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Chapter 29, Problem 1EF

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Problem

Let *F* be a field, and *K* a finite extension of *F*. Prove each of the following:

Any element algebraic over *K* is algebraic over *F*, and conversely.

Step-by-step solution

Step 1 of 3

Consider a field *F* and a finite extension *K* of *F*. Objective is to prove that any element is algebraic over *K* if and only if it is algebraic over *F*.

Suppose that *b* is algebraic over *K*. Then let $f(x)=x^n+a_{n-1}x^{n-1}+\cdots+a_0\in K[x]$, where $a_i\in K$ and hence the coefficients a_i's are algebraic over *F*.

Comment

Step 2 of 3

Since $F(a_0,\ldots,a_{n-1})$is a finite extension of *F*, therefore *b* is algebraic over $F(a_0,\ldots,a_{n-1})$. This is so because it is root of a nonzero polynomial with coefficients in $F(a_0,\ldots,a_{n-1})$. Thus,

$$F(a_0,\ldots,a_{n-1})(b)=F(a_0,\ldots,a_{n-1},b)$$

is a finite extension of $F(a_0,\ldots,a_{n-1})$. Also by the result, $F(a_0,\ldots,a_{n-1},b)$is a finite extension of *F*, thus an algebraic extension of *F*.

Hence, *b* is algebraic over *F*.

Comment

Step 3 of 3

Converse part is simple. If *a* is a root of a nonzero polynomial $f(x)\in F[x]$, then since $F[x]\subseteq K[x]$, *a* is also the root of polynomial $f(x)$viewed as an element of $K[x]$. Thus, *a* will be algebraic over *K*.

Hence, any element algebraic over *K* is algebraic over *F*, and conversely.

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