A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 5EB

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Problem

Solve the following systems of simultaneous Diophantine equations:

(*a*) $4x + 6y = 2$; $9x + 12y = 3$

(*b*) $3x + 4y = 2$; $5x + 6y = 2$; $3x + 10y = 8$.

Step-by-step solution

Step 1 of 6

Here, objective is to solve the given system of simultaneous linear Diophantine equations.

Comment

Step 2 of 6

Diophantine equation is in one or more unknowns, with the integer coefficients.

The Diophantine equation is of the form $ax + by = c; a, b, c$ are integers..

Comment

Step 3 of 6

(a)

Consider the system of equations $4x + 6y = 2, 9x + 12y = 3$

Consider the first two equations,

$$4x+6y=2,$$
$$2x+3y=1$$
$$2x=1 \bmod 3$$
$$x=2 \bmod 3..(1)$$
$$9x+12y=3$$
$$3x+4y=1$$
$$3x=1 \bmod 4$$
$$x=3 \bmod 4....(2)$$

Equation $....(1)=\text{Equation}...(2)$

$$2 \bmod 3=3 \bmod 4$$
$$2+3p=3 \bmod 4$$
$$3p=1 \bmod 4$$
$$p=3 \bmod 4$$
$$x=2+3(2+3p)$$
$$x=8 \bmod 9$$
$$x+9y=8$$

Comment

Step 4 of 6

Consider the congruence $x+9y=8$

$$\gcd(1,9)=1$$

$\gcd(1,9)=1$ is divisible by 8. So there is an integer pair solutions.

Apply Euclidian algorithm:

$$x+9y=8.................(1)$$
$$9=1\times 9+0$$

By applying extended Euclidian algorithm,

$$1=(1\times 1)+(9\times 0)$$
$$8=(1\times 8)+(9\times 0).............(2)$$

By comparing equations (1) and (2)

$$x=8, y=0$$

Hence, the solution of system of simultaneous Diophantine equations is $x=8, y=0$

Comment

Step 5 of 6

(b)

The system of equations $3x+4y=2, 5x+6y=2, 3x+10y=8$

Consider the first two equations,

$$3x+4y=2$$
$$3x=2\bmod 4$$
$$x=6\bmod 4..............(1)$$

$$5x+6y=2$$
$$5x=2\bmod 6$$
$$x=10\bmod 6...............(2)$$

Equation $....(1)=\text{Equation}...(2)$

$$6+4p=10\bmod 6$$
$$4p=4\bmod 6$$
$$2p=2\bmod 3$$

$$p=4\bmod 3$$
$$x=6+4(\bmod 3)$$
$$x=10\bmod 12.............(3)$$

Consider the third equation

$$3x+10y=8$$
$$3x=8\bmod 10$$
$$x=56\bmod 10...............(4)$$

$$10+12k=56\bmod 10$$
$$12k=46\bmod 10$$
$$6k=23\bmod 5$$
$$k=23\bmod 5$$

From Equation $....(3)$

$$x=10+12(23\bmod 5)$$
$$x=286\bmod 60$$
$$x+60y=286$$

Comment

Step 6 of 6

Consider the congruence $x+60y=286$

$$\gcd(1,60)=1$$

$\gcd(1,60)=1$ is divisible by 286 .So there is an integer pair solutions.

Apply Euclidian algorithm:

$$x+60y=286................(1)$$
$$60=1\times 60+0$$

By applying extended Euclidian algorithm,

$$x+60y=286$$
$$1=(1\times 1)+(60\times 0)$$
$$286=(1\times 286)+(60\times 0).............(2)$$

By comparing equations (1) and (2)

$x = 286, y = 0$

Hence, the solution of system of simultaneous Diophantine equations is $x = 286, y = 0$

Comment