A Book of Abstract Algebra | (2nd Edition)

Problem

Prove that the following are true for any integers a, b, and c:

If $a|d$ and $c|d$ and $\gcd(a, c) = 1$, then $ac|d$.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $if \;\; a|d \;\; and \;\; c|d \; and \;\; \gcd(a,c)=1, then \;\; ac|d.$

Comment

Step 2 of 2

Proof:-

Let suppose $a|d \;\; and \;\; c|d.$

Then there exist number *k* and *l* such that:-

$$d = ka \qquad\qquad ......(1)$$
$$d = lc \qquad\qquad ......(2)$$

Let $\gcd(a,c)=1$, then integers *a* and *c* are relatively prime.

Let us consider the theorem.

Theorem:-Any two nonzero integers *r* and *s* have a unique positive greatest common divisor *t*, Moreover, *t* is equal to a “Linear combination” of *r* and *s*. That is,

$$t = kr + ls \quad for \;\; some \;\; integer \;\; k \;\; and \;\; l$$

Let us suppose $\gcd(a,c)=1$. Then by above theorem:-

$$1 = ma + nc \quad for \quad some \quad integer \quad m \, and \quad n$$

$$1 = m\frac{d}{k} + n\frac{d}{l} \qquad \{from \quad equation\,(1) \quad and \quad (2)\}$$

$$1 = d\left(\frac{m}{k} + \frac{n}{l}\right)$$

$$1 = d\left(\frac{ml + nk}{lk}\right)$$

$$lk = d(ml + nk)$$

$$\frac{d}{a}\frac{\cancel{d}}{c} = \cancel{d}(ml + nk) \qquad \{from \quad equation \quad (1) \quad and \quad (2)\}$$

$$d = ac(ml + nk)$$

Thus, ac is a factor of $ac(ml + nk)$ that is a factor of *d*. Hence, ac divides *d* that is $ac \mid d$..

Proved

Comment