A Book of Abstract Algebra | (2nd Edition)

Problem

Let $a(x) = a_0 + a_1x + \cdots + a_nx^n \in F[x]$. The *derivative* of $a(x)$ is the following polynomial $a'(x) \in F[x]$:

$$a'(x) = a_1 + 2a_2x + \ldots + na_nx^{n-1}$$

(This is the same as the derivative of a polynomial in calculus.) We now prove the analogs of the formal rules of differentiation, familiar from calculus.

Let $a(x), b(x) \in F[x]$, and let $k \in F$.

Prove part:

Find the derivative of the following polynomials in $\mathbb{Z}_5[x]$:

$$x^6 + 2x^3 + x + 1 \quad x^5 + 3x^2 + 1 \quad x^{15} + 3x^{10} + 4x^5 + 1$$

Step-by-step solution

Step 1 of 4

Consider the arbitrary field F and let $a(x) = a_0 + a_1x + \cdots + a_nx^n \in F(x)$. The derivative of $a(x)$ will be given by

$a'(x) = a_1 + 2a_2x + \cdots + na_nx^{n-1} \in F(x)$.

Objective is to determine the derivative of the following polynomials in $Z_5[x]$:

The polynomial is $x^6 + 2x^3 + x + 1$.

Comment

Step 2 of 4

Let $a(x) = x^6 + 2x^3 + x + 1$. Then its derivative will be:

$a'(x) = 6x^5 + 6x + 1$.

And the derivative in $Z_5[x]$ will be:

$a'(x) = x^5 + x + 1$

because $6 \equiv 1$ in Z_5.

Comment

Step 3 of 4

Let $a(x) = x^5 + 3x^2 + 1$. Then its derivative will be:

$a'(x) = 5x^4 + 6x$.

And the derivative in $Z_5[x]$ will be:

$a'(x) = x$

because $5 \equiv 0, 6 \equiv 1$ in Z_5.

Comment

Step 4 of 4

The polynomial is $x^{15} + 3x^{10} + 4x^5 + 1$.

Let $a(x) = x^{15} + 3x^{10} + 4x^5 + 1$. Then its derivative will be:

$a'(x) = 15x^{14} + 30x^9 + 20x^4$.

And the derivative in $Z_5[x]$ will be $a'(x) = 0$, because $15, 20, 30 \equiv 0$ in Z_5.

Comment