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Chapter AC, Problem 4E

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Problem

Use mathematical induction to prove the following:

$1^3 + 2^3 + \cdots + n^3 = \frac{1}{4} n(n+1)^2$

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $1^3+2^3+\cdots+n^3=\frac{1}{4}n^2(n+1)^2$ using mathematical induction.

Comment

Step 2 of 2

Proof:-

$$p(n):1^3+2^3+\cdots+n^3=\frac{1}{4}n^2(n+1)^2$$

Let consider rule for $n=1.$

$$p(1):1^3=\frac{1}{4}\cdot 1^2(1+1)^2$$

$$p(1):1=\frac{1}{\cancel{4}}\cdot\cancel{4}$$

$$p(1):1=1$$

This rule is true for $n=1.$

Let this statement is true for $n=k.$

$$p(k):1^3+2^3+\cdots+k^3=\left(k\left(\frac{k+1}{2}\right)\right)^2 \qquad \cdots\cdots(1)$$

Let consider rule for $n=k+1$.

$$P(k+1):1^3+2^3+\cdots+k^3+(k+1)^3$$

Use the equation (1).

$$P(k+1):1^3+2^3+\cdots+k^3+(k+1)^3$$

$$P(k+1):\frac{1}{4}k^2(k+1)^2+(k+1)^3 \qquad \{from\ \ equation\ (1)\}$$

$$P(k+1):\ (k+1)^2\left(\frac{k^2}{4}+k+1\right)$$

$$P(k+1):(k+1)^2\left(\frac{k^2+4k+4}{4}\right)$$

$$P(k+1):(k+1)^2\frac{1}{4}(k+2)^2$$

$$P(k+1):\frac{1}{4}(k+1)^2((k+1)+1)^2$$

This result also true for $n=k+1$. Hence, by mathematical induction this rule is true for all positive integer *n*.

Proved

Comment