# A Book of Abstract Algebra | (2nd Edition)

Chapter 33, Problem 5EB

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## Problem

Let $G$ be a group. The symbol $H \triangleleft G$ is commonly used as an abbreviation of "$H$ is a *normal* subgroup of $G$." A *normal series* of $G$ is a finite sequence $H_0, H_1, \ldots, H_n$ of subgroups of $G$ such that

$$\{e\} = H_0 \triangleleft H_1 \triangleleft \cdots \triangleleft H_n = G$$

Such a series is called a *solvable series* if each quotient group $H_{i+1}/H_i$ is abelian. $G$ is called a *solvable group* if it has a solvable series.

Verify that $\{\varepsilon\} \subseteq \{\varepsilon, \beta, \delta\} \subseteq S_3$ is a solvable series for $S_3$. Conclude that $S_3$, and all of its subgroups, are solvable.

## Step-by-step solution

**Step 1** of 4

Here, objective is to verify that $\{\varepsilon\} \subseteq \{\varepsilon, \beta, \delta\} \subseteq S_3$ is a solvable series for $S_3$.

Comment

**Step 2** of 4

A group $G$ is solvable, if there exist a finite chain of successive subgroups.

Abelian groups are solvable.

Comment

**Step 3** of 4

Alternating group $A_3$ is the group of even permutations of finite set.

Symmetric group $S_3$ is the group of all permutations of three elements.

Consider in $S_3$ , $A_3$ be the group of even permutations.

Then, $A_3 \subseteq S_3$

Comment

**Step 4** of 4

Consider $\{\varepsilon,\beta,\delta\}$ has order three. Then $\{\varepsilon,\beta,\delta\} \subseteq S_3$

We know that,

$\{\varepsilon\} \subseteq \{\varepsilon,\beta,\delta\}$

So,

$\{\varepsilon\} \subseteq \{\varepsilon,\beta,\delta\} \subseteq S_3$

Is a subnormal sequence with Abelian quotients.

Therefore,

$\{\varepsilon\} \subseteq \{\varepsilon,\beta,\delta\} \subseteq S_3$ is a solvable series for $S_4$.

And all of its groups are solvable.

Hence, proved

Comment