A Book of Abstract Algebra | (2nd Edition)

Chapter AB, Problem 2E

Bookmark

Show all steps: ON

Problem

Prove that the following are true for any integers a, b, and c:

If $a|b$, then $a|(-b)$ and $(-a)|b$.

Step-by-step solution

Step 1 of 3

Objective:-

The objective is to prove $if\ a|b,\ then\ a|(-b)\ and\ (-a)|c.$

Comment

Step 2 of 3

Proof:-

Let suppose $a \mid b.$

Then there exist number *k* such that:-

$$b = ka \qquad \ldots\ldots(1)$$

Let us multiply both sides by $(-1).$

$$-b = -ka$$
$$-b = (-k)a$$
$$(-b) = la \qquad let\ \ (-k) = l$$

Thus, *a* is a factor of la that is a factor of $(-b)$. Hence, *a* divides $(-b)$ that is $a \mid (-b)$.

Comment

Step 3 of 3

Let suppose $a \mid b.$

Then there exist number *k* such that:-

$$b = ka \qquad \ldots\ldots(1)$$
$$b = k(a)(-1)(-1) \qquad \{\text{since}\ \ (-1)(-1) = 1\}$$
$$b = (-k)(-a)$$
$$b = m(-a) \qquad \{let\ \ (-k)\}$$

Thus, *a* is a factor of $m(-a)$ that is a factor of *b*. Hence, $(-a)$ divides $m(-a)$ that is $(-a) \mid b$.

Proved

Comment