A Book of Abstract Algebra | (2nd Edition)

Chapter 27, Problem 2EJ

Bookmark

Show all steps: ON

Problem

Suppose $a(x) \in F[x]$, and K is an extension of F. An element $c \in K$ is called a multiple root of $a(x)$ if $(x-c)^m | a(x)$ for some $m > 1$. It is often important to know if all the roots of a polynomial are different, or not.

We now consider a method for determining whether an arbitrary polynomial $a(x) \in F[x]$ has multiple roots in any extension of F.

Let K be any field containing all the roots of $a(x)$. Suppose $a(x)$ has a multiple root c.

Compute $a'(x)$, using part 1.

Step-by-step solution

Step 1 of 2

Consider that K is any field that contains all the roots of polynomial $a(x) = a_0 + a_1x + \cdots + a_nx^n$. Assume that $a(x)$ has a multiple root c. Then polynomial $a(x)$ will be

$$a(x) = (x-c)^2 q(x) \in K[x].$$

Objective is to determine the derivative of $a(x)$.

Use the following formula: for some $a(x), b(x) \in F[x]$,

$$[a(x)b(x)]' = a'(x)b(x) + a(x)b'(x).$$

Comment

Step 2 of 2

The required derivative $a'(x)$ will be:

$$\begin{aligned} a'(x) &= \left[(x-c)^2\right]' q(x) + (x-c)^2 q'(x) \\ &= 2(x-c)q(x) + (x-c)^2 q'(x) \\ &= (x-c)[2q(x) + (x-c)q'(x)]. \end{aligned}$$

Hence, $a'(x) = (x-c)[2q(x) + (x-c)q'(x)]$.

Comment