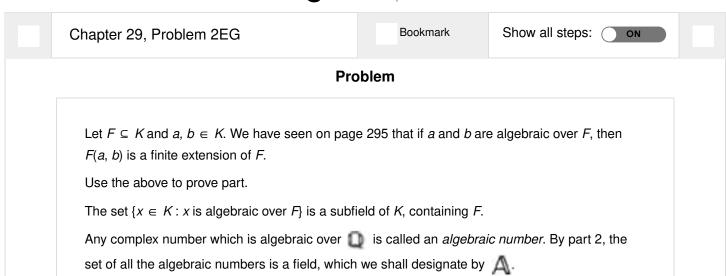
## A Book of Abstract Algebra (2nd Edition)



Let  $a(x) = a_0 + a_1 x + \dots + a_n x^n$  be in A[x], and let c be any root of a(x). We will prove that  $c \in A$ .

To begin with, all the coefficients of a(x) are in  $(a_0, a_1, \dots, a_n)$ .

## Step-by-step solution

## **Step 1** of 2

Consider a field F and an extension K of F. The objective is to prove that the set  $E = \{x \in K : x \text{ is algebraic over } F\}$  is a subfield of K, containing F.

Comment

## **Step 2** of 2

Take  $a, b \in E$ .

Consider the subfield F(a,b) of K.

F(a,b) is a finite extension  $\cdot$  since a and b are algebraic over F and hence an algebraic extension.

Thus all the elements of F(a,b) are algebraic over F and so  $F(a,b) \subseteq E$ .

The elements a+b, a-b, ab and  $1/a(a \ne 0)$  lie in F(a,b), and thus also in E.

So , E is a subfield of K.

Clearly,  $F \subseteq E$ , since any  $\alpha \in F$  is a zero of polynomial  $X - \alpha \in F[x]$  and therefore algebraic over F.

Comment