A Book of Abstract Algebra | (2nd Edition)

Chapter 29, Problem 6ED

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Problem

Let F be a field, and *K* a field extension of *F*. Prove the following:

If an irreducible polynomial $p(x) \in F[x]$ has a root in *K*, then deg $p(x)|[K:F]$.

Step-by-step solution

Step 1 of 2

Consider a field F and a field extension K of F. The objective is to prove that if an irreducible polynomial $p(x) \in F[x]$ has a root α in K, then $\deg p(x) | [K:F]$.

Comment

Step 2 of 2

Since $F \subseteq F(\alpha) \subseteq K$, $[K:F] = [K:F(\alpha)][F(\alpha):F]$.

But $[F(\alpha):F] = \deg p(x)$.

Therefore , $\deg p(x) | [K:F]$.

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