A Book of Abstract Algebra | (2nd Edition)

Chapter 24, Problem 3EC

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Problem

In $\mathbb{Z}_{10}[x]$,, (2x + 2)(2x + 2) = (2x + 2)(5x^3 + 2x + 2), yet (2x + 2) cannot be canceled in this equation. Explain why this is possible in 10[x], but not in $\mathbb{Z}_5[x]$.

Step-by-step solution

Step 1 of 2

Consider an equation

$(2x+2)(2x+2)=(2x+2)(5x^3+2x+2)$ ......(1)

Now prove $(2x+2)$ cannot be cancelled in this equation when ring in $\mathbb{Z}_{10}[x]$.

Suppose $(2x+2)$ can be cancelled in this equation in the ring $\mathbb{Z}_{10}[x]$.

Then,

$(2x+2)=(5x^3+2x+2)$

Bring $(2x+2)$ in to R.H.S

Then,

$$\begin{aligned} 5x^3+2x+2-2x-2&=0 \\ 5x^3&=0 \end{aligned} \quad \text{......(2)}$$

Which implies $5x^3$ is a zero polynomial.

But $5x^3$ is third degree polynomial in $\mathbb{Z}_{10}[x]$ and it cannot be a zero polynomial.

That contradicts the assumption $(2x+2)$ can be cancelled in this equation in the ring $\mathbb{Z}_{10}[x]$.

That implies $(2x+2)$ cannot be cancelled in $(2x+2)(2x+2)=(2x+2)(5x^3+2x+2)$ equation when ring in $\mathbb{Z}_{10}[x]$.

Comment

Step 2 of 2

Now consider the ring $\mathbb{Z}_5[x]$ and the equation.

$$(2x+2)(2x+2)=(2x+2)(5x^3+2x+2)$$

Theorem 1: If *A* is an integral domain then $A[x]$ is also an integral domain.

Theorem 2: If *p* is a prime number the ring $\mathbb{Z}_p$ is an integral domain.

Theorem 3: Let *a,b* and *c* belong to an integral domain. If $a \neq 0$ and $ab=ac$, then $b=c$.

Here, 5 is a prime number then by applying "Theorem 2" $\mathbb{Z}_5$ is an integral domain.

Then by applying "Theorem 1" $\mathbb{Z}_5[x]$ is an integral domain.

By using "Theorem 3", $(2x+2)$ can be cancelled in the equation.

Comment