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Chapter 27, Problem 3EI   Bookmark   Show all steps: ON

Problem

Let $a(x) = a_0 + a_1x + \cdots + a_nx^n \in F[x]$. The *derivative* of $a(x)$ is the following polynomial $a'(x) \in F[x]$:

$$a'(x) = a_1 + 2a_2x + \ldots + na_nx^{n-1}$$

(This is the same as the derivative of a polynomial in calculus.) We now prove the analogs of the formal rules of differentiation, familiar from calculus.

Let $a(x), b(x) \in F[x]$, and let $k \in F$.

Prove part:

$[ka(x)]' = ka'(x)$

Step-by-step solution

Step 1 of 3

Consider the arbitrary field F and let $a(x) = a_0 + a_1x + \cdots + a_nx^n \in F(x)$. The derivative of $a(x)$ will be given by

$$a'(x) = a_1 + 2a_2x + \cdots + na_nx^{n-1} \in F(x).$$

Objective is to prove that

$$[ka(x)]' = ka'(x).$$

where $a(x), b(x) \in F[x]$ and $k \in F$.

Use the formula:

$$[a(x)b(x)]' = a'(x)b(x) + a(x)b'(x)$$

Comment

Step 2 of 3

Consider the left hand side of $[ka(x)]' = ka'(x)$ and solve in the following manner:

$$\begin{aligned}[ka(x)]' &= k'a(x) + ka'(x)\\ &= 0 + ka'(x)\\ &= ka'(x).\end{aligned}$$

Note that, differentiation of any constant or scalar is always zero.

Comment

Step 3 of 3

Hence, $[ka(x)]' = ka'(x)$.

Comment