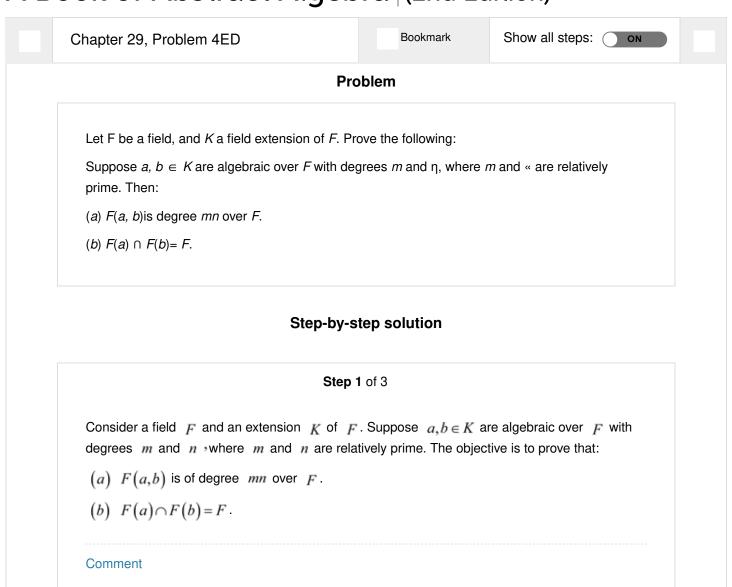
A Book of Abstract Algebra (2nd Edition)



Step 2 of 3

(a)

Suppose $a,b \in K$ are algebraic over F with minimal polynomials p(x) and q(x) of degrees m and n respectively.

Now
$$\cdot [F(a,b):F] = [F(a,b):F(a)][F(a):F] = m[F(a,b):F(a)].$$

Hence, $m \mid \lceil F(a,b) : F \rceil$.

Similarly, n | [F(a,b):F].

So, $mn \mid \lceil F(a,b) : F \rceil$ because m and n are relatively prime.

On the other hand, $[F(a,b):F(a)] \le n$ since the minimal polynomial for b over F(a) can have no greater degree than the minimal polynomial q(x) over F.

Therefore
$$\cdot [F(a,b):F] = [F(a,b):F(a)][F(a):F] \leq mn$$
.

Since
$$mn \mid \lceil F(a,b) : F \rceil$$
, $\lceil F(a,b) : F \rceil = mn$.

Comment

Step 3 of 3

Let $L = F(a) \cap F(b)$.

Then ,

$$\begin{split} & \big[F(a,b) \colon F \big] = \big[F(a) \colon F \big] \big[F(b) \colon F \big] \text{ *since } F(a) \text{ and } F(b) \text{ are intermediate fields} \\ & = \big[F(a) \colon L \big] \big[L \colon F \big] \big[F(b) \colon L \big] \big[L \colon F \big] \text{ *since } F \subseteq F(a) \cap F(b) \subseteq F(a) \subseteq K \text{ and } \\ & F \subseteq F(a) \cap F(b) \subseteq F(b) \subseteq K \end{split}$$

$$= [F(a):L][F(b):L][L:F]^{2}$$

$$= [F(a,b):L][L:F]^{2}$$

$$= [F(a,b):F][L:F]$$

Thus , [L:F]=1 and hence L=F , that is , $F(a)\cap F(b)=F$.

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Comment