

A Book of Abstract Algebra | (2nd Edition)



Chapter 30, Problem 5EC

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Problem

An angle α is called *constructible* iff there exist constructible points A , B , and C such that $\angle ABC = \alpha$.

Prove the following:

If α and β are constructible angles, so are $\alpha + \beta$, $\alpha - \beta$, $\frac{1}{2}\alpha$, and $n\alpha$ for any positive integer n .

Step-by-step solution

Step 1 of 5

Here, objective is to prove that $\alpha + \beta, \alpha - \beta, \frac{1}{2}\alpha$ and $n\alpha$ for any integer n are constructible.

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Step 2 of 5

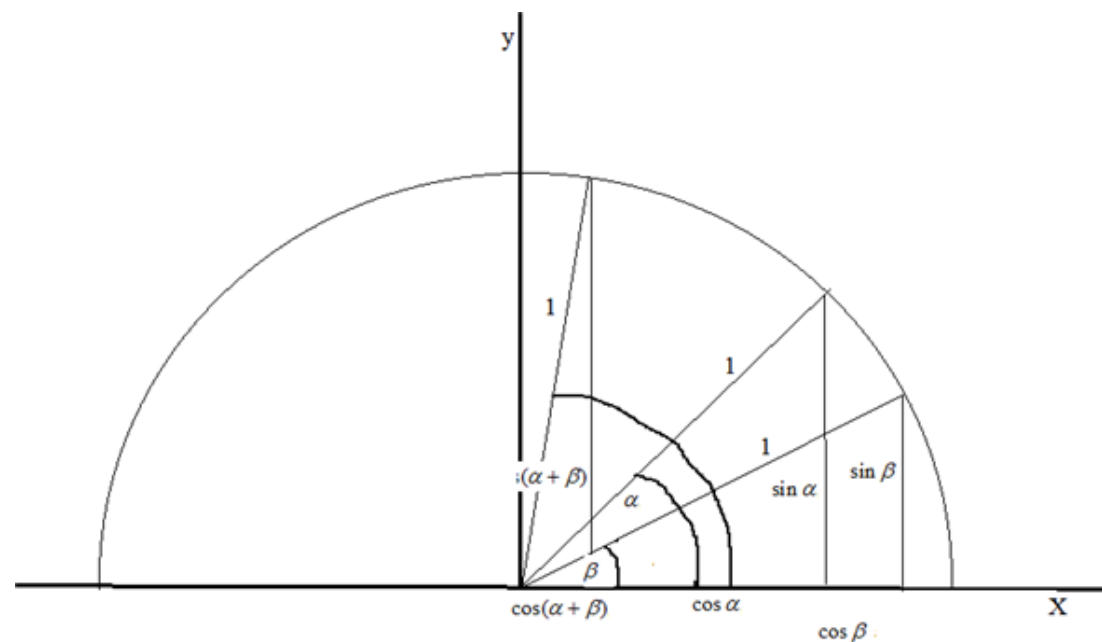
Constructible point is the end point of given unit segment or it is intersection of two lines

determined by constructional points.

Comment

Step 3 of 5

Consider the below figure:



Consider, the angles α, β are constructible from $\{O, I\}$

Then the lengths $\cos \alpha, \cos \beta$ are constructible.

If $(\cos \alpha, 0), (\cos \beta, 0)$ are constructible, then $\sin \alpha, \sin \beta$ are constructible, by using the identities

$$\sin \alpha = 1 - \cos^2 \alpha$$

$$\sin \beta = 1 - \cos^2 \beta$$

The lengths

$\sin \alpha, \sin \beta$ is constructible.

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Step 4 of 5

Consider

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

So,

$\cos(\alpha + \beta), \cos(\alpha - \beta)$ are constructible from $\cos \alpha, \cos \beta$ and $\sin \alpha, \sin \beta$.

Then draw the perpendicular lines from $(\cos(\alpha + \beta), 0), (\cos(\alpha - \beta), 0)$ to circumference of unit circle and draw the line joining from origin. The angles making with x-axis are $\alpha + \beta, \alpha - \beta$.

Therefore, $\alpha + \beta, \alpha - \beta$ are also constructible.

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Step 5 of 5

If α is constructible, then bisecting the angle we will construct $\frac{\alpha}{2}$

Similarly, $n\alpha$ is also constructible by multiplication of angle with any integer.

Therefore, $\alpha + \beta, \alpha - \beta, \frac{1}{2}\alpha$ and $n\alpha$ for any integer n are constructible, if α, β are constructible.

Hence, proved

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