

# A Book of Abstract Algebra | (2nd Edition)

Chapter AB, Problem 10E

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## Problem

Prove that the following are true for any integers  $a$ ,  $b$ , and  $c$ :

If  $d \mid ab$  and  $d \mid cb$  and  $\gcd(a, c) = 1$ , then  $d \mid b$ .

## Step-by-step solution

### Step 1 of 2

#### Objective:-

The objective is to prove *if  $d \mid ab$  and  $d \mid cb$  and  $\gcd(a, c) = 1$ , then  $d \mid b$ .*

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### Step 2 of 2

Proof:-

Let suppose  $d \mid ab$  and  $d \mid cb$ .

Then there exist number  $k$  and  $l$  such that:-

$$ab = kd \quad \dots\dots(1)$$

$$cb = ld \quad \dots\dots(2)$$

Let  $\gcd(a, c) = 1$ , then integers  $a$  and  $c$  are relatively prime.

Let us consider the theorem.

**Theorem:-**Any two nonzero integers  $r$  and  $s$  have a unique positive greatest common divisor  $t$ , Moreover,  $t$  is equal to a "Linear combination" of  $r$  and  $s$ . That is,

$$t = kr + ls \quad \text{for some integer } k \text{ and } l$$

Let us suppose  $\gcd(a, c) = 1$ . Then by above theorem:-

$1 = ma + nc$  for some integer  $m$  and  $n$

$$1 = m \frac{kd}{b} + n \frac{kd}{b} \quad \{ \text{from equation (1) and (2)} \}$$

$$1 = \frac{kd}{b}(m+n)$$

$$b = kd(m+n)$$

Thus,  $d$  is a factor of  $kd(m+n)$  that is a factor of  $b$ . Hence,  $d$  divides  $b$  that is  $d \mid b$ .

Proved

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