A Book of Abstract Algebra | (2nd Edition)

Chapter AA, Problem 17E

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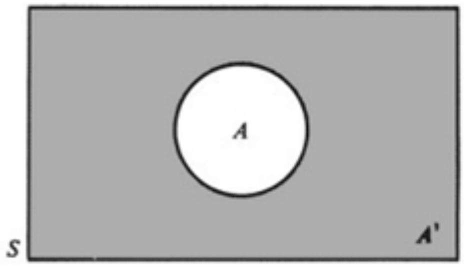
Problem

If *S* is a set, and *A* is a subset of *S*, then the *complement* of *A* in *S* is the set of all the elements of *S* which are not in *A*. The complement of *A* is denoted by *A*′:



$$A' = \{x \in S : x \notin A\}$$

Prove the following'.

$(A')' = A$.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $(A')' = A$.

Comment

Step 2 of 2

Proof:-

Let *A* and *B* are two sets.

If *S* is a set and *A* is a subset of *S*, then complementary of set *A* is defined as:-

$$A' = \{x \in S : x \notin A\}$$

Let S is a set and A is a subset of S. Let $x \in (A')'$.

$$x \in (A')'$$
$$\Rightarrow x \notin A'$$
$$\Rightarrow x \in A$$

So,

$$(A')' \subseteq A \qquad \text{......(1)}$$

Let $x \in A$.

$$x \in A$$
$$\Rightarrow x \notin A'$$
$$\Rightarrow x \in (A')'$$

So,

$$A \subseteq (A')' \qquad \text{......(2)}$$

Let us consider the equation (1) and (2).

$$(A')' = A$$

Proved

Comment