A Book of Abstract Algebra | (2nd Edition)

Chapter 16, Problem 3EJ

Bookmark

Show all steps: ON

Problem

Let *f* be a homomorphism from G onto *H* with kernel *K*:

$$f : G \underset{K}{\twoheadrightarrow} H$$

If *S* is any subgroup of *H*, let $S^* = \{x \in G: f(x) \in S\}$. Prove:

Let *g* be the restriction of *f* to *S*.*[That is, $g(x) = f(x)$ for every $x \in S^*$, and S^* is the domain of *g*.] Then *g* is a homomorphism from S^* onto *S*, and $K = \ker g$.

Step-by-step solution

Step 1 of 3

Suppose that *G* is any group. Let the mapping

$$f : G_K \to H$$

is a homomorphism from *G* onto *H* with kernel *K*. Assume that *S* is any subgroup of *H* and consider the following set:

$$S^* = \{x \in G : f(x) \in S\}.$$

Note that, the set S^* forms a subgroup of *G*.

Consider the following restriction map *g* of *f* to S^*

$$g : S^* \to S$$

defined as

$g(x) = f(x)$ for every $x \in S^*$.

Objective is to prove that restriction map *g* is a homomorphism from S^* onto *S* with $K = \ker g$.

Comment

Step 2 of 3

If *G* and *H* are two groups, a homomorphism from *G* to *H* is a function $f : G \to H$ such that for any two elements *a*, *b* in *G*,

$$f(ab) = f(a)f(b).$$

Assume that $x, y \in S^*$. Then use the homomorphism of mapping *f* in the following manner:

$$\begin{aligned} g(ab) &= f(ab) \\ &= f(a) \cdot f(b) \\ &= g(a) \cdot g(b). \end{aligned}$$

This shows that *g* is homomorphism also onto because *f* is onto.

Since codomain of *g* is same as mapping *f* and *K* is the kernel of *f*, therefore $K = \ker g$.

Comment

Step 3 of 3

Hence, the restriction map *g* is a homomorphism from S^* onto *S* with $K = \ker g$.

Comment