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Chapter 17, Problem 1EK

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Problem

is the only invertible element in *A*.

Step-by-step solution

Step 1 of 2

A Boolean ring is a ring in which every element is an idempotent element. That is, if *A* is Boolean ring then $a^2 = a$ for every $a \in A$.

Objective is to show that in any Boolean ring *A*, $a = -a$ for every $a \in A$.

Since $a^2 = a$ for every $a \in A$, therefore

$$a + a = (a+a)^2$$
$$= a^2 + a \cdot a + a \cdot a + a^2$$
$$= a^2 + a^2 + a^2 + a^2$$
$$= a + a + a + a.$$

Since *A* is a ring, therefore additive inverse of *a* will exist. Thus,

$a + a = 0$ for every $a \in A$.

It implies that $a = -a$.

Comment

Step 2 of 2

Hence, in any Boolean ring $a = -a$ for every $a \in A$.

Comment