A Book of Abstract Algebra | (2nd Edition)

Chapter AB, Problem 7E

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Problem

Prove that the following are true for any integers a, b, and c:

If gcd(*ab*, *c*) = 1, then gcd(*a*, *c*) = 1 and gcd(*b*, *c*) = 1.

Step-by-step solution

Step 1 of 3

Objective:-

The objective is to prove $if\ \gcd(ab,c)=1,\ then\ \gcd(a,c)=1\ and\ \gcd(b,c)=1.$

Comment

Step 2 of 3

Proof:-

Let us consider the theorem.

Theorem:-Any two nonzero integers *r* and *s* have a unique positive greatest common divisor *t*, Moreover, *t* is equal to a "Linear combination" of *r* and *s*. That is,

$t = kr + ls \quad for\ \ some\ \ integer\ \ k\ and\ \ l$

Let us suppose $\gcd(ab,c)=1.$ Then by above theorem:-

$1 = k(ab) + lc \quad for\ \ some\ \ integer\ \ k\ and\ \ l$

This can be written as:-

$1 = (kb)a + lc \quad for\ \ some\ \ integer\ \ k\ and\ \ l$

The integer 1 can be written as linear combination of the integer *a* and *c.*

Hence, by theorem:-

$$\boxed{\gcd(a,c)=1}$$

Proved

Comment

Step 3 of 3

Let us suppose $\gcd(ab,c)=1$. Then by above theorem:-

$$1=k(ab)+lc \quad for \ some \ integer \ k \ and \ l$$

This can be written as:-

$$1=(ka)b+lc \quad for \ some \ integer \ k \ and \ l$$

The integer 1 can be written as linear combination of the integer *b* and *c.*

Hence, by theorem:-

$$\boxed{\gcd(b,c)=1}$$

Proved

Comment