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Chapter 23, Problem 5EC | Bookmark | Show all steps: ON

Problem

Prove the following for all integers *a*, *b*, *c*, *d* and all positive integers *m* and *n:*

If $ab \equiv 0 \pmod{p}$, where *p* is a prime, then $a \equiv 0 \pmod{p}$ or $b \equiv 0 \pmod{p}$.

Step-by-step solution

Step 1 of 2

Consider the congruence equation

$ab \equiv 0 \pmod{p}$, where p is a prime

The object of the problem is to prove that if $ab \equiv 0 \pmod{p}$, where p is a prime then $a \equiv 0 \pmod{p}$ or $b \equiv 0 \pmod{p}$.

Use this definition, $a \equiv b \pmod{n}$ iff n divides $a - b$, to prove the given result.

Given that $ab \equiv 0 \pmod{p}$ so by the definition,

p divides ab

Comment

Step 2 of 2

Here p is a prime and by the result, if $p \mid cd$, where p is prime then $p \mid c$ or $p \mid d$

Thus, p divides a or p divides b.

Again by the definition of congruence equation,

$a \equiv 0 \pmod{p}$ or $b \equiv 0 \pmod{p}$

Therefore, if $ab \equiv 0 \pmod{p}$, where p is a prime then $a \equiv 0 \pmod{p}$ or $b \equiv 0 \pmod{p}$

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