

A Book of Abstract Algebra | (2nd Edition)

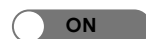


Chapter AA, Problem 7E



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Problem

Prove the following:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

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Step 2 of 2

Proof:-

Let A and B are two sets.

The union of two sets A and B is:-

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

The intersection of two sets A and B is:-

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Let $x \in A \cup (B \cap C)$.

$$x \in A \cup (B \cap C)$$

$$\Rightarrow x \in A \text{ or } x \in B \cap C$$

$$\begin{aligned}
&\Rightarrow x \in A \text{ or } x \in B \text{ and } x \in C \\
&\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C) \\
&\Rightarrow (x \in A \cup B) \text{ and } (x \in A \cup C) \\
&\Rightarrow x \in (A \cup B) \cap (A \cup C)
\end{aligned}$$

So,

$$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \quad \dots\dots(1)$$

Let $x \in A \cup (B \cap C)$.

$$\begin{aligned}
&x \in (A \cup B) \cap (A \cup C) \\
&\Rightarrow (x \in A \cup B) \text{ and } (x \in A \cup C) \\
&\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C) \\
&\Rightarrow x \in A \text{ or } x \in B \text{ and } x \in C \\
&\Rightarrow x \in A \cup (B \cap C)
\end{aligned}$$

So,

$$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \quad \dots\dots(2)$$

Let us consider the equation (1) and (2).

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Proved

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