A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 2EB

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Problem

Solve each of the following pairs of simultaneous congruences:

(a) $10x \equiv 2 \pmod{12}$; $6x \equiv 14 \pmod{20}$

(b)$4x \equiv 2 \pmod{6}$; $9x \equiv 3 \pmod{12}$

(c) $6x \equiv 2 \pmod{8}$; $10x \equiv 2 \pmod{12}$

Step-by-step solution

Step 1 of 5

Here, objective is to solve the given Pair of simultaneous congruence's.

Comment

Step 2 of 5

Consider a, b are integers, m is a positive integer.

If m divides $a-b$, then a is congruent to b modulo m which is represented by $a = b(\text{mod m})$

Consider the congruent equation $ax = b(\text{mod } n)$, has solutions if and only if $\gcd(a,n)$ is divisible by b. If $\gcd(a,n) = 1$, then the congruence has unique solution

Comment

Step 3 of 5

(a)

Consider the pair of congruence

$$10x \equiv 2 (\mathrm{mod}\, 12) \ldots\ldots\ldots\ldots\ldots (1)$$
$$6x \equiv 14 (\mathrm{mod}\, 20) \ldots\ldots\ldots\ldots (2)$$

From equation (1)

$$5x \equiv 1 (\mathrm{mod}\, 6).$$
$$x \equiv 1(5^{-1})(\mathrm{mod}\, 6).$$
$$x \equiv 5 (\mathrm{mod}\, 6)$$
$$x = 5 + 6p \ldots\ldots\ldots\ldots\ldots (3)$$

Substitute above equation in equation (2)

$$6(5+6p) \equiv 14 (\mathrm{mod}\, 20)$$
$$30 + 36p \equiv 14 (\mathrm{mod}\, 20)$$
$$9p \equiv -4 (\mathrm{mod}\, 5)$$
$$p \equiv -4(9^{-1})(\mathrm{mod}\, 5)$$
$$p \equiv -4(4)(\mathrm{mod}\, 5)$$
$$p \equiv 4 (\mathrm{mod}\, 5)$$
$$p = 4 + 5q$$

Substitute above equation in equation (3)

$$x = 5 + 6(4+5q)$$
$$x = 5 + 24 + 30q$$
$$x \equiv 29 (\mathrm{mod}\, 30)$$

Hence, the solution of set of pair of congruence's is $x \equiv 29 (\mathrm{mod}\, 30)$

Comment

Step 4 of 5

(b)

Consider the pair of congruence

$$4x \equiv 2 (\mathrm{mod}\, 6) \ldots\ldots\ldots\ldots\ldots (1)$$
$$9x \equiv 3 (\mathrm{mod}\, 12) \ldots\ldots\ldots\ldots (2)$$

From equation (1)

$$2x \equiv 1 (\mathrm{mod}\, 3)$$
$$x \equiv 1(2^{-1}) \,\mathrm{mod}\, 3$$
$$x \equiv 2 (\mathrm{mod}\, 3)$$
$$x = 2 + 3p \ldots\ldots (3)$$

Substitute above equation in equation (2)

$$9(2+3p) \equiv 3 (\mathrm{mod}\, 12)$$
$$18 + 27p \equiv 3 (\mathrm{mod}\, 12)$$
$$9p \equiv -5 (\mathrm{mod}\, 4)$$

Substitute above equation in equation (3)

$$x = 2 + 3p$$
$$3x = 6 + 9p$$
$$3x = 6 - 5(\bmod 4)$$
$$3x = 1 \bmod 4$$
$$x = 3 \bmod 4$$

Hence, the solution of set of pair of congruence's is $x = 3(\bmod 4)$

Comment

Step 5 of 5

(c)

Consider the pair of congruence

$$6x = 2(\bmod 8)\dots\dots\dots\dots\dots(1)$$
$$10x = 2(\bmod 12)\dots\dots\dots\dots(2)$$

From equation (1)

$$3x = 1(\bmod 4)$$
$$x = 3(\bmod 4)$$
$$x = 3 + 4p\dots\dots\dots\dots(3)$$

Substitute above equation in equation (2)

$$10x = 2(\bmod 12)$$
$$5x = 1(\bmod 6)$$
$$5(3 + 4p) = 1(\bmod 6)$$
$$10p = -7(\bmod 3)$$

Substitute above equation in equation (3)

$$10x = 30 + 4(2 \bmod 3).$$
$$10x = 38 \bmod 3$$
$$x = 38(10^{-1}) \bmod 3$$
$$x = 38(\bmod 3)$$

Hence, the solution of set of pair of congruence's is $x = 38(\bmod 3)$

Comment