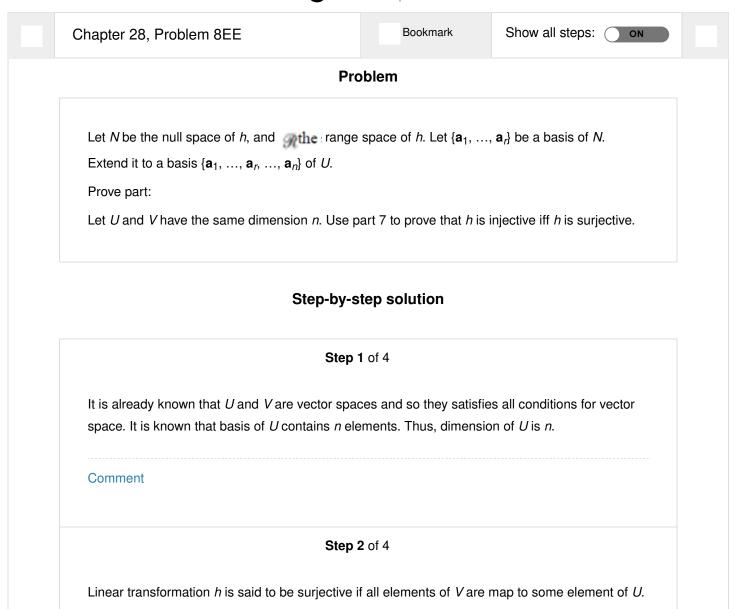
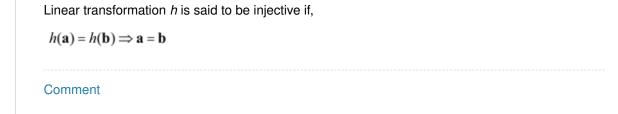
A Book of Abstract Algebra (2nd Edition)





Step 3 of 4

From question 7 of this section,

Hence dim(domain of h) = $n = r + n - r = \dim(\text{nullspace of } h) + \dim(\text{range space of } h)$

It is also given that,

$$\dim V = \dim U = n$$

Since *h* is surjective, rangespace of *h* is same as *V*. Thus,

$$\dim(\text{range space of } h) = \dim V = n$$

Substituting this in result of question 7,

 $\dim(\operatorname{domain} \operatorname{of} h) = n = r + n - r = \dim(\operatorname{nullspace} \operatorname{of} h) + \dim(\operatorname{range} \operatorname{space} \operatorname{of} h)$

$$\Rightarrow$$
 dim $U = n = n - r = dim(nullspace of h) + dim V$

$$\Rightarrow$$
 dim(nullspace of h)+dim $V = n$

$$\Rightarrow$$
 dim(nullspace of h) + n = n

$$\Rightarrow$$
 dim(nullspace of h) = 0

$$\Rightarrow$$
 nullspace of $h = \{0\}$

Thus, h is injective if h is surjective

Comment

Step 4 of 4

Now if *h* is injective, then

nullspace of $h = \{0\}$ $\Rightarrow \dim(\text{range space of } h) = n$

Or, h is surjective if h is injective

Hence it can be said that h is injective iff h is surjective

Comment