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## Problem

Throughout this set of questions, let *K* be a root field over *F*, let **G** = *Gal*(*K* : *F*), and let *I* be any intermediate field. Prove the following:

Let *I* be a normal extension of *F*. If **G** is a cyclic group, then *Gal*(*K* : *I*) and *Gal*(*I* : *F*) are cyclic groups.

## Step-by-step solution

### **Step 1** of 2

Consider a root field $K$ over $F$ ,let $G = Gal(K:F)$ ,and let $I$ be any intermediate field which is a normal extension of $F$. The objective is to prove that if $G$ is cyclic ,then $Gal(K:I)$ and $Gal(I:F)$ are cyclic.

Comment

### **Step 2** of 2

Because $K$ is cyclic over $F$ , $G(K:F)$ is a cyclic group.

Now , $G(K:I)$ is a subgroup of $G(K:F)$ ,and is thus cyclic as a subgroup of a cyclic group .

Therefore $K$ is cyclic over $I$ .

As $I$ is a normal extension of $F$ , $G(I:F) \cong \dfrac{G(K:F)}{G(K:I)}$ so $G(I:F)$ is isomorphic to a factor group of a cyclic group ,and is thus cyclic.

Comment