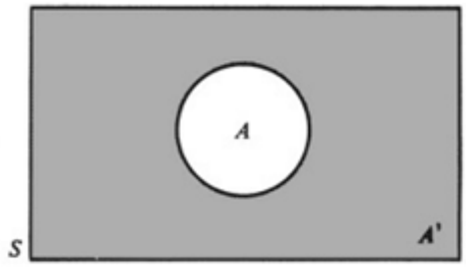
A Book of Abstract Algebra | (2nd Edition)

Chapter AA, Problem 16E | Bookmark | Show all steps: ON

Problem

If S is a set, and A is a subset of S, then the *complement* of A in S is the set of all the elements of S which are not in A. The complement of A is denoted by A':



$$A' = \{x \in S : x \notin A\}$$

Prove the following'.

$(A \cap B)' = A' \cap B'$.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $(A \cap B)' = A' \cup B'$.

Comment

Step 2 of 2

Proof:-

Let A and B are two sets.

If S is a set and A is a subset of S, then complementary of set A is defined as:-

$$A' = \{x \in S : x \notin A\}$$

Let *S* is a set and *A* and *B* are subset of *S*. Let $x \in (A \cup B)'$.

$$x \in (A \cap B)'$$
$$\Rightarrow x \notin (A \cap B)$$
$$\Rightarrow x \notin A \ \ or \ \ x \notin B$$
$$\Rightarrow x \in A' \ \ or \ x \in B'$$
$$\Rightarrow x \in A' \cup B'$$

So,

$$(A \cap B)' \subseteq A' \cup B' \qquad \text{......(1)}$$

Let $x \in A' \cup B'$

$$x \in A' \cup B'$$
$$\Rightarrow x \in A' \ \ or \ \ x \in B'$$
$$\Rightarrow x \notin A \ \ or \ \ x \notin B$$
$$\Rightarrow x \notin (A \cap B)$$
$$x \in (A \cup B)'$$

So,

$$A' \cup B' \subseteq (A \cap B)' \qquad \text{......(2)}$$

Let us consider the equation (1) and (2).

$$(A \cap B)' = A' \cup B'$$

Proved

Comment