA

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Problem

Solve the following linear congruences:

(*a*) $12x \equiv 7 \pmod{25}$

(*b*) $35x \equiv 8 \pmod{12}$

(*c*) $15x \equiv 9 \pmod{6}$

(*d*) $42x \equiv 12 \pmod{30}$

(*e*) $147x \equiv 49 \pmod{98}$

(*f*) $39x \equiv 26 \pmod{52}$

Step-by-step solution

Step 1 of 10

(a)

Consider the congruence equation

$12x \equiv 7(\bmod 25)$

Use the result, if $\gcd(a,n)=1$ then $ax \equiv b(\bmod n)$ has a solution modulo n , to solve the given equation.

The congruence equation $12x \equiv 7(\bmod 25)$ has a solution modulo 25 because

$\gcd(12,25)=1$

The congruence equation $12x \equiv 7(\bmod 25)$is equivalent to $\overline{12x} = \overline{7}$ in Z_{25}.

$$\bar{x} = \left(\overline{12}\right)^{-1} \bar{7} \text{ in } Z_{25}$$

$$\bar{x} = \left(\overline{23}\right) \bar{7} \text{ in } Z_{25}$$

$$\bar{x} = \overline{11} \text{ in } Z_{25}$$

Therefore, the solution of the congruence equation $12x \equiv 7 \pmod{25}$ is $\boxed{x \equiv 11 \pmod{25}}$.

Comment

Step 2 of 10

(b)

Consider the congruence equation

$$35x \equiv 8 \pmod{12}$$

Use the result, if $\gcd(a,n) = 1$ then $ax \equiv b \pmod{n}$ has a solution modulo n , to solve the given equation.

The congruence equation $35x \equiv 8 \pmod{12}$ has a solution modulo 12 because

$$\gcd(35,12) = 1.$$

The congruence equation $35x \equiv 8 \pmod{12}$ is equivalent to $\overline{35x} = \bar{8}$ in Z_{12}.

$$\bar{x} = \left(\overline{35}\right)^{-1} \bar{8} \text{ in } Z_{12}$$

$$\bar{x} = \left(\overline{11}\right)^{-1} \bar{8} \text{ in } Z_{12}$$

$$\bar{x} = \overline{118} \text{ in } Z_{12}$$

$$\bar{x} = \bar{4} \text{ in } Z_{12}$$

Therefore, the solution of the congruence equation $35x \equiv 8 \pmod{12}$ is $\boxed{x \equiv 4 \pmod{12}}$.

Comment

Step 3 of 10

(c)

Consider the congruence equation

$$15x \equiv 9 \pmod{6}$$

Use the result, the congruence $ax \equiv b \pmod{n}$ has a solution if and only if $\gcd(a,n) \mid b$ to solve the given equation.

The congruence equation $15x \equiv 9 \pmod{6}$ has a solution modulo 6 because

$$\gcd(15,9) = 3 \text{ and } 3 \mid 9.$$

Comment

Step 4 of 10

The solution of congruence equation $15x \equiv 9 \pmod{6}$ is same as the solution of

$5x \equiv 3 \pmod{2}$ $\left(\text{since } \frac{15}{3}x \equiv \frac{9}{3}\left(\bmod \frac{6}{3}\right)\right)$.

The congruence equation $5x \equiv 3 \pmod{2}$ is equivalent to $\overline{5x} = \overline{3}$ in Z_2.

$\overline{x} = \left(\overline{5}\right)^{-1}\overline{3}$ in Z_2

$\overline{x} = \overline{33}$ in Z_2

$\overline{x} = \overline{1}$ in Z_2

Therefore, the solution of the congruence equation $15x \equiv 9 \pmod{6}$ is $\boxed{x \equiv 1 \pmod{2}}$.

Comment

Step 5 of 10

(d)

Consider the congruence equation

$42x \equiv 12 \pmod{30}$

Use the result, the congruence $ax \equiv b \pmod{n}$ has a solution if and only if $\gcd(a,n) \mid b$ to solve the given equation.

The congruence equation $42x \equiv 12 \pmod{30}$ has a solution modulo 30 because

$\gcd(42,30) = 6$ and $6 \mid 12$.

Comment

Step 6 of 10

The solution of congruence equation $42x \equiv 12 \pmod{30}$ is same as the solution of

$7x \equiv 2 \pmod{5}$ $\left(\text{since } \frac{42}{6}x \equiv \frac{12}{6}\left(\bmod \frac{30}{6}\right)\right)$.

The congruence equation $7x \equiv 2 \pmod{5}$ is equivalent to $\overline{7x} = \overline{2}$ in Z_5.

$\overline{x} = \left(\overline{7}\right)^{-1}\overline{2}$ in Z_5

$\overline{x} = \overline{32}$ in Z_5

$\overline{x} = \overline{1}$ in Z_5

Therefore, the solution of the congruence equation $42x \equiv 12 \pmod{30}$ is $\boxed{x \equiv 1 \pmod 5}$.

Comment

Step 7 of 10

(e)

Consider the congruence equation

$147x \equiv 49 \pmod{98}$

Use the result, the congruence $ax \equiv b \pmod n$ has a solution if and only if $\gcd(a,n) \mid b$ to solve the given equation.

The congruence equation $147x \equiv 49 \pmod{98}$ has a solution modulo 98 because

$\gcd(147,98) = 49 \text{ and } 49 \mid 49$.

The solution of congruence equation $147x \equiv 49 \pmod{98}$ is same as the solution of

$3x \equiv 1 \pmod 2 \left(\text{since } \frac{147}{49} x \equiv \frac{49}{49} \left(\bmod \frac{98}{49} \right) \right)$.

Comment

Step 8 of 10

The congruence equation $3x \equiv 1 \pmod 2$ is equivalent to $\overline{3}\overline{x} = \overline{1} \text{ in } Z_2$.

$\overline{x} = \left(\overline{3}\right)^{-1} \overline{1} \text{ in } Z_2$

$\overline{x} = \left(\overline{1}\right)^{-1} \overline{1} \text{ in } Z_2$

$\overline{x} = \left(\overline{1}\right) \overline{1} \text{ in } Z_2$

$\overline{x} = \overline{1} \text{ in } Z_2$

Therefore, the solution of the congruence equation $147x \equiv 49 \pmod{98}$ is $\boxed{x \equiv 1 \pmod 2}$.

Comment

Step 9 of 10

(f)

Consider the congruence equation

$39x \equiv 26 \pmod{52}$

Use the result, the congruence $ax \equiv b \pmod n$ has a solution if and only if $\gcd(a,n) \mid b$ to solve the given equation.

The congruence equation $39x \equiv 26 \pmod{52}$ has a solution modulo 52 because

$\gcd(39,52) = 13$ and $13 \mid 26$.

The solution of congruence equation $39x \equiv 26 \pmod{52}$ is same as the solution of

$3x \equiv 2 \pmod{4}\left(\text{since } \frac{39}{13}x \equiv \frac{26}{13}\left(\bmod \frac{52}{13}\right)\right)$.

Comment

Step 10 of 10

The congruence equation $3x \equiv 2 \pmod{4}$ is equivalent to $\overline{3}\overline{x} = \overline{2}$ in Z_4.

$\overline{x} = \left(\overline{3}\right)^{-1}\overline{2}$ in Z_4

$\overline{x} = \left(\overline{3}\right)\overline{2}$ in Z_4

$\overline{x} = \overline{2}$ in Z_4

Therefore, the solution of the congruence equation $39x \equiv 26 \pmod{52}$ is $\boxed{x \equiv 2 \pmod{4}}$.

Comment