A Book of Abstract Algebra | (2nd Edition)

Problem

Let F be any field.

Prove part:

If c is algebraic over F, so are $c + 1$ and kc (where $k \in F$).

Step-by-step solution

Step 1 of 4

Let F be any field. If c is algebraic over F, so are $c+1$ and kc where $k \in F$.

Comment

Step 2 of 4

Since c is algebraic over F, there exists a polynomial $f(x) \in F[x]$ such that $f(c) = 0$.

We need to find a polynomial over F such that $c+1$ is root of that polynomial.

Consider the polynomial $g(x) = f(x-1)$ in $F[x]$. Then,

$$\begin{aligned} g(1+c) &= f(1+c-1) \\ &= f(c) \\ &= 0 \end{aligned}$$

Therefore, $1+c$ is also algebraic over F.

Comment

Step 3 of 4

Now for any $k \neq 0$ in F, k^{-1} exist in F as F is field.

Consider $h(x) = f(xk^{-1})$ in $F[x]$. Then,

$$\begin{aligned} h(ck) &= f(ckk^{-1}) \\ &= f(c \cdot 1) \quad [\, 1 \text{ is unity in } F\,] \\ &= f(c) \\ &= 0 \end{aligned}$$

Therefore, ck is also algebraic over F

Comment

Step 4 of 4

Comment