

A Book of Abstract Algebra | (2nd Edition)



Chapter 30, Problem 1EC

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Problem

An angle α is called *constructible* iff there exist constructible points A , B , and C such that $\angle ABC = \alpha$.

Prove the following:

The angle α is constructible iff $\sin \alpha$ and $\cos \alpha$ are constructible numbers.

Step-by-step solution

Step 1 of 4

Here, objective is to prove that the angle α is constructible, if and only if $\sin \alpha$ and $\cos \alpha$ are constructible numbers.

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Step 2 of 4

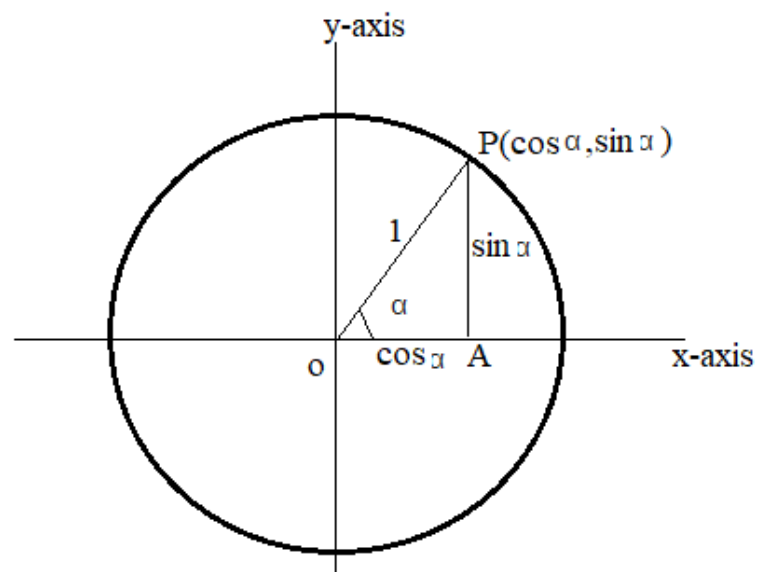
Constructible point is the end point of given unit segment or it is intersection of two lines determined by constructional points.

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Step 3 of 4

To prove $\sin \alpha$ and $\cos \alpha$ are constructible numbers, if the angle α is constructible.

Consider the below figure:



figureConstructionof α

Let us assume, the angle α is constructible

That is line segment OP is making an angle α with the x-axis and the point P is on the circumference of unit circle.

$\triangle OAP$ is a right angle triangle

$$\cos \alpha = \frac{OA}{OP}$$

$$\cos \alpha = OA \quad (\because OP=1)$$

$$\sin \alpha = \frac{AP}{OP}$$

$$\sin \alpha = AP \quad (\because OP=1)$$

Therefore,

$\sin \alpha$ and $\cos \alpha$ are constructible numbers.

Hence, proved

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Step 4 of 4

To prove, if $\sin \alpha$ and $\cos \alpha$ are constructible numbers, the angle α is constructible.

Consider

$$\cos \alpha = OA$$

$$\sin \alpha = AP$$

Then, join the line segment OP

$$OP^2 = OA^2 + AP^2$$

$$OP^2 = \cos^2 \alpha + \sin^2 \alpha$$

$$OP^2 = 1$$

Then, the line segment OP is making an angle α with the x-axis.

Therefore, the angle α is constructible, if and only if $\sin \alpha$ and $\cos \alpha$ are constructible numbers.

Hence,

The angle α is constructible, if and only if $\sin \alpha$ and $\cos \alpha$ are constructible numbers.

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