A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 5EA

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Problem

Solve the following congruences:

(a) $x^4 \equiv 4 \pmod 6$

(b) $2(x-1)^4 \equiv 0 \pmod 8$

(c) $x^3 + 3x^2 + 3x + 1 = 0 \pmod 8$

(d) $x^4 + 2x^2 + 1 \equiv 4 \pmod 5$

Step-by-step solution

Step 1 of 8

Here, objective is to solve the given congruence's .

Comment

Step 2 of 8

Consider the congruent equation $ax = b(\bmod n)$, has solutions if and only if $\gcd(a,n)$ is divisible by b. If $\gcd(a,n) = 1$, then the congruence has unique solution

Comment

Step 3 of 8

(a)

Consider the congruence $60x = 12 \pmod{24}$

$a = 60, b = 12, n = 24$
$\gcd(60, 24) = 6$

$\gcd(a,n)$ is divisible by b. Since, the congruence has solutions.

$60x = 12 \pmod{24}$
$60x = 12 + 24q$
$5x = 1 + 2q$
$5x = 1 \pmod{2}$

$a = 5, b = 1, n = 2$
$\gcd(5, 2) = 1$

Hence, the value of *m* for which the congruence equation has a unique solution $= 2$

Comment

Step 4 of 8

(b)

Consider the congruence $42x = 24 \pmod{30}$

$a = 42, b = 24, n = 30$
$\gcd(42, 30) = 6$

$\gcd(a,n)$ is divisible by b. Since, the congruence has solutions.

$42x = 24 \pmod{30}$
$42x = 24 + 30q$
$7x = 4 + 5q$
$7x = 4 \pmod{5}$

$a = 7, b = 4, m = 5$
$\gcd(7, 5) = 1$

Hence, the value of *m* for which the congruence equation has a unique solution $= 5$

Comment

Step 5 of 8

(c)

Consider the congruence $49x = 30 \pmod{25}$

$a = 49, b = 30, n = 25$
$\gcd(49, 25) = 1$

Hence, the value of *m* for which the congruence equation has a unique solution $= 25$

Comment

Step 6 of 8

(d)

Consider the congruence $39x = 14(\mathrm{mod}\ 52)$

$$a = 39, b = 14, n = 52$$
$$\gcd(39,52) = 13$$

$\gcd(a,n)$ is not divisible by b

Hence, the congruence has no solutions.

Comment

Step 7 of 8

(e)

Consider the congruence $147x = 47(\mathrm{mod}\ 98)$

$$a = 147, b = 47, n = 98$$
$$\gcd(147,98) = 49$$

$\gcd(a,n)$ is not divisible by b

Hence, the congruence has no solutions.

Comment

Step 8 of 8

(f)

Consider the congruence $39x = 26(\mathrm{mod}\ 52)$

$$a = 39, b = 26, n = 52$$
$$\gcd(39,52) = 13$$

$\gcd(a,n)$ is divisible by b .Since, the congruence has solutions.

$$39x = 26(\mathrm{mod}\ 52)$$
$$3x = 2 + 4q$$
$$3x = 2(\mathrm{mod}\,4)$$

$$a = 3, b = 2, m = 4$$
$$\gcd(3,4) = 1$$

Hence, the value of *m* for which the congruence equation has a unique solution $= 4$

Comment