A Book of Abstract Algebra | (2nd Edition)

Chapter 27, Problem 3EE | Bookmark | Show all steps: ON

Problem

Recall the definition of *F*(*a*). It is a field such that (i) $F \subseteq F(a)$; (ii) $a \in F(a)$; (iii) any field containing *F* and *a* contains *F*(*a*).

Use this definition to prove parts 1–5, where $F \subseteq K$, $c \in F$, and $a \in K$:

$a + c$ is a root of $p(x)$ iff a is a root of $p(x + c)$; *ca* is a root of $p(x)$ iff *a* is a root of $p(cx)$.

Step-by-step solution

Step 1 of 3

Using definition of $F(a)$ and $F \subseteq K$, $c \in F$, $a \in K$. Prove that $a+c$ is a root of $p(x)$ iff a is a root of $p(x+c)$; ca is a root of $p(x)$ iff a is root of $p(cx)$.

Comment

Step 2 of 3

Assume $a+c$ is root of $p(x)$. Then $p(a+c)=0$. And this implies a is a root of $p(x+c)$ because if we put $x=a$ in $p(x+c)$ and use the assumption. Then $p(a+c)=0$.

Now assume a is root of $p(x+c)$. Then $p(a+c)=0$. And this implies $a+c$ is a root of $p(x)$ because if we put $x=a+c$ in $p(x)$ and use the assumption. Then $p(a+c)=0$.

Comment

Step 3 of 3

Assume ca is root of $p(x)$.

Then $p(ca)=0$.

This implies a is a root of $p(cx)$ because if we put $x=a$ in $p(cx)$ and use the assumption. Then $p(ca)=0$.

Now assume a is root of $p(cx)$. Then $p(ca)=0$. And this implies ca is a root of $p(x)$ because if we put $x=ca$ in $p(x)$ and use the assumption. Then $p(ca)=0$.

Comment