

# Contents

## 1 Liouville (theorem 2.3)

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```
sage: v = matrix([[1, a, b, a*b, b^2, a*b^2]]).transpose()
sage: v
[ 1]
[ a]
[ b]
[ a*b]
[ b^2]
[ a*b^2]
sage: A = matrix([
.....: [0, 1, 0, 0, 0, 0],
.....: [2, 0, 0, 0, 0, 0],
.....: [0, 0, 0, 1, 0, 0],
.....: [0, 0, 2, 0, 0, 0],
.....: [0, 0, 0, 0, 0, 1],
.....: [0, 0, 0, 0, 2, 0]
.....: ])
sage: A*v
[ a]
[ 2]
[ a*b]
[ 2*b]
[ a*b^2]
[ 2*b^2]
sage: B = matrix([
.....: [0, 0, 1, 0, 0, 0],
.....: [0, 0, 0, 1, 0, 0],
.....: [0, 0, 0, 0, 1, 0],
.....: [0, 0, 0, 0, 0, 1],
.....: [2, 0, 0, 0, 0, 0],
.....: [0, 2, 0, 0, 0, 0]
.....: ])
sage: B*v
[ b]
[ a*b]
[ b^2]
[ a*b^2]
[ 2]
[ 2*a]
sage: A_B = A + B
sage: matrix.identity(6)
[1 0 0 0 0 0]
[0 1 0 0 0 0]
[0 0 1 0 0 0]
[0 0 0 1 0 0]
[0 0 0 0 1 0]
[0 0 0 0 0 1]
sage: x*matrix.identity(6)
[x 0 0 0 0 0]
[0 x 0 0 0 0]
[0 0 x 0 0 0]
[0 0 0 x 0 0]
[0 0 0 0 x 0]
[0 0 0 0 0 x]
sage: ( x*matrix.identity(6) - A_B ).determinant()
x^6 - 6*x^4 - 4*x^3 + 12*x^2 - 24*x - 4
```

## 1 Liouville (theorem 2.3)

$$|\alpha - p/q| > 1 > 1/q^n$$

So lets take  $|a - p/q| \leq 1$ .

Mean value theorem gives us  $f'(\gamma)$ .

$q^n f(p/q)$  is an integer means  $|f(p/q)| \geq 1/q^n$ .

$$\alpha < \gamma < p/q, |\alpha - p/q| \leq 1 \Rightarrow |\gamma - \alpha| < 1$$

Then observe

$$|f(\alpha) - f(p/q)| < C|\alpha - p/q| < C$$

So then  $f'(\gamma) < C = 1/c_0$ .

Combine these

$$\left| \alpha - \frac{p}{q} \right| = \left| \frac{p/q}{f'(\gamma)} \right| > \frac{c_0}{q^n}$$