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Chapter 33, Problem 5EC

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Problem

Let p be a prime number, and ω a primitive pth root of unity in the field F.

Explain why m and p are relatively prime. Explain why it follows that there are integers s and t such that $sm + tp = 1$.

Step-by-step solution

Step 1 of 4

Here, objective is to explain why m and p are relatively prime and prove that $sm + tp = 1$

Comment

Step 2 of 4

Consider the polynomial $x^p - a = p(x)f(x) \in F(x)$

Here, p is a prime and degree of $p(x) = m$

Comment

Step 3 of 4

Here, p is the degree of the polynomial $x^p - a$ and m is the degree of $f(x)$

Therefore, $p > m$

Here, p is a prime, then

It is clear that, there is no common factor between m and p.

Hence, m and p are said to be relatively prime.

Comment

Step 4 of 4

If *m* and *p* are relatively prime.

Then, there is no common factor between them.

So, the greatest common divisor of *m* and *p* is equal to one.

That is, $\gcd(p,m)=1.............(1)$

And also if *m* and *p* are relatively prime, then there exist some integers *s* and *t* such that

$\gcd(p,m)=sm+tp\ ;s,t\in Z..........(2)$

From the equations $..........(1)\ \&\ (2)$, we get

$sm+tp=1$

Hence, proved

Comment