# A Book of Abstract Algebra | (2nd Edition)

Chapter 33, Problem 4EA

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## Problem

Show that $ax^8 + bx^6 + cx^4 + dx^2 + e$ is solvable by radicals over any field. (HINT: Let $y = x$; use the fact that every fourth-degree polynomial is solvable by radicals.)

## Step-by-step solution

**Step 1** of 3

Here, objective is to prove that the given polynomial is solvable by radicals over any field.

Comment

**Step 2** of 3

A polynomial equation is solvable by radicals, if its roots are determined by applying finite number of additions, subtractions, multiplications, divisions, $n^{th}$ roots to the integers.

Galois Theory:

If the polynomial of degree is greater than or equal to $4$ are solvable by radicals.

Comment

**Step 3** of 3

Consider the polynomial $a(x) = ax^8 + bx^6 + +cx^4 + dx^2 + e$

Let $y = x^2$

Then, the equation becomes,

$$ax^8 + bx^6 + +cx^4 + dx^2 + e$$
$$= ay^4 + by^3 + +cy^2 + dy + e$$

The above polynomial is of degree $4$

Every polynomial of degree four is solvable by radicals.

So,

$ay^4 + by^3 + +cy^2 + dy + e$ is solvable by radicals.

Therefore, the polynomial $a(x) = ax^8 + bx^6 + +cx^4 + dx^2 + e$ is solvable by radicals over any field.

Hence, proved.

Comment