A Book of Abstract Algebra | (2nd Edition)

Problem

Let F be any field.

Prove part:

If the minimum polynomial of a over F is of degree 1, then $a \in F$, and conversely.

Step-by-step solution

Step 1 of 3

Objective is to prove that if the minimal polynomial of a over F is of degree 1, then $a \in F$, and conversely.

Firstly, suppose that the minimal polynomial of a over F is of degree 1. Let such minimal polynomial is $p(x)$. Since $p(x)$ is of degree 1, so it will be of the form:

$p(x) = x - a$.

In this case the solution of corresponding equation $p(x) = 0$ will be:

$$x - a = 0$$
$$x = a \in F.$$

Comment

Step 2 of 3

Conversely, let $a \in F$. Then the minimal polynomial of scalar or field element a will be:

$x = a$.

Or equivalently,

$x - a = 0$.

Therefore, the minimal polynomial will be $x - a$, which is of degree 1 (linear polynomial).

Comment

Step 3 of 3

Hence, the minimal polynomial of a over F is of degree 1 if and only if $a \in F$.

Comment