

A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 4EA

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Problem

Explain: $\mathbb{Q}(i, \sqrt{2})$ is the root field of $x^4 - 2x^2 + 9$ over \mathbb{Q} , and is the root field of $x^2 - 2\sqrt{2}x + 3$ over $\mathbb{Q}(\sqrt{2})$.

Step-by-step solution

Step 1 of 3

The objective is to find the root field of $x^4 - 2x^2 + 9$ over \mathbb{Q} and the root field of $x^2 - 2\sqrt{2}x + 3$ over $\mathbb{Q}(\sqrt{2})$.

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Step 2 of 3

Take $y = x^2$.

$$x^4 - 2x^2 + 9 = y^2 - 2y + 9$$

Use $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find roots of $y^2 - 2y + 9$.

Here, $a = 1$, $b = -2$, $c = 9$.

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 9}}{2 \cdot 1}$$

$$= \frac{2 \pm \sqrt{-32}}{2}$$

$$= 1 \pm 2\sqrt{2}i$$

$$x^2 = 1 \pm 2\sqrt{2}i$$

$$x = \sqrt{2} + i, x = -\sqrt{2} - i, x = -\sqrt{2} + i, x = \sqrt{2} - i$$

The roots of $x^4 - 2x^2 + 9$ are $\sqrt{2} + i$, $-\sqrt{2} - i$, $-\sqrt{2} + i$, $\sqrt{2} - i$.

Therefore, the root field of $x^4 - 2x^2 + 9$ over \mathbb{Q} is $\mathbb{Q}(\sqrt{2} + i, -\sqrt{2} - i, -\sqrt{2} + i, \sqrt{2} - i)$. This can be written simply as $\mathbb{Q}(\sqrt{2}, i)$.

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Step 3 of 3

Use $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find roots of $x^2 - 2\sqrt{2}x + 3$.

Here, $a = 1$, $b = -2\sqrt{2}$, $c = 3$.

$$\begin{aligned} x &= \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} \\ &= \frac{2\sqrt{2} \pm \sqrt{-4}}{2} \\ &= \sqrt{2} \pm i \end{aligned}$$

In $\mathbb{Q}(\sqrt{2})$, $\sqrt{2} \pm i$ are roots of $x^2 - 2\sqrt{2}x + 3$.

Therefore, the root field of $x^2 - 2\sqrt{2}x + 3$ over $\mathbb{Q}(\sqrt{2})$ is $\mathbb{Q}(\sqrt{2} \pm i)$. This can be written simply as $\mathbb{Q}(\sqrt{2}, i)$.

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