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Chapter 28, Problem 1EB

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Problem

Prove that $\{(a, b, c) : 2a - 3b + c = 0\}$ is a subspace of $\mathbb{R}^3$.

Step-by-step solution

Step 1 of 2

(a_1, a_2, a_3) represents a vector space in 3 dimension or $\mathbb{R}^3$ as it satisfies all conditions for vector space.

For 3 dimension, any subspace must be a plane or line or a point passing through origin. The reason for it lies in the fact that any linear combination of 2 vectors lying on plane and line also lies on that vector space.

Given condition for subspace is

$2a - 3b + c = 0$

This represents an equation of plane in $\mathbb{R}^3$ passing through origin. Hence it represents a vector space.

Comment

Step 2 of 2

Above mentioned method is useful in simple geometrical vector spaces but is not much useful in complex spaces. Here 2 step check is needed to determine if given subset is a subspace. First determine if any linear combination of vector in subspace is closed under given operation. Second, determine if 0 maps to 0.

STEP 1: For any 2 vectors (a,b,c) and (d,e,f),

$$2a - 3b + c = 0 \qquad (1)$$

$$2d - 3e + f = 0 \qquad (2)$$

Combining above 2 equations, $k(1) + l(2)$ gives

$$2(ka + ld) - 3(kb + le) + (kc + lf) = 0$$

Thus linear combination of 2 vectors in subspace lies in subspace.

STEP 2: Check if $(0,0,0)$ satisfies given condition,

$$2 \cdot 0 - 3 \cdot 0 + 0 = 0$$

$$\boxed{\text{Hence } \{(a,b,c) \mid 2a - 3b + c = 0\} \text{ represents a subspace}}$$

Comment