Contents

1 Exercise 2.2.8

$$f(x,y) = (bx - ay)^k g(x,y)$$

Differentiating f we get

$$f'(x,y) = k(bx - ay)^{k-1} g(x,y) + (bx - ay)^k g'(x,y)$$

Note that constants don't matter to us so we can abstract this system

$$f = u^4 g, \quad u' = 1, \quad A + A = A$$

$$f' = (u^k g)' = u^{k-1} g + u^k g'$$

Applying the rule recursively, we see that eventually g is a term. But we know that $g(P) \neq 0$ and $u(P) = 0$.

$$\begin{aligned}
f &= u^4 g \\
f' &= u^3 g + u^4 g' \\
f'' &= u^2 g + u^3 g' + u^4 g'' \\
f''' &= ug + u^2 g' + u^3 g'' + u^4 g''' \\
f'''' &= g + ug' + u^2 g'' + u^3 g''' + u^4 g''''
\end{aligned}$$

2 Exercise 2.2.12

$$\begin{aligned}
f(x,y,z) &= (t_0 s - s_0 t)^k g(a_1 s + b_1 t, a_2 s + b_2 t, a_3 s + b_3 t) \\
&= (t_0 s - s_0 t)^k g(c_1 u + d_1 v, c_2 u + d_2 v, c_3 u + d_3 v) \\
&= (u_0 v - v_0 u)^j g(c_1 u + d_1 v, c_2 u + d_2 v, c_3 u + d_3 v)
\end{aligned}$$

Because (s,t) and (u,v) define the same line, there is a transform between them defined by

$$\begin{aligned}
u &= \alpha_1 s + \beta_1 t \\
v &= \alpha_2 s + \beta_2 t
\end{aligned}$$

Now we saw that $(t_0 s - s_0 t)^k = (u_0 v - v_0 u)^j$ so

$$\begin{aligned}
(t_0 s - s_0 t)^k &= (u_0 v - v_0 u)^j \\
&= (u_0(\alpha_2 s + \beta_2 t) - v_0(\alpha_1 s + \beta_1 t))^j \\
&= (s(\alpha_2 u_0 - v_0 \alpha_1) + t(\beta_2 u_0 - \beta_1 v_0))^j
\end{aligned}$$

Taking $s(\alpha_2 u_0 - v_0 \alpha_1) + t(\beta_2 u_0 - \beta_1 v_0)$ which is a linear equation and substituting (s_0, t_0), see that that it's zero and so conclude

$$t_0 s - s_0 t = s(\alpha_2 u_0 - v_0 \alpha_1) + t(\beta_2 u_0 - \beta_1 v_0)$$

$$\Rightarrow k = j$$

Alternatively we can argue by expanding both sides out and observing the degrees of s and t that $k = j$.