

# A Book of Abstract Algebra | (2nd Edition)

Chapter 28, Problem 1EC

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## Problem

Prove that  $\{(0,0,0,1), (0,0,1,1), (0,1,1,1), (1,1,1,1)\}$  is a basis of  $\mathbb{R}^4$ .

## Step-by-step solution

### Step 1 of 2

Any set of basis is a set of vectors which are linearly independent and their number equals dimension of vector space. And any set is linearly independent if there exists no combination of these vectors which can give 0 vectors.

If  $u_1, u_2, \dots, u_n$  are  $n$  vectors of a vector space and these are linearly independent. Then for.

$$a_1u_1 + a_2u_2 + \dots + a_nu_n = 0$$

All  $a_i$  have to be zero.

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### Step 2 of 2

Any given basis is linearly independent if matrix formed with vectors as row of matrix same rank as number of rows of matrix.

Matrix formed by given set of vectors is,

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

It can be easily seen that this matrix has 4 pivots and thus have rank of 4, equal to number of vectors. Also numbers of vectors is equal to dimension of  $\mathbb{R}^4$  which is 4.

Hence given set of vectors are linearly independent and form basis of  $\mathbb{R}^4$

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