

A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 6EA

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Problem

Which of the following extensions are root fields over \mathbb{Q} ? Justify your answer:

$\mathbb{Q}(i)$; $\mathbb{Q}(\sqrt{2})$; $\mathbb{Q}(\sqrt[3]{2})$, where $\sqrt[3]{2}$ is the *real* cube root of 2;

$\mathbb{Q}(2+\sqrt{5})$; $\mathbb{Q}(i+\sqrt{3})$; $\mathbb{Q}(i, \sqrt{2}, \sqrt{3})$.

Step-by-step solution

Step 1 of 2

The objective is to find whether the following extensions are root fields over \mathbb{Q} or not, with justification.

$\mathbb{Q}(i)$; $\mathbb{Q}(\sqrt{2})$; $\mathbb{Q}(\sqrt[3]{2})$ where $\sqrt[3]{2}$ is the real cube root of 2; $\mathbb{Q}(2+\sqrt{5})$; $\mathbb{Q}(i+\sqrt{3})$; $\mathbb{Q}(i, \sqrt{2}, \sqrt{3})$.

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Step 2 of 2

$\mathbb{Q}(i)$ is the root field of $x^2 + 1$.

$\mathbb{Q}(\sqrt{2})$ is the root field of $x^2 - 2$.

$\mathbb{Q}(2+\sqrt{5})$ is same as $\mathbb{Q}(\sqrt{5})$. $\mathbb{Q}(\sqrt{5})$ is the root field of $x^2 - 5$.

$\mathbb{Q}(i, \sqrt{2}, \sqrt{3})$ is the root field of $x^6 - 4x^4 + x^2 + 6$.

$\mathbb{Q}(\sqrt[3]{2})$ is not a root field over \mathbb{Q} because it does not contains complex cube roots of 2.

$\mathbb{Q}(i+\sqrt{3})$ is not a root field over \mathbb{Q} because it does not contains the complex root $i-\sqrt{3}$.

Therefore, $\mathbb{Q}(i)$; $\mathbb{Q}(\sqrt{2})$; $\mathbb{Q}(2+\sqrt{5})$; and $\mathbb{Q}(i, \sqrt{2}, \sqrt{3})$ are root field over \mathbb{Q} , and

$\mathbb{Q}(\sqrt[3]{2})$ where $\sqrt[3]{2}$ is the real cube root of 2 and $\mathbb{Q}(i+\sqrt{3})$ are not root field over \mathbb{Q} .

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