

# A Book of Abstract Algebra | (2nd Edition)

Chapter 27, Problem 1EI

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Problem

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Let  $a(x) = a_0 + a_1x + \cdots + a_nx^n \in F[x]$ . The *derivative* of  $a(x)$  is the following polynomial  $a'(x) \in F[x]$ :

$$a'(x) = a_1 + 2a_2x + \cdots + na_nx^{n-1}$$

(This is the same as the derivative of a polynomial in calculus.) We now prove the analogs of the formal rules of differentiation, familiar from calculus.

Let  $a(x), b(x) \in F[x]$ , and let  $k \in F$ .

Prove part:

$[a(x) + b(x)]' = a'(x) + b'(x)$

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Step-by-step solution

Step 1 of 3

Consider the arbitrary field  $F$  and let  $a(x) = a_0 + a_1x + \cdots + a_nx^n \in F[x]$ . The derivative of  $a(x)$  will be given by

$$a'(x) = a_1 + 2a_2x + \cdots + na_nx^{n-1} \in F[x].$$

Objective is to prove that

$$[a(x) + b(x)]' = a'(x) + b'(x).$$

where  $a(x), b(x) \in F[x]$ .

Comment

Step 2 of 3

Suppose that  $a(x) = a_0 + a_1x + \cdots + a_nx^n$  and  $b(x) = b_0 + b_1x + \cdots + b_nx^n$ . Then their derivatives will be:

$$a'(x) = a_1 + 2a_2x + \cdots + na_nx^{n-1},$$

$$b'(x) = b_1 + 2b_2x + \cdots + nb_nx^{n-1}.$$

Now, consider the left hand side of  $[a(x) + b(x)]' = a'(x) + b'(x)$  as:

$$\begin{aligned} [a(x) + b(x)]' &= [(a_0 + b_0) + (a_1 + b_1)x + \cdots + (a_n + b_n)x^n]' \\ &= (a_1 + b_1) + \cdots + n(a_n + b_n)x^{n-1} \\ &= (a_1 + \cdots + na_nx^{n-1}) + (b_1 + \cdots + nb_nx^{n-1}) \\ &= a'(x) + b'(x). \end{aligned}$$

Comment

Step 3 of 3

Hence,  $[a(x) + b(x)]' = a'(x) + b'(x).$

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