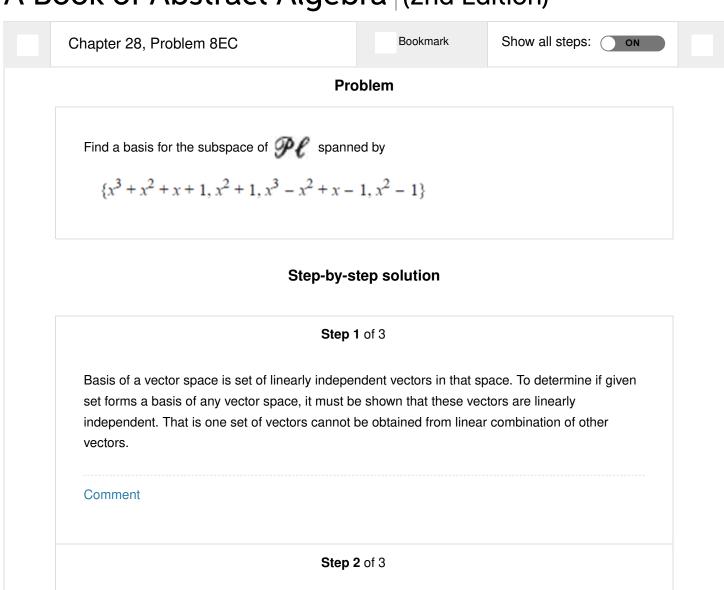
A Book of Abstract Algebra (2nd Edition)



In present case all $x^3, x^2, x, 1$ represents different dimension where one cannot be obtained from

other. Thus they can be considered 4 separate variables. And vectors with their combinations may be considered 4 linear equations. They will represent independent vectors if matrix formed by their coefficient is a full matrix or non-singular, which can be easily shown by reducing it to echelon form.

Comment

Step 3 of 3

Set given in question is $(x^3 + x^2 + x + 1, x^2 + 1, x^3 - x^2 + x - 1, x^2 - 1)$. Here observe that,

Matrix formed by coefficients of these vectors is

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Row reducing this matrix

$$\begin{pmatrix}
1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 \\
1 & -1 & 1 & -1 \\
0 & 0 & 1 & -1
\end{pmatrix}$$

$$\xrightarrow{R_3 \to R_3 - R_1}
\xrightarrow{R_4 \to R_4 - R_2}$$

$$\begin{pmatrix}
1 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 \\
0 & -2 & 0 & -2 \\
0 & 0 & 0 & -2
\end{pmatrix}$$

$$\xrightarrow{R_3 \leftrightarrow R_2}$$

$$\begin{pmatrix}
\boxed{1} & 1 & 1 & 1 \\
0 & \boxed{-2} & 0 & -2 \\
0 & 0 & \boxed{1} & 1
\end{pmatrix}$$

Clearly, this matrix is full matrix with 4 pivots. Consequently given 4 vectors are independent.

 $(x^3 + x^2 + x + 1, x^2 + 1, x^3 - x^2 + x - 1, x^2 - 1)$ is basis of given subspace

Comment