

# A Book of Abstract Algebra | (2nd Edition)

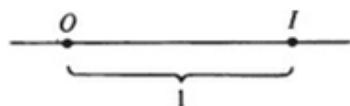
Chapter 30, Problem 2EA

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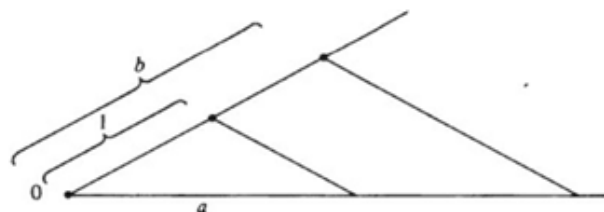
## Problem

If  $O$  and  $I$  are any two points in the plane, consider a coordinate system such that



the interval  $OI$  coincides with the unit interval on the  $x$  axis. Let  $\mathbb{D}$  be the set of real numbers such that  $a \in \mathbb{D}$  iff the point  $(a, 0)$  is constructible from  $\{O, I\}$ . Prove the following:

If  $a, b \in \mathbb{D}$ , then  $ab \in \mathbb{D}$ . (HINT: Use similar triangles. See the accompanying figure.)



## Step-by-step solution

### Step 1 of 4

Here, objective is to prove that, if  $a, b \in \mathbb{D}$ , then  $ab \in \mathbb{D}$ .

Consider  $a \in D$  if and only if the point  $(a,0)$  is constructible from  $\{O,I\}$

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### Step 2 of 4

Constructible point:

Constructible point is either the end point of given unit segment or it is the intersection of two lines determined by previous constructible points.

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### Step 3 of 4

$D$  is a set of real numbers. Therefore, we can add, subtract, multiply any two points of them.

Let  $a, b \in D$ , then the points  $(a,0)$  and  $(b,0)$  are constructible from  $\{O,I\}$

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### Step 4 of 4

Now, we have to prove that length  $ab$  is constructed from the lengths  $a$  and  $b$

Consider the below figure:

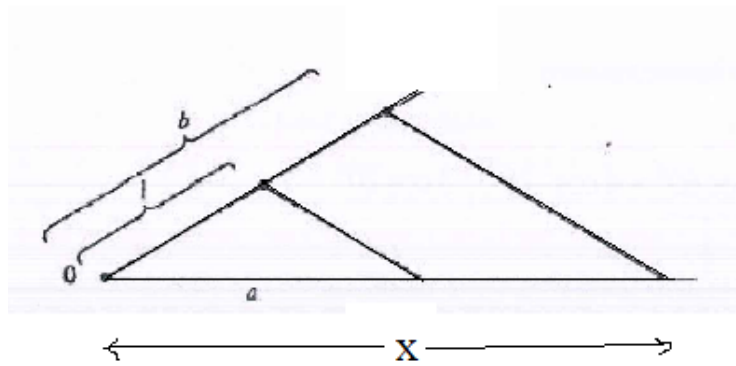


fig: multiplication

by observing, there exist two equal triangles.

$a$  and  $b$  are constructible lengths.

using the property of similar triangles, we have

$$\frac{1}{a} = \frac{b}{x}$$

$$x = ab$$

Then, the length  $ab$  is constructible from  $\{O, I\}$ , which implies  $ab \in D$

Therefore, if  $a, b \in D$ , then  $ab \in D$ .

Hence, proved

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