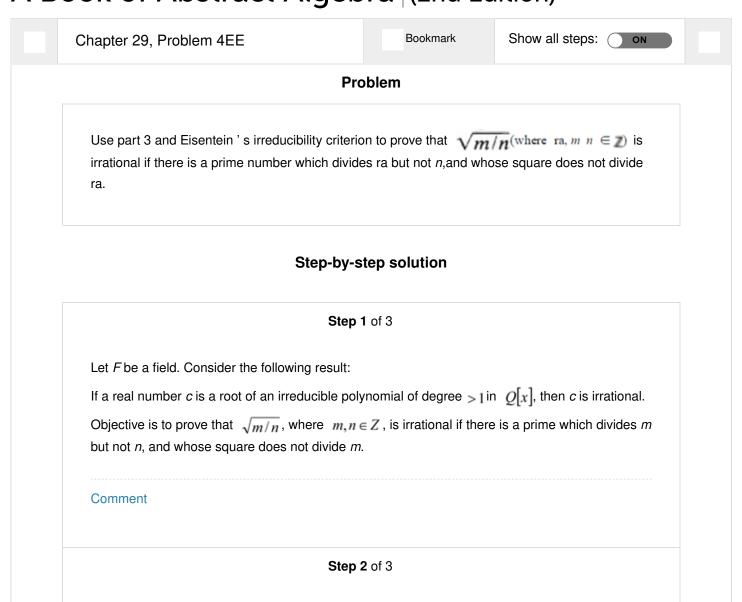
A Book of Abstract Algebra (2nd Edition)



Let $x = \sqrt{m/n}$. Then $x^2 = \frac{m}{n}$, and $nx^2 - m = 0$. Assume that $p(x) = nx^2 - m$. By Eisenstein's irreducible criterion, the polynomial p(x) will be irreducible if there is a prime number p such that $p \mid m$, $p \nmid n$ and $p^2 \nmid m$. Let these conditions holds and p(x) is irreducible.

Since $x = \sqrt{m/n}$ is a root of an irreducible polynomial p(x) of degree > 1 in Q[x], therefore by the above result $\sqrt{m/n}$ will be irrational.

Comment

Step 3 of 3

Hence, $\sqrt{m/n}$, where $m,n\in \mathbb{Z}$, is irrational if there is a prime which divides m but not n, and whose square does not divide m.

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