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Chapter 24, Problem 1EI

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Problem

Let A be an integral domain. By the closing part of Chapter 20, every integral domain can be extended to a "field of quotients." Thus, $A[x]$ can be extended to a field of polynomial quotients, which is denoted by $A(x)$. Note that $A(x)$ consists of all the fractions $a(x)/b(x)$ for $a(x)$ and $b(x) \neq 0$ in $A[x]$, and these fractions are added, subtracted, multiplied, and divided in the customary way.

Show that $A(x)$ has the same characteristic as A.

Step-by-step solution

Step 1 of 1

$A[x]$ is an integral domain where A is an integral domain. Then the field of polynomial quotients $A(x) = \left\{ \frac{f(x)}{g(x)} : f, g \in A[x], g \neq 0 \right\}$

Let $f(x) = a_0 + a_1 x + ... + a_n x^n$ be polynomial of degree n where $a_0, a_1, ..., a_n \in A$

$$\Rightarrow \quad A(x) = \frac{f(x)}{g(x)} = \frac{a_0 + a_1 x + ... + a_n x^n}{g(x)}$$

Suppose, p be the characteristic of A then

$$p \cdot A(x) = p \times \frac{f(x)}{g(x)}$$

$$= \frac{p \cdot a_0 + p \cdot a_1 x + ... + p \cdot a_n x^n}{g(x)} = 0$$

(Because p is the characteristic of A, so $p \cdot a_i = 0$, $0 \leq i \leq n$)

$\Rightarrow$ p is the characteristic of $A(x)$

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