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Chapter AA, Problem 10E

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Problem

Prove the following:

$A \cap (A \cup B) = A$.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $A \cup (A \cap B) = A$.

Comment

Step 2 of 2

Proof:-

Let *A* and *B* are two sets.

The union of two sets *A* and *B* is:-

$$A \cup B = \{x : x \in A \ \ or \ \ x \in B\}$$

The intersection of two sets *A* and *B* is:-

$$A \cap B = \{x : x \in A \ \ and \ \ x \in B\}$$

Let $x \in A \cap (B \cup C)$.

$$x \in A \cap (B \cup C)$$
$$\Rightarrow x \in A \ \ and \ \ (\ x \in B \cap C)$$

$$\Rightarrow x \in A \ \ and \ \ x \in B \ \ or \ \ x \in C$$
$$\Rightarrow (x \in A \ \ and \ \ x \in B) \ \ and \ (\ x \in A \ \ or \ \ x \in C\ )$$
$$\Rightarrow (x \in A \cap B) \ \ or \ (\ x \in A \cap C\ )$$
$$\Rightarrow x \in (A \cap B) \cup (A \cap C\ )$$

So,

$$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \qquad \qquad ......(1)$$

Let $x \in A \cup (B \cap C)$.

$$x \in (A \cap B) \cup (A \cap C)$$
$$\Rightarrow (x \in A \cap B) \ \ or \ (\ x \in A \cap C\ )$$
$$\Rightarrow (x \in A \ \ and \ \ x \in B) \ \ or \ (\ x \in A \ \ and \ \ x \in C\ )$$
$$\Rightarrow x \in A \ \ and \ \ (x \in B \ \ or \ \ x \in C)$$
$$\Rightarrow x \in A \cap (B \cup C)$$

So,

$$(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \qquad \qquad ......(2)$$

Let us consider the equation (1) and (2).

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

According to this theorem:-

$$A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$$
$$A \cap (A \cup B) = A \cap (A \cap B)$$
$$A \cap (A \cup B) = A$$

Since the common elements in set *A* and union of set and *B* are elements of set *A*.

Proved

Comment