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Chapter 24, Problem 2EG

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Problem

Let A and B be rings and let $h : A \to B$ be a homomorphism with kernel K. Define $\bar{h} : A[x] \to B[x]$ by

$$\bar{h}(a_0 + a_1x + \cdots + a_nx^n) = h(a_0) + h(a_1)x + \cdots + h(a_n)x^n$$

(We say that $\bar{h}$ is *induced by h*.)

Describe the kernel $\bar{K}$ of $\bar{h}$

Step-by-step solution

Step 1 of 1

Let $\ker(\bar{h}) = \bar{k}$ $\Rightarrow$ all $a(x) \in A(x)$ such that $\bar{h}(a(x)) = 0$ are in $\bar{k}$ where $a(x) = a_0 + a_1x + \ldots\ldots + a_nx^n$

$\Rightarrow \; h(a_0) + h(a_1)x + \ldots + h(a_n)x^n = 0$

$\Rightarrow \; h(a_0) = h(a_1) = \ldots = h(a_n) = 0$

$\Rightarrow$all $a_i, 0 \le i \le n$ are in k

$\Rightarrow \; \bar{k} = \{k[x] \mid k$ is kernel of $h : A \to B\}$

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