A Book of Abstract Algebra | (2nd Edition)

Problem

An angle α is called *constructible* iff there exist constructible points *A, B*, and *C* such that $\angle LABC = \alpha$.

Prove the following:

The following angles are constructible: $30°,\ 75°,\ 22\frac{1}{2}°$.

Step-by-step solution

Step 1 of 5

Here, objective is to prove that, the given angles are constructible.

Comment

Step 2 of 5

Constructible angle:

An angle $\frac{2\pi}{N}$ is constructible if and only if *N* is either a power of two or power of two times a set

of Fermat points.

Comment

Step 3 of 5

Consider the angle 30^0

$\frac{360}{12} = 30^0$

$12 = 2^2 \times 3$

2^2 is a power of two and 3 is a Fermat prime.

Hence, 30^0 is constructible angle.

Comment

Step 4 of 5

Consider the angle 75^0

$75^0 = 30^o + 45^o$

$45^o = \frac{360}{8}$

8 is a power of two

$30^0 = \frac{360}{12}$

$12 = 2^2 \times 3$

2^2 is a power of two and 3 is a Fermat prime.

Then, the angles 30^o and 45^o are constructible.

Therefore, the angle 75^0 is constructed by summing of 30^o and 45^o

Hence, 75^0 is constructible angle.

Comment

Step 5 of 5

Consider the angle $22\frac{1}{2}^{0}$

$22\frac{1}{2}^{0}=\frac{45^{0}}{2}$

$\frac{360}{16}=\frac{45^{0}}{2}$

$16=2^{4}$

16 is a power of two

Hence, $22\frac{1}{2}^{0}$ is constructible angle.

Comment