

A Book of Abstract Algebra | (2nd Edition)

Chapter 32, Problem 1E

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Problem

Throughout this set of questions, let K be a root field over F , let $\mathbf{G} = \text{Gal}(K : F)$, and let I be any intermediate field. Prove the following:

$I^* = \text{Gal}(K : I)$ is a subgroup of \mathbf{G} .

Step-by-step solution

Step 1 of 2

Consider a root field K over F , let $G = \text{Gal}(K : F)$, and let I be any intermediate field. The objective is to prove that $I^* = \text{Gal}(K : I)$ is a subgroup of G .

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Step 2 of 2

Let I be any intermediate field.

Then $\text{Gal}(K : I) = \{\sigma \in \text{Aut}(F) : \sigma(b) = b \forall b \in I\} \subseteq \{\sigma \in \text{Aut}(F) : \sigma(a) = a \forall a \in K\} = \text{Gal}(K : F)$.

Therefore, $\text{Gal}(K : I)$ is a subgroup of $\text{Aut}(F)$ contained in $\text{Gal}(K : F)$.

Hence, $I^* = \text{Gal}(K : I)$ is a subgroup of G .

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