A Book of Abstract Algebra | (2nd Edition)

Chapter 30, Problem 4EC

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Problem

An angle α is called *constructible* iff there exist constructible points *A, B*, and *C* such that $\angle LABC = \alpha$.

Prove the following:

$\cos(2\alpha) \in \mathbb{D}$ iff $\cos\alpha \in \mathbb{D}$.

Step-by-step solution

Step 1 of 3

Here, objective is to prove that $\cos 2\alpha \in D$, if and only if $\cos\alpha \in D$

Comment

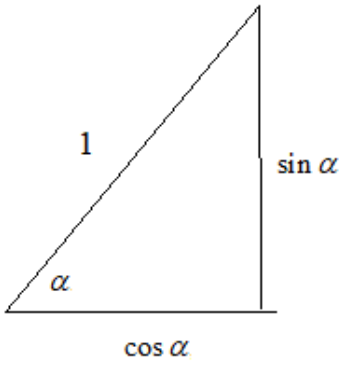
Step 2 of 3

Constructible point is the end point of given unit segment or it is intersection of two lines determined by constructional points.

Comment

Step 3 of 3

Consider the below figure:



Consider $\cos\alpha \in D$, then the point $(\cos\alpha, 0)$ is constructible from $\{O, I\}$

$\cos(2\alpha) = 2\cos^2\alpha - 1$

If $\cos\alpha$ is constructible then $\cos 2\alpha$ is also constructible.

If $\cos 2\alpha$ is constructible then $\cos 2\alpha \in D$.

Therefore, $\cos(\alpha+\beta), \cos(\alpha-\beta) \in D$, if $\cos\alpha, \cos\beta \in D$.

Hence, proved

Comment