A Book of Abstract Algebra | (2nd Edition)

Chapter 30, Problem 6EF

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Problem

By de Moivre's theorem,

$$\omega = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$$

is a complex seventh root of unity. Since

$$x^7 - 1 = (x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$$

ω is a root of $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$.

Prove that the regular polygon of seven sides is not constructible.

Step-by-step solution

Step 1 of 4

Here, objective is to prove that regular polygon of seven sides is not constructible.

Comment

Step 2 of 4

Regular *n*-gon has *n* equal length of sides and all angles are equal.

Regular *n* sided polygon is constructible if and only if the angle $\frac{2\pi}{n}$ is constructible

An angle $\frac{2\pi}{N}$ is constructible if and only if *N* is either a power of two or power of two and a set of distinct Fermat points

Comment

Step 3 of 4

Consider regular seven sided Polygon or $7-\text{gon}$

Number of sides $n=7$

Comment

Step 4 of 4

To verify $\frac{2\pi}{7}$ is constructible or not:

$$\frac{2\pi}{N}=\frac{2\pi}{7}$$
$$N=7$$

Here,

7 cannot be written in the form of $2^{2^n}+1$. So 7 is not a Fermat prime

Therefore, $\frac{2\pi}{7}$ is not a constructible angle.

Regular $7-\text{gon}$ is constructible if and only if the angle $\frac{2\pi}{7}$ is constructible

Hence,

Regular polygon of seven sides is not constructible.

Comment