# A Book of Abstract Algebra | (2nd Edition)

Chapter 30, Problem 4EB

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## Problem

Prove each of the following:

If a point *P* is constructible from $\mathbb{Q} \times \mathbb{Q}$, it is constructible from {*O, I*}.

By combining parts 2 and 4, we get the following important fact: Any point *P* is constructible from $\mathbb{Q} \times \mathbb{Q}$ iff *P* is constructible from {*O,I*}. Thus, we may define a point to be *constructible* iff it is constructible from {*O, I*}.

## Step-by-step solution

**Step 1** of 4

Here, objective is to prove that if a point *p* is constructible from $Q \times Q$, then it is constructible from $(I, O)$.

Comment

**Step 2** of 4

Constructible point:

The point is either the end point of given unit segment or it is the intersection of two lines determined by previous constructible points is called as constructible point.

Comment

**Step 3** of 4

$Q \times Q$ is a set of all rational numbers

Let $p,q$ are rational numbers. Then the point $P(p,q) \in Q \times Q$

Let $p = \frac{a}{b}(\text{rational})$

But as per the definition of *D*, $(a,0)$ and $(0,b)$ are constructible from $(O,I)$

Comment

**Step 4** of 4

Consider the below figure:

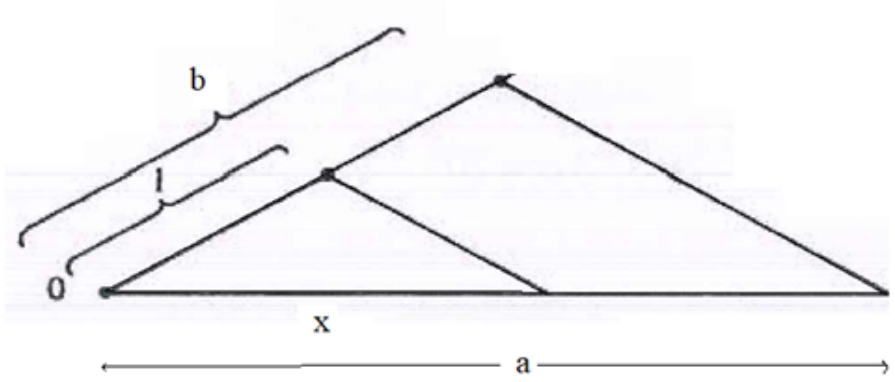


figure: construction of a/b

by observing there exist two similar triangles.

$a$ and $b$ are constructible lengths.

using the property of equal triangles, we have

$$\frac{b}{a}=\frac{1}{x}$$

$$x=\frac{a}{b}$$

Then,

The length $p=\frac{a}{b}$ is constructible from $\{O,I\}$

Similarly,

$q$ is also constructible from $\{O,I\}$

Therefore, the point *P* is constructible from $(I,O)$.

Comment