

# A Book of Abstract Algebra | (2nd Edition)

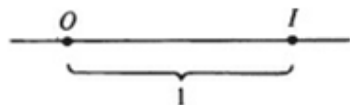
Chapter 30, Problem 6EA

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## Problem

If  $O$  and  $I$  are any two points in the plane, consider a coordinate system such that the interval  $OI$  coincides with the unit interval on the  $x$  axis. Let  $\mathbb{D}$  be the set of real numbers such that  $a \in \mathbb{D}$  iff the point  $(a, 0)$  is constructible from  $\{O, I\}$ . Prove the following:



If  $a$  is a real root of any quadratic polynomial with coefficients in  $\mathbb{D}$ , then  $a \in \mathbb{D}$ . (HINT: Complete the square and use part 4.)

## Step-by-step solution

### Step 1 of 4

Here, objective is to prove that  $a \in \mathbb{D}$ , if  $a$  is a real root of any quadratic polynomial with coefficients in  $\mathbb{D}$ .

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### Step 2 of 4

Constructible point:

This point is either the end point of given unit segment or it is the intersection of two lines determined by previous constructible points.

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### Step 3 of 4

Consider  $a \in D$  if and only if the point  $(a, 0)$  is constructible from  $\{O, I\}$

Consider the below figure:

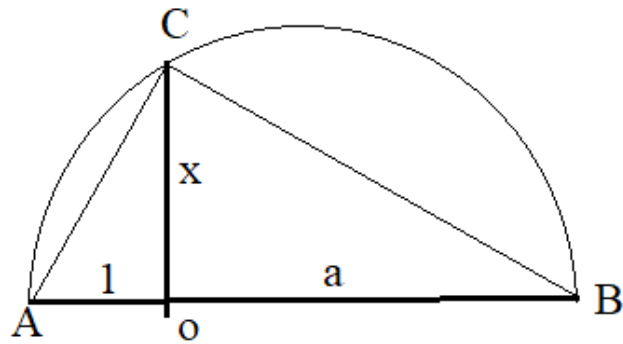


figure:construction of sqare root of a

By observing there exist two equal triangles

$\triangle AOB$  and  $\triangle BOC$

Using the property of similar triangles, we have

$$\frac{1}{x} = \frac{x}{a}$$

$$x^2 = a$$

$$x = \sqrt{a}$$

Then, the length  $\sqrt{a}$  is constructed from  $\{O, I\}$  which implies  $\sqrt{a} \in D$ .

So,  $D$  is the field of constructible numbers.

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#### Step 4 of 4

Consider  $a$  is a root of the quadratic polynomial

$$x^2 + bx + c = 0$$

$$a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$$

$$b^2 - 4ac \geq 0$$

$D$  is a field closed under square roots of positive elements.

Then,  $a \in D$

Hence, proved

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