A Book of Abstract Algebra | (2nd Edition)

Chapter 24, Problem 2EI

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Problem

Let *A* be an integral domain. By the closing part of Chapter 20, every integral domain can be extended to a “field of quotients.” Thus, *A*[*x*] can be extended to a field of polynomial quotients, which is denoted by *A*(*x*). Note that *A*(*x*) consists of all the fractions *a*(*x*)/*b*(*x*) for *a*(*x*) and *b*(*x*) ≠ 0 in *A*[*x*], and these fractions are added, subtracted, multiplied, and divided in the customary way.

Using part 1, explain why there is an infinite field of characteristic *p*, for every prime *p*.

Step-by-step solution

Step 1 of 1

Let p is prime then

$$ {}_p(X)=\left\{\frac{f(X)}{g(X)}:f,g\in \ {}_p[X],g\neq 0\right\} $$

Rational functions in X with coefficients in ${}_p$ (we can assume ${}_p = \ /p$)

The field ${}_p(X)$ is infinite because, it contains $1,X,X^2,.....$ all distinct and characteristic of ${}_p(X)$ is p because ${}_p$ has characteristic p

$\Rightarrow$ There is an infinite field of characteristic p, for every prime p.

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