

A Book of Abstract Algebra | (2nd Edition)

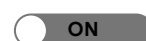


Chapter 23, Problem 1EB



Bookmark

Show all steps:



ON

Problem

Solve each of the following pairs of simultaneous congruences:

(a) $x \equiv 7 \pmod{8}$; $x \equiv 11 \pmod{12}$

(b) $x \equiv 12 \pmod{18}$; $x \equiv 30 \pmod{45}$

(c) $x \equiv 8 \pmod{15}$; $x \equiv 11 \pmod{14}$

Step-by-step solution

Step 1 of 5

Here, objective is to solve the given Pair of simultaneous congruence's.

[Comment](#)

Step 2 of 5

Consider a, b are integers, m is a positive integer.

If m divides $a - b$, then a is congruent to b modulo m which is represented by $a \equiv b \pmod{m}$

Consider the congruent equation $ax \equiv b \pmod{n}$, has solutions if and only if $\gcd(a, n)$ is divisible by b . If $\gcd(a, n) = 1$, then the congruence has unique solution

[Comment](#)

Step 3 of 5

(a)

Consider the pair of congruence

$$x = 7(\text{mod } 8) \dots \dots \dots (1)$$

$$x = 11(\text{mod } 12) \dots \dots \dots (2)$$

From equation (1)

$$x = 7 + 8p \dots \dots \dots (3)$$

Substitute above equation in equation (2)

$$7 + 8p = 11(\text{mod } 12)$$

$$8p = 4(\text{mod } 12)$$

$$2p = 1(\text{mod } 3)$$

$$p = 1(2^{-1})(\text{mod } 3)$$

$$p = 2(\text{mod } 3)$$

$$p = 2 + 3q$$

Substitute above equation in equation (3)

$$x = 7 + 8(2 + 3q)$$

$$x = 23 + 24q$$

$$x = 23(\text{mod } 24)$$

Hence, the solution of set of pair of congruence's is $x = 23(\text{mod } 24)$

[Comment](#)

Step 4 of 5

(b)

Consider the pair of congruence

$$x = 12(\text{mod } 18) \dots \dots \dots (1)$$

$$x = 30(\text{mod } 45) \dots \dots \dots (2)$$

From equation (1)

$$x = 12 + 18p \dots \dots \dots (3)$$

Substitute above equation in equation (2)

$$12 + 18p = 30(\text{mod } 45)$$

$$18p = 18(\text{mod } 45)$$

$$2p = 2(\text{mod } 5)$$

$$p = 1(2^{-1})(\text{mod } 5)$$

$$p = 3(\text{mod } 5)$$

$$p = 3 + 5q$$

Substitute above equation in equation (3)

$$x = 12 + 18(3 + 5q)$$

$$x = 12 + 54 + 90q$$

$$x = 66 + 90q$$

$$x = 66(\text{mod } 90)$$

Hence, the solution of set of pair of congruence's is $x = 66(\text{mod } 90)$

[Comment](#)

Step 5 of 5

(c)

Consider the pair of congruence

$$x = 8(\text{mod } 15) \dots \dots \dots (1)$$

$$x = 11(\text{mod } 14) \dots \dots \dots (2)$$

From equation (1)

$$x = 8 + 15p \dots \dots \dots (3)$$

Substitute above equation in equation (2)

$$8 + 15p = 11(\text{mod } 14)$$

$$15p = 3(\text{mod } 14)$$

$$p = 3(15^{-1})(\text{mod } 14)$$

$$p = 3(1)(\text{mod } 14)$$

$$p = 3(\text{mod } 14)$$

$$p = 3 + 14q$$

Substitute above equation in equation (3)

$$x = 8 + 15(3 + 14q)$$

$$x = 8 + 45 + 210q$$

$$x = 53 + 210q$$

$$x = 53(\text{mod } 210)$$

Hence, the solution of set of pair of congruence's is $x = 53(\text{mod } 210)$

[Comment](#)

