

A Book of Abstract Algebra | (2nd Edition)

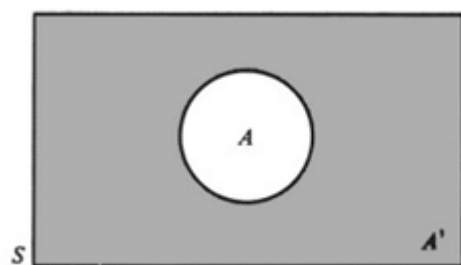
Chapter AA, Problem 18E

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Problem

If S is a set, and A is a subset of S , then the *complement* of A in S is the set of all the elements of S which are not in A . The complement of A is denoted by A' :



$$A' = \{x \in S : x \notin A\}$$

Prove the following'.

$$A \cap A' = \emptyset.$$

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $A \cap A' = \emptyset$.

[Comment](#)

Step 2 of 2

Proof:-

Let A and B are two sets.

If S is a set and A is a subset of S , then complementary of set A is defined as:-

$$A' = \{x \in S : x \notin A\}$$

Let S is a set and A is a subset of S . Let $x \in A \cap A'$.

$$x \in A \cap A'$$

$$\Rightarrow x \in A \text{ and } x \in A'$$

$$\Rightarrow x \in A \text{ and } x \notin A$$

$$\Rightarrow x \in \emptyset$$

Since these two operations can never occur simultaneously.

So,

$$A \cap A' \subseteq \emptyset \quad \dots\dots(1)$$

The empty set is subset of each set.

So,

$$\emptyset \subseteq A \cap A' \quad \dots\dots(2)$$

Let us consider the equation (1) and (2).

$$A \cap A' = \emptyset$$

Proved

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