A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 5ED

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Problem

Prove the following for an integers *a*, *b*, *c* and all positive integers *m* and *n*:

If $a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$ where gcd(*m*, *n*) = 1, then $a \equiv b \pmod{mn}$.

Step-by-step solution

Step 1 of 3

Here, objective is to prove that $a = b(\mathrm{mod}\, mn)$

Comment

Step 2 of 3

Consider a, b are integers, m, n are positive integer.

If *m* divides $a - b$, then *a* is congruent to *b* modulo *m* which is represented by $a = b(\mathrm{mod}\ \mathrm{m})$

Comment

Step 3 of 3

Consider $a = b(\mathrm{mod}\, m)$

$a - b$ is divided by m

Consider $a = b(\mathrm{mod}\, n)$

$a-b$ is divided by n ,

Then,

$a-b$ is divided by mn

That is, *a* is congruent to *b* modulo mn

The above statement can be represented as,

$a = b(\mathrm{mod}\, mn)$

Therefore, if $a \equiv b(\mathrm{mod}\, m)$ and $a \equiv b(\mathrm{mod}\, n)$, Then $a \equiv b(\mathrm{mod}\, mn)$

Hence, proved

Comment