

A Book of Abstract Algebra | (2nd Edition)

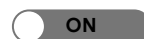


Chapter 16, Problem 3EI



Bookmark

Show all steps:



Problem

Let H and K be normal subgroups of a group G , with $H \subseteq K$. Define $\phi: G/H \rightarrow G/K$ by $\phi(Ha) = Ka$.
Prove part:
 ϕ is surjective.

Step-by-step solution

Step 1 of 3

Suppose that G is any group and let H, K are normal subgroups of G with $H \subseteq K$.

Consider a mapping $\phi: G/H \rightarrow G/K$ defined by

$$\phi(Ha) = Ka, \text{ for all } a \in G.$$

Objective is to prove that function ϕ is surjective, that is, onto.

A mapping $f: G \rightarrow H$ is said to be surjective if for every $y \in H$ there exists $x \in G$ such that $f(x) = y$.

[Comment](#)

Step 2 of 3

Assume that $Kx \in G/K$, for some $x \in G$. An element x is the member of G because of the coset of G/K . So from there it implies that

$$Hx \in G/H$$

such that

$$\phi(Hx) = Kx.$$

This satisfies the definition of surjective map.

[Comment](#)

Step 3 of 3

Hence, function ϕ is surjective.

[Comment](#)