

# A Book of Abstract Algebra | (2nd Edition)

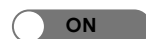


Chapter AA, Problem 10E



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## Problem

*Prove the following:*

$$A \cap (A \cup B) = A.$$

## Step-by-step solution

### Step 1 of 2

#### Objective:-

The objective is to prove  $A \cup (A \cap B) = A$ .

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### Step 2 of 2

Proof:-

Let  $A$  and  $B$  are two sets.

The union of two sets  $A$  and  $B$  is:-

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

The intersection of two sets  $A$  and  $B$  is:-

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Let  $x \in A \cap (B \cup C)$ .

$$x \in A \cap (B \cup C)$$

$$\Rightarrow x \in A \text{ and } (x \in B \cap C)$$

$$\begin{aligned}
&\Rightarrow x \in A \text{ and } x \in B \text{ or } x \in C \\
&\Rightarrow (x \in A \text{ and } x \in B) \text{ and } (x \in A \text{ or } x \in C) \\
&\Rightarrow (x \in A \cap B) \text{ or } (x \in A \cap C) \\
&\Rightarrow x \in (A \cap B) \cup (A \cap C)
\end{aligned}$$

So,

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \quad \dots\dots(1)$$

Let  $x \in A \cup (B \cap C)$ .

$$\begin{aligned}
&x \in (A \cap B) \cup (A \cap C) \\
&\Rightarrow (x \in A \cap B) \text{ or } (x \in A \cap C) \\
&\Rightarrow (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C) \\
&\Rightarrow x \in A \text{ and } (x \in B \text{ or } x \in C) \\
&\Rightarrow x \in A \cap (B \cup C)
\end{aligned}$$

So,

$$(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \quad \dots\dots(2)$$

Let us consider the equation (1) and (2).

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

According to this theorem:-

$$\begin{aligned}
A \cap (A \cup B) &= (A \cap A) \cup (A \cap B) \\
A \cap (A \cup B) &= A \cap (A \cap B) \\
A \cap (A \cup B) &= A
\end{aligned}$$

Since the common elements in set  $A$  and union of set  $A$  and  $B$  are elements of set  $A$ .

Proved

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