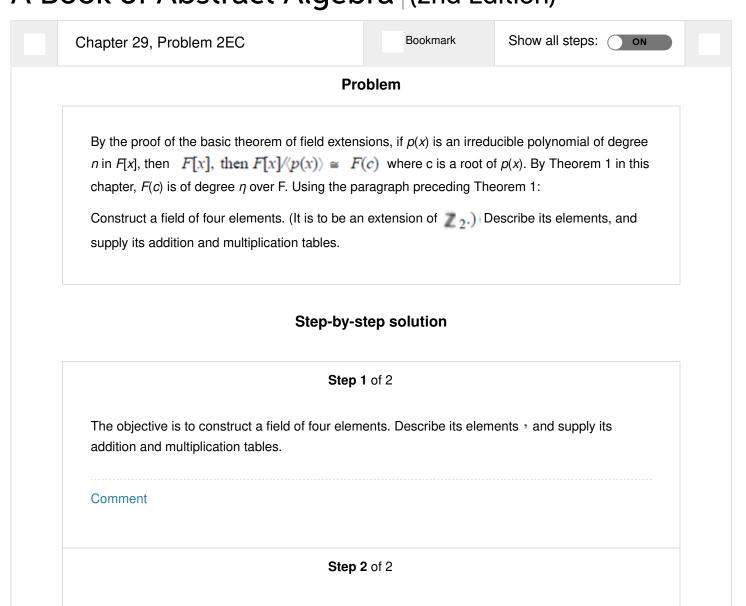
## A Book of Abstract Algebra (2nd Edition)



Let 
$$f(x) = x^2 + x + 1 \in {}_{2}[x].$$

Then 
$$f(0)=1$$
 and  $f(1)=1+1+1=1$ 

Then there exist an extension field E of  $_2$  containing a zero  $\alpha$  of f(x).

Since every element  $\beta$  of a simple extension  $E = F(\alpha)$  can be uniquely expressed in the form  $\beta = b_0 + b_1 \alpha + ... + b_{n-1} \alpha^{n-1}$  where  $b_i \in F$  and  $\alpha$  is algebraic over F,  $_2(\alpha)$  has elements  $0, 1, \alpha, 1 + \alpha$ .

This gives a field of four elements.

## Addition Tables: Multiplication Tables:

	+	0	1	$\alpha$	$1+\alpha$		0	1	$\alpha$	$1+\alpha$
	0	0	1	α	$1+\alpha$	0	0	0	0	0
	1	1	0	$1 + \alpha$	$\alpha$	1	0	1	$\alpha$	$1+\alpha$
	α	α	$1+\alpha$ $\alpha$	0	1	$\alpha$	0	$\alpha$	$1+\alpha$	1
1	+α	1+α	$\alpha$	1	0	$1 + \alpha$	0	$1+\alpha$	1	$\alpha$

Comment