

A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 2EA

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Problem

Prove that $x^2 - 3$ and $x^2 - 2x - 2$ are both irreducible over \mathbb{Q} . Then find their root fields over \mathbb{Q} and show they are the same.

Step-by-step solution

Step 1 of 3

The objective is to prove that $x^2 - 3$ and $x^2 - 2x - 2$ are irreducible over \mathbb{Q} . Also, find their roots fields over \mathbb{Q} and show that they are same.

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Step 2 of 3

Eisenstein's irreducibility criteria- "Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$ be a polynomial with integer coefficients. If there exists a prime number p such that $p \nmid a_n$, $p \mid a_{n-1}, a_{n-2}, \dots, a_1, a_0$, and $p^2 \nmid a_0$, then $f(x)$ is irreducible over \mathbb{Q} ".

For $f(x) = x^2 - 3$, $a_2 = 1, a_1 = 0, a_0 = -3$.

$3 \nmid 1$, $3 \mid 0, -3$, and $9 \nmid -3$. Therefore, by Eisenstein's irreducibility criteria, $x^2 - 3$ is irreducible over \mathbb{Q} .

For $f(x) = x^2 - 2x - 2$, $a_2 = 1, a_1 = -2, a_0 = -2$.

$2 \nmid 1$, $2 \mid -2, -2$, and $4 \nmid -2$. Therefore, by Eisenstein's irreducibility criteria, $x^2 - 2x - 2$ is irreducible over \mathbb{Q} .

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The roots of $x^2 - 3$ are $\pm\sqrt{3}$. Therefore, the root field of $x^2 - 3$ over \mathbb{Q} is $\mathbb{Q}(\pm\sqrt{3})$. This can be written simply as $\mathbb{Q}(\sqrt{3})$.

Use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find roots of $x^2 - 2x - 2$.

Here, $a = 1$, $b = -2$, and $c = -2$.

$$\begin{aligned} x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} \\ &= \frac{2 \pm \sqrt{12}}{2} \\ &= 1 \pm \sqrt{3} \end{aligned}$$

Therefore, the root field of $x^2 - 2x - 2$ over \mathbb{Q} is $\mathbb{Q}(1 \pm \sqrt{3})$. This can be written simply as $\mathbb{Q}(\sqrt{3})$.

Therefore, it is shown that $x^2 - 3$ and $x^2 - 2x - 2$ have same root fields over \mathbb{Q} .

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