Dook of Abstract Algobra (and Edition)

AD	OOK OI	ADSITAC	i Aigeb	ia (2nd	i Edition)

Chapter 33, Problem 2EA

Bookmark

Show all steps: ON

Problem

Show that the following polynomials in $\mathbb{Q}[X]$ are not solvable by radicals:

(a)
$$2x^5 - 5x^4 + 5$$

(b)
$$x^5 - 4x^2 + 2$$

(c)
$$x^5 - 4x^4 + 2x + 2$$

Step-by-step solution

Step 1 of 6

Here, objective is to prove that the polynomials in Q(x) are not solvable by radicals.

Comment

Step 2 of 6

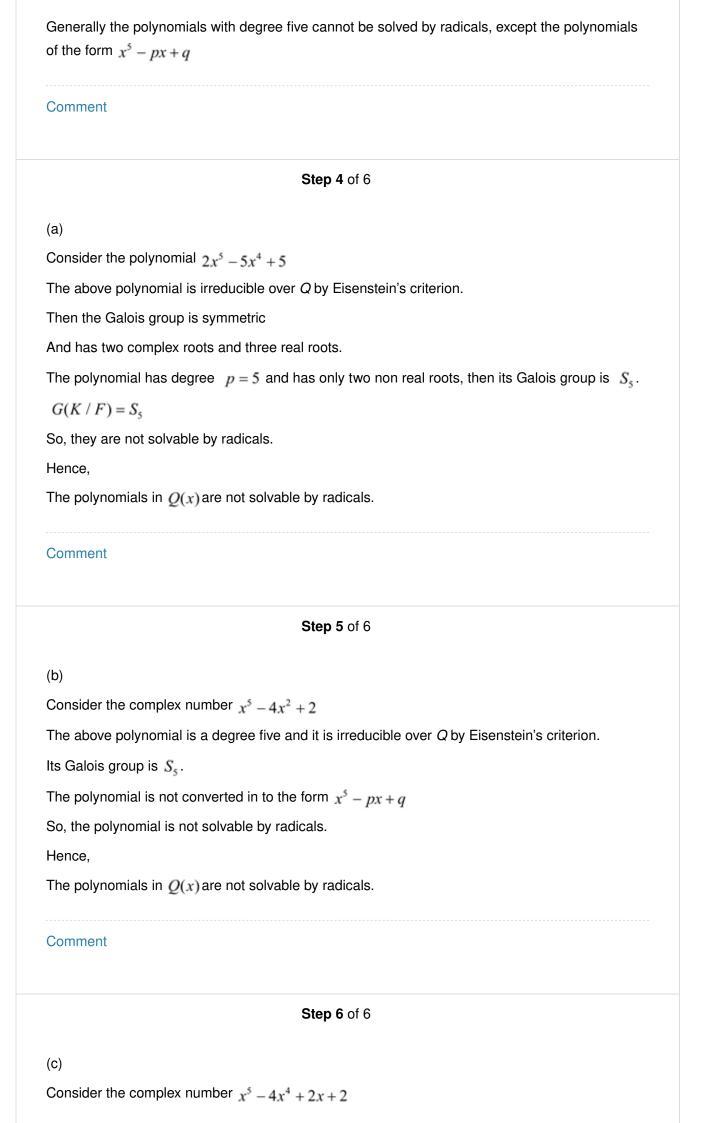
A polynomial equation is solvable by radicals, if its roots are determined by applying finite number of additions, subtractions, multiplications, divisions, n^{th} roots to the integers.

Galois group:

If a polynomial $f(x) \in Q(x)$ has degree p and has only two non real roots, then its Galois group

If the polynomial whose Galois group is S_5 they are not solvable by radicals.

Comment



The above polynomial is having degree five and it is irreducible over Q by	y Eisenstein's criterion.
Its Galois group is S_5 .	
$G(K/F) = S_5$	

The polynomial is not converted in to the form $x^5 - px + q$

So, the polynomial is not solvable by radicals.

Hence,

The polynomials in Q(x) are not solvable by radicals.

Comment