

# A Book of Abstract Algebra | (2nd Edition)

Chapter AC, Problem 3E

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## Problem

Use mathematical induction to prove the following:

$$1^2 + 2^2 + \dots + n^2 = \frac{1}{6} n(n+1)(2n+1)$$

## Step-by-step solution

### Step 1 of 2

#### Objective:-

The objective is to prove  $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n(n+1)(2n+1)$  using mathematical induction.

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### Step 2 of 2

Proof:-

$$p(n): 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{1}{6} n(n+1)(2n+1)$$

Let consider rule for  $n = 1$ .

$$P(1): 1^2 = \frac{1}{6} \cdot 1(1+1)(1+2)$$

$$P(1): 1 = \frac{1}{6} \cdot 6$$

$$p(1): 1 = 1$$

This rule is true for  $n = 1$ .

Let this rule is true for  $n = k$ .

$$p(k): 1^2 + 2^2 + 3^2 + \dots + k^2 = \frac{1}{6} k(k+1)(2k+1) \dots \dots \dots (1)$$

Let consider rule for  $n = k + 1$ .

$$P(k+1): 1^2 + 2^2 + 3^2 + \dots + k^2 + (k+1)^2$$

Use the equation (1).

$$P(k+1): \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$$

$$P(k+1): (k+1) \left[ \frac{1}{6}k(2k+1) + k+1 \right]$$

$$P(k+1): (k+1) \left[ \frac{2k^2 + k + 6k + 6}{6} \right]$$

$$P(k+1): \frac{1}{6}(k+1)(2k^2 + 7k + 6)$$

$$P(k+1): \frac{(k+1)}{6} [2k^2 + 4k + 3k + 6]$$

$$P(k+1): \frac{(k+1)}{6} [2k(k+2) + 3(k+2)]$$

$$P(k+1): \frac{1}{6}(k+1)(k+2)(2k+3)$$

$$P(k+1): \frac{[(k+1)((k+1)+1)\{2(k+1)+1\}]}{6}$$

This result also true for  $n = k + 1$ . Hence, by mathematical induction this rule is true for all positive integer  $n$ .

Proved

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