

A Book of Abstract Algebra | (2nd Edition)



Chapter 30, Problem 2EE



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Problem

We will show that $2\pi/5$ is a constructible angle, and it will follow that the regular pentagon is constructible.

Prove that $\omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2} = 0$.

Step-by-step solution

Step 1 of 3

Here, objective is to prove that $\omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2} = 0$.

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Step 2 of 3

De Moivre's theorem:

$\omega = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$ is a complex fifth root of unity.

Since $x^5 - 1 = (x - 1)(x^4 + x^3 + x^2 + x + 1)$

ω is a root of $P(x) = (x^4 + x^3 + x^2 + x + 1)$

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Step 3 of 3

Consider ω is a root of $P(x) = (x^4 + x^3 + x^2 + x + 1)$

Then,

$$P(\omega) = 0$$

$$(\omega^4 + \omega^3 + \omega^2 + \omega + 1) = 0$$

$$\omega^2 \cdot \omega^2 + \omega^2 \cdot \omega + \omega^2 \cdot 1 + \omega^2 \cdot \omega^{-1} + \omega^2 \cdot \omega^{-2} = 0$$

$$\omega^2(\omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2}) = 0$$

$$\omega^2 = 0 \text{ or}$$

$$\omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2} = 0$$

Hence, proved

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