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Chapter 28, Problem 6EC

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Problem

Find a basis for the subspace of $\mathbb{R}^3$ spanned by the set of vectors (x, y, z) such that $x^2 + y^2 + z^2 = 1$.

Step-by-step solution

Step 1 of 2

(a_1, a_2, a_3) represents a vector space in 3 dimension or $\mathbb{R}^3$ as it satisfies all conditions for vector space.

For 3 dimension, any subspace must be a plane or line or a point passing through origin. The reason for it lies in the fact that any linear combination of 2 vectors lying on plane and line also lies on that vector space.

Comment

Step 2 of 2

Given condition for subspace is

$$x^2 + y^2 + z^2 = 1$$

This does not represents an equation of plane or line or point but represents an equation of a sphere of radius 1. Thus it is not a vector space.

Also points $u = (1,0,0)$ and $v = (-1,0,0)$ lies on vector space, but their combination,

$u + v = (0,0,0)$ does not lie on sphere

$\boxed{\text{Hence given set is not a subspace and there is no basis}}$

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