# A Book of Abstract Algebra | (2nd Edition)

Chapter 24, Problem 2EC

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## Problem

Give examples of divisors of zero, of degrees 0, 1, and 2, in $\mathbb{Z}_4[x]$.

## Step-by-step solution

**Step 1** of 1

Consider the ring $\mathbb{Z}_4[x]$. The set of elements of $\mathbb{Z}_4$ are $\{0,1,2,3\}$.

Theorem: A non zero element *a* in $\mathbb{Z}_n$ is a zero divisor if and only if $\gcd(a,n)\neq 1$, and there is a $b=\dfrac{n}{\gcd(a,n)}$ such that $ab=0$

Apply the above theorem to find zero divisors of $\mathbb{Z}_4$.

$$\gcd(1,4)=1$$
$$\gcd(2,4)=2$$
$$\gcd(3,4)=1$$

Hence zero divisor in $\mathbb{Z}_4$ is 2.

Now construct polynomials whose all coefficients are 2.

Example of zero divisor of zero degree polynomial in $\mathbb{Z}_4[x]$ is $p(x)=2$.

Examples of zero divisors of first degree polynomial in $\mathbb{Z}_4[x]$ are following.

$$p(x)=2x$$
$$p(x)=2x+2$$

Examples of zero divisors of second degree polynomial in $\mathbb{Z}_4[x]$ are following.

$p(x) = 2x^2$

$q(x) = 2x^2 + 2x$

$r(x) = 2x^2 + 2$

$s(x) = 2x^2 + 2x + 2$

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