

# A Book of Abstract Algebra | (2nd Edition)



Chapter 29, Problem 1EA



Bookmark

Show all steps: ☒ ON



## Problem

Find a basis for  $\mathbb{Q}(i\sqrt{2})$  over  $\mathbb{Q}$ , and describe the elements of  $\mathbb{Q}(i\sqrt{2})$ . (See the two examples immediately following Theorem 1.)

## Step-by-step solution

### Step 1 of 2

The objective is to find a basis for  $\mathbb{Q}(i\sqrt{2})$  over  $\mathbb{Q}$ , and describe the elements of  $\mathbb{Q}(i\sqrt{2})$ .

[Comment](#)

### Step 2 of 2

The minimal polynomial of  $i\sqrt{2}$  over  $\mathbb{Q}$  is  $x^2 + 2$  (It is monic and irreducible (2-Eisenstein) with  $i\sqrt{2}$  as a root).

Hence  $\left[ \begin{pmatrix} i\sqrt{2} \end{pmatrix} \right] = 2$  and a basis is  $\{1, i\sqrt{2}\}$ .

Every element in  $\left[ \begin{pmatrix} i\sqrt{2} \end{pmatrix} \right]$  is therefore a linear combination of  $1$  and  $i\sqrt{2}$ , that is, a number of the form  $a + bi\sqrt{2}$  where  $a, b \in \mathbb{R}$ .

---

[Comment](#)