A Book of Abstract Algebra | (2nd Edition)

Chapter 17, Problem 2EB

Bookmark

Show all steps: ON

Problem

Describe the divisors of zero in $\mathcal{F}(\mathbb{R})$.

Step-by-step solution

Step 1 of 3

Consider that the set $F(R)$ of all the function from real number *R* to *R*, with the following addition and multiplication:

$$(f+g)(x)=f(x)+g(x),$$
$$(fg)(x)=f(x)g(x),$$

for every real number *x*.

Note that, $F(R)$ satisfies all the axioms to be a commutative ring with unity. The zero element is the zero function, the unity is constant function 1, and the negative of any *f* is $-f$ in $F(R)$.

Objective is to describe the divisors of zero in $F(R)$.

Comment

Step 2 of 3

A nonzero function $f \in F(R)$ is a divisor of zero if there exists some nonzero function $g \in F(R)$ such that $fg=0$, here 0 is the zero element which is zero function in $F(R)$.

The product $fg=0$ means that for all *x* in real numbers,

$$(fg)(x)=0$$
$$f(x)g(x)=0.$$

Suppose that

$$f(x)=\begin{cases} 1, & x \text{ is even} \\ 0, & x \text{ is odd} \end{cases},$$

and

$$g(x)=\begin{Bmatrix} 0, & x \text{ is even} \\ 1, & x \text{ is odd} \end{Bmatrix}.$$

Both the functions are nonzero but their product is

$$(fg)(x)=0$$

for all real *x*.

Comment

Step 3 of 3

Thus, $F(R)$ contains the divisor of zero.

Comment