

A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 3EA

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Problem

(a) Explain why $2x^2 \equiv 8 \pmod{10}$ has the same solutions as $x^2 \equiv 4 \pmod{5}$. (b) Explain why $x \equiv 2 \pmod{5}$ and $x \equiv 3 \pmod{5}$ are all the solutions of $2x^2 \equiv 8 \pmod{10}$.

Step-by-step solution

Step 1 of 3

(a)

Consider the congruence equation

$$2x^2 \equiv 8 \pmod{10}$$

Take $x^2 = y$ then $2y \equiv 8 \pmod{10}$

Use the result, the congruence $ax \equiv b \pmod{n}$ has a solution if and only if $\gcd(a, n) \mid b$ to solve the given equation.

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Step 2 of 3

The congruence equation $2y \equiv 8 \pmod{10}$ has a solution modulo 10 because

$$\gcd(2, 10) = 2 \text{ and } 2 \mid 8.$$

The solution of congruence equation $2y \equiv 8 \pmod{10}$ is same as the solution of

$$y \equiv 4 \pmod{5} \left(\text{since } \frac{2}{2}y \equiv \frac{8}{2} \pmod{\frac{10}{2}} \right).$$

Therefore, the solution of congruence equation $2x^2 \equiv 8 \pmod{10}$ is same as the solution of $x^2 \equiv 4 \pmod{5}$

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Step 3 of 3

(b)

The congruence equation $x^2 \equiv 4 \pmod{5}$ is equivalent to $(\bar{x})^2 = \bar{4}$ in Z_5 .

Need to find the values for x in Z_5 such that $(\bar{x})^2 = 4$ in Z_5

The solutions of $(\bar{x})^2 = 4$ in Z_5 are $\bar{x} = \bar{2}$ and $\bar{x} = \bar{3}$.

Verify that

$$\begin{aligned}(\bar{2})^2 &= (\bar{2})(\bar{2}) \\ &= \bar{4} \text{ in } Z_5\end{aligned}$$

$$\begin{aligned}(\bar{3})^2 &= (\bar{3})(\bar{3}) \\ &= \bar{9} \\ &= \bar{4} \text{ in } Z_5\end{aligned}$$

Therefore, the solutions of the congruence equation $2x^2 \equiv 8 \pmod{10}$ are

$$\boxed{x \equiv 2 \pmod{5} \text{ and } x \equiv 3 \pmod{5}}.$$

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