

A Book of Abstract Algebra | (2nd Edition)

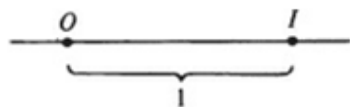
Chapter 30, Problem 3EA

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Problem

If O and I are any two points in the plane, consider a coordinate system such that



the interval OI coincides with the unit interval on the x axis. Let \mathbb{D} be the set of real numbers such that $a \in \mathbb{D}$ iff the point $(a, 0)$ is constructible from $\{O, I\}$. Prove the following:

If $a, b \in \mathbb{D}$, then $a/b \in \mathbb{D}$. (Use the same figure as in part 2.)

Step-by-step solution

Step 1 of 4

Here, objective is to prove that, if $a, b \in \mathbb{D}$, then $a/b \in \mathbb{D}$.

Consider $a \in \mathbb{D}$ if and only if the point $(a, 0)$ is constructible from $\{O, I\}$

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Step 2 of 4

Constructible point:

This point is either the end point of given unit segment or it is the intersection of two lines determined by previous constructible points.

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Step 3 of 4

D is a set of real numbers. Therefore, we can add, subtract, multiply any two points of them.

Let $a, b \in D$

Consider $a \in D$ if and only if the point $(a, 0)$ is constructible from $\{O, I\}$

Similarly,

$b \in D$ if and only if the point $(b, 0)$ is constructible from $\{O, I\}$

Therefore, the points $(a, 0)$, $(b, 0)$ are constructible lengths from $\{O, I\}$

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Step 4 of 4

Now, we have to prove that length a/b is constructed from the lengths a and b

Consider the below figure:

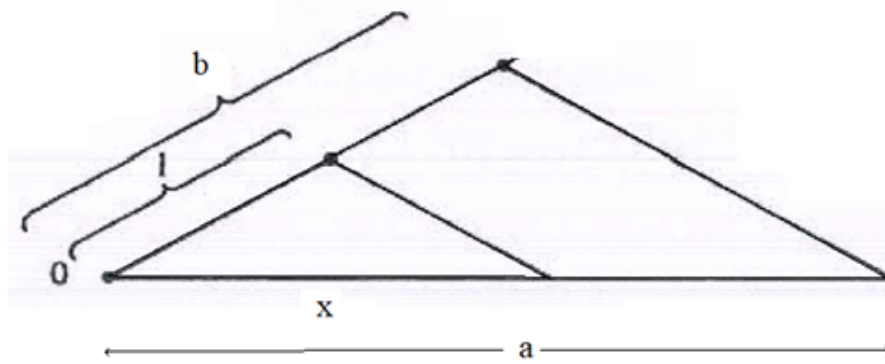


figure: construction of a/b

by observing there exist two similar triangles.

a and b are constructible lengths.

using the property of similar triangles, we have

$$\frac{1}{x} = \frac{b}{a}$$

$$x = \frac{a}{b}$$

Then, the point $(a/b, 0)$ is constructible from $\{O, I\}$ which implies $a/b \in D$

Therefore, if $a, b \in D$, then $a/b \in D$.

Hence, proved

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