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Chapter 16, Problem 3EI Bookmark Show all steps: ON

Problem

Let H and K be normal subgroups of a group G, with $H \subseteq k$ Define ϕ: $G/H \to G/K$ by $\phi(Ha) = Ka$. Prove part:

ϕ is surjective.

Step-by-step solution

Step 1 of 3

Suppose that G is any group and let H, K are normal subgroups of G with $H \subseteq K$.

Consider a mapping $\phi : G/H \to G/K$ defined by

$\phi(Ha) = Ka$, for all $a \in G$.

Objective is to prove that function ϕ is surjective, that is, onto.

A mapping $f : G \to H$ is said to be surjective if for every $y \in H$ there exists $x \in G$ such that $f(x) = y$.

Comment

Step 2 of 3

Assume that $Kx \in G/K$, for some $x \in G$. An element *x* is the member of *G* because of the coset of G/K. So from there it implies that

$$Hx \in G/H$$

such that

$$\phi(Hx) = Kx.$$

This satisfies the definition of surjective map.

Comment

Step 3 of 3

Hence, function ϕ is surjective.

Comment