

A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 3EA

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Problem

Find the root field of $x^4 - 2$, first over \mathbb{Q} , then over \mathbb{R} .

Step-by-step solution

Step 1 of 3

The objective is to find the root field of $x^4 - 2$, over \mathbb{Q} and \mathbb{R} .

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Step 2 of 3

$$\begin{aligned} x^4 - 2 &= (x^2 - \sqrt{2})(x^2 + \sqrt{2}) \\ &= (x - 2^{1/4})(x + 2^{1/4})(x - 2^{1/4}i)(x + 2^{1/4}i) \end{aligned}$$

The roots of $x^4 - 2$ are $\pm 2^{1/4}$, $\pm 2^{1/4}i$.

Therefore, the root field of $x^4 - 2$ over \mathbb{Q} is $\mathbb{Q}(\pm 2^{1/4}, \pm 2^{1/4}i)$. This can be written simply as $\mathbb{Q}(2^{1/4}, i)$.

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Step 3 of 3

$$\begin{aligned} x^4 - 2 &= (x^2 - \sqrt{2})(x^2 + \sqrt{2}) \\ &= (x - 2^{1/4})(x + 2^{1/4})(x - 2^{1/4}i)(x + 2^{1/4}i) \end{aligned}$$

In \mathbb{R} , $\pm 2^{1/4}$ are roots of $x^4 - 2$.

Therefore, the root field of $x^4 - 2$ over \mathbb{R} is $\mathbb{R}(\pm 2^{1/4})$. This can be written simply as $\mathbb{R}(2^{1/4})$.

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