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Chapter 29, Problem 2EE

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Problem

Let *F* be a field.

Prove part:

a is of degree 1 over *F* iff $a \in F$.

Step-by-step solution

Step 1 of 3

Let *F* be a field. Objective is to prove that *a* is of degree 1 over *F* if and only if $a \in F$.

First consider that *a* is of degree 1 over *F*. Note that degree 1 polynomial is linear and always an irreducible. Therefore, $p(x) = x - a$ will be minimum polynomial of *a*. Also, $p(x) = 0$ implies that $x - a = 0$ and then $x = a$. Thus, $a \in F$.

Comment

Step 2 of 3

Conversely, assume that $a \in F$. Then degree of $F(a)$ over *F* will be 1, this is so because in this case $F(a)$ will be same as *F*, or $F(a) = F$. This shows that minimal polynomial of *a* will be linear. Thus, degree of *a* will be 1.

Comment

Step 3 of 3

Hence, *a* is of degree 1 over *F* if and only if $a \in F$.

Comment