

# A Book of Abstract Algebra | (2nd Edition)

Chapter 29, Problem 1EC

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## Problem

By the proof of the basic theorem of field extensions, if  $p(x)$  is an irreducible polynomial of degree  $n$  in  $F[x]$ , then  $F[x]/\langle p(x) \rangle \cong F(c)$  where  $c$  is a root of  $p(x)$ . By Theorem 1 in this chapter,  $F(c)$  is of degree  $n$  over  $F$ . Using the paragraph preceding Theorem 1:

Prove that every element of  $F(c)$  can be written *uniquely* as  $a_0 + a_1c + \cdots + a_{n-1}c^{n-1}$ , for some  $a_0, \dots, a_{n-1} \in F$ .

## Step-by-step solution

### Step 1 of 3

Consider that  $F$  is a field. Objective is to prove that every element of  $F(c)$  can be written uniquely as  $a_0 + a_1c + \cdots + a_{n-1}c^{n-1}$ , for some  $a_0, \dots, a_{n-1} \in F$ .

Note that,  $F(c)$  denotes the smallest field which contains  $F$  and  $c$ , where  $c$  is the root of some polynomial of  $F[x]$ . Since  $c$  is algebraic over  $F$ , therefore  $F(c)$  consists of all the elements of the form  $a(c)$ , for all  $a(x) \in F[x]$ .

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### Step 2 of 3

Since  $F(c)$  is an extension of  $F$ , so  $F(c)$  can be regarded as a vector space over  $F$ . According to the question details,  $F(c)$  is of degree  $n$  over  $F$ . So, let  $p(x)$  is the minimal polynomial of degree  $n$  of  $c$  over  $F$ . Then, the  $n$  elements

$$1, c, c^2, \dots, c^{n-1}$$

are linearly independent and span  $F(c)$ . It shows that the set of  $n$  vectors  $\{1, c, c^2, \dots, c^{n-1}\}$  is a

basis of  $F(c)$ .

By the basis property, each element of  $F(c)$  can be written uniquely as a linear combination of basis elements,  $1, c, c^2, \dots, c^{n-1}$ .

[Comment](#)

**Step 3 of 3**

Thus, every element of  $F(c)$  can be written uniquely as  $a_0 + a_1c + \dots + a_{n-1}c^{n-1}$ , for some  $a_0, \dots, a_{n-1} \in F$ .

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