

A Book of Abstract Algebra | (2nd Edition)

Chapter 30, Problem 5EA

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Problem

If O and I are any two points in the plane, consider a coordinate system such that the interval OI coincides with the unit interval on the x axis. Let \mathbb{D} be the set of real numbers such that $a \in \mathbb{D}$ iff the point $(a, 0)$ is constructible from $\{O, I\}$. Prove the following:



$$\mathbb{Q} \subseteq \mathbb{D}.$$

Step-by-step solution

Step 1 of 4

Here, objective is to prove that $\mathbb{Q} \subseteq \mathbb{D}$.

Consider \mathbb{D} is the field of constructible numbers.

\mathbb{Q} is the set of rational numbers.

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Step 2 of 4

Constructible point:

This point is either the end point of given unit segment or it is the intersection of two lines determined by previous constructible points.

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Step 3 of 4

Consider the below figure:

$a \in D$ then, the point $(a,0)$ is constructible from $\{O, P\}$ and $(p,0), (0,q)$ are constructible lengths.

Unit point $B = (0,1)$

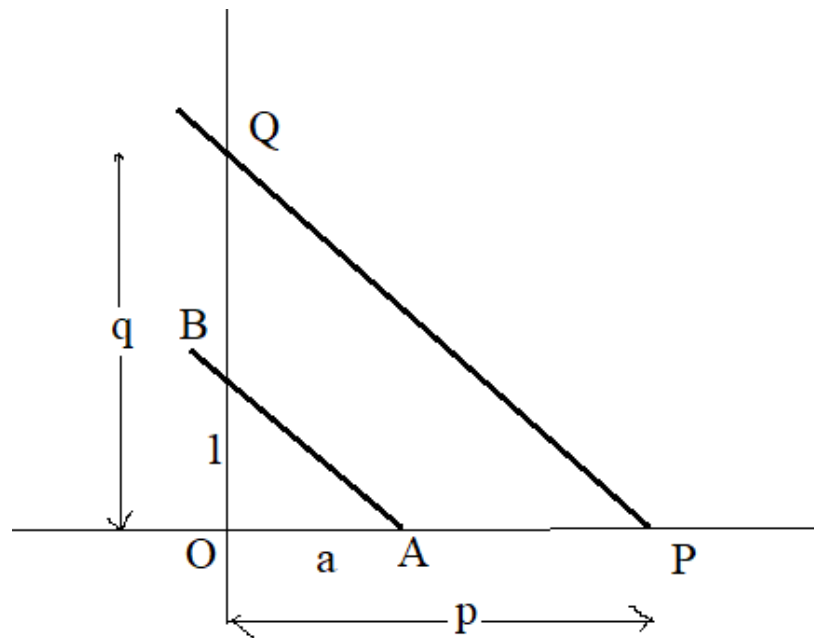


fig:construction of p/q

By observing there exist two similar triangles

$\triangle AOB$ and $\triangle POQ$

Using the property of similar triangles, we have

$$\frac{a}{1} = \frac{p}{q}$$

$$a = \frac{p}{q}$$

Therefore, $a \in Q$, where $a = \frac{p}{q} (q \neq 0)$,

Since $a = \frac{p}{q}$ is a rational number.

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Step 4 of 4

That means, the element $a = \frac{p}{q}$ is belongs to Q is the element of D , which implies $Q \subseteq D$.

Hence, proved

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