# A Book of Abstract Algebra | (2nd Edition)

Chapter AC, Problem 2E

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## Problem

*Use mathematical induction to prove the following:*

$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$

## Step-by-step solution

**Step 1** of 2

**Objective:-**

The objective is to prove $1^3+2^3+\cdots+n^3=\left(1+2+\cdots+n\right)^2$ using mathematical induction.

Comment

**Step 2** of 2

Proof:-

$$p(n):1^3+2^3+\cdots+n^3=\left(1+2+\cdots+n\right)^2$$

$$p(n):1^3+2^3+\cdots+n^3=\left(\sum_1^n n\right)^2$$

$$p(n):1^3+2^3+\cdots+n^3=\left(n\left(\frac{n+1}{2}\right)\right)^2 \qquad \left\{\sin ce \quad 1+2+\cdots+n=\frac{n(n+1)}{2}\right\}$$

Let consider rule for $n=1.$

$$p(1):1^3=\left(1\left(\frac{1+1}{2}\right)\right)^2$$

$$p(1):1=1$$

This rule is true for $n=1.$

Let this statement is true for $n=k.$

$$p(k):1^3+2^3+\cdots+k^3=\left(k\left(\frac{k+1}{2}\right)\right)^2 \qquad \cdots\cdots(1)$$

Let consider statement for $n=k+1.$

$$p(k+1):1^3+2^3+\cdots+k^3+(k+1)^3$$

Use the equation (1).

$$p(k+1):\left(\frac{k(k+1)}{2}\right)^2+(k+1)^3$$

$$p(k+1):k^2\left(\frac{k+1}{2}\right)^2+(k+1)^3$$

$$p(k+1):k^2\frac{(k+1)^2}{4}+(k+1)^3$$

$$p(k+1):(k+1)^2\left(\frac{k^2}{4}+k+1\right)$$

$$P(k+1):(k+1)^2\left(\frac{k^2+4k+4}{4}\right)$$

$$P(k+1):(k+1)^2\left(\frac{k+2}{4}\right)^2$$

$$P(k+1):\left[\frac{(k+1)(k+2)}{2}\right]^2$$

This result also true for $n=k+1$. Hence, by mathematical induction this rule is true for all positive integer *n*.

Proved

Comment