

# A Book of Abstract Algebra | (2nd Edition)

Chapter AC, Problem 2E

Bookmark

Show all steps: ☒ ON

## Problem

Use mathematical induction to prove the following:

$$1^3 + 2^3 + \dots + n^3 = (1 + 2 + \dots + n)^2$$

## Step-by-step solution

### Step 1 of 2

#### Objective:-

The objective is to prove  $1^3 + 2^3 + \dots + n^3 = (1 + 2 + \dots + n)^2$  using mathematical induction.

[Comment](#)

### Step 2 of 2

Proof:-

$$p(n): 1^3 + 2^3 + \dots + n^3 = (1 + 2 + \dots + n)^2$$

$$p(n): 1^3 + 2^3 + \dots + n^3 = \left( \sum_{i=1}^n i \right)^2$$

$$p(n): 1^3 + 2^3 + \dots + n^3 = \left( n \left( \frac{n+1}{2} \right) \right)^2 \quad \left\{ \text{since } 1 + 2 + \dots + n = \frac{n(n+1)}{2} \right\}$$

Let consider rule for  $n = 1$ .

$$p(1): 1^3 = \left( 1 \left( \frac{1+1}{2} \right) \right)^2$$

$$p(1): 1 = 1$$

This rule is true for  $n = 1$ .

Let this statement is true for  $n = k$ .

$$p(k): 1^3 + 2^3 + \dots + k^3 = \left( k \left( \frac{k+1}{2} \right) \right)^2 \quad \dots\dots(1)$$

Let consider statement for  $n = k + 1$ .

$$p(k+1): 1^3 + 2^3 + \dots + k^3 + (k+1)^3$$

Use the equation (1).

$$p(k+1): \left( \frac{k(k+1)}{2} \right)^2 + (k+1)^3$$

$$p(k+1): k^2 \left( \frac{k+1}{2} \right)^2 + (k+1)^3$$

$$p(k+1): k^2 \frac{(k+1)^2}{4} + (k+1)^3$$

$$p(k+1): (k+1)^2 \left( \frac{k^2}{4} + k + 1 \right)$$

$$P(k+1): (k+1)^2 \left( \frac{k^2 + 4k + 4}{4} \right)$$

$$P(k+1): (k+1)^2 \left( \frac{k+2}{4} \right)^2$$

$$P(k+1): \left[ \frac{(k+1)(k+2)}{2} \right]^2$$

This result also true for  $n = k + 1$ . Hence, by mathematical induction this rule is true for all positive integer  $n$ .

Proved

---

[Comment](#)

**COMPANY**

About Chegg  
Chegg For Good  
College Marketing  
Corporate  
Development  
Investor Relations  
Jobs  
Join Our Affiliate  
Program  
Media Center  
Site Map

**LEGAL & POLICIES**

Advertising  
Choices  
Cookie Notice  
General Policies  
Intellectual  
Property Rights  
Terms of Use  
Global Privacy  
Policy  
Honor Code  
Honor Shield

**CHEGG PRODUCTS AND SERVICES**

Cheap Textbooks  
Chegg Coupon  
Chegg Play  
Chegg Study Help  
College Textbooks  
eTextbooks  
Flashcards  
Learn  
Chegg Math  
Solver

Mobile Apps  
Sell Textbooks  
Solutions Manual  
Study 101  
Textbook Rental  
Used Textbooks  
Digital Access  
Codes  
Chegg Money

**CHEGG NETWORK**

EasyBib  
Internships.com  
Thinkful

**CUSTOMER  
SERVICE**

Customer Service  
Give Us Feedback  
Help with  
eTextbooks  
Help to use  
EasyBib Plus  
Manage Chegg  
Study Subscription  
Return Your Books  
Textbook Return  
Policy