A Book of Abstract Algebra | (2nd Edition)

Chapter AA, Problem 20E

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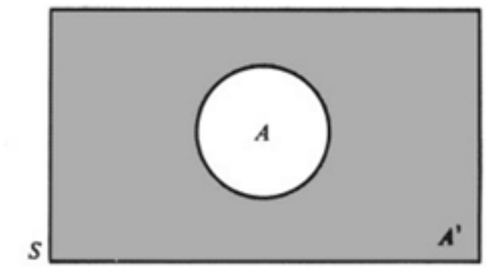
Problem

If *S* is a set, and *A* is a subset of *S*, then the *complement* of *A* in *S* is the set of all the elements of *S* which are not in *A*. The complement of *A* is denoted by *A*′:



$A' = \{x \in S : x \notin A\}$

Prove the following’.

If $A \subseteq B$ and $C = B - A$, then $A = B - C$.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove that if $A \subseteq B, and \;\; C = B - A, then\; A = B - C.$

Comment

Step 2 of 2

Proof:-

Let *A* and *B* are two sets. Let $x \in A \subseteq B$.

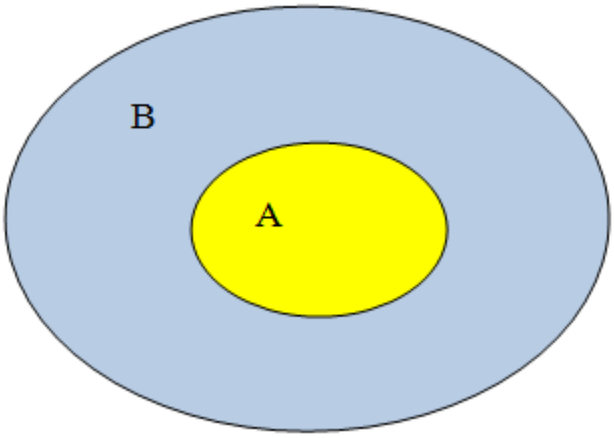
Subsets:-If sets *A* and *B* are such that every elements of *A* are also elements of *B*, then *A* is said to be subset of *B*.

$$A \subseteq B \Leftrightarrow \{x \in A \Rightarrow x \in B\}$$

So the set *B* contains the set *A* and set *A* completely lies within set *B*.

Let the set B is denoted by Blue color and set *A* is denoted by Yellow color.

Figure (1)

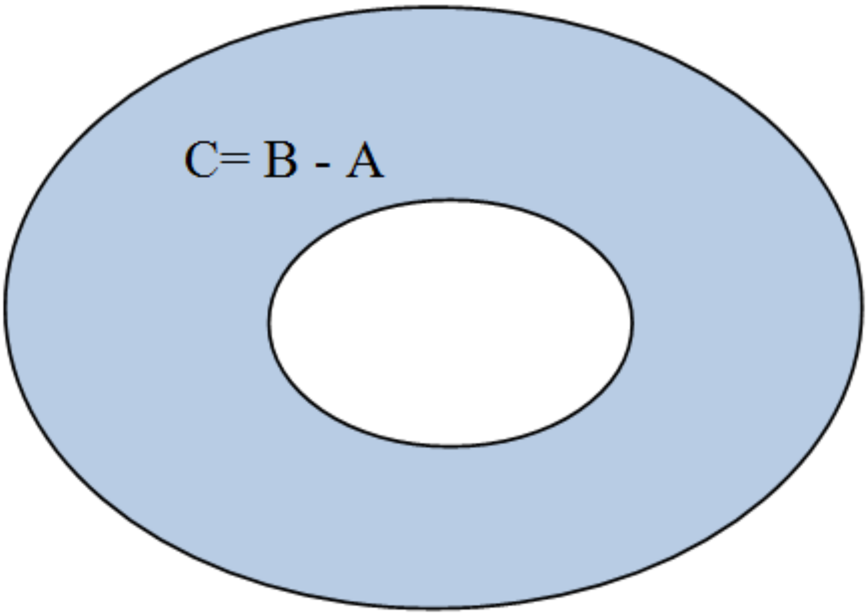


If *S* is a set and *A* is a subset of *S*, subtraction of *B* and *A* is defined as:-

$$B - A = \{x : x \in B \;\; and \;\; x \notin A\}$$

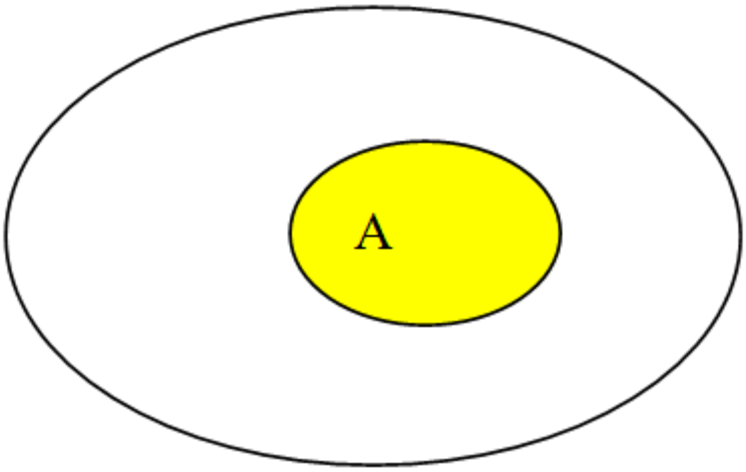
Thus, the set $C = B - A$ is obtained by deleting the Yellow color circle from above figure.

Figure (2)



Let us now subtract set *C* from the set B. The difference $B - C$ is obtained by deleting the set blue coor in figrue 3 from figrue (1). Now the remaining figure is:-

Figure (3)



This figure denotes the set *A*.

Hence,

$$if\ A \subseteq B, and\ \ C = B - A, then\ A = B - C.$$

Comment