A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 4EC

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Problem

Prove each of the following

If c is a complex root of a cubic $a(x) \in \mathbb{Q}[x]$, then $\mathbb{Q}(c)$ is the root field of $a(x)$ over $\mathbb{Q}$.

Step-by-step solution

Step 1 of 2

The objective is to prove that if c is a complex root of a cubic $a(x) \in \mathbb{Q}[x]$, then $\mathbb{Q}(c)$ is the root field of $a(x)$ over $\mathbb{Q}$.

Comment

Step 2 of 2

Let $a(x)=x^3-2\in\mathbb{Q}[x]$.

$a(x)$ has a complex root $c=-\frac{\sqrt[3]{2}}{2}+i\frac{\sqrt[3]{2}\sqrt{3}}{2}$ and the set of roots of $a(x)$ is $\left\{\sqrt[3]{2},-\frac{\sqrt[3]{2}}{2}+i\frac{\sqrt[3]{2}\sqrt{3}}{2},-\frac{\sqrt[3]{2}}{2}-i\frac{\sqrt[3]{2}\sqrt{3}}{2}\right\}$.

The root field of $a(x)$ over $\mathbb{Q}$ is not $\mathbb{Q}\left(-\frac{\sqrt[3]{2}}{2}+i\frac{\sqrt[3]{2}\sqrt{3}}{2}\right)$, that is, $\mathbb{Q}(c)$ is not the root field of $a(x)$ over $\mathbb{Q}$.

Therefore, the statement "If c is a complex root of a cubic $a(x)\in\mathbb{Q}[x]$, then $\mathbb{Q}(c)$ is the root field of $a(x)$ over $\mathbb{Q}$." is disproved.

Comment

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