A Book of Abstract Algebra | (2nd Edition)

Chapter 16, Problem 4EI

Problem

Let H and K be normal subgroups of a group G, with $H \subseteq k$ Define ϕ: $G/H \to G/K$ by $\phi(Ha) = Ka$. Prove part:

ker ϕ K/H

Step-by-step solution

Step 1 of 3

Suppose that G is any group and let H, K are normal subgroups of G with $H \subseteq K$.

Consider a mapping $\phi : G/H \to G/K$ defined by

$\phi(Ha) = Ka$, for all $a \in G$.

Objective is to prove that $\ker\phi = K/H$.

Comment

Step 2 of 3

Let $Ha \in G/H$, for some $a \in G$. If $Ha \in Ker\phi$ then by the define function,

$$\phi(Ha) = K$$
$$Ka = K.$$

By the coset property, the last step implies that $a \in K$. The condition $a \in K$ corresponds that $Ha \in K/H$. Thus,

$\ker\phi \subseteq K/H$.

Now let $Ha \in K/H$. Then $a \in K$. Also it implies that

$$Ka = K$$
$$\phi(Ha) = K$$

And thus

$K/H \subseteq \ker\phi$.

On combining both the equations of containment, one get

$\ker\phi = K/H$.

Comment

Step 3 of 3

Hence, $\ker\phi = K/H$ as required.

Comment