


A Book of Abstract Algebra | (2nd Edition)



Chapter 30, Problem 4EE

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Problem

We will show that $2\pi/5$ is a constructible angle, and it will follow that the regular pentagon is constructible.

Use part 3 and A6 to prove that $\cos (2\pi/5)$ is a constructible number.

Step-by-step solution

Step 1 of 5

Here, objective is to prove that $\cos \frac{2\pi}{5}$ is a constructible number.

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Step 2 of 5

if the point $(a,0)$ is constructible from $\{O,I\}$, then $a \in D$

$a \in D$, if a is a real root of any quadratic polynomial with coefficients in D .

De Moivre's theorem: $\omega = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$ is a complex fifth root of unity.

Since $x^5 - 1 = (x-1)(x^4 + x^3 + x^2 + x + 1)$, ω is a root of $P(x) = (x^4 + x^3 + x^2 + x + 1)$

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Step 3 of 5

Consider $\omega = \cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$

$$\frac{1}{\omega} = \cos \frac{2\pi}{5} - i \sin \frac{2\pi}{5}$$

$$\left(\omega + \frac{1}{\omega} \right) = 2 \cos \frac{2\pi}{5}$$

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Step 4 of 5

Consider ω is a root of $P(x) = (x^4 + x^3 + x^2 + x + 1)$

Then,

$$(\omega^4 + \omega^3 + \omega^2 + \omega + 1) = 0$$

$$\omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2} = 0$$

$$\left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega + \frac{1}{\omega}\right) - 1 = 0$$

$$\left(2\cos\frac{2\pi}{5}\right)^2 + \left(2\cos\frac{2\pi}{5}\right) - 1 = 0$$

$$4\cos^2\frac{2\pi}{5} + 2\cos\frac{2\pi}{5} - 1 = 0$$

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Step 5 of 5

Consider the polynomial $4x^2 - 2x - 1$

put $x = 2\cos\frac{2\pi}{5}$ in above equation , then

$$4x^2 - 2x - 1 =$$

$$= 4\cos^2\frac{2\pi}{5} + 2\cos\frac{2\pi}{5} - 1$$

$$= 0$$

Hence, $\cos\frac{2\pi}{5}$ is a root of quadratic polynomial $4x^2 - 2x - 1 = 0$, then $\cos\frac{2\pi}{5} \in D$

Therefore, $\cos\frac{2\pi}{5}$ is a constructible from $\{O, I\}$.

Hence, proved.

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