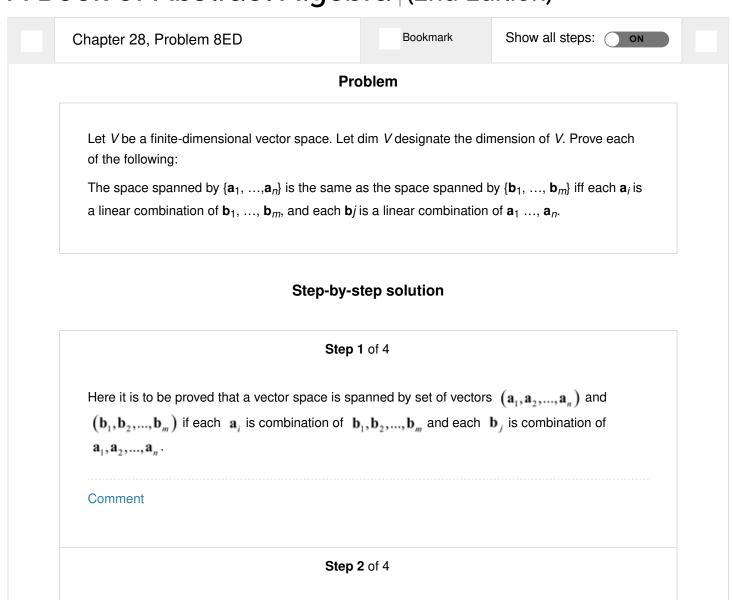
## A Book of Abstract Algebra (2nd Edition)



To prove this we assume that given vector space is spanned by both  $(\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_n)$  and  $(\mathbf{b}_1, \mathbf{b}_2, ..., \mathbf{b}_m)$ . Then prove that each  $\mathbf{a}_i$  is combination of  $\mathbf{b}_1, \mathbf{b}_2, ..., \mathbf{b}_m$  and each  $\mathbf{b}_j$  is combination of  $\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_n$ .

Comment

## **Step 3** of 4

Let  $\mathbf{v}$  be any vector in vector space V, then

$$\mathbf{v} = t_1 \mathbf{a}_1 + t_2 \mathbf{a}_2 + \dots + t_n \mathbf{a}_n$$

$$\mathbf{v} = k_1 \mathbf{b}_1 + k_2 \mathbf{b}_2 + \dots + k_m \mathbf{b}_m$$

Since, all  $(a_1, a_2, ..., a_n)$  lies in vector space V, it can be said that for all  $a_i$ ,

$$\mathbf{a}_{i} = k_{1}\mathbf{b}_{1} + k_{2}\mathbf{b}_{2} + ... + k_{m}\mathbf{b}_{m} ; i \in (1, 2, ..., n)$$

Hence each  $\mathbf{a}_i$  is a combination of  $\mathbf{b}_1, \mathbf{b}_2, ..., \mathbf{b}_m$ 

Comment

## Step 4 of 4

Also, all  $(\mathbf{b}_1, \mathbf{b}_2, ..., \mathbf{b}_m)$  lies in vector space V, it can be said that for all  $\mathbf{b}_I$ ,

$$\mathbf{b}_{j} = t_{1}\mathbf{a}_{1} + t_{2}\mathbf{a}_{2} + ... + t_{n}\mathbf{a}_{n} \; ; j \in (1, 2, ..., m)$$

Hence each  $\mathbf{b}_j$  is a combination of  $\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_n$ 

.....

Comment