# A Book of Abstract Algebra | (2nd Edition)



## Problem

Prove each of the following:

The line $ax + by + c = 0$ is constructible if $a, b, c \in \mathbb{D}$.

## Step-by-step solution

**Step 1** of 4

Here, objective is to prove that the line $ax + by + c = 0$ is constructible, if $a, b, c \in D$.

Consider $a \in D$ if and only if the point $(a, 0)$ is constructible from $\{O, I\}$

Comment

**Step 2** of 4

A line is constructible if it is passing through two constructible points..

Comment

**Step 3** of 4

Find the two points on the line:

Consider the line $ax+by+c=0$

put $x=0$

$by+c=0$

$y=-\frac{c}{b}$

put $y=0$

$ax+c=0$

$x=-\frac{c}{a}$

So, the line $ax+by+c=0$ is passing through $\left(0,-\frac{c}{b}\right),\left(-\frac{c}{a},0\right)$

Comment

**Step 4** of 4

The two points $\left(0,-\frac{c}{b}\right),\left(-\frac{c}{a},0\right)$ are constructible if and only if $a,b,c\in D$

If the two points $\left(0,-\frac{c}{b}\right),\left(-\frac{c}{a},0\right)$ are constructible, then the line $ax+by+c=0$ is also constructible

Therefore,, the line $ax+by+c=0$ is constructible, if $a,b,c\in D$.

Hence, proved

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