Contents

1 $K[x,y]/\langle x-a_1, y-a_2\rangle \cong K$

Let $P = (a_1, a_2)$ and ϕ be the evaluation map

$$\phi : K[x,y] \to K$$

$$\phi(f(x,y)) = f(P)$$

then $\phi(K) = K$ so we see ϕ is surjective.

We also see $K[x,y]/\ker\phi \cong K$. Since the map is surjective, and K is a field, therefore $\ker\phi$ is maximal.

Now we prove $\ker\phi = \langle x-a_1, y-a_2\rangle$. We can easily see $\langle x-a_1, y-a_2\rangle \subseteq \ker\phi$, so now we prove the reverse inclusion. We write $f(x,y) = \sum f_i(x)y^i$ and then see $f(x,a_2) = \sum f_i(x)a_2^i$.

$$f(x,y) - f(x,a_2) = \sum f_i(x)(y^i - a_2^i)$$

but $(y^i - a_2^i) = (y-a_2)(y^{i-1} + \cdots + a_2)$

$$\Rightarrow f(x,y) - f(x,a_2) \in \langle y - a_2\rangle$$

Continuing with the same argument for x, and noting $f(a_1,a_2) = 0$, we see

$$f(x,y) \in \langle x-a_1, y-a_2\rangle \Rightarrow \ker\phi = \langle x-a_1, y-a_2\rangle$$