A Book of Abstract Algebra | (2nd Edition)

Chapter 28, Problem 1ED

Problem

Let V be a finite-dimensional vector space. Let dim V designate the dimension of V. Prove each of the following:

If U is a subspace of V, then dim $U \leq$ dim V.

Step-by-step solution

Step 1 of 3

By definition of subspace, it is known that subspace is some subset of any vector space which itself is a vector space or follows properties of subspace.

Step 2 of 3

Dimension of a subspace is a measure of how large it is. It can also be thought of as maximum numbers of independent vectors in a subspace.

Step 3 of 3

Here V is a given vector space and U is a subspace of V. As largest possible subset is vector space itself, we find that,

$$\max(\dim U) = \dim V$$

Other than that trivial subset of any subset is null set or just 0 vector.

Here, $\dim U = 0$

All other subsets are between *V* and null-sets. Thus dimension of all other subspaces – which are subsets with special properties will be between null-space and *V*.

$\boxed{\text{Hence } \dim U \le \dim V}$

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