

A Book of Abstract Algebra | (2nd Edition)

Chapter AC, Problem 1E

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Problem

$$+ 3 + 5 + \cdots + (2n - 1) = n^2$$

(That is, the sum of the first n odd integers is equal to n^2 .)

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ using mathematical induction.

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Step 2 of 2

Proof:-

$$P(n) = 1 + 3 + 5 + \cdots + (2n - 1) = n^2$$

Let consider rule for $n = 1$.

$$P(1) = 1$$

$$P(1) = 1^2$$

Thus, this rule is true for $n = 1$.

Let this rule is true for $n = k$.

$$P(k) = 1 + 3 + 5 + \cdots + (2k - 1) = k^2 \quad \text{.....(1)}$$

Let consider rule for $n = k + 1$.

$$P(k + 1) = 1 + 3 + 5 + \cdots + (2k - 1) + (2(k + 1) - 1)$$

$$P(k + 1) = 1 + 3 + 5 + \cdots + (2k - 1) + (2k + 2 - 1)$$

$$P(k + 1) = 1 + 3 + 5 + \cdots + (2k - 1) + (2k + 1)$$

Use the equation (1).

$$P(k+1) = k^2 + (2k+1)$$

$$P(k+1) = k^2 + 2k + 1^2$$

$$P(k+1) = (k+1)^2$$

This rule also true for $n = k + 1$. Hence, by mathematical induction this rule is true for all positive integer n .

Proved

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