A Book of Abstract Algebra | (2nd Edition)

Chapter 32, Problem 2EH

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Problem

Prove: Any automorphism of $\mathbb{R}$ sends squares of numbers to squares of numbers, hence positive numbers to positive numbers.

Step-by-step solution

Step 1 of 2

The objective is to prove that any automorphism of $\mathbb{R}$ sends squares of numbers to squares of numbers, hence positive numbers to positive numbers.

Comment

Step 2 of 2

Let $s \in \mathbb{R}$ and σ be an automorphism of $\mathbb{R}$.

Then s^2 is a square in $\mathbb{R}$.

Since σ is a homomorphism, $\sigma(s^2) = \sigma(s)\sigma(s)$, a square in $\mathbb{R}$.

Now, let $p \in \mathbb{R}$ be a positive real number.

$\sigma(p) = \sigma(\sqrt{p}\sqrt{p})$, since $\sqrt{p} \in \mathbb{R}$

$= \sigma(\sqrt{p})\sigma(\sqrt{p})$, a square which is in particular a positive real number.

Hence, any automorphism of $\mathbb{R}$ sends squares of numbers to squares of numbers, hence positive numbers to positive numbers.

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