

A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 3EC

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Problem

Prove the following for all integers a, b, c, d and all positive integers m and n :

If $a \equiv b \pmod{n}$, then $ac \equiv bc \pmod{n}$.

Step-by-step solution

Step 1 of 2

Consider the congruence equation

$$a \equiv b \pmod{n}$$

Object of the problem is to prove that if $a \equiv b \pmod{n}$ then $ac \equiv bc \pmod{n}$.

Use the definition, $a \equiv b \pmod{n}$ iff n divides $a - b$ to prove the result.

By the definition of congruence equation, n divides $a - b$

There is integer p such that

$$a - b = np$$

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Step 2 of 2

Multiply both sides of the equation with c .

$$c(a - b) = ncp$$

$$ac - (bc) = n(cp)$$

$$ac - (bc) = ns \quad \text{take } cp = s$$

Again by the definition of congruence equation, $ac \equiv bc \pmod{n}$

Therefore, if $a \equiv b \pmod{n}$ then $ac \equiv bc \pmod{n}$

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