

A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 3EB

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Problem

Find the root field of $x^3 + x^2 + 1 \in \mathbb{Z}_2[x]$ over \mathbb{Z}_2 . Write its addition and multiplication tables.

Step-by-step solution

Step 1 of 4

The objective is to find the root field of $x^3 + x^2 + 1$ over \mathbb{Z}_2 , and write its addition and multiplication tables.

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Step 2 of 4

Let $a(x) = x^3 + x^2 + 1 \in \mathbb{Z}_2$.

$$a(0) = 0^3 + 0^2 + 1$$

$$= 1$$

$$\neq 0$$

$$a(1) = 1^3 + 1^2 + 1$$

$$= 1 + 1 + 1$$

$$= 1$$

$$\neq 0$$

Therefore, $a(x)$ has no roots in \mathbb{Z}_2 .

This implies that $x^3 + x^2 + 1$ is irreducible over \mathbb{Z}_2 .

Therefore, the root field of $x^3 + x^2 + 1$ over \mathbb{Z}_2 is $\mathbb{Z}_2[x]/(x^3 + x^2 + 1)$.

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Step 3 of 4

Let k be any root of $a(x) = x^3 + x^2 + 1 \in \mathbb{Z}_2$.

Then $k^3 + k^2 + 1 = 0$.

$$k^3 = -k^2 - 1$$

$$= k^2 + 1$$

The root field is of size $2^3 = 8$.

The addition table is given as follows:

+	0	1	k	$1+k$	k^2	$1+k^2$	$k+k^2$	1
0	0	1	k	$1+k$	k^2	$1+k^2$	$k+k^2$	1
1	1	0	$1+k$	k	$1+k^2$	k^2	$1+k+k^2$	1
k	k	$1+k$	0	1	$k+k^2$	$1+k+k^2$	k^2	1
$1+k$	$1+k$	k	1	0	$1+k+k^2$	$k+k^2$	$1+k^2$	1
k^2	k^2	$1+k^2$	$k+k^2$	$1+k+k^2$	0	1	k	1
$1+k^2$	$1+k^2$	k^2	$1+k+k^2$	$k+k^2$	1	0	$1+k$	1
$k+k^2$	$k+k^2$	$1+k+k^2$	k^2	$1+k^2$	k	$1+k$	0	1

$1+k+k^2$	$1+k+k^2$	$k+k^2$	$1+k^2$	k^2	$1+k$	k	1	0
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Step 4 of 4

The multiplication table is given as follows:

\times	0	1	k	$1+k$	k^2	$1+k^2$	$k+k^2$	$1+k+k^2$
0	0	0	0	0	0	0	0	0
1	0	1	k	$1+k$	k^2	$1+k^2$	$k+k^2$	$1+k+k^2$
k	0	k	k^2	$k+k^2$	$1+k^2$	$1+k+k^2$	1	$1+k$
$1+k$	0	$1+k$	$k+k^2$	$1+k^2$	1	k	$1+k+k^2$	k^2
k^2	0	k^2	$1+k^2$	1	$1+k+k^2$	$1+k$	k	$k+k^2$
$1+k^2$	0	$1+k^2$	$1+k+k^2$	k	$1+k$	$k+k^2$	k^2	1
$k+k^2$	0	$k+k^2$	1	$1+k+k^2$	k	k^2	$1+k$	$1+k^2$
$1+k+k^2$	0	$1+k+k^2$	$1+k$	k^2	$k+k^2$	1	$1+k^2$	k

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