

A Book of Abstract Algebra | (2nd Edition)

Chapter 30, Problem 3EF

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Problem

By de Moivre's theorem,

$$\omega = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$$

is a complex seventh root of unity. Since

$$x^7 - 1 = (x - 1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$$

ω is a root of $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$.

Prove that $8x^3 + 4x^2 - 4x - 1$ has no rational roots. Conclude that it is irreducible over \mathbb{Q} .

Step-by-step solution

Step 1 of 4

Here, objective is to prove that $8x^3 + 4x^2 - 4x - 1$ has no rational roots.

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Step 2 of 4

Rational root theorem:

Consider the polynomial $a_n x^n + a_{n-1} x^{n-1} + \dots + a_0 = 0$, there is a rational solution and that could be determined by checking all the numbers $= \pm \frac{\text{dividers of } a_0}{\text{dividers of } a_n}$

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Step 3 of 4

Consider the polynomial $8x^3 + 4x^2 - 4x - 1$

$$a_n = 8, a_0 = -1$$

$$\text{Rational numbers} = \pm \frac{1}{1, 2, 4, 8}$$

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Step 4 of 4

Validate all the rational roots:

For $x = 1$

$$\begin{aligned} 8x^3 + 4x^2 - 4x - 1 \\ = 8 + 4 - 4 - 1 \\ = 7 \\ \neq 0 \end{aligned}$$

For $x = -1$

$$\begin{aligned} 8x^3 + 4x^2 - 4x - 1 = -1 \\ \neq 0 \end{aligned}$$

For $x = -1/2$

$$8x^3 + 4x^2 - 4x - 1 = 1$$

$$\neq 0$$

For $x = 1/2$

$$8x^3 + 4x^2 - 4x - 1 = -1$$

$$\neq 0$$

For $x = -1/4$

$$8x^3 + 4x^2 - 4x - 1 = 1/8$$

$$\neq 0$$

For $x = 1/4$

$$8x^3 + 4x^2 - 4x - 1 = -13/8$$

$$\neq 0$$

For $x = 1/8$

$$8x^3 + 4x^2 - 4x - 1 = -91/64$$

$$\neq 0$$

For $x = -1/8$

$$8x^3 + 4x^2 - 4x - 1 = -29/64$$

$$\neq 0$$

Therefore, $8x^3 + 4x^2 - 4x - 1$ has no rational roots and it is irreducible over \mathbb{Q} .

Hence, proved

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