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Chapter 16, Problem 2EC

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Problem

Let G be an abelian group. Let $H = \{x^2 : x \in G\}$ and $K = \{x \in G : x^2 = e\}$.

Find the kernel of f.

Step-by-step solution

Step 1 of 3

Suppose that G be an abelian group. Consider the following sets:

$$H = \{x^2 : x \in G\},$$
$$K = \{x \in G : x^2 = e\}.$$

Then the function given by

$$f(x) = x^2$$

forms a homomorphism of G onto H. objective is to determine the kernel of mapping f.

Comment

Step 2 of 3

The kernel of any mapping $f : G \to H$ is defined by:

$\ker f = \{x \in G : f(x) = e\}$

Where *e* is the identity element of group *H*.

According to this definition the kernel of *f* will be:

$\ker f = \{x \in G : f(x) = e\}$.

Since $f(x) = x^2$, so equivalently

$\ker f = \{x \in G : x^2 = e\}$.

That is, the defined set *K* is nothing but the kernel of mapping *f*.

Comment

Step 3 of 3

Hence, $\ker f = K$.

Comment