

# A Book of Abstract Algebra | (2nd Edition)



Chapter 30, Problem 3EC



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ON



## Problem

An angle  $\alpha$  is called *constructible* iff there exist constructible points  $A$ ,  $B$ , and  $C$  such that  $\angle ABC = \alpha$ .

Prove the following:

If  $\cos \alpha, \cos \beta \in \mathbb{D}$ , then  $\cos(\alpha + \beta), \cos(\alpha - \beta) \in \mathbb{D}$ .

## Step-by-step solution

### Step 1 of 3

Here, objective is to prove that  $\cos \alpha, \cos \beta \in D$ , then  $\cos(\alpha + \beta), \cos(\alpha - \beta) \in D$

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### Step 2 of 3

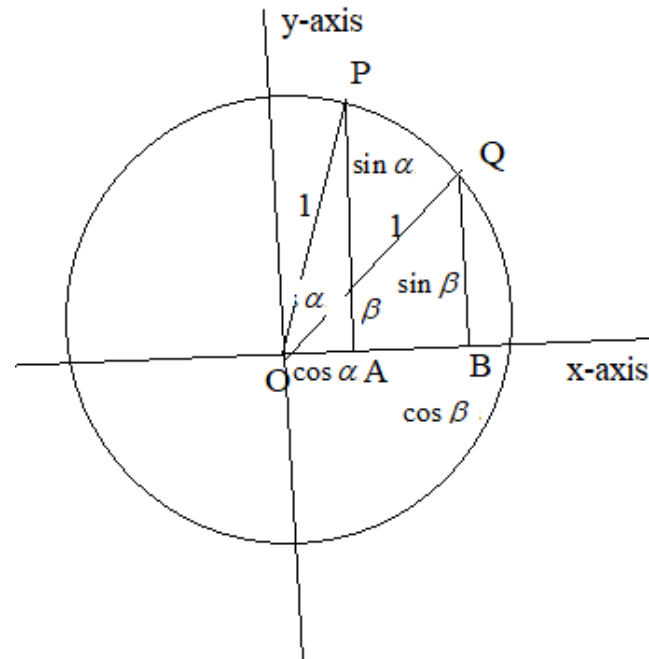
Constructible point is the end point of given unit segment or it is intersection of two lines determined by constructional points.

Consider  $\cos \alpha, \cos \beta \in D$ , then the points  $(\cos \alpha, 0), (\cos \beta, 0)$  are constructible from  $\{O, I\}$

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### Step 3 of 3

Consider the below figure:



If  $(\cos \alpha, 0), (\cos \beta, 0)$  are constructible, then  $\sin \alpha, \sin \beta$  are constructible, by using the identities

$$\sin \alpha = 1 - \cos^2 \alpha$$

$$\sin \beta = 1 - \cos^2 \beta$$

Consequently,

$\cos(\alpha + \beta), \cos(\alpha - \beta)$  are also constructible. Since

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

if  $\cos(\alpha + \beta), \cos(\alpha - \beta)$  are constructible then  $\cos(\alpha + \beta), \cos(\alpha - \beta) \in D$ .

Therefore,  $\cos(\alpha + \beta), \cos(\alpha - \beta) \in D$ , if  $\cos \alpha, \cos \beta \in D$ .

Hence, proved

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