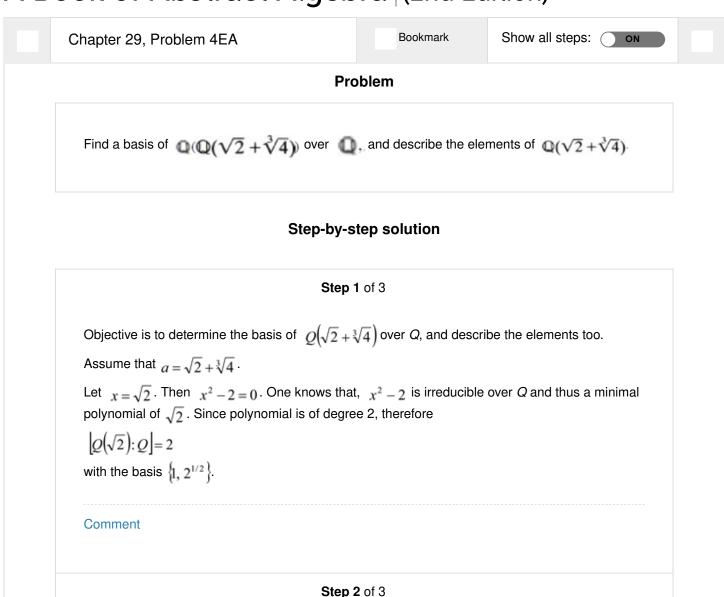
## A Book of Abstract Algebra (2nd Edition)



Let  $y = \sqrt[3]{4}$ . Then  $y^3 - 4 = 0$ . Substitute y = y + 1 and get,

$$(y+1)^3-4=0$$

$$y^3 + 1 + 3y^2 + 3y - 4 = 0$$

$$y^3 + 3y^2 + 3y - 3 = 0.$$

The last polynomial is irreducible by Eisenstein's criterion. Thus,  $y^3 - 4$  is a minimal polynomial of  $\sqrt[3]{4}$ . Since polynomial is of degree 3, therefore

$$\left[Q(\sqrt[3]{4}):Q\right]=3$$

The basis for this will be:

$$\{1, 4^{1/3}, 4^{2/3}\}$$
, or  $\{1, 2^{2/3}, 2^{4/3}\}$ .

Comments (1)

## **Step 3** of 3

Also  $a = \sqrt{2} + \sqrt[3]{4}$ , it implies that  $\sqrt{2}$ ,  $\sqrt[3]{4} \in Q(a)$ . Next, in  $Q(\sqrt{2}, \sqrt[3]{4})$ , a satisfies  $a = \sqrt{2} + \sqrt[3]{4}$ . So,  $Q(\sqrt{2}, \sqrt[3]{4}) = Q(a)$ .

Then, the required basis, with the help of theorem, will be:

$$\{1, 2^{1/2}, 2^{2/3}, 2^{4/3}, 2^{2/3} \cdot 2^{1/2}, 2^{4/3} \cdot 2^{1/2}\}$$

And the elements of Q(a) will be of the form:

$$Q(a) = \left\{ p + q \cdot 2^{1/2} + r \cdot 2^{2/3} + s \cdot 2^{4/3} + t \cdot 2^{2/3} \cdot 2^{1/2} + u \cdot 2^{4/3} \cdot 2^{1/2} : p, q, r, s, t, u \in Q \right\}$$

Comment