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Chapter 16, Problem 4EN

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Problem

Let G be a finite group, and K a p-Sylow subgroup of G. Let X be the set of all the conjugates of K. See Exercise M2. If C_1, $C_2 \in X$, let $C_1 \sim C_2$ iff $C_1 = aC_2a^{-l}$ for some $\alpha \in K$

Use parts 2 and 3 to prove that the number of elements in X is $k_p + 1$, for some integer k.

Step-by-step solution

Step 1 of 3

Assume that G is a finite group and K a p-Sylow subgroup of G. Consider the set X as the set of all the conjugates of K. Define an equivalence relation as:

If $C_1, C_2 \in X$, let $C_1 \approx C_2$ if and only if $C_1 = aC_2a^{-1}$ for some $a \in K$.

Objective is to prove that the number of elements in X is $kp + 1$, for some integer k.

Comment

Step 2 of 3

From the previous result one have:

(1) The number of elements in $[C]$ is either 1 or a power of *p*.

(2) There is only one single element class, that is, the only class with a single element is $[K]$.

Now, by using the equivalence relation, one have partitioned the order of *X* into some equivalence classes of size p^i, for some integer *i*, with exactly one class of size 1. This size one class is of identity class, $p^0 = 1$.

From this argument, it implies that the number of elements in *X* is one more than a multiple of *p*.

Comment

Step 3 of 3

Hence, the number of elements in *X* is $kp + 1$, for some integer *k*.

Comment