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Chapter 24, Problem 2EB	Bookmark	Show all steps: ⬤ ON

Problem

Is there any ring A such that in $A[x]$, some polynomial of degree 2 is equal to a polynomial of degree 4? Explain.

Step-by-step solution

Step 1 of 1

Consider any ring $A[x]$. Let a general second degree polynomial $a(x) = a_2x^2 + a_1x + a_0$ and a general fourth degree polynomial $b(x) = b_4x^4 + b_3x^3 + b_2x^2 + b_1x + b_0$ in $A[x]$.

Objective of this question is to check these two polynomials will be equal in any of the ring.

Now recall the rule for equality of two polynomials.

$a(x) = a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0$ and $b(x) = b_nx^n + b_{n-1}x^{n-1} + ... + b_1x + b_0$ are said to be equal when $a_n = b_n, a_{n-1} = b_{n-1}..., a_1 = b_1, a_0 = b_0$ in the given ring.

$a(x) = a_2x^2 + a_1x + a_0$ ......(1)

$b(x) = b_4x^4 + b_3x^3 + b_2x^2 + b_1x + b_0$ ......(2)

Rewrite equation (1) as follows.

$a(x) = 0x^4 + 0x^3 + a_2x^2 + a_1x + a_0$

Now compare the coefficients of corresponding terms.

If $a(x) = b(x)$ in any ring $A[x]$.

Then,

$b_4 = 0, b_3 = 0, b_2 = a_2, b_1 = a_1, b_0 = a_0$

Polynomial $b(x)$ is a 4th degree polynomial hence b_4 cannot be equal to zero.

Hence, any of the second degree polynomial is not equal to fourth degree polynomial in any of

the ring.

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