

A Book of Abstract Algebra | (2nd Edition)

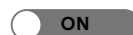


Chapter 28, Problem 7EC



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Problem

Let U be the subspace of $\mathcal{F}(\mathbb{R})$ spanned by $\{\cos^2 x, \sin^2 x, \cos 2x\}$. Find the dimension of U , and then find a basis of U .

Step-by-step solution

Step 1 of 3

Basis of a vector space is set of linearly independent vectors in that space. To determine if given set forms a basis of any vector space, it must be shown that these vectors are linearly independent. That is one set of vectors cannot be obtained from linear combination of other vectors.

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Step 2 of 3

Dimension of vector space is number of independent vectors that span a subspace

Set given in question is $(\sin^2 x, \cos^2 x, \cos 2x)$. Here observe that,

$$\cos^2 x = 1 - \sin^2 x$$

$$\cos 2x = 1 - 2\sin^2 x$$

1 is not in given set of basis. So $\sin^2 x$ and $\cos^2 x$ are independent vectors. But,

$$\cos 2x = \cos^2 x - \sin^2 x$$

In other words, $\cos 2x$ is linear combination of other vectors. Hence only $\sin^2 x$ and $\cos^2 x$ are independent. consequently dimension of given subspace is 2

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Step 3 of 3

As $\sin^2 x$ and $\cos^2 x$ are linearly independent and $\cos 2x$ is not,

one possible set of basis is $(\sin^2 x, \cos^2 x)$.

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