A Book of Abstract Algebra | (2nd Edition)

Problem

Let F be any field.

Prove part:

If cd is algebraic over F, then c is algebraic over $F(d)$. If $c + d$ is algebraic over F, then c is algebraic over $F(d)$ (Assume $c \neq 0$ and $d \neq 0$.)

Step-by-step solution

Step 1 of 4

If cd is algebraic over F, then c is algebraic over $F(d)$. If $c+d$ is algebraic over F, then c is algebraic over $F(d)$. (assume $c \neq 0$ and $d \neq 0$)

Comment

Step 2 of 4

Since cd is algebraic over F, there exists a polynomial $f(x) \in F[x]$ such that $f(cd) = 0$.

We need to find a polynomial over $F(d)$ such that c is root of that polynomial.

Consider the polynomial $g(x) = f(xd)$ in $F(d)[x]$.

Then,

$g(c) = f(cd) \ = 0$.

Therefore, c is also algebraic over $F(d)$.

Comment

Step 3 of 4

Now since $c, d \neq 0$ is algebraic over F, then there exists $f(x) \in F[x]$ such that $f(c+d) = 0$

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Consider $h(x) = f(x+d)$ in $F(d)$. Then,

$h(c) = f(c+d) = 0$

Therefore, c is also algebraic over $F(d)$.

Comment

Step 4 of 4

Comment