# A Book of Abstract Algebra | (2nd Edition)

Chapter 33, Problem 1EA

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## Problem

Find radical extensions of $\mathbb{Q}$ containing the following complex numbers:

(*a*) $(\sqrt{5}-\sqrt[5]{2})/(\sqrt[4]{3}+\sqrt[3]{4})$

(*b*) $\sqrt{(1-\sqrt[9]{2})/\sqrt[3]{1-\sqrt{5}}}$

(*c*) $\sqrt[5]{(\sqrt{3}-2i)^3/(i-\sqrt{11})}$

## Step-by-step solution

**Step 1** of 5

Here, objective is to find the radical extensions of *Q* containing the given complex numbers.

Comment

**Step 2** of 5

*Q* is the field of rational numbers.

Radical Extension:

A field extension $E:Q$ is called as radical extension, if there is a tower of field extensions $E=Q(u_1,u_2,.....u_n):Q(u_1,u_2,....u_{n-1}):Q(u_1):Q$

Comment

**Step 3** of 5

(a)

Consider the complex number $(\sqrt{5}-\sqrt[5]{2})/(\sqrt[4]{3}+\sqrt[3]{4})$

The radical extensions are

$$E: Q(\sqrt{5}, \sqrt[5]{2}, \sqrt[4]{3}, \sqrt[3]{4})$$
$$(\sqrt{5} - \sqrt[5]{2}) / (\sqrt[4]{3} + \sqrt[3]{4})$$

Comment

**Step 4** of 5

(b)

Consider the complex number $\sqrt{(1-\sqrt[9]{2}) / (\sqrt[3]{1-\sqrt{5}})}$

The radical extensions are

$$E: Q(\sqrt[9]{2}, \sqrt[18]{2}, \sqrt{5}, \sqrt[6]{5}, \sqrt[12]{5})$$
$$\sqrt{(1-\sqrt[9]{2}) / (\sqrt[3]{1-\sqrt{5}})}$$

Comment

**Step 5** of 5

(c)

Consider the complex number $\sqrt[5]{(\sqrt{3}-2i)^3 / (i-\sqrt{11})}$

$$\sqrt[5]{(\sqrt{3}-2i)^3 / (i-\sqrt{11})}$$
$$= \left( \frac{(\sqrt{3}-2i)^3}{(i-\sqrt{11})} \times \frac{(i+\sqrt{11})}{(i-\sqrt{11})} \right)^{1/5}$$
$$= \left( \frac{(\sqrt{3}-2i)^3 (i+\sqrt{11})}{(-1-11)} \right)^{1/5}$$
$$= \left( \frac{(9\sqrt{33} - 10 + 10i\sqrt{11} + 9\sqrt{3})}{12} \right)^{1/5}$$

The radical extensions are

$$E: Q(\sqrt{33}.\sqrt{11}, i, \sqrt[10]{33}, \sqrt[10]{11}, \sqrt[10]{3}, i^{1/5})$$

Comment