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Chapter AC, Problem 3E

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Problem

Use mathematical induction to prove the following:

$1^2 + 2^2 + \cdots + n^2 = \frac{1}{6} n(n + 1)(2n + 1)$

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $1^2+2^2+3^2+\cdots+n^2=\frac{1}{6}n(n+1)(2n+1)$ using mathematical induction.

Comment

Step 2 of 2

Proof:-

$$p(n):1^2+2^2+3^2+\cdots+n^2=\frac{1}{6}n(n+1)(2n+1)$$

Let consider rule for $n=1.$

$$P(1):1^2=\frac{1}{6}\cdot 1(1+1)(1+2)$$

$$P(1):1=\frac{1}{6}\cdot 6$$

$$p(1):1=1$$

This rule is true for $n=1.$

Let this rule is true for $n=k.$

$$p(k):1^2+2^2+3^2+\cdots+k^2=\frac{1}{6}k(k+1)(2k+1).....(1)$$

Let consider rule for $n=k+1$.

$$p(k+1):1^2+2^2+3^2+\cdots+k^2+(k+1)^2$$

Use the equation (1).

$$P(k+1):\frac{1}{6}k(k+1)(2k+1)+(k+1)^2$$

$$P(k+1):(k+1)\left[\frac{1}{6}k(2k+1)+k+1\right]$$

$$P(k+1):(k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$$

$$P(k+1):\frac{1}{6}(k+1)(2k^2+7k+6)$$

$$P(k+1):\frac{(k+1)}{6}\left[2k^2+4k+3k+6\right]$$

$$P(k+1):\frac{(k+1)}{6}\left[2k(k+2)+3(k+2)\right]$$

$$P(k+1):\frac{1}{6}(k+1)(k+2)(2k+3)$$

$$P(k+1):\frac{\left[(k+1)\left((k+1)+1\right)\left\{2(k+1)+1\right\}\right]}{6}$$

This result also true for $n=k+1$. Hence, by mathematical induction this rule is true for all positive integer *n*.

Proved

Comment