

A Book of Abstract Algebra | (2nd Edition)



Chapter AA, Problem 24E



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Problem

Prove the following identities involving cartesian products:

$$A \times (B - D) = (A \times B) - (A \times D).$$

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $A \times (B - D) = (A \times B) - (A \times D)$.

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Step 2 of 2

Proof:-

Let A and B are two sets.

The Cartesian product of two sets A and B is:-

$$A \times B = \{(x, y) : x \in A, y \in B\}$$

The difference of two sets A and B is:-

$$A - B = \{x : x \in A, x \notin B\}$$

Let $(x, y) \in A \times (B - D)$.

$$(x, y) \in A \times (B - D)$$

$$\Rightarrow x \in A, y \in (B - D)$$

$$\Rightarrow x \in A, (y \in B, y \notin D)$$

$$\Rightarrow (x \in A, y \in B), (x \in A, y \notin D)$$

$$\Rightarrow (x, y) \in (A \times B), (x, y) \notin (A \times D)$$

$$\Rightarrow (x, y) \in (A \times B) - (A \times D)$$

So,

$$A \times (B - D) \subseteq (A \times B) - (A \times D) \quad \dots\dots(1)$$

Let $(x, y) \in (A \times B) - (A \times D)$.

$$(x, y) \in (A \times B) - (A \times D)$$

$$\Rightarrow (x, y) \in (A \times B), (x, y) \notin (A \times D)$$

$$\Rightarrow (x \in A, y \in B), (x \notin A, y \notin D)$$

$$\Rightarrow x \in A, (y \in B, y \notin D)$$

$$\Rightarrow x \in A, y \in (B - D)$$

$$\Rightarrow (x, y) \in A \times (B - D)$$

So,

$$(A \times B) - (A \times D) \subseteq A \times (B - D) \quad \dots\dots(2)$$

Let us consider the equation (1) and (2).

$$A \times (B - D) = (A \times B) - (A \times D)$$

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