

A Book of Abstract Algebra | (2nd Edition)

Chapter AC, Problem 4E

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Problem

Use mathematical induction to prove the following:

$$1^3 + 2^3 + \cdots + n^3 = \frac{1}{4} n(n+1)^2$$

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $1^3 + 2^3 + \cdots + n^3 = \frac{1}{4} n^2 (n+1)^2$ using mathematical induction.

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Step 2 of 2

Proof:-

$$p(n): 1^3 + 2^3 + \cdots + n^3 = \frac{1}{4} n^2 (n+1)^2$$

Let consider rule for $n = 1$.

$$p(1): 1^3 = \frac{1}{4} \cdot 1^2 (1+1)^2$$

$$p(1): 1 = \frac{1}{4} \cdot 4$$

$$p(1): 1 = 1$$

This rule is true for $n = 1$.

Let this statement is true for $n = k$.

$$P(k): 1^3 + 2^3 + \dots + k^3 = \left(k \left(\frac{k+1}{2} \right) \right)^2 \quad \dots\dots(1)$$

Let consider rule for $n = k + 1$.

$$P(k+1): 1^3 + 2^3 + \dots + k^3 + (k+1)^3$$

Use the equation (1).

$$P(k+1): 1^3 + 2^3 + \dots + k^3 + (k+1)^3$$

$$P(k+1): \frac{1}{4} k^2 (k+1)^2 + (k+1)^3 \quad \{ \text{from equation (1)} \}$$

$$P(k+1): (k+1)^2 \left(\frac{k^2}{4} + k + 1 \right)$$

$$P(k+1): (k+1)^2 \left(\frac{k^2 + 4k + 4}{4} \right)$$

$$P(k+1): (k+1)^2 \frac{1}{4} (k+2)^2$$

$$P(k+1): \frac{1}{4} (k+1)^2 ((k+1)+1)^2$$

This result also true for $n = k + 1$. Hence, by mathematical induction this rule is true for all positive integer n .

Proved

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