A Book of Abstract Algebra | (2nd Edition)

Problem

Recall the definition of $F(a)$. It is a field such that (i) $F \subseteq F(a)$; (ii) $a \in F(a)$; (iii) any field containing F and a contains $F(a)$.

Use this definition to prove parts 1–5, where $F \subseteq K$, $c \in F$, and $a \in K$:

$F(a) = F(a + c)$ and $F(a) = F(ca)$. (Assume $c \neq 0$.)

Step-by-step solution

Step 1 of 6

Using definition of $F(a)$ and $F \subseteq K$, $c \in F$, $a \in K$. Prove that $F(a) = F(a+c)$ and $F(a) = F(ca)$.(Assume $c \neq 0$)

Comment

Step 2 of 6

$F(a)$ is a filed such that $F \subseteq F(a)$, $a \in F(a)$ and any field containing F and a contains $F(a)$.

Comment

Step 3 of 6

By above definition of , $F \subseteq F(a+c)$ and $a+c \in F(a+c)$. Since $c \in F$, therefore $a \in F(a+c)$.

Hence, $F(a+c)$ is field containing F and a. Therefore by above definition $F(a) \subseteq F(a+c)$.---------- **(i)**

Also, $F(a)$ contains F and a. Since $c \in F$, therefore $a+c \in F(a)$. Again by above definition, $F(a+c) \subseteq F(a)$.---------- **(ii)**

Comment

Step 4 of 6

By **(i)** and **(ii)**,

$F(a) = F(a+c)$.

By above definition of , $F \subseteq F(ca)$ and $ca \in F(ca)$. Since $c \in F$, therefore $a \in F(ca)$.

Hence, $F(ca)$ is field containing F and a. Therefore by above definition $F(a) \subseteq F(ca)$.---------- **(i)**

Also, $F(a)$ contains F and a. Since $c \in F$, therefore $ca \in F(a)$. Again by above definition, $F(ca) \subseteq F(a)$.---------- **(ii)**

Comment

Step 5 of 6

By **(i)** and **(ii)**,

$F(a) = F(ca)$.

Comment

Step 6 of 6

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Comment