A Book of Abstract Algebra | (2nd Edition)

Chapter 17, Problem 1EH   Bookmark   Show all steps: ON

Problem

Prove part:

In any ring, $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$.

Step-by-step solution

Step 1 of 3

Objective is to prove that in any ring,

$$a(b-c)=ab-ac,$$
$$(b-c)a=ba-ca.$$

In any ring, distributive property holds, that is,

$$a(b+c)=ab+ac,$$
$$(b+c)a=ba+ca,$$

where $a,b,c\in R$ (ring). Since c is the member of some ring R, so negative of c will definitely exist, say $-c\in R$.

Comment

Step 2 of 3

Replace the c by its negative $-c$ in the distributive property $a(b+c)=ab+ac$ of ring and get,

$$a\{b+(-c)\}=ab+a(-c)$$

Since product in ring is associative, so

$$\begin{aligned} a(-c)&=a(-1\cdot c)\\ &=-1(a\cdot c)\\ &=-(a\cdot c)\end{aligned}$$

Thus,

$$a\{b+(-c)\}=ab+a(-c)$$
$$a(b-c)=ab-ac.$$

Similarly replace *c* by $-c$ in the right distributive law, and get

$$\{b+(-c)\}a=ba+(-c)a$$
$$(b-c)a=ba-ca.$$

Comment

Step 3 of 3

Hence, in any ring $a(b-c)=ab-ac, (b-c)a=ba-ca.$

Comment