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Chapter AC, Problem 1E

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Problem

+ 3 + 5 + ··· + (2*n* – 1) = n^2

(That is, the sum of the first *n* odd integers is equal to n^2.)

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $1+3+5+\cdots+(2n-1)=n^2$ using mathematical induction.

Comment

Step 2 of 2

Proof:-

$$P(n)=1+3+5+\cdots+(2n-1)=n^2$$

Let consider rule for $n=1$.

$$P(1)=1$$
$$P(1)=1^2$$

Thus, this rule is true for $n=1$.

Let this rule is true for $n=k$.

$$P(k)=1+3+5+\cdots+(2k-1)=k^2 \qquad \ldots\ldots(1)$$

Let consider rule for $n=k+1$.

$$P(k+1)=1+3+5+\cdots+(2k-1)+(2(k+1)-1)$$
$$P(k+1)=1+3+5+\cdots+(2k-1)+(2k+2-1)$$
$$P(k+1)=1+3+5+\cdots+(2k-1)+(2k+1)$$

Use the equation (1).

$$P(k+1)=k^2+(2k+1)$$
$$P(k+1)=k^2+2k+1^2$$
$$P(k+1)=(k+1)^2$$

This rule also true for $n=k+1$. Hence, by mathematical induction this rule is true for all positive integer *n*.

Proved

Comment