A Book of Abstract Algebra | (2nd Edition)

Chapter 27, Problem 6EC

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Problem

Let $p(x)$ be an irreducible polynomial of degree n over F. Let c denote a root of $p(x)$ in some extension of F (as in the basic theorem on field extensions).

Describe $\mathbb{Z}_3[x]/\langle x^3+x^2+2\rangle$, as in part 4.

Step-by-step solution

Step 1 of 2

Objective is to determine the elements of $Z_3[x]/\langle x^3+x^2+2\rangle$ with their addition and multiplication tables.

The elements of $Z_3[x]/\langle x^3+x^2+2\rangle$ has the following form:

$$\frac{Z_3[x]}{\langle x^3+x^2+2\rangle}=\{ax^2+bx+c+\langle x^3+x^2+2\rangle : a,b,c\in Z_3\}.$$

The polynomial is quadratic because all higher degree polynomials will get absorb by $\langle x^3+x^2+2\rangle$. Since $a,b,c\in Z_3$, so all three have only 3 choices. Thus, there are total $3^3=27$ elements.

Comment

Step 2 of 2

And the elements will be:

$$\frac{Z_3[x]}{\langle x^3+x^2+2\rangle}=\left\{\begin{array}{l}0,1,2,x,x+1,x+2,2x,2x+1,2x+2,\\ x^2,x^2+1,x^2+2,x^2+x,x^2+x+1,x^2+x+2,x^2+2x,\\ x^2+2x+1,x^2+2x+2,2x^2,2x^2+1,2x^2+2,2x^2+x,2x^2+x+1,\\ 2x^2+x+2,2x^2+2x,2x^2+2x+1,2x^2+2x+2\end{array}\right\}.$$

For the addition table and multiplication tables, use the properties of elements in Z_3 and the condition $x^3+x^2+2=0$:

$+$	0	1	2	x	$\cdots$
0	0	1	2	x	$\cdots$
1	1	2	$3\equiv 0$	$1+x$	$\cdots$
2	2	$3\equiv 0$	$4\equiv 1$	$2+x$	$\cdots$
x	x	$x+1$	$x+2$	$2x$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

$*$	0	1	2	x	$\cdots$
0	0	0	0	0	$\cdots$
1	0	1	2	x	$\cdots$
2	0	2	$4\equiv 1$	$2x$	$\cdots$
x	0	x	$2x$	x^2	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

Comment