

A Book of Abstract Algebra | (2nd Edition)

Chapter 27, Problem 4EE

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Problem

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Recall the definition of $F(a)$. It is a field such that (i) $F \subseteq F(a)$; (ii) $a \in F(a)$; (iii) any field containing F and a contains $F(a)$.
Use this definition to prove parts 1–5, where $F \subseteq K$, $c \in F$, and $a \in K$:
Let $p(x)$ be irreducible, and let a be a root of $p(x + c)$. Then
$$F[x]/\langle p(x + c) \rangle \cong F(a) \quad \text{and} \quad F[x]/\langle p(x) \rangle \cong F(a + c)$$

Conclude that $F[x]/\langle p(x + c) \rangle \cong F[x]/\langle p(x) \rangle$.

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Step-by-step solution

Step 1 of 3

Consider that F is any arbitrary field and K is its extension field. Let $c \in F$, and $a \in K$. Assume that $p(x) \in F[x]$ be some irreducible polynomial, and a be the root of $p(x + c)$.
Objective is to prove that
$$F[x]/\langle p(x + c) \rangle \cong F(a),$$
$$F[x]/\langle p(x) \rangle \cong F(a + c).$$

Consider the following result:
Let F is any arbitrary field. If $p(x) \in F[x]$ is an irreducible polynomial and c is some root of $p(x)$, then
$$\frac{F[x]}{\langle p(x) \rangle} \cong F(c).$$

Comment

Step 2 of 3

Note that, if $p(x) \in F[x]$ is some irreducible polynomial, then $p(x + c)$ will also be irreducible. Because substitution x by $x + c$ does not change the behaviour of polynomial.
Since a is the root of $p(x + c)$, so one can apply the above result and get that
$$F[x]/\langle p(x + c) \rangle \cong F(a).$$

Again, since a is the root of $p(x + c)$, therefore $a + c$ will be the root of $p(x)$. Use the above mentioned result, and obtained that
$$F[x]/\langle p(x) \rangle \cong F(a + c).$$

Comment

Step 3 of 3

One knows that, for some nonzero c ,
$$F(a) = F(a + c).$$

Use this fact, and conclude that
$$F[x]/\langle p(x + c) \rangle \cong F[x]/\langle p(x) \rangle.$$

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