

A Book of Abstract Algebra | (2nd Edition)



Chapter 30, Problem 1EG



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Problem

Prove each of the following:

If the number a is a root of an irreducible polynomial $p(x) \in \mathbb{Q}[x]$ whose degree is not a power of 2, then a is not a constructible number.

Step-by-step solution

Step 1 of 3

Here, objective is to prove that a is not a constructible number.

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Step 2 of 3

Consider a is a root of a irreducible polynomial $p(x) \in \mathbb{Q}(x)$ whose degree is not a power of two.
That is the root a is not a real. So a is a complex root.

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Step 3 of 3

Complex number is constructible if and only if there exist a tower of complex field extensions.

$$\mathbb{Q}(x) = F_0 \leq F_1 \leq \dots \leq F_n$$

Where,

$$a \in F_n$$

$$[F_{i+1} : F_i] \leq 2 \quad ; \forall 0 \leq i < n$$

a is constructible if the degree of $\mathbb{Q}(x)$ is power of two.

Therefore, a is not a constructible number, if a is a root of a irreducible polynomial $p(x) \in \mathbb{Q}(x)$ whose degree is not a power of two.

Hence, proved

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