

A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 2EB

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Problem

Find the root field of $x^2 + x + 2$ over \mathbb{Z}_3 , and write its addition and multiplication tables.

Step-by-step solution

Step 1 of 4

The objective is to find the root field of $x^2 + x + 2$ over \mathbb{Z}_3 , and write its addition and multiplication tables.

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Step 2 of 4

The polynomial $x^2 + x + 2$ is of degree 2 and the root field is of at most degree 2 extension of \mathbb{Z}_3 .

Since $x^2 + x + 2$ is an irreducible polynomial over \mathbb{Z}_3 , therefore, the root field is of degree 2 extension and hence, the root field is of size 3^2 , that is, 9.

Let a be a root of the polynomial.

Then $a^2 + a + 2 = 0$.

$$\begin{aligned} a^2 &= -a - 2 \\ &= 2a + 1 \end{aligned}$$

By hit and trial, it is found that $0, 1, 2, a, 2a, 1+a, 2+a, 1+2a$, and $2+2a$ are the nine elements of $\mathbb{Z}_3(a)$.

Therefore, $\mathbb{Z}_3(0, 1, 2, a, 2a, 1+a, 2+a, 1+2a, 2+2a)$ is the root field of $x^2 + x + 2$ over \mathbb{Z}_3 .

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Step 3 of 4

The addition table is as follows:

+	0	1	2	a	$2a$	$1+a$	$2+a$	$1+2a$	$2+2a$
0	0	1	2	a	$2a$	$1+a$	$2+a$	$1+2a$	$2+2a$
1	1	2	0	$1+a$	$1+2a$	$2+a$	a	$2+2a$	$2a$
2	2	0	1	$2+a$	$2+2a$	a	$1+a$	$2a$	$1+2a$
a	a	$1+a$	$2+a$	$2a$	0	$1+2a$	$2+2a$	1	2
$2a$	$2a$	$1+2a$	$2+2a$	0	a	1	2	$1+a$	$2+a$
$1+a$	$1+a$	$2+a$	a	$1+2a$	1	$2+2a$	$2a$	2	0
$2+a$	$2+a$	a	$1+a$	$2+2a$	2	$2a$	$1+2a$	0	1
$1+2a$	$1+2a$	$2+2a$	$2a$	1	$1+a$	2	0	$2+a$	a
$2+2a$	$2+2a$	$2a$	$1+2a$	2	$1+2a$	0	1	a	$1+a$

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Step 4 of 4

The multiplication table is as follows:

\times	0	1	2	a	$2a$	$1+a$	$2+a$	$1+2a$	$2+2a$
0	0	0	0	0	0	0	0	0	0
1	0	1	2	a	$2a$	$1+a$	$2+a$	$1+2a$	$2+2a$
2	0	2	1	$2a$	a	$2+2a$	$1+2a$	$2+a$	$1+a$

a	0	a	$2a$	$1+2a$	$2+a$	1	$1+a$	$2+2a$	2
$2a$	0	$2a$	a	$2+a$	$1+2a$	2	$2+2a$	$1+a$	1
$1+a$	0	$1+a$	$2+2a$	1	2	$2+a$	$2a$	a	$1+2a$
$2+a$	0	$2+a$	$1+2a$	$1+a$	$2+2a$	$2a$	2	1	a
$1+2a$	0	$1+2a$	$2+a$	$2+2a$	$1+a$	a	1	2	$2a$
$2+2a$	0	$2+2a$	$1+a$	2	1	$1+2a$	a	$2a$	$2+a$

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