

# A Book of Abstract Algebra | (2nd Edition)

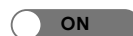


Chapter 30, Problem 6EG



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## Problem

Prove each of the following:

The circle  $x^2 + y^2 + ax + by + c = 0$  is constructible if  $a, b, c \in \mathbb{D}$ .

## Step-by-step solution

### Step 1 of 4

Here, objective is to prove that the circle  $x^2 + y^2 + ax + by + c = 0$  is constructible, if  $a, b, c \in D$ .

Consider  $a \in D$  if and only if the point  $(a, 0)$  is constructible from  $\{O, I\}$

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### Step 2 of 4

A circle is constructible if its centre and radius are constructible.

The standard form of circle equation is

$$(x - A)^2 + (y - B)^2 = R^2$$

centre- $(A, B)$

radius- $R$

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### Step 3 of 4

Consider the equation of the circle  $x^2 + y^2 + ax + by + c = 0$

Transform in to standard form of circle,

$$x^2 + y^2 + ax + by + c = 0$$

$$x^2 + y^2 + ax + by + \frac{a^2}{4} + \frac{b^2}{4} = \frac{a^2}{4} + \frac{b^2}{4} - c$$

$$\left(x + \frac{a}{2}\right)^2 + \left(y + \frac{b}{2}\right)^2 = \frac{a^2}{4} + \frac{b^2}{4} - c$$

$$\text{centre} = \left(-\frac{a}{2}, -\frac{b}{2}\right)$$

$$\text{radius} = \frac{a^2}{4} + \frac{b^2}{4} - c$$


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### Step 4 of 4

The centre and radius are constructible if and only if  $a, b, c \in D$

If the centre and radius are constructible, then circle  $x^2 + y^2 + ax + by + c = 0$  is constructible

Therefore, the circle  $x^2 + y^2 + ax + by + c = 0$  is constructible, if  $a, b, c \in D$ .

Hence, proved

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