A Book of Abstract Algebra | (2nd Edition)

Problem

$A[x_1, x_2]$ denotes the ring of all the polynomials in *two letters* x_1 and x_2 with coefficients in A. For example, $x^2 - 2xy + y^2 + x - 5$ is a quadratic polynomial in $\mathbb{Q}[x, y]$. More generally, $A[x_1, \ldots, x_n]$ is the ring of the polynomials in *n letters* $x_1, \ldots, x_n$ with coefficients in A. Formally it is defined as follows:Let $A[x_1]$ be denoted by A_1; then $A_1[x_2]$ is $A[x_1, x_2]$. Continuing in this fashion, we may adjoin one new letter *xi* at a time, to get $A[x_1, \ldots, x_n]$.

Give a reasonable definition of the *degree* of any polynomial $p(x, y)$ in $A[x, y]$ and then list all the polynomials of degree ≤ 3 in $Z_3[x, y]$.

Step-by-step solution

Step 1 of 1

Polynomials $p(x,y)$ in $A[x,y]$ are algebraic expressions consisting of terms in the form $a_{ij}x^i y^j$ .The degree of the polynomial $p(x,y)$ is largest sum of exponents of x and y .

All the polynomials of degree ≤ 3 in ${}_3[x,y]$ are listed below:

Degree(0) $\rightarrow$ $a_{00} \in {}_3$ but $a_{00} \neq 0$ # of polynomials = 2

Degree(1) $\rightarrow$ $a_{00} + a_{01}y + a_{10}x$ here all the coefficient belong to ${}_3$ but coefficients of x and y are not simultaneously zero, so # of polynomials $= 3\times 3\times 3 - 3 = 24$. Here, we minus 3 for excluding the cases when $a_{01} = a_{10} = 0$

Degree(2) $\rightarrow$ $a_{00} + a_{10}x + a_{01}y + a_{11}xy + a_{20}x^2 + a_{02}y^2$

Coefficients of xy, x^2 and y^2 are not simultaneously zero

# of polynomials $= 3\times 3\times 3\times 3\times 3\times 3 - 3\times 3\times 3 = 702$

Degree(3) $\rightarrow$ $a_{00} + a_{10}x + a_{01}y + a_{11}xy + a_{20}x^2 + a_{02}y^2 + a_{21}x^2y + a_{12}xy^2 + a_{30}x^3 + a_{03}y^3$

Coefficients of x^2y, xy^2, x^3 and y^3 are not simultaneously zero

# of polynomials $= 3^{10} - 3^{6} = 3^{6} \times 80 = 729 \times 80 = 58320$

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