A Book of Abstract Algebra | (2nd Edition)

Chapter 29, Problem 1EE

Bookmark

Show all steps: ON

Problem

Let *F* be a field.

Prove part:

The degree of *a* over *F* is the same as the degree of 1/*a* over *F*. It is also the same as the degrees of *a* + *c* and *ac* over *F*, for any $c \in F$.

Step-by-step solution

Step 1 of 3

Objective is to prove that the degree of *a* over field *F* is the same as the degree of $1/a$ over *F*. Also show that degrees of $a+c, ac$ over *F*, for any $c \in F$.

Let degree of *a* over field *F* is *n*, that is,

$[F(a):F]=n$.

Since *F* is a field and $a \in F$, therefore its inverse also belong to *F*. That is, $a^{-1} \in F$. Thus, $F(a^{-1}) \subseteq F(a)$. And since $\frac{1}{a^{-1}} = a$, one have that

$F(a) \subseteq F(a^{-1})$.

Comment

Step 2 of 3

Hence, $F(a) = F(a^{-1})$.

Since both are the same extension, therefore they have the same degrees. Thus, the degree of *a* over field *F* is the same as the degree of $1/a$ over *F*.

Since $[F(a):F]=n$, so there is a minimal polynomial of *a* of degree *n*, say $p(x)$. One knows that $p(x)$is irreducible. Then substituting *x* by $x+c$, or xc , $p(x)$will remain irreducible.

Comment

Step 3 of 3

Thus, the degree of *a* over field *F* is the same as the degrees of $a+c, ac$ over *F*, for any $c \in F$.

Comment

COMPANY

About Chegg
Chegg For Good
College Marketing
Corporate Development
Investor Relations
Jobs
Join Our Affiliate Program
Media Center
Site Map

LEGAL & POLICIES

Advertising Choices
Cookie Notice
General Policies
Intellectual Property Rights
Terms of Use
Global Privacy Policy
Honor Code
Honor Shield

CHEGG PRODUCTS AND SERVICES

Cheap Textbooks
Chegg Coupon
Chegg Play
Chegg Study Help
College Textbooks
eTextbooks
Flashcards
Learn
Chegg Math Solver
Mobile Apps
Sell Textbooks
Solutions Manual
Study 101
Textbook Rental
Used Textbooks
Digital Access Codes
Chegg Money

CHEGG NETWORK

EasyBib
Internships.com
Thinkful

CUSTOMER SERVICE

Customer Service
Give Us Feedback
Help with eTextbooks
Help to use EasyBib Plus
Manage Chegg Study Subscription
Return Your Books
Textbook Return Policy