

A Book of Abstract Algebra | (2nd Edition)

Chapter 27, Problem 1EG

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Problem

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Let F be a field, and let c be transcendental over F . Prove the following:
 $\{a(c):a(x) \in F[x]\}$ is an integral domain isomorphic to $F[x]$.

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Step-by-step solution

Step 1 of 3 ^

Consider that F is any arbitrary field and let $c \in F$ is transcendental over F . Objective is to prove that the set $\{a(c):a(x) \in F[x]\}$ is an integral domain isomorphic to $F[x]$.

Assume the substitution function $\sigma_c:F[x] \rightarrow E$ as follows:

For every polynomial $a(x) \in F[x]$,

$$\sigma_c(a(x)) = a(c),$$

where E is a field, F is a subfield of E .

Comment

Step 2 of 3 ^

It is known that substitution function is a homomorphism. The range of this homomorphism is the set of all the elements $a(c)$, for all $a(x)$ in $F[x]$, that is,

$$\text{Range}\sigma_c = \{a(c):a(x) \in F[x]\}.$$

Since $\text{Range}\sigma_c$ is a field and every field is an integral domain, therefore $\{a(c):a(x) \in F[x]\}$ is an integral domain.

Consider the following result:

Element c is transcendental over F if and only if kernel of homomorphism σ_c is trivial, that is,

$$\ker\sigma_c = \{0\}.$$

Thus, the mapping from $F[x]$ to $\text{Range}\sigma_c$ is bijective homomorphism.

Comment

Step 3 of 3 ^

Hence, $\{a(c):a(x) \in F[x]\}$ is isomorphic to $F[x]$.

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