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Chapter AA, Problem 9E

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Problem

Prove the following:

$A \cup (A \cap B) = A$.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $A \cup (A \cap B) = A$.

Comment

Step 2 of 2

Proof:-

Let *A* and *B* are two sets.

The union of two sets *A* and *B* is:-

$$A \cup B = \{x : x \in A \ \ or \ \ x \in B\}$$

The intersection of two sets *A* and *B* is:-

$$A \cap B = \{x : x \in A \ \ and \ \ x \in B\}$$

Let $x \in A \cup (A \cap B)$.

$$x \in A \cup (B \cap C)$$
$$\Rightarrow x \in A \ \ or \ (\ x \in B \cap C)$$

$$\Rightarrow x \in A \ \ or \ \ x \in B \ \ and \ \ x \in C$$

$$\Rightarrow (x \in A \ \ or \ \ x \in B) \ \ and \ (\ x \in A \ \ or \ \ x \in C\ )$$

$$\Rightarrow (x \in A \cup B) \ \ and \ (\ x \in A \cup C\ )$$

$$\Rightarrow x \in (A \cup B) \cap (A \cup C\ )$$

So,

$$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \qquad \qquad ......(1)$$

Let $x \in A \cup (B \cap C)$.

$$x \in (A \cup B) \cap (A \cup C\ )$$

$$\Rightarrow (x \in A \cup B) \ \ and \ (\ x \in A \cup C\ )$$

$$\Rightarrow (x \in A \ \ or \ \ x \in B) \ \ and \ (\ x \in A \ \ or \ \ x \in C\ )$$

$$\Rightarrow x \in A \ \ or \ \ x \in B \ \ and \ \ x \in C$$

$$\Rightarrow x \in A \cup (B \cap C)$$

So,

$$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \qquad \qquad ......(2)$$

Let us consider the equation (1) and (2).

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

According to this theorem:-

$$A \cup (A \cap B) = (A \cup A) \cap (A \cup B)$$

$$A \cup (A \cap B) = A \cap (A \cup B)$$

$$A \cup (A \cap B) = A$$

Since the common elements in set *A* and *B* are also elements of set *A*.

Proved

Comment