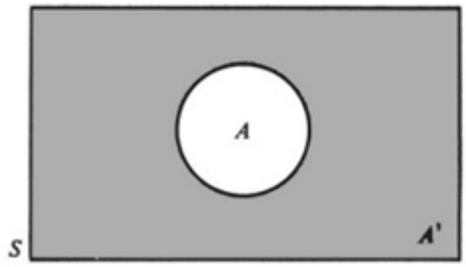
A Book of Abstract Algebra | (2nd Edition)

Problem

If *S* is a set, and *A* is a subset of *S*, then the *complement* of *A* in *S* is the set of all the elements of *S* which are not in *A*. The complement of *A* is denoted by *A*′:



$$A' = \{x \in S : x \notin A\}$$

Prove the following'.

If $A \subseteq B$, then $A \cap B' = 0$, and conversely.

Step-by-step solution

Step 1 of 4

Objective:-

The objective is to prove that if $A \subseteq B$, *then* $A \cup B' = 0$. Conversely, if $A \cup B' = 0$, *then* $A \subseteq B$.

Comment

Step 2 of 4

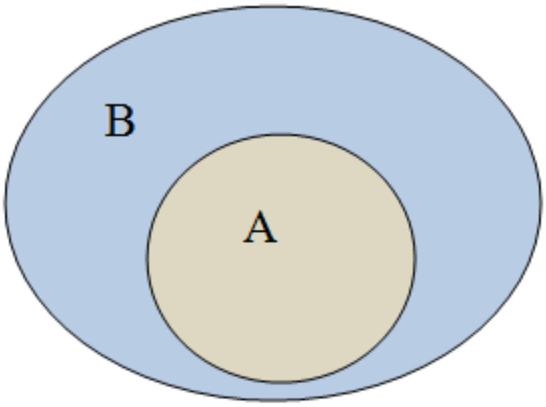
Proof:-

Let *A* and *B* are two sets. Let $x \in A \subseteq B$.

Subsets:-If sets *A* and *B* are such that every elements of *A* are also elements of *B*, then *A* is said to be subset of *B*.

$$A \subseteq B \Leftrightarrow \{x \in A \Rightarrow x \in B\}$$

So the set *B* contains the set *A* and set *A* completely lies within set *B*.

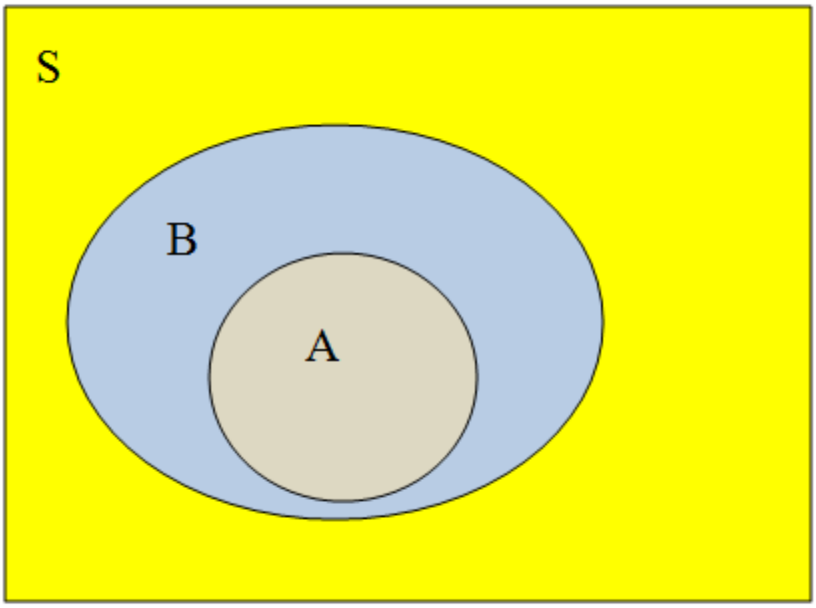


If *S* is a set and *A* is a subset of *S*, then complementary of set *A* is defined as:-

$$A' = \{x \in S : x \notin A\}$$

According to this definition:-

$$B' = \{x \in S : x \notin B\}$$



The B' is shown by the yellow color in the figure.

