A Book of Abstract Algebra | (2nd Edition)

Chapter 33, Problem 2ED

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Problem

Let *G* be a group. The symbol $H \lhd G$ should be read, "*H* is a normal subgroup of *G*." A *maximal* normal subgroup of *G* is an $H \lhd G$ such that, if $H \lhd J \lhd G$, then necessarily $J = H$ or $J = G$. Prove the following:

Let $f: G \to H$ be a homomorphism. If $J \lhd H$, then $f^{-1}(J) < G$.

Step-by-step solution

Step 1 of 4

Here, objective is to prove that $f^{-1}(J) < G$

Comment

Step 2 of 4

Finite group is a group which contains finite number of elements.

If *G* is a finite group. Then *H* is normal subgroup of *G* is denoted by $H \lhd G$

Consider $f : G \to H$ is a homomorphism, then $f(xy) = f(x).f(y); \forall x, y \in G.$

Comment

Step 3 of 4

Consider $f : G \to H$ is a homomorphism and $J \lhd H$

That is *J* is any subgroup of *H*.

Let $x, y \in f^{-1}(J)$

Then, $f(x), f(y) \in J$

$f(xy) = f(x).f(y) \in J$

Since, J is a group which is closed under multiplication.

So $f(xy) \in J$

$xy \in f^{-1}(J)$

$f^{-1}(J)$ is closed under multiplication.

Comment

Step 4 of 4

Let $z \in f^{-1}(J)$

$f(z) \in J$

$f(c)^{-1} \in J$

$f(c^{-1}) \in J$

$c^{-1} \in f^{-1}(J)$

$f^{-1}(J)$ is closed under inversion.

So, $f^{-1}(J)$ is closed under multiplication and inversion.

Therefore, $f^{-1}(J) < G$ is a subgroup of *G*, implies $f^{-1}(J) < G$.

Hence, proved

Comment