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Chapter 28, Problem 3ED

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Problem

Let *V* be a finite-dimensional vector space. Let dim *V* designate the dimension of *V*. Prove each of the following:

Any set of vectors containing **0** is linearly dependent.

Step-by-step solution

Step 1 of 3

A set of vectors which is said to be linearly independent if there exists no combination of these vectors which can give 0 vector apart from a combination in which all coefficients are 0.

Comment

Step 2 of 3

If $u_1, u_2, \ldots, u_n$ are *n* vectors of a vector space and these are linearly independent. Then for.

$a_1u_1 + a_2u_2 + \ldots + a_nu_n = 0$

All a_i have to be zero.

Comment

Step 3 of 3

Now consider any set which includes 0 vector among them. Let this set be $(u_1, u_2, \ldots, u_n, 0)$. One particular combination of these vectors which give 0 is

$$0 \cdot u_1 + 0 \cdot u_2 + \ldots + 0 \cdot u_n + a \cdot 0 = 0$$

Here *a* can have any value not necessarily equal to 0. Thus this combination fails to satisfy condition for being linearly independent. $\boxed{\text{Hence any set with } \mathbf{0} \text{vector is linearly dependent}}$.

Comment