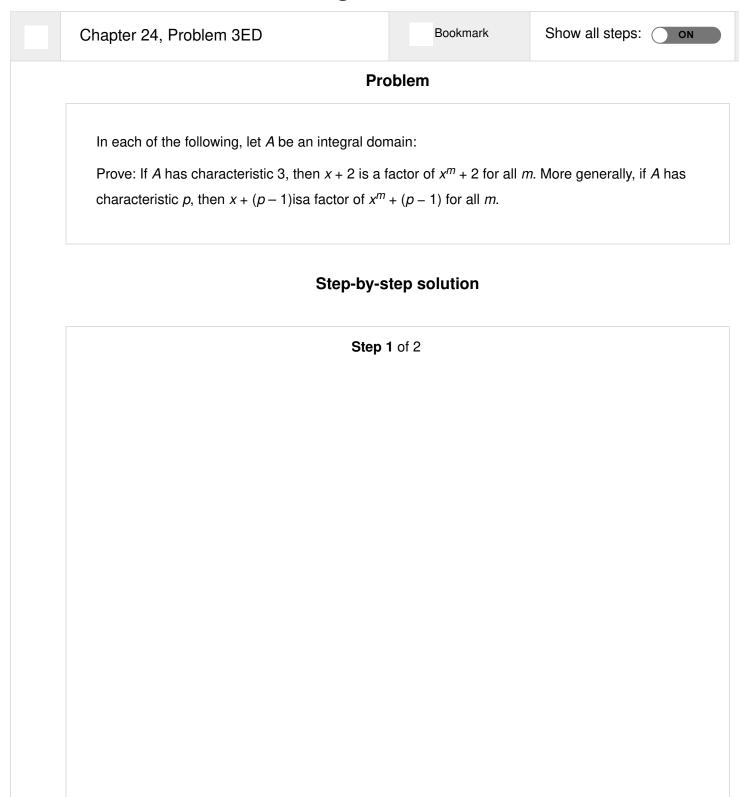
## A Book of Abstract Algebra (2nd Edition)



Step 1 of 2

Here, we use induction method to prove

**Basic case**: for m=1

 $x^m + 2 = x + 2$ , so obviously true

**Hypothesis**: assume x+2 is factor of  $x^m+2$  for m=k

 $\Rightarrow x+2$  is a factor of  $x^k+2$ 

**To prove:** x+2 is a factor of  $x^m+2$  for m=k+1

As 3 is characteristic of A so

$$x^{k+1} + 2 \equiv x^{k+1} + 3x^k + 2$$
$$\equiv x^{k+1} + 2x^k + x^k + 2$$
$$\equiv x^k (x+2) + x^k + 2$$

 $\Rightarrow$  x+2 is a factor of  $x^{k+1}+2$  so true for m=k+1

Hence, x+2 is a factor of  $x^m+2$  for all m

Comment

## **Step 2** of 2

Step 2 of 2

Just like in step 1, here also, we use induction method to prove.

**Basic case**: for m=1

$$x^m + (p-1) = x + (p-1)$$
, so obviously true

**Hypothesis**: assume x+(p-1) is a factor of  $x^m+(p-1)$  for m=k

$$\Rightarrow x + (p-1)$$
 is a factor of  $x^k + (p-1)$ 

**To prove:** x+(p-1) is a factor of  $x^m+(p-1)$  for m=k+1

As p is characteristic of A

$$x^{k+1} + (p-1) \equiv x^{k+1} + px^k + (p-1)$$
$$\equiv x^{k+1} + (p-1)x^k + x^k + (p-1)$$
$$\equiv x^k (x + (p-1)) + x^k + (p-1)$$

 $\Rightarrow$  x + (p-1) is a factor of  $x^{k+1} + (p-1)$  so true for m = k+1

Therefore, x+(p-1) is a factor of  $x^m+(p-1)$  for all m

Comment