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A Book of Abstract Algebra | (2nd Edition)



Chapter 24, Problem 1EB



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Problem

Is $x^8 + 1 = x^3 + 1$ in $\mathbb{Z}_5[x]$? Explain your answer.

Step-by-step solution

Step 1 of 1

Consider two polynomials $a(x) = x^8 + 1$ and $b(x) = x^3 + 1$ in $\mathbb{Z}_5[x]$. Objective of this question is to find these two polynomials are equal or not.

Rewrite these polynomials as follows.

$$a(x) = x^8 + 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 1 \dots\dots(1)$$

$$b(x) = 0x^8 + 0x^7 + 0x^6 + 0x^5 + 0x^4 + x^3 + 0x^2 + 0x + 1 \dots\dots(2)$$

Now recall the rule for equality of two polynomials.

$a(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ and $b(x) = b_n x^n + b_{n-1} x^{n-1} + \dots + b_1 x + b_0$ are said to be equal when $a_n = b_n, a_{n-1} = b_{n-1}, \dots, a_1 = b_1, a_0 = b_0$ in the given ring.

Compare equation (1) and (2).

Coefficient of x^8 of $a(x) = 1$

Coefficient of x^8 of $b(x) = 0$

Then, its coefficients of corresponding terms are not equal.

Hence these polynomials are not equal in $\mathbb{Z}_5[x]$.

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