A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 2EB

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Problem

Find the root field of $x^2 + x + 2$ over $\mathbb{Z}_3$, and write its addition and multiplication tables.

Step-by-step solution

Step 1 of 4

The objective is to find the root field of x^2+x+2 over $\mathbb{Z}_3$, and write its addition and multiplication tables.

Comment

Step 2 of 4

The polynomial x^2+x+2 is of degree 2 and the root field is of at most degree 2 extension of $\mathbb{Z}_3$.

Since x^2+x+2 is an irreducible polynomial over $\mathbb{Z}_3$, therefore, the root field is of degree 2 extension and hence, the root field is of size 3^2, that is, 9.

Let a be a root of the polynomial.

Then $a^2+a+2=0$.

$$\begin{aligned} a^2 &= -a-2 \\ &= 2a+1 \end{aligned}$$

By hit and trial, it is found that 0, 1, 2, a, $2a$, $1+a$, $2+a$, $1+2a$, and $2+2a$ are the nine elements of $\mathbb{Z}_3(a)$.

Therefore, $\mathbb{Z}_3(0,1,2,a,2a,1+a,2+a,1+2a,2+2a)$ is the root field of x^2+x+2 over $\mathbb{Z}_3$.

Comment

Step 3 of 4

The addition table is as follows:

+	0	1	2	a	$2a$	$1+a$	$2+a$	$1+2a$	$2+2a$
0	0	1	2	a	$2a$	$1+a$	$2+a$	$1+2a$	$2+2a$
1	1	2	0	$1+a$	$1+2a$	$2+a$	a	$2+2a$	$2a$
2	2	0	1	$2+a$	$2+2a$	a	$1+a$	$2a$	$1+2a$
a	a	$1+a$	$2+a$	$2a$	0	$1+2a$	$2+2a$	1	2
$2a$	$2a$	$1+2a$	$2+2a$	0	a	1	2	$1+a$	$2+a$
$1+a$	$1+a$	$2+a$	a	$1+2a$	1	$2+2a$	$2a$	2	0
$2+a$	$2+a$	a	$1+a$	$2+2a$	2	$2a$	$1+2a$	0	1
$1+2a$	$1+2a$	$2+2a$	$2a$	1	$1+a$	2	0	$2+a$	a
$2+2a$	$2+2a$	$2a$	$1+2a$	2	$1+2a$	0	1	a	$1+a$

Comment

Step 4 of 4

The multiplication table is as follows:

×	0	1	2	a	$2a$	$1+a$	$2+a$	$1+2a$	$2+2a$
0	0	0	0	0	0	0	0	0	0
1	0	1	2	a	$2a$	$1+a$	$2+a$	$1+2a$	$2+2a$
2	0	2	1	$2a$	a	$2+2a$	$1+2a$	$2+a$	$1+a$

a	0	a	$2a$	$1+2a$	$2+a$	1	$1+a$	$2+2a$	2
$2a$	0	$2a$	a	$2+a$	$1+2a$	2	$2+2a$	$1+a$	1
$1+a$	0	$1+a$	$2+2a$	1	2	$2+a$	$2a$	a	$1+2a$
$2+a$	0	$2+a$	$1+2a$	$1+a$	$2+2a$	$2a$	2	1	a
$1+2a$	0	$1+2a$	$2+a$	$2+2a$	$1+a$	a	1	2	$2a$
$2+2a$	0	$2+2a$	$1+a$	2	1	$1+2a$	a	$2a$	$2+a$

Comment