A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 1EA

Bookmark

Show all steps: ON

Problem

Show that $\mathbb{Q}(\sqrt{3}, i)$ is the root field of $(x^2 - 2x - 2)(x^2 + 1)$ over $\mathbb{Q}$.

Comparing part 1 with the example, we note that different polynomials may have the same root field. This is true even if the polynomials are irreducible.

Step-by-step solution

Step 1 of 2

The objective is to show that $\mathbb{Q}(\sqrt{3}, i)$ is the root field of $(x^2 - 2x - 2)(x^2 + 1)$ over $\mathbb{Q}$.

Comment

Step 2 of 2

Let $a(x) = (x^2 - 2x - 2)(x^2 + 1)$.

Use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find roots of $x^2 - 2x - 2$.

Here, $a=1$, $b=-2$, and $c=-2$.

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4\cdot1\cdot(-2)}}{2\cdot1}$$
$$=\frac{2\pm\sqrt{12}}{2}$$
$$=1\pm\sqrt{3}$$

The roots of x^2+1 are $\pm i$.

The roots of $a(x)=(x^2-2x-2)(x^2+1)$ are $1\pm\sqrt{3}$, $\pm i$.

Therefore, the root field is $\mathbb{Q}(1\pm\sqrt{3},\pm i)$. This can be written simplify as $\mathbb{Q}(\sqrt{3},i)$.

Comment