

# A Book of Abstract Algebra | (2nd Edition)

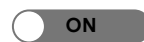


Chapter 32, Problem 3EG



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## Problem

Let  $F$  be a field, and  $K$  a finite extension of  $F$ . Suppose  $a, b \in K$ . Prove part:

Aside from the identity function, there are no  $\mathbb{Q}$ -fixing automorphisms of  $\mathbb{Q}(\sqrt[3]{2})$ . [HINT:

Note that  $\mathbb{Q}(\sqrt[3]{2})$  contains only real numbers.]

## Step-by-step solution

### Step 1 of 2

The objective is to prove that aside from the identity function, there are no  $\mathbb{Q}$ -fixing automorphism of  $\mathbb{Q}(\sqrt[3]{2})$ .

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### Step 2 of 2

Because  $\mathbb{Q}(\sqrt[3]{2})$  lies in  $\mathbb{R}$  and the other two conjugates  $\sqrt[3]{2} \cdot \frac{-1+\sqrt{3}i}{2}$  and  $\sqrt[3]{2} \cdot \frac{-1-\sqrt{3}i}{2}$  of  $\sqrt[3]{2}$  do not lie in  $\mathbb{R}$ , no map of  $\sqrt[3]{2}$  into any conjugate other than  $\sqrt[3]{2}$  itself can give rise to an automorphism of  $\mathbb{Q}(\sqrt[3]{2})$ .

The other two maps give rise to isomorphism of  $\mathbb{Q}(\sqrt[3]{2})$  onto a subfield of  $\bar{\mathbb{Q}}$ .

Because any automorphism of  $\mathbb{Q}(\sqrt[3]{2})$  must leave the prime field  $\mathbb{Q}$  fixed, the identity is the only automorphism of  $\mathbb{Q}(\sqrt[3]{2})$ .

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