

A Book of Abstract Algebra | (2nd Edition)

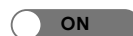


Chapter 30, Problem 2EC



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Problem

An angle α is called *constructible* iff there exist constructible points A , B , and C such that $\angle ABC = \alpha$.

Prove the following:

$$\cos \alpha \in \mathbb{D} \text{ iff } \sin \alpha \in \mathbb{D}.$$

Step-by-step solution

Step 1 of 5

Here, objective is to prove that $\cos \alpha \in D$, if and only if $\sin \alpha \in D$

Consider $a \in D$ if and only if the point $(a, 0)$ is constructible from $\{O, I\}$

Consider $\sin \alpha \in D$ if and only if the point $(0, \sin \alpha)$ is constructible from $\{O, I\}$

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Step 2 of 5

Constructible point is the end point of given unit segment or it is intersection of two lines determined by constructional points.

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Step 3 of 5

Consider $a \in D$ if and only if the point $(a, 0)$ is constructible from $\{O, I\}$

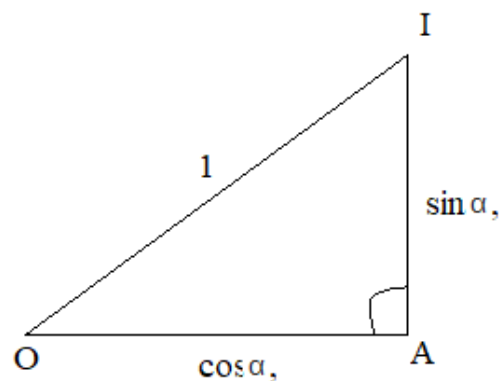
Similarly, $\sin \alpha \in D$, if and only if the point $(0, \sin \alpha)$ is constructible from $\{O, I\}$

And $\cos \alpha \in D$, if and only if the point $(\cos \alpha, 0)$ is constructible from $\{O, I\}$

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Step 4 of 5

Consider the below figure:



Consider the point $(0, \sin \alpha)$ is constructible from $\{O, I\}$

$\triangle OAI$ is a right angle triangle. Apply Pythagorean Theorem, then

$$OA^2 + AI^2 = 1$$

$$x^2 + \sin^2 \alpha = 1$$

$$x^2 = 1 - \sin^2 \alpha$$

$$x^2 = \cos^2 \alpha$$

$$x = \cos \alpha$$

So, the point $(\cos \alpha, 0)$ is constructible from $\{O, I\}$, which implies, $\cos \alpha \in D$.

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Step 5 of 5

Consider the point $(\cos \alpha, 0)$ is constructible from $\{O, I\}$

$\triangle OAI$ is a right angle triangle. Apply Pythagorean Theorem, then

$$OA^2 + AI^2 = 1$$

$$\cos^2 \alpha + y^2 = 1$$

$$y^2 = 1 - \cos^2 \alpha$$

$$y^2 = \sin^2 \alpha$$

$$y = \sin \alpha$$

So, the point $(0, \sin \alpha)$ is constructible from $\{O, I\}$, which implies, $\sin \alpha \in D$.

Hence,

$$\cos \alpha \in D, \text{ if and only if } \sin \alpha \in D$$

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