A Book of Abstract Algebra | (2nd Edition)

Problem

Prove the following:

$A \cup A = A$ and $A \cap A = A$.

Step-by-step solution

Step 1 of 3

Objective:-

The objective is to prove $A \cup A = A \;\; and \;\; A \cap A = A.$

Comment

Step 2 of 3

Exercise (a):-

Let *A* and *B* are two sets. Let $x \in A \cup A$.

The union of two sets *A* and *B* is:-

$$A \cup B = \{x : x \in A \;\; or \;\; x \in B\}$$

According to this definition:-

$$\begin{aligned} A \cup A &\Rightarrow x \in A \;\; or \;\; x \in A \\ &\Rightarrow x \in A \end{aligned}$$

So,

$$A \cup A \subseteq A \qquad \ldots\ldots(1)$$

Let $x \in A$.

$$\begin{aligned} A &\Rightarrow x \in A \;\; or \;\; x \in A \\ &\Rightarrow x \in A \cup A \end{aligned}$$

So,

$$A \subseteq A \cup A \qquad ......(2)$$

Let us consider the equation (1) and (2).

$$A \cup A = A$$

Proved

Comment

Step 3 of 3

Exercise (b):-

Let *A* and *B* are two sets. Let $x \in A \cap A$.

The intersection of two sets *A* and *B* is:-

$$A \cap B = \{x : x \in A \ \ and \ \ x \in B\}$$

According to this definition:-

$$\begin{aligned} A \cap A &\Rightarrow x \in A \ \ and \ \ x \in A \\ &\Rightarrow x \in A \end{aligned}$$

So,

$$A \cap A \subseteq A \qquad ......(3)$$

Let $x \in A$.

$$\begin{aligned} A &\Rightarrow x \in A \ \ and \ \ x \in A \\ &\Rightarrow x \in A \cap A \end{aligned}$$

So,

$$A \subseteq A \cap A \qquad ......(4)$$

Let us consider the equation (3) and (4).

$$A \cap A = A$$

Proved

Comment