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Chapter 16, Problem 1EF

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Problem

Let *G* be a group; let *H* and *K* be subgroups of *G*, with *H* a normal subgroup of *G*. Prove the following:

$H \cap K$ is a normal subgroup of *K*

Step-by-step solution

Step 1 of 3

Suppose that *G* is any group and let *H, K* are the subgroups of *G*, with *H* a normal subgroup of *G*. Objective is to prove that $H \quad K$ is a normal subgroup of *K*.

Since *H* and *K* both are subgroups of *G*, therefore their intersection $H \quad K$ will also be a subgroup of *G*. Also $H \quad K$ is a subset of *K*. Task is to show that $H \quad K$ is a normal subgroup of *K*. That is, there is a need to show that

$kak^{-1} \in H \quad K$

for all $k \in K$, and $a \in H \quad K$.

Comment

Step 2 of 3

Let $a \in H \quad K$. Then $a \in H$ and $a \in K$. Since *H* is a normal subgroup of *G*, therefore for some $k \in G$ and *a* in *H*,

$kak^{-1} \in H$.

Being *K* as a subgroup, the condition $a \in K$ and $k \in K$ implies that

$kak^{-1} \in K$.

Since $kak^{-1} \in H$ and $kak^{-1} \in K$. Therefore, $kak^{-1} \in H \quad K$, for some $k \in K$, and $a \in H \quad K$.

Comment

Step 3 of 3

Hence, $H \quad K$ is a normal subgroup of *K*.

Comment