

A Book of Abstract Algebra | (2nd Edition)

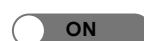


Chapter 23, Problem 2EB



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Problem

Solve each of the following pairs of simultaneous congruences:

(a) $10x \equiv 2 \pmod{12}$; $6x \equiv 14 \pmod{20}$

(b) $4x \equiv 2 \pmod{6}$; $9x \equiv 3 \pmod{12}$

(c) $6x \equiv 2 \pmod{8}$; $10x \equiv 2 \pmod{12}$

Step-by-step solution

Step 1 of 5

Here, objective is to solve the given Pair of simultaneous congruence's.

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Step 2 of 5

Consider a, b are integers, m is a positive integer.

If m divides $a - b$, then a is congruent to b modulo m which is represented by $a \equiv b \pmod{m}$

Consider the congruent equation $ax \equiv b \pmod{n}$, has solutions if and only if $\gcd(a, n)$ is divisible by b . If $\gcd(a, n) = 1$, then the congruence has unique solution

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Step 3 of 5

(a)

Consider the pair of congruence

$$10x = 2(\text{mod } 12) \dots\dots\dots(1)$$

$$6x = 14(\text{mod } 20) \dots\dots\dots(2)$$

From equation (1)

$$5x = 1(\text{mod } 6).$$

$$x = 1(5^{-1})(\text{mod } 6).$$

$$x = 5(\text{mod } 6)$$

$$x = 5 + 6p \dots\dots\dots(3)$$

Substitute above equation in equation (2)

$$6(5 + 6p) = 14(\text{mod } 20)$$

$$30 + 36p = 14(\text{mod } 20)$$

$$9p = -4(\text{mod } 5)$$

$$p = -4(9^{-1})(\text{mod } 5)$$

$$p = -4(4)(\text{mod } 5)$$

$$p = 4(\text{mod } 5)$$

$$p = 4 + 5q$$

Substitute above equation in equation (3)

$$x = 5 + 6(4 + 5q)$$

$$x = 5 + 24 + 30q$$

$$x = 29(\text{mod } 30)$$

Hence, the solution of set of pair of congruence's is $x = 29(\text{mod } 30)$

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Step 4 of 5

(b)

Consider the pair of congruence

$$4x = 2(\text{mod } 6) \dots\dots\dots(1)$$

$$9x = 3(\text{mod } 12) \dots\dots\dots(2)$$

From equation (1)

$$2x = 1(\text{mod } 3)$$

$$x = 1(2^{-1}) \text{mod } 3$$

$$x = 2(\text{mod } 3)$$

$$x = 2 + 3p \dots\dots\dots(3)$$

Substitute above equation in equation (2)

$$9(2 + 3p) = 3(\text{mod } 12)$$

$$18 + 27p = 3(\text{mod } 12)$$

$$9p = -5(\text{mod } 4)$$

Substitute above equation in equation (3)

$$x = 2 + 3p$$

$$3x = 6 + 9p$$

$$3x = 6 - 5(\text{mod } 4)$$

$$3x = 1 \text{ mod } 4$$

$$x = 3 \text{ mod } 4$$

Hence, the solution of set of pair of congruence's is $x = 3(\text{mod } 4)$

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Step 5 of 5

(c)

Consider the pair of congruence

$$6x = 2(\text{mod } 8) \dots \dots \dots (1)$$

$$10x = 2(\text{mod } 12) \dots \dots \dots (2)$$

From equation (1)

$$3x = 1(\text{mod } 4)$$

$$x = 3(\text{mod } 4)$$

$$x = 3 + 4p \dots \dots \dots (3)$$

Substitute above equation in equation (2)

$$10x = 2(\text{mod } 12)$$

$$5x = 1(\text{mod } 6)$$

$$5(3 + 4p) = 1(\text{mod } 6)$$

$$10p = -7(\text{mod } 3)$$

Substitute above equation in equation (3)

$$10x = 30 + 4(2 \text{ mod } 3).$$

$$10x = 38 \text{ mod } 3$$

$$x = 38(10^{-1}) \text{ mod } 3$$

$$x = 38(\text{mod } 3)$$

Hence, the solution of set of pair of congruence's is $x = 38(\text{mod } 3)$

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