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Chapter 29, Problem 6EE

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Problem

If *a* and *b* are algebraic over *F*, prove that F(*a*, *b*) is a finite extension of *F*.

Step-by-step solution

Step 1 of 2

Consider a field F. Suppose a and b are algebraic over F. The objective is to prove that $F(a,b)$ is a finite extension of F.

Comment

Step 2 of 2

Since b is algebraic over F, it is algebraic over $F(a)$.

Thus, $[F(a,b):F(a)]$ is finite.

Then $[F(a,b):F]=[F(a,b):F(a)][F(a):F]<\infty$.

Therefore , $F(a,b)$ is a finite extension of F.

Comment