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Chapter 33, Problem 5EA

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Problem

Explain why parts 3 and 4 do not contradict the principal finding of this chapter: that polynomial equations of degree $n \geq 5$ do not have a general solution by radicals.

Step-by-step solution

Step 1 of 5

Here, objective is to explain why the given polynomials are do not contradict the principal "polynomial equation of degree $n \geq 5$ do not have a general solution by radicals".

Comment

Step 2 of 5

A polynomial equation is solvable by radicals, if its roots are determined by applying finite number of additions, subtractions, multiplications, divisions, n^{th} roots to the integers.

Galois group:

If the polynomial degree is greater than or equal to 4 are solvable by radicals.

Comment

Step 3 of 5

Consider the polynomial $a(x) = x^5 - 10x^4 + +40x^3 - 80x^2 + 79x - 30$

$a(x) = (x-2)^5 - (x-2) = 0$

Let $y = x - 2$

Then, the equation becomes, $y^5 - y = 0$

The above equation is of the form $x^5 - px + q$. So it can be solved by radicals.

Comment

Step 4 of 5

Consider the polynomial $a(x) = ax^8 + bx^6 + +cx^4 + dx^2 + e$

Let $y = x^2$

Then, the equation becomes,

$$ax^8 + bx^6 + +cx^4 + dx^2 + e$$
$$= ay^4 + by^3 + +cy^2 + dy + e$$

The above polynomial is of degree 4

Every polynomial of degree four is solvable by radicals.

Therefore, the polynomial $a(x) = ax^8 + bx^6 + +cx^4 + dx^2 + e$ is solvable by radicals.

Comment

Step 5 of 5

The above polynomials are of degree five and eight.

As per the principal “If the polynomial degree is greater than or equal to 4 are solvable by radicals” they are not solved by radicals. But it’s not true. They are solved by radicals.

Since,

The principal does not assert that higher degree polynomials have no solution.

Every non constant polynomial in one unknown, with complex or real coefficients has at least one complex number as a solution.

There is no general solution in radicals which can apply to all polynomials with degree $n \geq 5$.

Therefore, the polynomials are do not contradict the principal.

Comment