A Book of Abstract Algebra | (2nd Edition)

Chapter AB, Problem 12E

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Problem

Prove that the following are true for any integers a, b, and c:

If lcm(*a*, *b*) = *c*, then gcd(*a*, *b*) = 1.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove *if* $lcm(a,b)=ab, then \gcd(a,b)=1$.

Comment

Step 2 of 2

Proof:-

Let us consider the theorem.

Theorem:-If *p* and *q* are two integers with greatest common divisor $\gcd(p,q)$ and least common multiple $lcm(p,q)$, then

$$p \times q = \gcd(p,q) \times lcm(p,q)$$

Let us suppose $lcm(a,c) = ab.$ Then by above theorem:-

$$\cancel{a \times b} = \cancel{ab} \times \gcd(a,b)$$

$$1 = \gcd(a,b)$$

$$\gcd(a,b) = 1$$

Proved

Comment