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Chapter 28, Problem 4EB | Bookmark | Show all steps: ON

Problem

Prove that { *f* : *f* is a constant on the interval [0,1]} is a subspace of $\mathcal{F}(\mathbb{R})$.

Step-by-step solution

Step 1 of 2

It is already shown that $f(\mathbb{R})$ represents a vector space as it satisfies all conditions for vector space.

Given subset for $f(\mathbb{R})$ is set of all functions which are constant in closed interval $[1,0]$.

Or given condition for subspace is

$$\{f \mid f \text{ is constant in interval } [1,0]\}$$

Comment

Step 2 of 2

A 2 step check is needed to determine if given subset is a subspace. First determine if any linear combination of functions in subset is closed under given operation. Second, determine if 0 satisfies given conditions.

STEP 1: For any 2 functions *f* and *g*,

$$f(x)=0 \quad \forall x\in[0,1] \qquad (1)$$
$$g(x)=0 \quad \forall x\in[0,1] \qquad (2)$$

Combining above 2 equations, $s(1)+t(2)$ gives

$$sf(x)+tg(y)=0$$

As functions are vector space in themselves, any constant multiple of function is also a function. Also sum of 2 functions is also a function. Thus,

$$\begin{aligned} & sf(x)+tg(x)=0 && \forall x\in[0,1] \\ & \Rightarrow f'(x)+g'(x)=0 && \forall x\in[0,1] \\ & \Rightarrow F(x)=0 && \forall x\in[0,1] \end{aligned}$$

Thus linear combination of 2 functions in subset lies in subset.

STEP 2: Check if 0 function (which is 0 everywhere) satisfies given condition,

$$f_0(x)=0 \qquad \forall x\in[0,1]$$

$$\boxed{\text{Hence given set represents a subspace}}$$

Comment