A Book of Abstract Algebra (2nd Edition)

Chapter AA, Problem 15E

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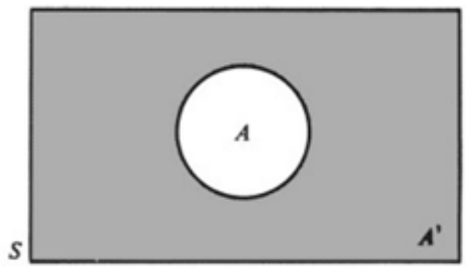
Problem

If S is a set, and A is a subset of S, then the *complement* of A in S is the set of all the elements of S which are not in A. The complement of A is denoted by A':



$$A' = \{x \in S : x \notin A\}$$

Prove the following'.

$(A \cup B)' = A' \cap B'$.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $(A \cup B)' = A' \cap B'$.

Comment

Step 2 of 2

Proof:-

Let A and B are two sets.

If S is a set and A is a subset of S, then complementary of set A is defined as:-

$$A' = \{x \in S : x \notin A\}$$

Let *S* is a set and *A* and *B* are subset of *S*. Let $x \in (A \cup B)'$.

$$x \in (A \cup B)'$$
$$\Rightarrow x \notin (A \cup B)$$
$$\Rightarrow x \notin A \;\; and \;\; x \notin B$$
$$\Rightarrow x \in A' \;\; and \;\; x \in B'$$
$$\Rightarrow x \in A' \cap B'$$

So,

$$(A \cup B)' \subseteq A' \cap B' \qquad \text{......(1)}$$

Let $x \in A' \cap B'$

$$x \in A' \cap B'$$
$$\Rightarrow x \in A' \;\; and \;\; x \in B'$$
$$\Rightarrow x \notin A \;\; and \;\; x \notin B$$
$$\Rightarrow x \notin (A \cup B)$$

$$x \in (A \cup B)'$$

So,

$$A' \cap B' \subseteq (A \cup B)' \qquad \text{......(2)}$$

Let us consider the equation (1) and (2).

$$(A \cup B)' = A' \cap B'$$

Proved

Comment