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Chapter 28, Problem 5EC

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Problem

Find a basis for each of the following subspaces of $\mathbb{R}^3$:

(a) $S_1 = \{(x, y, z) : 3x - 2y + z = 0\}$ (b) $S_2 = \{(x, y, z) : x + y - z = 0 \text{ and } 2x - y + z = 0\}$

Step-by-step solution

Step 1 of 4

All 3 subspaces represents a plane in $\mathbb{R}^3$. Consequently, all these subspaces have dimension of 2 in 3-dimensional vector space.

To determine basis for these, any 2 of components are assigned values which in themselves form independent 2- dimensional vectors and 3rd component is calculated.

Comment

Step 2 of 4

(a) Subspace is represented by $3x - 2y + z = 0$

2 vectors with first 2 components being linearly independent are $(x=1, y=0; x=0, y=1)$.

Substituting these in plane equation 3rd component of these 2 vector are obtained.

For $(x=1, y=0)$, $z=-3$

For $(x=0, y=1)$, $z=2$

$$\boxed{\text{Hence basis for given subspace is } \begin{pmatrix}1\\0\\-3\end{pmatrix}, \begin{pmatrix}0\\1\\2\end{pmatrix}}$$

Comment

Step 3 of 4

(b) Subspace is represented by $x+y-z=0$

2 vectors with first 2 components being linearly independent are $(x=1, y=0; x=0, y=1)$.

Substituting these in plane equation 3rd component of these 2 vector are obtained.

For $(x=1, y=0)$, $z=1$

For $(x=0, y=1)$, $z=1$

$$\boxed{\text{Hence basis for given subspace is } \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}}$$

Comment

Step 4 of 4

(c) subspace is represented by $2x-y+z=0$

2 vectors with first 2 components being linearly independent are $(x=1, y=0; x=0, y=1)$.

Substituting these in plane equation 3rd component of these 2 vector are obtained.

For $(x=1, y=0)$, $z=-2$

For $(x=0, y=1)$, $z=1$

$$\boxed{\text{Hence basis for given subspace is } \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}$$

Comment