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Chapter 32, Problem 1EA

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Problem

Show that $\mathbb{Q}(i, \sqrt{2})$ is the root field of $(x^2 + 1)(x^2 - 2)$ over $\mathbb{Q}$.

Step-by-step solution

Step 1 of 2

The objective is to show that $\mathbb{Q}\left(i, \sqrt{2}\right)$ is the root field of $\left(x^2+1\right)\left(x^2-2\right)$ over $\mathbb{Q}$.

Comment

Step 2 of 2

The polynomial $f(x)=\left(x^2+1\right)\left(x^2-2\right)$ has roots $\left\{i,-i,\sqrt{2},-\sqrt{2}\right\}$.

Since $\mathbb{Q}\left(i,\sqrt{2}\right)$ contains the roots of $f(x)=\left(x^2+1\right)\left(x^2-2\right)$, $\mathbb{Q}\left(i,\sqrt{2}\right)$ is the root field of $\left(x^2+1\right)\left(x^2-2\right)$ over $\mathbb{Q}$.

Comment