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Chapter 33, Problem 4ED

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Problem

Let G be a group. The symbol $H \lhd G$ should be read, "H is a normal subgroup of G." A *maximal* normal subgroup of G is an $H \lhd G$ such that, if $H \lhd J \lhd G$, then necessarily $J = H$ or $J = G$. Prove the following:

If K is a maximal normal subgroup of G, then G/K has no nontrivial normal subgroups. (Use part 3.)

Step-by-step solution

Step 1 of 4

Here, objective is to prove that $\frac{G}{K}$ has no nontrivial normal subgroups.

Comment

Step 2 of 4

Consider G is a finite group. H is normal subgroup of G is denoted by $H \lhd G$

A maximal normal subgroup of G is given by

$H \lhd G$, if $H \lhd J \lhd G$ then, necessarily $J = H$ or $J = G$

Comment

Step 3 of 4

If G is a finite group, then the group H is normal subgroup of G is denoted by $H \lhd G$

A simple group is a nontrivial, which have the subgroups trivial group and group itself.

If the group is not a simple , then it has nontrivial group and quotient group.

Comment

Step 4 of 4

Consider K is maximal normal subgroup of G which is represented by $K \triangleleft G$,

Then, $H \triangleleft K \triangleleft G$

$\frac{G}{H}$ is not a simple.

Since, H is not a maximal subgroup.

Then, $\frac{G}{K}$ is simple.

Since, K is maximal normal subgroup of G

So it has no nontrivial subgroups.

Hence, proved

Comment