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Chapter 29, Problem 1ED

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Problem

Let F be a field, and *K* a field extension of *F*. Prove the following:

[*K*:*F*] = *1* iff *K* = *F*.

Step-by-step solution

Step 1 of 3

Consider a field F, and an extension K of F. The objective is to prove that $[K:F]=1$ if and only if $K=F$.

Comment

Step 2 of 3

Suppose $[K:F]=1$.

Show that $K=F$.

$[K:F]=1$ implies that K is a vector space over F of dimension 1.

Then $\{1\}$ is a basis of K over F.

Thus, $K=F$.

Comment

Step 3 of 3

Conversely , suppose that $K = F$.

Show that $[K : F] = 1$.

Since $K = F$, $[K : F] = [F : F] = 1$.

Comment