

# A Book of Abstract Algebra | (2nd Edition)



Chapter 29, Problem 2ED



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## Problem

Let  $F$  be a field, and  $K$  a field extension of  $F$ . Prove the following:

If  $[K : F]$  is a prime number, there is no field properly between  $F$  and  $K$  (that is, there is no field  $L$  such that  $F \subsetneq L \subsetneq K$ ).

## Step-by-step solution

### Step 1 of 2

Consider a field  $F \subseteq K$  with  $[K : F] = p$  for some prime number  $p$ . The objective is to show that there is no field  $L$  properly between  $F$  and  $K$ .

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### Step 2 of 2

Suppose there is such a field  $L$ .

$$p = [K : F] = [K : L][L : F].$$

This implies that either  $[K:E]=1$  or  $[E:F]=1$ .

That is, either  $K=E$  or  $E=F$ , a contradiction.

Thus, no such field  $L$  can exist.

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