A Book of Abstract Algebra | (2nd Edition)

Chapter 16, Problem 4EF

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Problem

Let *G* be a group; let *H* and *K* be subgroups of *G*, with *H* a normal subgroup of *G*. Prove the following:

Every member of the quotient group *HK*/*H* may be written in the form *Hk* for some $k \in K$.

Step-by-step solution

Step 1 of 3

Suppose that *G* is any group and let *H, K* are the subgroups of *G*, with *H* a normal subgroup of *G*. Consider the following set:

$$HK = \{xy : x \in H, y \in K\}.$$

Objective is to prove that elements of quotient group HK/H can be written in the form Hk for some *k* in *K*.

According to the definition of factor group, if *G* be a group and *H* is its normal subgroup then G/H is defined. The elements of quotient group G/H will be of the form:

$$G/H = \{Ha : a \in G\}$$

with the operation of multiplication as $HaHb = Hab$.

Comment

Step 2 of 3

By using the definition of elements of quotient group, one get

$$HK/H = \{Hx : x \in HK\}.$$

Since $x \in HK$, therefore it will be the product of elements of *H* and *K*. That is,

$$x = hk,$$

for some $h \in H$ and $k \in K$.

Consider the following property of coset that states that,

$Ha = H$ if and only if $a \in H$.

Apply this here and get that if $h \in H$ then $Hh = H$. Thus,

$$\begin{aligned} HK/H &= \{Hx : x \in HK\} \\ &= \{Hhk : h \in H, k \in K\} \\ &= \{Hk : k \in K\}. \end{aligned}$$

Comment

Step 3 of 3

Hence, elements of quotient group HK/H can be written in the form Hk for some k in K.

Comment