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Chapter 33, Problem 4EC

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Problem

Let p be a prime number, and ω a primitive pth root of unity in the field F.

Use part 3 to prove that $b^p = a^m$.

Step-by-step solution

Step 1 of 4

Here, objective is to prove that $b^p = a^m$

Consider $\text{degree } p(x) = m$

Comment

Step 2 of 4

Consider the polynomial $x^p - a$.

$x^p - a = 0$

$x = \sqrt[p]{a}\ \omega$

Then, the root $d = \sqrt[p]{a}$, ω is the p^{th} root of unity

Comment

Step 3 of 4

Consider the polynomial $x^p - a \in F(x)$

p is a prime and $x^p - a$ is reducible in $F(x)$

Let us assume $d_1, d_2, \ldots\ldots\ldots\ldots d_p$ are the roots of $x^p - a$

Then,

$$x^p - a = (x-d_1)(x-d_2).......(x-d_p)$$

$p(x) = (x-d_1)(x-d_2).......(x-d_m)$ $p(x)$ is equal to the product of *m* number of these factors. Since, $\text{degree}\ p(x) = m$

Comment

Step 4 of 4

Let the Constant term of above equation is *b,* is the product of $d_1, d_2, ...d_m$

$$b = (d_1 d_2 .....d_m)$$

$$b = \sqrt[p]{a}\ ......\sqrt[p]{a}$$

$$b = \omega^k (\sqrt[p]{a})^m$$

$$b = \omega^k d^m$$

$$b = \left(\sqrt[p]{a}\right)^m \qquad (\because \omega^k = 1)$$

$$b = a^{m/p}$$

$$b = \left(a^m\right)^{1/p}$$

$$b^p = a^m$$

Hence, proved

Comment