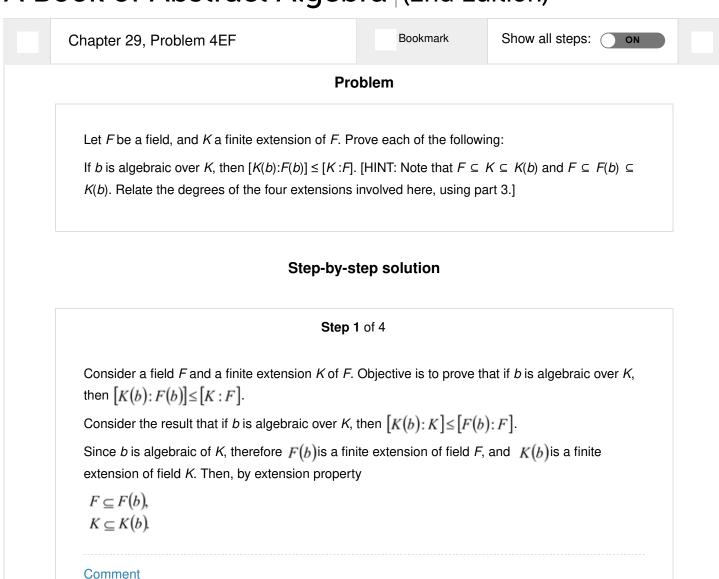
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## **Step 2** of 4

On combining the above relationship, one gets

$$F \subseteq F(b) \subseteq K(b)$$

Also K is a finite extension of F, so  $F \subseteq K$ . And then,

$$F \subseteq K \subseteq K(b)$$

Now by the formula for degree calculation of field,

$$[K(b):F] = [K(b):F(b)] \cdot [F(b):F],$$

and

$$[K(b):F]=[K(b):K]\cdot [K:F].$$

Comment

## **Step 3** of 4

Equate both the relations solve as follows:

$$\begin{split} \big[ K(b) \colon & F(b) \big] \cdot \big[ F(b) \colon F \big] &= \big[ K(b) \colon K \big] \cdot \big[ K \colon F \big] \\ &\frac{\big[ K(b) \colon F(b) \big]}{\big[ K \colon F \big]} &= \frac{\big[ K(b) \colon K \big]}{\big[ F(b) \colon F \big]}, \\ &\frac{\big[ K(b) \colon F(b) \big]}{\big[ K \colon F \big]} &\leq 1, \\ &\big[ K(b) \colon F(b) \big] \leq \big[ K \colon F \big]. \end{split}$$

The third step is obtained from the condition that  $[K(b):K] \leq [F(b):F]$ , because it implies that  $[K(b):K] \leq 1$ .

Comment

Hence, if $b$ is algebraic over $K$ , then $[K(b):F(b)] \leq [K:F]$ .
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