

A Book of Abstract Algebra | (2nd Edition)

Chapter 27, Problem 4EB

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Problem

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For each of the following polynomials $p(x)$, find a number a such that $p(x)$ is the minimum polynomial of a over \mathbb{Q} :
(a) $x^2 + 2x - 1$
(b) $x^4 + 2x^2 - 1$
(c) $x^4 - 10x^2 + 1$

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Step-by-step solution

Step 1 of 5

(a)

Objective is to determine a number a such that $p(x) = x^2 + 2x - 1$ is the minimum polynomial of a is over \mathbb{Q} .
The required such number a will be the solution of equation $x^2 + 2x - 1 = 0$.
Let $ax^2 + bx + c = 0$ be some quadratic equation where a is nonzero, then
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

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Step 2 of 5

Use the method of solving the quadratic equation and get
$$x^2 + 2x - 1 = 0$$
$$x = \frac{-2 \pm \sqrt{4 + 4}}{2}$$
$$x = -1 \pm \sqrt{2}.$$
Thus, the required a will be $-1 \pm \sqrt{2}$.

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Step 3 of 5

(b)

Objective is to determine a number a such that $p(x) = x^4 + 2x^2 - 1$ is the minimum polynomial of a is over \mathbb{Q} .
Let $x^2 = y$, then equation will become $y^2 + 2y - 1 = 0$. Use the method of solving the quadratic equation and get
$$y^2 + 2y - 1 = 0$$
$$y = \frac{-2 \pm \sqrt{4 + 4}}{2}$$
$$y = -1 \pm \sqrt{2}$$
$$x^2 = -1 \pm \sqrt{2}$$
Thus, the required a will be $\sqrt{-1 \pm \sqrt{2}}$.

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Step 4 of 5

(c)

Objective is to determine a number a such that $p(x) = x^4 - 10x^2 + 1$ is the minimum polynomial of a is over \mathbb{Q} .
Let $x^2 = y$, then equation will become $y^2 - 10y + 1 = 0$. Use the method of solving the quadratic equation and get
$$y^2 - 10y + 1 = 0$$
$$y = \frac{10 \pm \sqrt{100 - 4}}{2}$$
$$y = 5 \pm 2\sqrt{6}$$
$$x^2 = 5 \pm 2\sqrt{6}.$$

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Step 5 of 5

Thus, the required a will be $\sqrt{5 \pm 2\sqrt{6}}$.

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