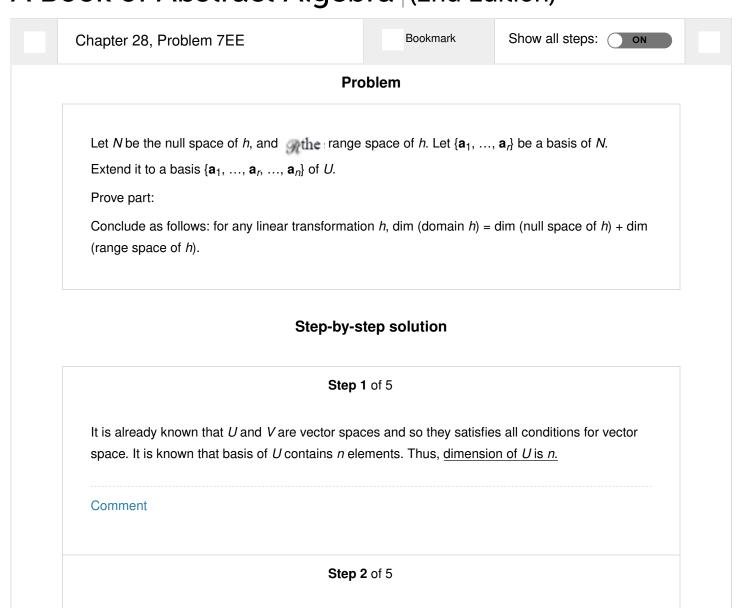
A Book of Abstract Algebra (2nd Edition)



Range space of h is subspace of V is set of all elements of V which are map of vectors of U. Comment **Step 3** of 5 Or given subspace is $\{\mathbf{r} \in V \mid h(\mathbf{u}) = \mathbf{r} \text{ for } \mathbf{u} \in U\}$ Comment **Step 4** of 5 Thus any element in range is a map of some vector in *U* Comment **Step 5** of 5 For any element r in range of h, we can find a element u in U such that $h(\mathbf{u}) = \mathbf{r}$ Since *U* is a vector space, every element in *U* can be expressed as linear combination of basis of U. So, $\mathbf{u} = t_1 \mathbf{a}_1 + t_2 \mathbf{a}_2 + ... t_r \mathbf{a}_r + ... + t_n \mathbf{a}_n$ Taking linear transformation $h(\mathbf{u}) = h(t_1\mathbf{a}_1 + t_2\mathbf{a}_2 + ...t_r\mathbf{a}_r + ... + t_n\mathbf{a}_n)$

...(1)

 $\Rightarrow h(\mathbf{u}) = t_1 h(\mathbf{a}_1) + t_2 h(\mathbf{a}_2) + \dots + t_r h(\mathbf{a}_r) + \dots + t_n h(\mathbf{a}_n)$ Since $(\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_r)$ is null basis of h,

$$h(\mathbf{a}_r) = \mathbf{0} \ \forall r \in (0, 1, ..., r)$$

Also number of elements in basis of *h* is *r*. Thus, dimension of null-space is *r*.

Therefore (1) can be rewritten as,

$$h(\mathbf{u}) = t_{r+1}h(\mathbf{a}_{r+1}) + \dots + t_nh(\mathbf{a}_n)$$

 $h(\mathbf{u})$ represents a subspace, all element of which can be expressed as linear combinations of $h(\mathbf{a}_{r+1}),...,h(\mathbf{a}_n)$. In other words $h(\mathbf{a}_{r+1}),...,h(\mathbf{a}_n)$ forms basis of range of h. Dimension of subspace is number of elements is basis of a subspace. Here range subspace have n-relements in its basis. In other words dimension of range space is (n-r)

Hence dim(domain of h) =
$$n = r + n - r = \dim(\text{nullspace of } h) + \dim(\text{range space of } h)$$

Comment