A Book of Abstract Algebra | (2nd Edition)

Chapter 33, Problem 6ED

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Problem

Let G be a group. The symbol $H \lhd G$ should be read, "H is a normal subgroup of G." A *maximal* normal subgroup of G is an $H \lhd G$ such that, if $H \lhd J \lhd G$, then necessarily $J = H$ or $J = G$. Prove the following:

If $H \lhd K \lhd G$, then G/K is a homomorphic image of G/H.

Step-by-step solution

Step 1 of 4

Here, objective is to prove that $\frac{G}{K}$ is homomarphic image of. $\frac{G}{H}$

Comment

Step 2 of 4

Consider G is a finite group. H is normal subgroup of G is denoted by $H \lhd G$

A maximal normal subgroup of G is given by

$H \lhd G$, if $H \lhd J \lhd G$ then, necessarily $J = H$ or $J = G$

Comment

Step 3 of 4

Consider $H \lhd K \lhd G$

Then, necessarily $K = H$ or $K = G$

ϕ :is the homomorphism

Comment

Step 4 of 4

Let $\phi : H \to K$ is a homomorpism. Then

$$\phi(H) = K$$
$$\phi(gH) = gK \quad \forall gH \in G/H$$
$$g_1H = g_2H$$
$$\begin{aligned} \text{Consider } \phi(g_1Hg_2H) &= \phi(g_1g_2H) \qquad (\because H \triangleleft G) \\ &= g_1g_2K \\ &= g_1Kg_2K \qquad (\because K \triangleleft G) \\ &= \phi(g_1)\phi(g_2) \end{aligned}$$

We know that,

$$gK \in G/K$$
$$gH \in G/H$$

So, $\phi : \frac{G}{H} \to \frac{G}{K}$

Therefore,

$\frac{G}{K}$ is homomorphic image of. $\frac{G}{H}$

Hence, proved.

Comment