# A Book of Abstract Algebra | (2nd Edition)

Chapter 33, Problem 1ED

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## Problem

Let *G* be a group. The symbol $H \lhd G$ should be read, "*H* is a normal subgroup of *G*." A *maximal* normal subgroup of *G* is an $H \lhd G$ such that, if $H \lhd J \lhd G$, then necessarily $J = H$ or $J = G$. Prove the following:

If *G* is a *finite* group, every normal subgroup of *G* is contained in a maximal normal subgroup.

## Step-by-step solution

**Step 1** of 4

Here, objective is to prove that every normal subgroup of *G* is contained in a maximal normal subgroup.

Comment

**Step 2** of 4

Finite group is a group which contains finite number of elements.

Comment

**Step 3** of 4

Consider *G* is a finite group. *H* is normal subgroup of *G* is denoted by $H \lhd G$

A maximal normal subgroup of *G* is given by

$H \lhd G$, if $H \lhd J \lhd G$ then, necessarily $J = H$ or $J = G$

Maximal subgroup is a proper subgroup that is not containing in any other proper subgroup.

Comment

**Step 4** of 4

Consider *H* is a proper subgroup of *G*.

Let *H* is a maximal normal subgroup of *G* or consider $H < H_1$

Where $H_1$ is a proper subgroup of *G* , which has larger order.

Similarly, we can create a chain of proper subgroups of *G.*

$H < H_1 < H_2$..................

The above relation is a chain of integers in increasing order.

Then, it is clear that the maximal subgroup containing *H.*

Therefore, every normal subgroup *H* of *G* is contained in a maximal normal subgroup.

Hence, proved

Comment