The intersection of two sets *A* and *B* is:-

$$A \cap B = \{x : x \in A \ \ and \ \ x \in B\}$$

According to this definition:-

$$A \cap B' = \{x : x \in A \ \ and \ \ x \in B'\}$$

Hence,

$$If \ \ A \subseteq B, then \ \ A \cup B' = 0.$$

Proved

Comment

Step 3 of 4

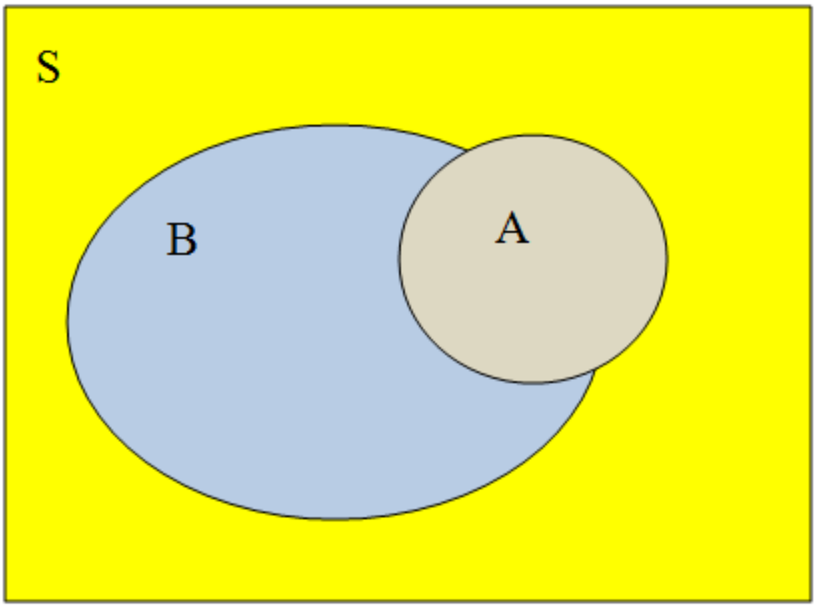
Conversely:-

The union of two sets *A* and *B* is:-

$$A \cup B = \{x : x \in A \ \ or \ \ x \in B\}$$

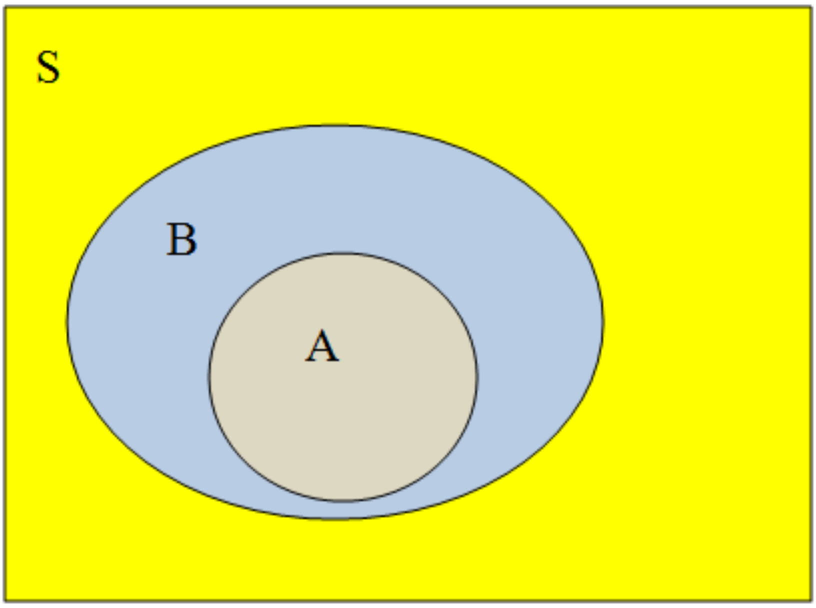
According to this definition:-

$$A \cap B' = \{x : x \in A \ \ and \ \ x \in B'\}$$



The B' is shown by the yellow color in the figure.

If $A \cup B' = 0$, then there is no common elements in set *A* and B'. So set B completely contains the set *A*.



Comment

Step 4 of 4

Subsets:-If sets *A* and *B* are such that every elements of *A* are also elements of *B*, then *A* is said to be subset of *B*.

$$A \subseteq B \Leftrightarrow \{x \in A \Rightarrow x \in B\}$$

According to this definition *A* is subset of *B*.

Hence,

$$A \cup B' = 0, \textit{then} \quad A \subseteq B.$$

Proved

Comment