

# A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 5EB

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## Problem

Find the root field of  $x^3 + x^2 + x + 2$  over  $\mathbb{Z}_3$ . Find a basis for this root field over  $\mathbb{Z}_3$ .

## Step-by-step solution

### Step 1 of 3

The objective is to find the root field of  $x^3 + x^2 + x + 2$  over  $\mathbb{Z}_3$ , and a basis for this root field over  $\mathbb{Z}_3$ .

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### Step 2 of 3

Let  $a(x) = x^3 + x^2 + x + 2 \in \mathbb{Z}_3$ .

$$\begin{aligned} a(0) &= 0^3 + 0^2 + 0 + 2 \\ &= 2 \\ &\neq 0 \end{aligned}$$

$$\begin{aligned} a(1) &= 1^3 + 1^2 + 1 + 2 \\ &= 1 + 1 + 1 + 2 \\ &= 2 \\ &\neq 0 \end{aligned}$$

$$\begin{aligned} a(2) &= 2^3 + 2^2 + 2 + 2 \\ &= 8 + 4 + 2 + 2 \\ &= 1 \\ &\neq 0 \end{aligned}$$

Therefore,  $a(x)$  has no roots in  $\mathbb{Z}_3$ .

This implies that  $x^3 + x^2 + x + 2$  is irreducible over  $\mathbb{Z}_3$ .

Therefore, the root field of  $x^3 + x^2 + x + 2$  over  $\mathbb{Z}_3$  is  $\mathbb{Z}_3[x]/(x^3 + x^2 + x + 2)$ .

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Step 3 of 3

The root field has  $3^3 = 27$  times.

If  $k$  is any root of  $a(x) = x^3 + x^2 + x + 2$ , then the basis for this root field is

$$\left\{ 0, 1, 2, k, k+1, k+2, k^2, k^2+1, k^2+2, k^2+k, k^2+k+1, k^2+k+2, k^3, \right. \\ \left. k^3+1, k^3+2, k^3+k, k^3+k+1, k^3+k+2, k^3+k^2, k^3+k^2+1, k^3+k^2+2, \right. \\ \left. k^3+k^2+k, k^3+k^2+k+1, k^3+k^2+k+2 \right\}.$$

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