

A Book of Abstract Algebra | (2nd Edition)

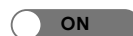


Chapter 29, Problem 3ED



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Problem

Let F be a field, and K a field extension of F . Prove the following:

If $[K:F]$ is a prime, then $K = F(a)$ for every $a \in K - F$.

Step-by-step solution

Step 1 of 2

Consider a field F and an extension K of F . The objective is to prove that if $[K:F]$ is a prime, then $K = F(a)$ for every $a \in K - F$.

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Step 2 of 2

Note that $F \subseteq F(a) \subseteq K$.

Since $a \notin F$, $F \neq F(a)$, so $[F(a):F] > 1$.

But also , $[F(a):F] | [K:F] = p$.

Now , $[F(a):F] \neq 1$, implies that $[F(a):F] = p$.

$$[K:F] = [K:F(a)][F(a):F]$$

$$p = [K:F(a)] \cdot p$$

Therefore , $[K:F(a)] = 1$. Hence , $K = F(a)$.

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