

# A Book of Abstract Algebra | (2nd Edition)

Chapter 27, Problem 2EC

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Problem

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Let  $p(x)$  be an irreducible polynomial of degree  $n$  over  $F$ . Let  $c$  denote a root of  $p(x)$  in some extension of  $F$  (as in the basic theorem on field extensions).  
  
If  $s(c) = t(c)$  in  $F(c)$ , where  $s(x)$  and  $t(x)$  have degree  $< n$ , prove that  $s(x) = t(x)$ .

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Step-by-step solution

Step 1 of 3 ^

Consider any arbitrary field  $F$ . Suppose  $s(x), t(x) \in F[x]$  are the polynomials having degree  $< n$ . Objective is to prove that if  $s(c) = t(c)$  in some extension  $F(c)$  of  $F$ , then  
$$s(x) = t(x).$$
Consider the following result:  
  
If any polynomial  $a(x)$  has degree  $n$ , it has at most  $n$  roots.

Comment

Step 2 of 3 ^

It is known that,  $s(c) = t(c)$  in  $F(c)$ . That is,  
$$s(c) - t(c) = 0 \text{ in } F(c).$$
Let  $p(x) = s(x) - t(x)$ . Then, the degree of this polynomial will be  $< n$  for sure, because  $\deg s(x), \deg t(x) < n$ . According to the result, the roots of  $p(x)$  will be less than  $n$  in  $F$ . But  
$$p(c) = 0$$
in  $F(c)$  ( $n$ -values). It implies that,  $p(x) = 0$  for all values of  $x$  in extension field  $F(c)$ . Thus,  
$$s(c) = t(c)$$
 for all values of  $c$ .

Comments (2)

Step 3 of 3 ^

Hence,  $s(x) = t(x)$ .

Comment

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