A Book of Abstract Algebra | (2nd Edition)

	Chapter 32, Problem 1EB	Bookmark	Show all steps: ⬤ ON

Problem

Show that $\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ is the root field of $(x^2 - 2)(x^2 - 3)(x^2 - 5)$ over $\mathbb{Q}$.

Step-by-step solution

Step 1 of 2

The objective is to show that $\mathbb{Q}\left(\sqrt{2}, \sqrt{3}, \sqrt{5}\right)$ is the root field of $\left(x^2 - 2\right)\left(x^2 - 3\right)\left(x^2 - 5\right)$ over $\mathbb{Q}$.

Comment

Step 2 of 2

The polynomial $f(x) = \left(x^2 - 2\right)\left(x^2 - 3\right)\left(x^2 - 5\right)$ has roots $\left\{\pm\sqrt{2}, \pm\sqrt{3}, \pm\sqrt{5}\right\}$.

Since $\mathbb{Q}\left(\sqrt{2}, \sqrt{3}, \sqrt{5}\right)$ contains the roots of $f(x) = \left(x^2 - 2\right)\left(x^2 - 3\right)\left(x^2 - 5\right)$,

$\mathbb{Q}\left(\sqrt{2}, \sqrt{3}, \sqrt{5}\right)$ is the root field of $\left(x^2 - 2\right)\left(x^2 - 3\right)\left(x^2 - 5\right)$.

Comment