A Book of Abstract Algebra | (2nd Edition)

Chapter 16, Problem 5EA

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Problem

In each of the following, use the fundamental homomorphism theorem to prove that the two given groups are isomorphic. Then display their tables.

$\mathbb{Z}_3$ and $(\mathbb{Z}_3 \times \mathbb{Z}_3)/K$, where $K = \{(0, 0), (1, 1), (2, 2)\}$.[HINT: Consider the function $f(a, b) = a - b$ from $\mathbb{Z}_3 \times \mathbb{Z}_3$ to $\mathbb{Z}_3$.]

Step-by-step solution

Step 1 of 4

Consider the two groups Z_3 and $(Z_3 \times Z_3)/K$, where $K = \{(0,0),(1,1),(2,2)\}$. Objective is to prove that these two groups are isomorphic by using the fundamental homomorphism theorem.

According to the fundamental homomorphism theorem, if $f : G \to H$ is a homomorphism of G onto H, with kernel K then

$H \cong G/K$.

Comment

Step 2 of 4

Consider the function $f : Z_3 \times Z_3 \to Z_3$ defined by

$f(a,b) = a - b$

for all $(a,b) \in Z_3 \times Z_3$. Then,

(a,b)	$f(a,b)$	$f(a,b)$ in Z_3
$(0,0)$	$0-0=0$	0

$(0,1)$	$0-1=-1$	2
$(0,2)$	-2	1
$(1,0)$	1	1
$(1,1)$	0	0
$(1,2)$	-1	2
$(2,0)$	2	2
$(2,1)$	1	1
$(2,2)$	0	0

Comment

Step 3 of 4

Since 0 is the zero element or additive identity in Z_3, therefore the elements of kernel will be:

$K=\{(0,0),(1,1),(2,2)\}$.

From the table it implies that map *f* is onto, also subtraction operation is homomorphism. Therefore, the map *f* is homomorphism from $Z_3 \times Z_3$ onto Z_3 with kernel *K*.

The addition table of Z_3 will be:

$+$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

Comment

Step 4 of 4

Hence, by the fundamental homomorphism theorem it concludes that

$Z_3 \cong (Z_3 \times Z_3)/K$.

Comment