# A Book of Abstract Algebra | (2nd Edition)

Chapter AA, Problem 23E

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## Problem

*Prove the following identities involving cartesian products*:

$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

## Step-by-step solution

**Step 1** of 2

**Objective:-**

The objective is to prove $(A\times B)\cap(C\times D)=(A\cap C)\times(B\cap D)$.

Comment

**Step 2** of 2

Proof:-

Let $A$ and $B$ are two sets.

The Cartesian product of two sets $A$ and $B$ is:-

$$A\times B=\{(x,y):x\in A, y\in B\}$$

The intersection of two sets $A$ and $B$ is:-

$$A\cap B=\{x:x\in A \text{ and } x\in B\}$$

Let $(x,y)\in(A\times B)\cap(C\times D)$.

$$(x,y)\in(A\times B)\cap(C\times D)$$
$$\Rightarrow(x,y)\in(A\times B) \text{ and } (x,y)\in(C\times D)$$
$$\Rightarrow(x\in A, y\in B) \text{ and } (x\in C, y\in D)$$
$$\Rightarrow(x\in A \text{ and } x\in C),(y\in B \text{ and } y\in D)$$

$\Rightarrow (x \in A \cap C), (y \in B \cap D)$

$\Rightarrow (x, y) \in (A \cap C) \times (B \cap D)$

So,

$$(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D) \quad \ldots\ldots(1)$$

Let $(x, y) \in (A \cap C) \times (B \cap D)$.

$(x, y) \in (A \cap C) \times (B \cap D)$

$\Rightarrow (x \in A \cap C), (y \in B \cap D)$

$\Rightarrow (x \in A \text{ and } x \in C), (y \in B \text{ and } y \in D)$

$\Rightarrow (x \in A, y \in B) \text{ and } (x \in C, y \in D)$

$\Rightarrow (x, y) \in (A \times B) \text{ and } (x, y) \in (C \times D)$

$\Rightarrow (x, y) \in (A \times B) \cap (C \times D)$

So,

$$(A \cap C) \times (B \cap D) \subseteq (A \times B) \cap (C \times D) \quad \ldots\ldots(2)$$

Let us consider the equation (1) and (2).

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D).$$

Proved

Comment