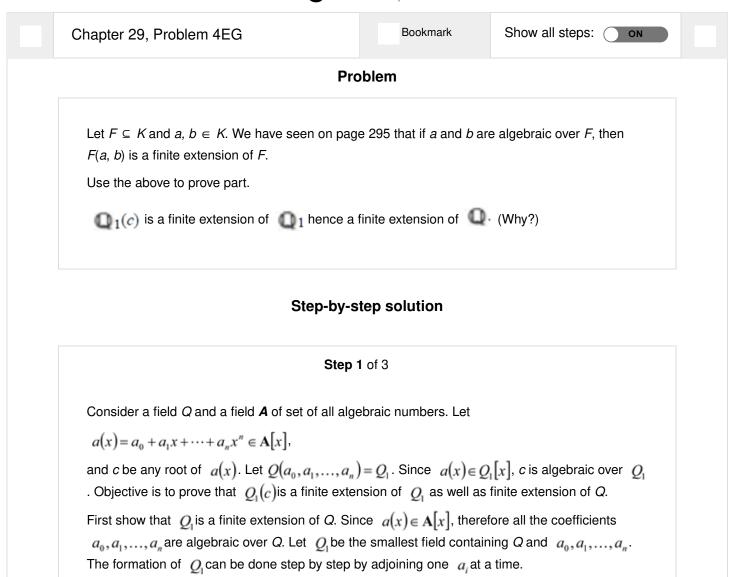
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Step 2 of 3

Note that, the degree of F(c) over F is equal to the degree of the minimal polynomial of c over F.

If a_0, a_1, \dots, a_n are algebraic over Q, then by this result, each extension in

$$Q \subseteq Q(a_0) \subseteq Q(a_0, a_1) \subseteq Q(a_0, a_1, a_2) \subseteq \cdots \subseteq Q(a_0, a_1, \dots, a_n)$$

is a finite extension. Again by the theorem of finite extension, $Q(a_0, a_1)$ is a finite extension of Q. Also, $Q(a_0, a_1, a_2)$ is a finite extension of Q, and so on.

Thus, Q_i is a finite extension of Q.

Comment

Step 3 of 3

Since c is algebraic over Q_1 , then an extension $Q_1(c)$ can be formed by adjoining a single element to Q_1 . This is known as simple extension. Since simple extensions are always finite, therefore $Q_1(c)$ is a finite extension of Q_1 .

Since $Q_1(c)$ is a finite extension of Q_1 and Q_1 is a finite extension of Q_2 , therefore $Q_1(c)$ is also a finite extension of Q_2 .

Comment