A Book of Abstract Algebra | (2nd Edition)

Chapter 28, Problem 4ED

Bookmark

Show all steps: ON

Problem

Let *V* be a finite-dimensional vector space. Let dim *V* designate the dimension of *V*. Prove each of the following:

The set {**a**}, containing only one nonzero vector **a**, is linearly independent.

Step-by-step solution

Step 1 of 3

A set of vectors which is said to be linearly independent if there exists no combination of these vectors which can give **0** vector apart from a combination in which all coefficients are 0.

Comment

Step 2 of 3

If $u_1, u_2, \ldots, u_n$ are *n* vectors of a vector space and these are linearly independent. Then for.

$a_1u_1 + a_2u_2 + \ldots + a_nu_n = \mathbf{0}$

All a_i have to be zero.

Comment

Step 3 of 3

Now consider any set which includes just one vector. Let this set be $(\mathbf{a})$. Now there is only one combination involving this single vector, which is

$$k \cdot \mathbf{a}$$

Here *k* can have any value not necessarily equal to 0. As **a** is non-zero vector, this combination will never be equal to **0**, unless *k* is equal to 0, which is condition for being linearly independent.

Thus this combination satisfy condition for being linearly independent.

$\boxed{\text{Hence any set withjust one non-zero vector } \vec{\mathbf{a}} \text{ is linearly independent}}$.

Comment