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Chapter 24, Problem 1EC

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Problem

Prove: If A is not an integral domain, neither is $A[x]$.

Step-by-step solution

Step 1 of 1

Definition: A commutative ring with unity is said to be an integral domain if it has no zero divisors.

Consider a ring *A* which is not an integral domain.

Then by using above definition, *A* is not an integral domain implies there exists two numbers $a \neq 0, b \neq 0 \in A$ such that $ab = 0$ (That is there exist zero divisors).

Now consider a polynomial ring $A[x]$.

Now show that $A[x]$ is not integral domain.

For this find two non zero polynomials in $A[x]$ which are zero divisors.

Let $p(x) = ax$ and $q(x) = bx^2 + bx$.

Then,

$$\begin{aligned} p(x)q(x) &= (ax)(bx^2 + bx) \\ &= abx^3 + abx^2 \end{aligned}$$

Since $ab = 0$ implies,

$$\begin{aligned} p(x)q(x) &= 0x^3 + 0x^2 \\ &= 0 \end{aligned}$$

That implies there is zero divisors. It implies $A[x]$ is not an integral domain.

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