A Book of Abstract Algebra | (2nd Edition)

Chapter 30, Problem 2EA

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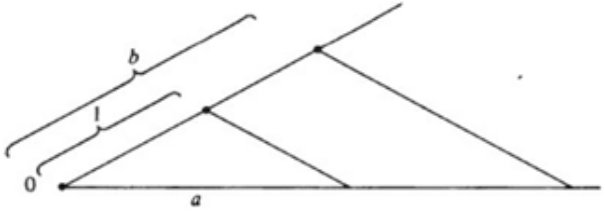
Problem

If *O* and *I* are any two points in the plane, consider a coordinate system such that

the interval *OI* coincides with the unit interval on the *x* axis. Let $\mathbb{D}$ be the set of real numbers such that $a \in \mathbb{D}$ iff the point (*a*, 0) is constructible from {*O, I*}. Prove the following:

If $a, b \in \mathbb{D}$, then $ab \in \mathbb{D}$. then. (HINT: Use similar triangles. See the accompanying figure.)



Step-by-step solution

Step 1 of 4

Here, objective is to prove that, if $a, b \in D$, then $ab \in D$.

Consider $a \in D$ if and only if the point $(a,0)$ is constructible from $\{O,I\}$

Comment

Step 2 of 4

Constructible point:

Constructible point is either the end point of given unit segment or it is the intersection of two lines determined by previous constructible points.

Comment

Step 3 of 4

D is a set of real numbers. Therefore, we can add, subtract, multiply any two points of them.

Let $a,b \in D$, then the points $(a,0)$ and $(b,0)$ are constructible from $\{O,I\}$

Comment

Step 4 of 4

Now, we have to prove that length ab is constructed from the lengths a and b

Consider the below figure:

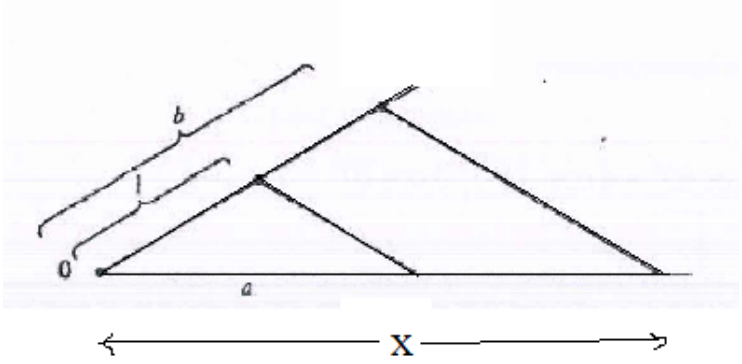


fig: multiplication

by observing, there exist two equal triangles.

a and b are constructible lengths.

using the property of similar triangles, we have

$$\frac{1}{a} = \frac{b}{x}$$

$$x = ab$$

Then, the length ab is constructible from $\{O, I\}$, which implies $ab \in D$

Therefore, if $a, b \in D$, then $ab \in D$.

Hence, proved

Comment