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Chapter 28, Problem 3EB

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Problem

Prove that $\{f : f(l) = 0\}$ is a subspace of $\mathscr{F}(\mathbb{R})$.

Step-by-step solution

Step 1 of 2

It is already shown that $f(\mathbb{R})$ represents a vector space as it satisfies all conditions for vector space.

Given subset for $f(\mathbb{R})$ is set of all functions which passes through a fixed point $(1,0)$.

Or given condition for subspace is

$$\{f \mid f(1) = 0\}$$

Comment

Step 2 of 2

A 2 step check is needed to determine if given subset is a subspace. First determine if any linear combination of functions in subset is closed under given operation. Second, determine if 0 satisfies given conditions.

STEP 1: For any 2 functions *f* and *g*,

$$f(1) = 0 \qquad (1)$$
$$g(1) = 0 \qquad (2)$$

Combining above 2 equations, $s(1)+t(2)$ gives

$$sf(1)+tg(1)=0$$

As functions are vector space in themselves, any constant multiple of function is also a function. Also sum of 2 functions is also a function. Thus,

$$sf(1)+tg(1)=0$$
$$\Rightarrow f'(1)+g'(1)=0$$
$$\Rightarrow F(1)=0$$

Thus linear combination of 2 functions in subset lies in subset.

STEP 2: Check if 0 function (which is 0 everywhere) satisfies given condition,

$$f_0=0 \qquad \forall x$$
$$\Rightarrow f_0(1)=0$$

$$\boxed{\text{Hence given set represents a subspace}}$$

Comment