A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 3ED

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Problem

Prove the following for an integers *a*, *b*, *c* and all positive integers *m* and *n*:

If $a \equiv b \pmod{p}$ for every prime *p*, then $a = b$.

Step-by-step solution

Step 1 of 3

Here, objective is to prove that $a = b$

Comment

Step 2 of 3

Consider a, b are integers, *m* is a positive integer.

If *m* divides $a - b$, then *a* is congruent to *b* modulo *m* which is represented by

$$a = b(\text{mod m})$$
$$a = b + mq$$

Comment

Step 3 of 3

Consider $a = b(\text{mod } p)$; *p* is a prime.

$a = b(\text{mod } p)$

$a = b + pk$

$a - b = pk$; when $-1 \le k \le 1$

P divides equivalently $a - b$, that is $p \mid a - b$;

$a - b \in \{\ldots\ldots, -3p, -2p, -p, 0, p, 2p, 3p, \ldots\ldots\ldots\ldots\}$

$a - b = 0(\text{mod } p) \qquad \left(\because -\frac{p}{2} < a, b < \frac{p}{2}\right)$

$a - b = 0$

$a = b$

Hence, proved

Comment