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Chapter 29, Problem 3EE

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Problem

Let *F* be a field.

Prove part:

If a real number *c* is a root of an irreducible polynomial of degree >1 in $\mathbb{Q}[x]$, then *c* is irrational.

Step-by-step solution

Step 1 of 2

Let *F* be a field. Objective is to prove that if a real number *c* is a root of an irreducible polynomial of degree >1 in $Q[x]$, then *c* is irrational.

Consider that real number *c* is a root of an irreducible polynomial $p(x)$ of degree >1 in $Q[x]$. Assume, if possible, that *c* is rational number (not irrational).

Note that, $Q[x]$ contains all the polynomial with rational coefficients. Also by the assumed condition, $p(c)=0$ where $c \in Q$. That is, the polynomial $p(x)$ has a rational root in $Q[x]$. But it cannot be possible because polynomial $p(x)$ is irreducible in $Q[x]$. Thus, assume assumption was wrong, and *c* must be irrational.

Comment

Step 2 of 2

Hence, if a real number *c* is a root of an irreducible polynomial of degree >1 in $Q[x]$, then *c* is irrational.

Comment