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Chapter 23, Problem 2EC

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Problem

Prove the following for all integers *a*, *b*, *c*, *d* and all positive integers *m* and *n:*

If $a \equiv b \pmod{n}$, then $a + c \equiv b + c \pmod{n}$.

Step-by-step solution

Step 1 of 2

Consider the congruence equation

$$a \equiv b \pmod{n}$$

Object of the problem is to prove that if $a \equiv b \pmod{n}$ then $a+c \equiv b+c \pmod{n}$.

Use the definition $a \equiv b \pmod{n}$ iff n divides $a-b$ to prove the result.

By the definition of congruence equation, n divides $a-b$

There is an integer p such that

$$a-b=np$$

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Step 2 of 2

Add and subtract c to left side of the equation.

$$a-b+c-c=np$$
$$a+c-(b+c)=np$$

Again by the definition of congruence equation, $a+c \equiv b+c \pmod{n}$

Therefore, if $a \equiv b \pmod{n}$ then $a + c \equiv b + c \pmod{n}$

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