A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 9EC

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Problem

Prove the following for all integers *a*, *b*, *c*, *d* and all positive integers *m* and *n:*

If $a \equiv b \pmod{n}$, then $a \equiv b \pmod{m}$ for any *m* which is a factor of *n*.

Step-by-step solution

Step 1 of 2

Consider the congruence equation

$a \equiv b(\bmod n)$

The object of the problem is to prove that if $a \equiv b(\bmod n)$ then $a \equiv b(\bmod m)$ for any m which is a factor of n

Use the definition, $a \equiv b(\bmod n)$ iff n divides $a-b$ to prove the given result.

By the definition, n divides $a-b$

This implies that there is an integer q such that

$a-b=nq$

Comment

Step 2 of 2

Let m be factor of n then for any integer r, $n=mr$

Then,

$a-b=mrq$

$a-b=mq'$ take $rq=q'$

This implies that m divides $a-b$.

By the definition of congruence equation, $a \equiv b \pmod{m}$, m is factor of n

Therefore, if $a \equiv b \pmod{n}$ then $a \equiv b \pmod{m}$ for any m which is a factor of n

Comment