A Book of Abstract Algebra | (2nd Edition)

Chapter AA, Problem 8E

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Problem

Prove the following:

$A \cup 0 = A$ and $A \cap 0 = 0$.

Step-by-step solution

Step 1 of 3

Objective:-

The objective is to prove $A \cup \Phi = A \;\; and \;\; A \cap \Phi = \Phi$.

Comment

Step 2 of 3

Exercise (a):-

Let A and B are two sets. Let $x \in A \cup \Phi$.

The union of two sets A and B is:-

$$A \cup B = \{x : x \in A \;\; or \;\; x \in B\}$$

According to this definition:-

$$\begin{aligned} A \cup \Phi &\Rightarrow x \in A \;\; or \;\; x \in \Phi \\ &\Rightarrow x \in A \end{aligned}$$

So,

$$A \cup \Phi \subseteq A \qquad ......(1)$$

Let $x \in A$.

$$\begin{aligned} A &\Rightarrow x \in A \;\; or \;\; x \in \Phi \\ &\Rightarrow x \in A \cup \Phi \end{aligned}$$

So,

$$A \subseteq A \cup \Phi \qquad \text{......}(2)$$

Let us consider the equation (1) and (2).

$$A \cup \Phi = A$$

Proved

Comment

Step 3 of 3

Exercise (b):-

Let *A* and *B* are two sets. Let $x \in A \cap \Phi$.

The intersection of two sets *A* and *B* is:-

$$A \cap B = \{x : x \in A \ \ and \ \ x \in B\}$$

According to this definition:-

$$\begin{aligned} A \cap \Phi &\Rightarrow x \in A \ \ and \ \ x \in \Phi \\ &\Rightarrow x \in \Phi \end{aligned}$$

Since there is only empty set common in set *A* and empty set.

So,

$$A \cap \Phi \subseteq \Phi \qquad \text{......}(3)$$

Let $x \in \Phi$.

$$\begin{aligned} A &\Rightarrow x \in A \ \ and \ \ x \in \Phi \\ &\Rightarrow x \in A \cap \Phi \end{aligned}$$

So,

$$\Phi \subseteq A \cap \Phi \qquad \text{......}(4)$$

Let us consider the equation (3) and (4).

$$A \cap \Phi = \Phi$$

Proved

Comment