

A Book of Abstract Algebra | (2nd Edition)

Chapter 30, Problem 2EF

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Problem

By de Moivre's theorem,

$$\omega = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$$

is a complex seventh root of unity. Since

$$x^7 - 1 = (x - 1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$$

ω is a root of $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$.

Prove that

$$8 \cos^3 \frac{2\pi}{7} + 4 \cos^2 \frac{2\pi}{7} - 4 \cos \frac{2\pi}{7} - 1 = 0$$

(Use part 1 and Exercise E1.) Conclude that $\cos(2\pi/7)$ is a root of $8x^3 + 4x^2 - 4x - 1$.

Step-by-step solution

Step 1 of 5

Here, objective is to prove that $8\cos^3\frac{2\pi}{7} + 4\cos^2\frac{2\pi}{7} - 4\cos\frac{2\pi}{7} - 1 = 0$ and $\cos\frac{2\pi}{7}$ is a root of $8x^3 + 4x^2 - 4x - 1 = 0$

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Step 2 of 5

De Moivre's theorem:

$\omega = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$ is a complex seventh root of unity.

Since $x^7 - 1 = (x - 1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$

ω is a root of $P(x) = (x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$

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Step 3 of 5

Consider $\omega = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$

$$\frac{1}{\omega} = \cos\frac{2\pi}{7} - i\sin\frac{2\pi}{7}$$

$$\left(\omega + \frac{1}{\omega}\right) = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7} + \cos\frac{2\pi}{7} - i\sin\frac{2\pi}{7}$$

$$\left(\omega + \frac{1}{\omega}\right) = 2\cos\frac{2\pi}{7}$$

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Step 4 of 5

Consider ω is a root of $P(x) = (x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$

Then, $P(\omega) = 0$

$$(\omega^4 + \omega^3 + \omega^2 + \omega + 1) = 0$$

$$\omega^2(\omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2}) = 0$$

$$\omega^2 = 0 \text{ or}$$

$$\omega^3 + \omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2} + \omega^{-3} = 0$$

$$\left(\omega + \frac{1}{\omega}\right)^3 + \left(\omega + \frac{1}{\omega}\right)^2 - 2\left(\omega + \frac{1}{\omega}\right) - 1 = 0$$

$$\left(2\cos\frac{2\pi}{7}\right)^3 + \left(2\cos\frac{2\pi}{7}\right)^2 - 2\left(2\cos\frac{2\pi}{7}\right) - 1 = 0$$

$$8\cos^3\frac{2\pi}{7} + 4\cos^2\frac{2\pi}{7} - 4\cos\frac{2\pi}{7} - 1 = 0$$

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Step 5 of 5

put $x = \cos\frac{2\pi}{7}$ in above equation, then

$$8x^3 + 4x^2 - 4x - 1 = 0$$

$$\text{Hence, } 8\cos^3\frac{2\pi}{7} + 4\cos^2\frac{2\pi}{7} - 4\cos\frac{2\pi}{7} - 1 = 0$$

$$\text{and } \cos\frac{2\pi}{7} \text{ is a root of } 8x^3 + 4x^2 - 4x - 1 = 0$$

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