A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 1EB

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Problem

Solve each of the following pairs of simultaneous congruences:

(*a*) $x \equiv 7 \pmod 8$; $x \equiv 11 \pmod{12}$

(*b*) $x \equiv 12 \pmod{18}$; $x \equiv 30 \pmod{45}$

(*c*) $x \equiv 8 \pmod{15}$; $x = 11 \pmod{14}$

Step-by-step solution

Step 1 of 5

Here, objective is to solve the given Pair of simultaneous congruence's.

Comment

Step 2 of 5

Consider a, b are integers, *m* is a positive integer.

If *m* divides $a-b$, then *a* is congruent to *b* modulo *m* which is represented by $a = b(\text{mod m})$

Consider the congruent equation $ax = b(\text{mod } n)$, has solutions if and only if $\gcd(a,n)$ is divisible by b. If $\gcd(a,n)=1$, then the congruence has unique solution

Comment

Step 3 of 5

(a)

Consider the pair of congruence

$$x = 7 (\bmod 8) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$
$$x = 11 (\bmod 12) \ldots\ldots\ldots\ldots\ldots (2)$$

From equation (1)

$$x = 7 + 8p \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Substitute above equation in equation (2)

$$7 + 8p = 11 (\bmod 12)$$
$$8p = 4 (\bmod 12)$$
$$2p = 1 (\bmod 3)$$
$$p = 1(2^{-1}) (\bmod 3)$$
$$p = 2 (\bmod 3)$$
$$p = 2 + 3q$$

Substitute above equation in equation (3)

$$x = 7 + 8(2 + 3q)$$
$$x = 23 + 24q$$
$$x = 23 (\bmod 24)$$

Hence, the solution of set of pair of congruence's is $x = 23 (\bmod 24)$

Comment

Step 4 of 5

(b)

Consider the pair of congruence

$$x = 12 (\bmod 18) \ldots\ldots\ldots\ldots\ldots\ldots (1)$$
$$x = 30 (\bmod 45) \ldots\ldots\ldots\ldots\ldots (2)$$

From equation (1)

$$x = 12 + 18p \ldots\ldots (3)$$

Substitute above equation in equation (2)

$$12 + 18p = 30 (\bmod 45)$$
$$18p = 18 (\bmod 45)$$
$$2p = 2 (\bmod 5)$$
$$p = 1(2^{-1}) (\bmod 5)$$
$$p = 3 (\bmod 5)$$
$$p = 3 + 5q$$

Substitute above equation in equation (3)

$$x = 12 + 18(3 + 5q)$$
$$x = 12 + 54 + 90q$$
$$x = 66 + 90q$$
$$x = 66 (\bmod 90)$$

Hence, the solution of set of pair of congruence's is $x = 66 (\bmod 90)$

Comment

Step 5 of 5

(c)

Consider the pair of congruence

$$x = 8 (\bmod 15) \ldots\ldots\ldots\ldots\ldots (1)$$
$$x = 11 (\bmod 14) \ldots\ldots\ldots\ldots (2)$$

From equation (1)

$$x = 8 + 15p \ldots\ldots\ldots\ldots\ldots (3)$$

Substitute above equation in $\text{equation } (2)$

$$8 + 15p = 11 (\bmod 14)$$
$$15p = 3 (\bmod 14)$$

$$p = 3(15^{-1})(\bmod 14)$$
$$p = 3(1)(\bmod 14)$$
$$p = 3 (\bmod 14)$$

$$p = 3 + 14q$$

Substitute above equation in $\text{equation } (3)$

$$x = 8 + 15(3 + 14q)$$
$$x = 8 + 45 + 210q$$
$$x = 53 + 210q$$
$$x = 53 (\bmod 210)$$

Hence, the solution of set of pair of congruence's is $x = 53 (\bmod 210)$

Comment