

# A Book of Abstract Algebra | (2nd Edition)



Chapter 23, Problem 7EC



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## Problem

Prove the following for all integers  $a, b, c, d$  and all positive integers  $m$  and  $n$ :

If  $a \equiv b \pmod{m}$ , then  $a + km \equiv b \pmod{m}$ , for any integer  $k$ . In particular,  $a + km \equiv a \pmod{m}$ .

## Step-by-step solution

### Step 1 of 5

Here, objective is to prove that,  $a + km \equiv b \pmod{m}$ , if  $a \equiv b \pmod{m}$

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### Step 2 of 5

Consider  $a, b$  are integers,  $m$  is a positive integer.

If  $m$  divides  $a - b$ , then  $a$  is congruent to  $b$  modulo  $m$  which is represented by  $a \equiv b \pmod{m}$

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### Step 3 of 5

Consider  $a \equiv b \pmod{m}$ .....(1)

Then,  $m$  divides  $a - b$ ,

$$\frac{a-b}{m} = k; k \text{ is integer}$$

$$a - b = km$$

$$a = b + km$$

$$b - a = -km$$

$$b = a + (-k)m$$

$$b = a + pm$$

$$b = a(\text{mod } m) \dots \dots \dots (2)$$

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#### Step 4 of 5

To prove  $a + km \equiv b(\text{mod } m)$  :

Substitute equation ..(2) in equation..(1)

$$a = a(\text{mod } m)(\text{mod } m)$$

$$a = a(\text{mod } m)$$

$$a = a + km$$

Then, from equation..(1)

$$a + km = b(\text{mod } m)$$

Hence, proved

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#### Step 5 of 5

To prove  $a + km = a(\text{mod } m)$  :

$$a \equiv b(\text{mod } m)$$

$$b - a = km$$

$$b = a + km$$

From equation..(2)

$$a + km = a(\text{mod } m)$$

Hence, proved

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