# A Book of Abstract Algebra | (2nd Edition)

Chapter 24, Problem 3EG

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## Problem

Let *A* and *B* be rings and let $h : A \to B$ be a homomorphism with kernel *K*. Define $\bar{h} : A[x] \to B[x]$ by

$$\bar{h}(a_0 + a_1x + \cdots + a_nx^n) = h(a_0) + h(a_1)x + \cdots + h(a_n)x^n$$

(We say that $\bar{h}$ is *induced by h*.)

Prove that $\bar{h}$ is surjective iff *h* is surjective.

## Step-by-step solution

**Step 1** of 2

Step 1 of 2

**To prove:** if $\bar{h}$ is surjective $\Rightarrow h$ is surjective

Let $\bar{h}$ is surjective then every element in $b(x) \in B[x]$ there exist an element $a(x) \in A[x]$ such that $\bar{h}(a(x)) = b(x)$

$a(x) = a_0 + a_1x + \ldots\ldots + a_nx^n$ and

$b(x) = b_0 + b_1x + \ldots\ldots + b_nx^n$

$\Rightarrow \bar{h}(a_0 + a_1x + \ldots\ldots + a_nx^n) = b_0 + b_1x + \ldots\ldots + b_nx^n$

$\Rightarrow h(a_0) + h(a_1)x + \ldots + h(a_n)x^n = b_0 + b_1x + \ldots\ldots + b_nx^n$

$\Rightarrow h(a_0) = b_0, h(a_1) = b_1, \ldots, h(a_n) = b_n$

$b_0, b_1, \ldots b_n \in B$ and are arbitrary

$\Rightarrow$ For every element in $b \in B$ there exist a element $a \in A$ such that $h(a) = b$

$\Rightarrow$ $h$ is surjective

Comment

**Step 2** of 2

Step 2 of 2

**To prove:** if $h$ is surjective $\Rightarrow \bar{h}$ is surjective

Let $h$ is surjective then for every element in $b \in B$ there exist a element $a \in A$ such that $h(a) = b$. Suppose $b(x) = b_0 + b_1 x + ...... + b_n x^n, b(x) \in B[x]$ and $b_0, b_1, ... b_n \in B$ then there exist $a_0, a_1, ... a_n \in A$ such that $h(a_0) = b_0, h(a_1) = b_1, ..., h(a_n) = b_n$ so

$$\begin{aligned} b(x) &= h(a_0) + h(a_1)x + ... + h(a_n)x^n \\ &= \bar{h}(a_0 + a_1 x + ... + a_n x^n) \\ &= \bar{h}(a(x)) \end{aligned}$$

$\Rightarrow$ $\bar{h}$ is surjective

Comment