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Chapter 23, Problem 9EE | Bookmark | Show all steps: ON

Problem

Find the following integers *x:*

(*a*) $x \equiv 8^{38} \pmod{210}$

(*b*) $x \equiv 7^{57} \pmod{133}$

(*c*) $x \equiv 5^{73} \pmod{66}$

Step-by-step solution

Step 1 of 3

Consider any two distinct prime numbers *p* and *q*. Suppose $(p-1) \mid m$ and $(q-1) \mid m$. Then

$$a^m \equiv 1 \pmod{pq},$$

where $p \nmid a$ and $q \nmid a$. Also

$$a^{m+1} \equiv a \pmod{pq},$$

for integers *a*.

(a)

Objective is to determine the integer *x*, where $x \equiv 8^{38} (\bmod 210)$.

The $210 = 2 \times 3 \times 5 \times 7$. Taking the notation from the result, let $m = 36$.

Note that, 36 is divisible by $(2-1), (3-1), (5-1)$ and $(7-1)$. So, by using the second part of result, it implies that $8^{36+1} \equiv 8 (\bmod 210)$. Also, then

$$8^{37} \equiv 8 (\bmod 210)$$
$$8^{38} \equiv 8 \times 8 (\bmod 210).$$

Thus, $8^{38} \equiv 64 (\bmod 210)$.

Comment

Step 2 of 3

(b)

Objective is to determine the integer *x*, where $x \equiv 7^{57} \pmod{133}$.

The $133 = 7 \times 19$. Taking the notation from the result, let $m = 54$.

Note that, 54 is divisible by $(7-1)$ and $(19-1)$. So, by using the second part of result, it implies that $7^{54+1} \equiv 7 \pmod{133}$. Also, then

$$\begin{aligned}
7^{55} &\equiv 7 \pmod{133} \\
7^{56} &\equiv 49 \pmod{133} \\
7^{57} &\equiv 343 \pmod{133} \\
&\equiv 77 \pmod{133}
\end{aligned}$$

Thus, $7^{57} \equiv 77 \pmod{133}$.

Comment

Step 3 of 3

(c)

Objective is to determine the integer *x*, where $x \equiv 5^{73} \pmod{66}$.

The $66 = 2 \times 3 \times 11$. Let $m = 70$.Note that, 70 is divisible by $(2-1), (3-1)$ and $(11-1)$.

So, by using the result, it implies that $5^{71} \equiv 5 \pmod{66}$. Also, then

$$\begin{aligned}
5^{72} &\equiv 25 \pmod{66} \\
5^{73} &\equiv 125 \pmod{66} \\
&\equiv 59 \pmod{66}.
\end{aligned}$$

Thus, $5^{73} \equiv 59 \pmod{66}$.

Comment