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Chapter 30, Problem 2EB

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Problem

Prove each of the following:

If a point *P* is constructible from {*O,I*} [that is, from (0, 0) and (1, 0)], then *P* is constructible from

$\mathbb{Q} \times \mathbb{Q}$

Step-by-step solution

Step 1 of 4

Here, objective is to prove that if a point *P* is constructible from $\{O, I\}$, then *P* is constructible from $Q \times Q$.

Comment

Step 2 of 4

Constructible point:

The point is simply produced by Euclidian constructions.

The point is either the end point of given unit segment or it is the intersection of two lines determined by previous constructible points is called as constructible point.

Comment

Step 3 of 4

Let the point $P(a,b)$ is constructible from (O,I).

As per the definition of *D*, $(a,0)$ and $(0,b)$ are constructible from (O,I)

By using compass we can construct the point $(0,b)$ along y-axis.

Draw the perpendicular line to the x-axis passing through $(a,0)$ and perpendicular line to the y-axis passing through $(0,b)$.Then the perpendicular lines intersect at the point (a,b).

So, the point $P(a,b)$ is constructible from (O,I),

Comment

Step 4 of 4

$Q\times Q$ is a set of all rational numbers

The points which are constructed from *D* are belongs to $Q\times Q$

Then $(a,b)\in Q\times Q$

Therefore, that if a point *P* is constructible from $\{O,I\}$, then *P* is constructible from $Q\times Q$.

Hence, proved

Comment