

A Book of Abstract Algebra | (2nd Edition)

Chapter AA, Problem 11E

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Problem

Prove the following:

$$A \cap (B - C) = (A \cap B) - C.$$

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $A \cap (B - C) = (A \cap B) - C$.

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Step 2 of 2

Proof:-

Let A and B are two sets.

The difference of two sets A and B is:-

$$A - B = \{x : x \in A, x \notin B\}$$

The intersection of two sets A and B is:-

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Let $x \in A \cap (B - C)$

$$x \in A \cap (B - C)$$

$$\Rightarrow x \in A \text{ and } (x \in B - C)$$

$$\Rightarrow x \in A \text{ and } (x \in B, x \notin C)$$

$$\Rightarrow (x \in A \text{ and } x \in B), (x \notin C)$$

$$\Rightarrow (x \in A \cap B), (x \notin C)$$

$$\Rightarrow (x \in A \cap B, x \notin C)$$

$$\Rightarrow x \in A \cap B - C$$

So,

$$A \cap (B - C) \subseteq (A \cap B) - C \quad \dots\dots(1)$$

Let $x \in A \cap B - C$

$$x \in A \cap B - C$$

$$\Rightarrow (x \in A \cap B, x \notin C)$$

$$\Rightarrow (x \in A \cap B), (x \notin C)$$

$$\Rightarrow (x \in A \text{ and } x \in B), (x \notin C)$$

$$\Rightarrow x \in A \text{ and } (x \in B, x \notin C)$$

$$\Rightarrow x \in A \text{ and } (x \in B - C)$$

$$\Rightarrow x \in A \cap (B - C)$$

So,

$$(A \cap B) \cup (A \cap B) - C \subseteq A \cap (B - C) \quad \dots\dots(2)$$

Let us consider the equation (1) and (2).

$$A \cap (B - C) = (A \cap B) - C$$

Proved

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