A Book of Abstract Algebra | (2nd Edition)

Chapter 28, Problem 7EC

Bookmark

Show all steps: ON

Problem

Let U be the subspace of $\mathcal{F}(\mathbb{R})$ spanned by $\{\cos^2 x, \sin^2 x, \cos 2x\}$. Find the dimension of U, and then find a basis of U.

Step-by-step solution

Step 1 of 3

Basis of a vector space is set of linearly independent vectors in that space. To determine if given set forms a basis of any vector space, it must be shown that these vectors are linearly independent. That is one set of vectors cannot be obtained from linear combination of other vectors.

Comment

Step 2 of 3

Dimension of vector space is number of independent vectors that span a subspace

Set given in question is $\left(\sin^2 x, \cos^2 x, \cos 2x\right)$. Here observe that,

$$\cos^2 x = 1 - \sin^2 x$$

$$\cos 2x = 1 - 2\sin^2 x$$

1 is not in given set of basis. So $\sin^2 x$ and $\cos^2 x$ are independent vectors. But,

$$\cos 2x = \cos^2 x - \sin^2 x$$

In other words, $\cos 2x$ is linear combination of other vectors. Hence only $\sin^2 x$ and $\cos^2 x$ are independent. consequently dumension of given subspace is 2

Comment

Step 3 of 3

As $\sin^2 x$ and $\cos^2 x$ are linearly independent and $\cos 2x$ is not,

one possible set of basis is $\left(\sin^2 x, \cos^2 x\right)$.

Comment