

A Book of Abstract Algebra | (2nd Edition)

Chapter AB, Problem 14E

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Problem

Prove that the following are true for any integers a , b , and c :

$$\text{lcm}(a, ab) = ab.$$

Step-by-step solution

Step 1 of 4

Objective:-

The objective is to prove $\text{lcm}(a, ab) = ab$.

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Step 2 of 4

Proof:-

Let us first prove a theorem which helps in prove this result.

Let us suppose $\text{gcd}(b, c) = t$.

Let us consider the theorem.

Theorem:-Any two nonzero integers r and s have a unique positive greatest common divisor t , Moreover, t is equal to a "Linear combination" of r and s . That is,

$$t = kr + ls \text{ for some integer } k \text{ and } l$$

According to this definition:-

$$t = mb + nc \text{ for some integer } m \text{ and } n \quad \text{.....(1)}$$

Let us multiply by a both sides.

$$ta = mba + nca \text{ for some integer } m \text{ and } n$$

$$ta = mab + nac \text{ for some integer } m \text{ and } n$$

Thus, according to the definition ta is greatest common divisor of ab and ac .

$$\gcd(ab, ac) = at \quad \dots\dots(2)$$

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Step 3 of 4

Let us consider the theorem.

Theorem:-If p and q are two integers with greatest common divisor $\gcd(p, q)$ and least common multiple $\text{lcm}(p, q)$, then

$$p \times q = \gcd(p, q) \times \text{lcm}(p, q)$$

According to this theorem:-

$$b \times c = \gcd(b, c) \times \text{lcm}(b, c)$$

$$b \times c = t \times \text{lcm}(b, c) \quad \dots\dots(3)$$

And,

$$ab \times ac = at \times \text{lcm}(ab, ac) \quad \dots\dots(4)$$

Let us divide the equation (3) by (4)

$$\frac{b \times c}{ab \times ac} = \frac{t \times \text{lcm}(b, c)}{at \times \text{lcm}(ab, ac)}$$

$$\frac{\cancel{b} \times \cancel{c}}{a^2 \cancel{(b \times c)}} = \frac{\cancel{t} \times \text{lcm}(b, c)}{a \cancel{t} \times \text{lcm}(ab, ac)}$$

$$\frac{1}{a \times \cancel{t}} = \frac{\text{lcm}(b, c)}{\cancel{t} \times \text{lcm}(ab, ac)}$$

$$\frac{1}{a} = \frac{\text{lcm}(b, c)}{\text{lcm}(ab, ac)}$$

$$\text{lcm}(ab, ac) = a \cdot \text{lcm}(b, c)$$

Proved

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Step 4 of 4

According to this theorem:-

$$\text{lcm}(a, ab) = \text{lcm}(a \cdot 1, a \cdot b)$$

$$\text{lcm}(a, ab) = a \cdot \text{lcm}(1, b)$$

The least common multiply of 1 and any integer is always equal to that integer.

So,

$$lcm(a,ab) = a \cdot b$$

$$lcm(a,ab) = ab$$

Proved

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