A Book of Abstract Algebra | (2nd Edition)

Chapter 17, Problem 1EI

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Problem

Prove that parts are true in a nontrivial ring with unity.

If *a* is invertible and *ab* = *ac*, then *b* = *c*.

Step-by-step solution

Step 1 of 3

Consider an arbitrary nontrivial ring *R* with unity. Suppose that $a \in R$ is an invertible element, that is, multiplicative inverse of *a* exists in *R*. Objective is to show that

if $ab = ac$, then $b = c$.

Since $a \in R$ is an invertible element, so there exists $a^{-1} \in R$ such that

$$aa^{-1} = 1, a^{-1}a = 1,$$

where 1 stands for the unity of the ring.

Comment

Step 2 of 3

Pre-multiply of both the sides of condition $ab = ac$ by a^{-1} and use that $a^{-1}a = 1$ as:

$$\begin{aligned} ab &= ac \\ a^{-1}(ab) &= a^{-1}(ac) \\ a^{-1}ab &= a^{-1}ac \\ 1b &= 1c \end{aligned}$$

The last equation $1b = 1c$ implies that $b = c$ because 1 is the unity of the ring.

Comment

Step 3 of 3

Hence, if $ab = ac$ then $b = c$ for some invertible element $a \in R$.

Comment