A Book of Abstract Algebra | (2nd Edition)

Chapter 33, Problem 2EB

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Problem

Let G be a group. The symbol $H \lhd G$ is commonly used as an abbreviation of "H is a *normal* subgroup of G." A *normal series* of G is a finite sequence $H_0, H_1, \ldots, H_n$ of subgroups of G such that

$$\{e\} = H_0 \lhd H_1 \lhd \cdots \lhd H_n = G$$

Such a series is called a *solvable series* if each quotient group H_{i+1}/H_i is abelian. G is called a *solvable group* if it has a solvable series.

Let G be a solvable group, with a solvable series $H_0, \ldots, H_n$. Let K be a subgroup of G. Show that $J_0 = K \cap H_0, \ldots, J_n = K \cap H_n$ is a normal series of K.

Step-by-step solution

Step 1 of 4

Here, objective is to prove that $J_0 = K \cap H_0, \ldots\ldots\ldots, J_n = K \cap H_n$ is a normal series of K.

Comment

Step 2 of 4

Consider G is a group and normal series of G is a finite sequence $H_0, H_1, \ldots\ldots\ldots, H_n$ of subgroups of G, such that $\{e\} = H_0 \lhd H_1 \lhd \ldots\ldots\ldots \lhd H_n = G$ such type of series is called solvable series.

H is normal subgroup of G is represented by $H \lhd G$.

Comment

Step 3 of 4

Let *G* is a solvable group with series $H = H_0, H_1, \ldots\ldots\ldots, H_n$ and

K be the subgroup of *G.*

Consider the finite sequence $H = H_0, H_1, \ldots\ldots\ldots, H_n$ is a subgroup of G,

That is *K* and *H* are subgroups of *G.*

Then, $KH = HK$ for every H. So, HK is a subgroup of *G.*

Comment

Step 4 of 4

If, HK is a subgroup of *G.* Then, *K* is normal to HK and

$K \bigcap H$ is normal to *K.*

So, the series $K \bigcap H_0, K \bigcap H_1, \ldots\ldots\ldots\ldots, K \bigcap H_n$ is normal to *K.*

Therefore,

$J_0 = K \bigcap H_0, \ldots\ldots\ldots\ldots, J_n = K \bigcap H_n$ is a normal series of *K.*

Hence, proved

Comment