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Chapter 29, Problem 2EA

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Problem

Show that every element of $\mathbb{R}(2 + 3i)$ can be written as *a* + *bi*, where $a, b \in \mathbb{R}$ Conclude that $\mathbb{R}(2 + 3i) = \mathbb{C}$.

Step-by-step solution

Step 1 of 2

Objective is to prove that every element of $R(2+3i)$ can be written as $a+bi$, where $a, b \in R$ and then draw a conclusion that $R(2+3i) = C$.

By the definition of extension field, the elements of $R(2+3i)$ will be of the following form:

$$R(2+3i) = \{x + y(2+3i) : x, y \in R\}.$$

Or in most simplified form:

$$\begin{aligned} R(2+3i) &= \{x + 2y + 3yi : x, y \in R\} \\ &= \{(x+2y) + (3y)i : x, y \in R\} \\ &= \{a + bi : a, b \in R\} \end{aligned}$$

where $a = x + 2y \in R$, and $b = 3y \in R$.

Comment

Step 2 of 2

Thus, $R(2+3i)$ consists of all the linear combinations of 1 and *i* with real coefficients, that is, all the $a+bi$, where $a,b \in R$.

Also, the set of all complex numbers consists of the all the elements of the form:

$C = \{a+bi : a,b \in R\}$.

Clearly, then $R(2+3i) = C$, as required.

Comment