A Book of Abstract Algebra | (2nd Edition)

Chapter AA, Problem 5E

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Problem

Prove the following:

$A \cap B = B \cap A$.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $A \cap B = B \cap A$.

Comment

Step 2 of 2

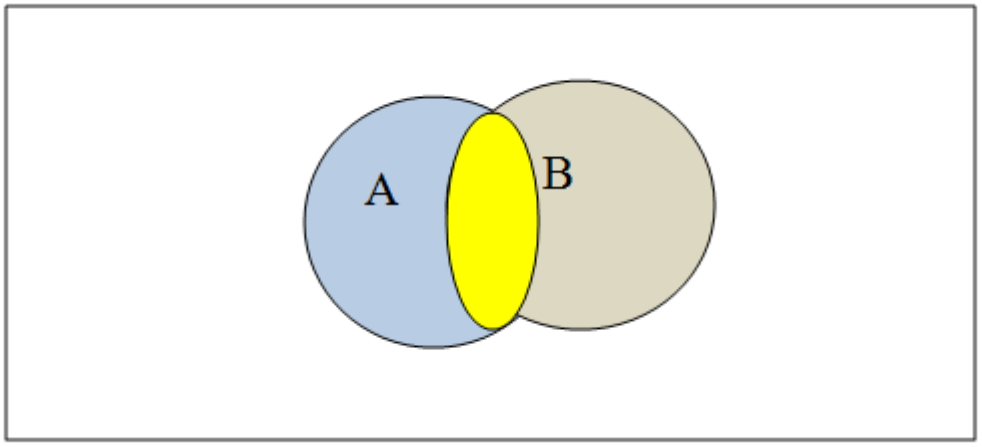
Proof:-

Let A and B are two sets

The set of common elements of sets A and B is called intersection of two sets. The intersection of two sets A and B is:-

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Let us show graphically the intersection two sets A and B.



According to this definition:-

$$A \cap B \Rightarrow x \in A \ \ and \ \ x \in B$$
$$\Rightarrow x \in B \ \ and \ \ x \in A$$
$$\Rightarrow x \in B \cap A$$

So,

$$A \cap B \subseteq B \cap A \qquad \ldots\ldots(1)$$

Let $x \in B \cap A$.

$$B \cap A \Rightarrow x \in B \ \ and \ x \in A$$
$$\Rightarrow x \in A \ \ and \ \ x \in B$$
$$\Rightarrow x \in A \cap B$$

So,

$$B \cap A \subseteq A \cap B \qquad \ldots\ldots(2)$$

Let us consider the equation (1) and (2).

$$A \cap B = B \cap A.$$

Proved

Comment