# A Book of Abstract Algebra | (2nd Edition)

Chapter 32, Problem 3EG

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## Problem

Let *F* be a field, and *K* a finite extension of *F*. Suppose *a*, *b* ∈ *K*. Prove part:

Aside from the identity function, there are no $\mathbb{Q}$-fixing automorphisms of $\mathbb{Q}(\sqrt[3]{2})$. [HINT: Note that $\mathbb{Q}(\sqrt[3]{2})$ contains only real numbers.]

## Step-by-step solution

**Step 1** of 2

The objective is to prove that aside from the identity function, there are no $\mathbb{Q}-$fixing automorphism of $\mathbb{Q}\left(\sqrt[3]{2}\right)$.

Comment

**Step 2** of 2

Because $\mathbb{Q}\left(\sqrt[3]{2}\right)$ lies in $\mathbb{R}$ and the other two conjugates $\sqrt[3]{2}\cdot\frac{-1+\sqrt{3}i}{2}$ and $\sqrt[3]{2}\cdot\frac{-1-\sqrt{3}i}{2}$ of $\sqrt[3]{2}$ do not lie in $\mathbb{R}$, no map of $\sqrt[3]{2}$ into any conjugate other than $\sqrt[3]{2}$ itself can give rise to an automorphism of $\mathbb{Q}\left(\sqrt[3]{2}\right)$.

The other two maps give rise to isomorphism of $\mathbb{Q}\left(\sqrt[3]{2}\right)$ onto a subfield of $\bar{\mathbb{Q}}$.

Because any automorphism of $\mathbb{Q}\left(\sqrt[3]{2}\right)$ must leave the prime field $\mathbb{Q}$ fixed, the identity is the only automorphism of $\mathbb{Q}\left(\sqrt[3]{2}\right)$.

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