

A Book of Abstract Algebra | (2nd Edition)

Chapter 32, Problem 1EE

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Problem

Describe the root field K of $x^7 - 1$ over \mathbb{Q} . Explain why $[K : \mathbb{Q}] = 6$.

Step-by-step solution

Step 1 of 2

The objective is to describe the root field K of $x^7 - 1$ over \mathbb{Q} and to explain why $[K : \mathbb{Q}] = 6$.

[Comment](#)

Step 2 of 2

If ω is the primitive root of unity, then $1, \omega, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6$ are the seven distinct elements of $\mathbb{Q}(\omega)$, and $(\omega^k)^7 = (\omega^7)^k = 1^k = 1$ shows that these seven elements are seven zeros of $x^7 - 1$.

Thus, $x^7 - 1$ splits in $\mathbb{Q}(\omega)$.

Hence, $K = \mathbb{Q}(\omega)$ is the root field of $x^7 - 1$ over \mathbb{Q} .

Since 7 is prime, the degree $[K : \mathbb{Q}] = 7 - 1 = 6$.

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