

A Book of Abstract Algebra | (2nd Edition)

Chapter 32, Problem 3EA

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Problem

List the elements of $Gal(\mathbb{Q}(i, \sqrt{2}) : \mathbb{Q})$ and exhibit its table.

Step-by-step solution

Step 1 of 2

The objective is to list the elements of $Gal(\mathbb{Q}(i, \sqrt{2}) : \mathbb{Q})$ and exhibit its table.

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Step 2 of 2

The root field $\mathbb{Q}(i, \sqrt{2})$ is of degree 4 over \mathbb{Q} .

Therefore, there are four automorphism of $\mathbb{Q}(i, \sqrt{2})$ which fix \mathbb{Q} , since the number of automorphism is equal to the degree of $\mathbb{Q}(i, \sqrt{2})$ over \mathbb{Q} .

Since an automorphism is determined by its effect on $\sqrt{2}$ and i , there are four possibilities, namely,

$$\epsilon: \begin{cases} \sqrt{2} \mapsto \sqrt{2} \\ i \mapsto i \end{cases} \quad \alpha: \begin{cases} \sqrt{2} \mapsto -\sqrt{2} \\ i \mapsto i \end{cases} \quad \beta: \begin{cases} \sqrt{2} \mapsto \sqrt{2} \\ i \mapsto -i \end{cases} \quad \gamma: \begin{cases} \sqrt{2} \mapsto -\sqrt{2} \\ i \mapsto -i \end{cases}.$$

Thus, the Galois group of $\mathbb{Q}(i, \sqrt{2})$ over \mathbb{Q} is $Gal(\mathbb{Q}(i, \sqrt{2}) : \mathbb{Q}) = \{\epsilon, \alpha, \beta, \gamma\}$.

The operation is composition, giving the table:

\circ	ε	α	β	γ
ε	ε	α	β	γ
α	α	ε	γ	β
β	β	γ	ε	α
γ	γ	β	α	ε

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