A Book of Abstract Algebra | (2nd Edition)

Chapter 30, Problem 1EC

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Problem

An angle α is called *constructible* iff there exist constructible points *A, B*, and *C* such that $\angle LABC = \alpha$.

Prove the following:

The angle α is constructible iff sin α and cos α are constructible numbers.

Step-by-step solution

Step 1 of 4

Here, objective is to prove that the angle α is constructible, if and only if $\sin\alpha$ and $\cos\alpha$ are constructible numbers.

Comment

Step 2 of 4

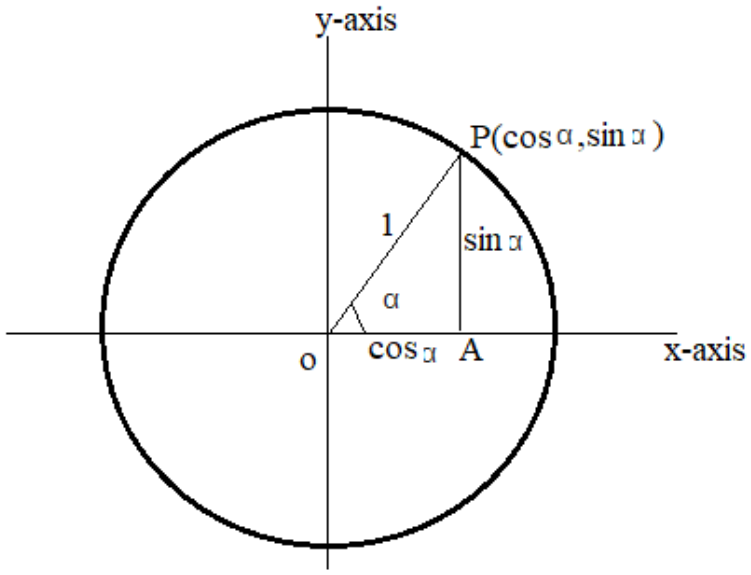
Constructible point is the end point of given unit segment or it is intersection of two lines determined by constructional points.

Comment

Step 3 of 4

To prove $\sin\alpha$ and $\cos\alpha$ are constructible numbers, if the angle α is constructible.

Consider the below figure:



figureConstructionof α

Let us assume, the angle α is constructible

That is line segment OP is making an angle α with the x-axis and the point *P* is on the circumference of unit circle.

ΔOAP is a right angle triangle

$$\cos\alpha = \frac{OA}{OP}$$

$$\cos\alpha = OA \qquad (\because \text{OP}=1)$$

$$\sin\alpha = \frac{AP}{OP}$$

$$\sin\alpha = AP \qquad (\because \text{OP}=1)$$

Therefore,

$\sin\alpha$ and $\cos\alpha$ are constructible numbers.

Hence, proved

Comment

Step 4 of 4

To prove, if $\sin\alpha$ and $\cos\alpha$ are constructible numbers, the angle α is constructible.

Consider

$$\cos\alpha = OA$$

$$\sin\alpha = AP$$

Then, join the line segment OP

$$OP^2 = OA^2 + OB^2$$

$$OP^2 = \cos^2\alpha + \sin^2\alpha$$

$$OP^2 = 1$$

Then, the line segment OP is making an angle α with the x-axis.

Therefore, the angle α is constructible, if and only if $\sin\alpha$ and $\cos\alpha$ are constructible numbers.

Hence,

The angle α is constructible, if and only if $\sin\alpha$ and $\cos\alpha$ are constructible numbers.

Comment