A Book of Abstract Algebra | (2nd Edition)
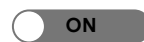
Chapter 32, Problem 2EC

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Problem

Show that $[\mathbb{Q}(\sqrt[3]{2}):\mathbb{Q}]=3$.

Step-by-step solution

Step 1 of 2

The objective is to show that $\left[\mathbb{Q}\left(\sqrt[3]{2}\right):\mathbb{Q}\right]=3$.

Comment

Step 2 of 2

Clearly, $\sqrt[3]{2}$ is the root of x^3-2.

Also, x^3-2 is irreducible polynomial of lowest degree 3 over $\mathbb{Q}$ by Eisenstein $(p=2)$.

Therefore, $\left[\mathbb{Q}\left(\sqrt[3]{2}\right):\mathbb{Q}\right]=\deg\left(x^3-2\right)=3$.

Comment