A Book of Abstract Algebra | (2nd Edition)

Chapter AB, Problem 8E

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Problem

Prove that the following are true for any integers a, b, and c:

If there are integers *k* and *l* such that *ka* + *lb* = 1, then *a* and are relatively prime.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove that if there are integers *k* and *l* such that $ka+lb=1,$ then *a* and *b* are relatively prime.

Comment

Step 2 of 2

Proof:-

Let there are integers *k* and *l* such that:-

$$ka+lb=1 \qquad \ldots\ldots(1)$$

Let us consider the theorem.

Theorem:-Any two nonzero integers *r* and *s* have a unique positive greatest common divisor *t*, Moreover, *t* is equal to a “Linear combination” of *r* and *s*. That is,

$$t=kr+ls \quad for \ \ some \ \ \text{integer} \ \ k \ and \ \ l \qquad \ldots\ldots(2)$$

Let us compare the equation (1) and (2).

$$t=1.$$

According to the above theorem 1 is greatest common divisor of *a* and *b*. The integers *a* and *b* has not common divisor except 1. Thus, *a* and *b* are relatively prime.

Proved

Comment