

A Book of Abstract Algebra | (2nd Edition)

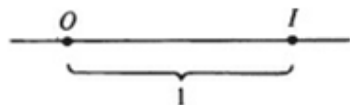
Chapter 30, Problem 4EA

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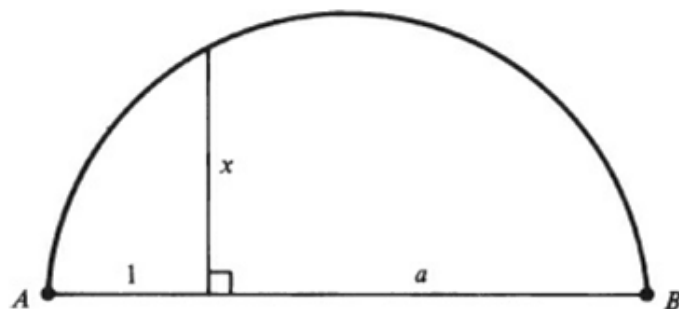
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Problem

If O and I are any two points in the plane, consider a coordinate system such that the interval OI coincides with the unit interval on the x axis. Let \mathbb{D} be the set of real numbers such that $a \in \mathbb{D}$ iff the point $(a, 0)$ is constructible from $\{O, I\}$. Prove the following:



If $a > 0$ and $a \in \mathbb{D}$, then $\sqrt{a} \in \mathbb{D}$. (HINT: In the accompanying figure, AB is the diameter of a circle. Use an elementary property of chords of a circle to show that $x = \sqrt{a}$.)



It follows from parts 1 to 4 that \mathbb{D} is a field, closed with respect to taking square roots of positive numbers. \mathbb{D} is called the *field of constructible numbers*.

Step-by-step solution

Step 1 of 3

Here, objective is to prove that, if $a \in D, a > 0$, then $\sqrt{a} \in D$.

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Step 2 of 3

Constructible point:

This point is either the end point of given unit segment or it is the intersection of two lines determined by previous constructible points.

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Step 3 of 3

Consider $a \in D$ if and only if the point $(a,0)$ is constructible from $\{O,I\}$

Now, we have to prove that length \sqrt{a} is constructed from the length a

Consider the below figure:

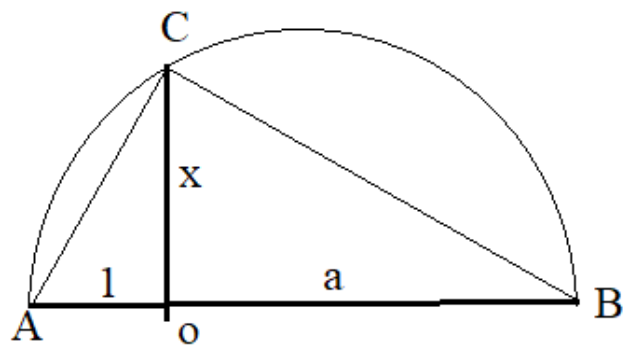


figure:construction of sqare root of a

AB is the diameter of a circle.

By observing there exist two similar triangles

$\triangle AOB$ and $\triangle BOC$

Using the property of similar triangles, we have

$$\frac{1}{x} = \frac{x}{a}$$

$$x^2 = a$$

$$x = \sqrt{a}$$

Then, the length \sqrt{a} is constructed from $\{O, I\}$ which implies $\sqrt{a} \in D$

Therefore, if $a \in D, a > 0$, then $\sqrt{a} \in D$.

Hence, proved

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