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Chapter 28, Problem 2ED

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Problem

Let V be a finite-dimensional vector space. Let dim V designate the dimension of V. Prove each of the following:

If U is a subspace of V, and dim U = dim V, then $U = V$.

Step-by-step solution

Step 1 of 3

By definition of subspace, it is known that subspace is some subset of any vector space which itself is a vector space or follows properties of subspace.

Comment

Step 2 of 3

Dimension of a subspace is a measure of how large it is. It can also be thought of as maximum numbers of independent vectors in a subspace.

Comment

Step 3 of 3

Here V is a given vector space and U is a subspace of V. Let vector space has dimension n, then there is set of maximum n possible linearly independent vectors. These vectors are said to span V.

Now consider any subspace *U*. If $\dim U = \dim V$, then set of *n* independent vectors span *U*. But we also know that any set of *n* independent vectors span *V*. This is only possible if *U* is coincident with *V*.

$\boxed{\text{Hence } \dim U = \dim V}$

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