A Book of Abstract Algebra | (2nd Edition)

Chapter AA, Problem 12E

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Problem

Prove the following:

$(A \cup B) - C = (A - C) \cup (B - C)$.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $(A \cup B) - C = (A - C) \cup (B - C)$.

Comment

Step 2 of 2

Proof:-

Let A and B are two sets.

The difference of two sets A and B is:-

$$A - B = \{x : x \in A, x \notin B\}$$

The union of two sets A and B is:-

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Let $x \in (A \cup B) - C$

$$x \in (A \cup B) - C$$
$$\Rightarrow x \in A \cup B, x \notin C$$
$$\Rightarrow x \in A \text{ or } x \in B, x \notin C$$
$$\Rightarrow (x \in A, x \notin C) \text{ or } (x \in B, x \notin C)$$

$\Rightarrow (x \in A - C)\, or\, (x \in B - C)$

$\Rightarrow (x \in A - C) \cup (x \in B - C)$

$\Rightarrow x \in (A - C) \cup (B - C)$

So,

$$(A \cup B) - C \subseteq (A - C) \cup (B - C) \qquad \ldots\ldots(1)$$

Let $x \in (A - C) \cup (B - C)$

$x \in (A - C) \cup (B - C)$

$\Rightarrow (x \in A - C) \cup (x \in B - C)$

$\Rightarrow (x \in A - C)\, or\, (x \in B - C)$

$\Rightarrow (x \in A, x \notin C)\, or\, (\ x \in B, x \notin C)$

$\Rightarrow x \in A \ \ or \ \ x \in B, x \notin C$

$\Rightarrow x \in A \cup B, x \notin C$

$x \in (A \cup B) - C$

So,

$$(A - C) \cup (B - C) \subseteq (A \cup B) - C \qquad \ldots\ldots(2)$$

Let us consider the equation (1) and (2).

$$(A \cup B) - C = (A - C) \cup (B - C)$$

Proved

Comment