A Book of Abstract Algebra | (2nd Edition)

Chapter 29, Problem 4EE

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Problem

Use part 3 and Eisentein ' s irreducibility criterion to prove that $\sqrt{m/n}$(where ra, m n $\in \mathbb{Z}$) is irrational if there is a prime number which divides ra but not *n*,and whose square does not divide ra.

Step-by-step solution

Step 1 of 3

Let *F* be a field. Consider the following result:

If a real number *c* is a root of an irreducible polynomial of degree >1 in $Q[x]$, then *c* is irrational.

Objective is to prove that $\sqrt{m/n}$, where $m,n \in Z$, is irrational if there is a prime which divides *m* but not *n*, and whose square does not divide *m*.

Comment

Step 2 of 3

Let $x=\sqrt{m/n}$. Then $x^2=\frac{m}{n}$, and $nx^2-m=0$. Assume that $p(x)=nx^2-m$. By Eisenstein's irreducible criterion, the polynomial $p(x)$ will be irreducible if there is a prime number *p* such that $p\mid m,\ p\nmid n$ and $p^2\nmid m$. Let these conditions holds and $p(x)$ is irreducible.

Since $x=\sqrt{m/n}$ is a root of an irreducible polynomial $p(x)$ of degree >1 in $Q[x]$, therefore by the above result $\sqrt{m/n}$ will be irrational.

Comment

Step 3 of 3

Hence, $\sqrt{m/n}$, where $m,n\in Z$, is irrational if there is a prime which divides *m* but not *n*, and whose square does not divide *m*.

Comment