

A Book of Abstract Algebra | (2nd Edition)

Chapter AC, Problem 5E

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Problem

Use mathematical induction to prove the following:

$$\frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{n}{(n+1)!} = \frac{n! - 1}{n!}$$

Step-by-step solution

Step 1 of 3

The objective is to prove that $\frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{n}{(n+1)!} = \frac{(n+1)! - 1}{(n+1)!}$ using mathematical induction.

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Step 2 of 3

Step 1: For $n = 1$,

$$\begin{aligned} \frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{n}{(n+1)!} &= \frac{1}{2!} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \frac{(n+1)! - 1}{(n+1)!} &= \frac{(1+1)! - 1}{(1+1)!} \\ &= \frac{1}{2} \end{aligned}$$

The theorem holds for $n = 1$.

Assume that the theorem is true for $n = k$.

$$\text{Then, } \frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{k}{(k+1)!} = \frac{(k+1)! - 1}{(k+1)!} \quad \dots \quad (1)$$

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Step 3 of 3

For $n = k + 1$,

$$\begin{aligned}\frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{k+1}{((k+1)+1)!} &= \left(\frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{k}{(k+1)!} \right) + \frac{k+1}{(k+2)!} \\ &= \frac{(k+1)! - 1}{(k+1)!} + \frac{k+1}{(k+2)!} \quad [\text{using (1)}] \\ &= \frac{(k+2)((k+1)! - 1) + k+1}{(k+2)!} \\ &= \frac{(k+2)! - 1}{(k+2)!}\end{aligned}$$

The theorem is true for $n = k + 1$.

Therefore, it is proved that $\frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{n}{(n+1)!} = \frac{(n+1)! - 1}{(n+1)!}$ using mathematical induction.

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