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## A Book of Abstract Algebra (2nd Edition)

Chapter 28, Problem 4EE

Problem  Let N be the null space of h, and  the range space of h. Let {a₁,, a₂} be a basis of N. Extend it to a basis {a₁,, aₙ, aₙ} of U. Prove part:  Every vector b∈  is a linear combination of h(aₙ+1),, h(aₙ).  Step-by-step solution  Step 1 of 5  It is already known that U and V are vector spaces and so they satisfies all conditions for vecto space.  Comment  Step 2 of 5  Range space of h is subspace of V is set of all elements of V which are map of vectors of U.  Comment  Step 3 of 5  Or given subspace is		
Extend it to a basis {a <sub>1</sub> ,, a <sub>n</sub> ,, a <sub>n</sub> } of <i>U</i> .  Prove part:  Every vector <b>b</b> is a linear combination of <i>h</i> (a <sub>n+1</sub> ),, <i>h</i> (a <sub>n</sub> ).  Step-by-step solution  Step 1 of 5  It is already known that <i>U</i> and <i>V</i> are vector spaces and so they satisfies all conditions for vecto space.  Comment  Step 2 of 5  Range space of <i>h</i> is subspace of <i>V</i> is set of all elements of <i>V</i> which are map of vectors of <i>U</i> .  Comment		Problem
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## **Step 4** of 5

Thus any element in range is a map of some vector in *U* 

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## **Step 5** of 5

For any element r in range of h, we can find a element u in U such that

$$h(\mathbf{u}) = \mathbf{r}$$

Since U is a vector space, every element in U can be expressed as linear combination of basis of U. So,

$$\mathbf{u} = t_1 \mathbf{a}_1 + t_2 \mathbf{a}_2 + ... t_r \mathbf{a}_r + ... + t_n \mathbf{a}_n$$

Taking linear transformation

$$h(\mathbf{u}) = h(t_1 \mathbf{a}_1 + t_2 \mathbf{a}_2 + \dots t_r \mathbf{a}_r + \dots + t_n \mathbf{a}_n)$$
  

$$\Rightarrow h(\mathbf{u}) = t_1 h(\mathbf{a}_1) + t_2 h(\mathbf{a}_2) + \dots + t_r h(\mathbf{a}_r) + \dots + t_n h(\mathbf{a}_n) \qquad \dots (1)$$

Since  $(\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_r)$  is null basis of h,

$$h(\mathbf{a}_r) = \mathbf{0} \forall r \in (0, 1, ..., r)$$

Therefore (1) can be rewritten as,

$$h(\mathbf{u}) = t_{r+1}h(\mathbf{a}_{r+1}) + ... + t_nh(\mathbf{a}_n)$$

Hence every vector **b** in range can be written as linear combination of  $h(\mathbf{a}_{r+1}),...,h(\mathbf{a}_n)$ 

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