A Book of Abstract Algebra | (2nd Edition)

Problem

Let F be any field.

Prove part:

Name a field $(\neq \mathbb{R} \text{ or } \mathbb{C})$ which contains a root of $x^5 + 2x^3 + 4x^2 + 6$.

Step-by-step solution

Step 1 of 3

Objective is to find the field $(\neq R \text{ or } C)$ that contains a root of $x^5+2x^3+4x^2+6$. That is, find an algebraically closed field for polynomial $x^5+2x^3+4x^2+6$.

The one way to find such field is calculate the roots of this polynomial. Since polynomial $x^5+2x^3+4x^2+6$ is of degree 5, so root calculation is not so easy task. One can use any software or programmable calculator for finding such roots.

Comment

Step 2 of 3

The roots of $x^5+2x^3+4x^2+6=0$, by using software, are:

$$-1.52365$$
$$0.0943563 \pm 1.25534i$$
$$0.667471 \pm 1.42804i$$

Note that, number are rationals along with the complex number i. So, the required field will be extension of Q with i, that is, $Q(i)$.

Comment

Step 3 of 3

Hence, the field $Q(i)$ contain the roots of $x^5+2x^3+4x^2+6$.

Comment