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Chapter 29, Problem 3ED

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Problem

Let F be a field, and K a field extension of F. Prove the following:

If $[K:F]$ is a prime, then $K = F(a)$ for every $a \in K - F$.

Step-by-step solution

Step 1 of 2

Consider a field F and an extension K of F. The objective is to prove that if $[K:F]$ is a prime, then $K = F(a)$ for every $a \in K - F$.

Comment

Step 2 of 2

Note that $F \subseteq F(a) \subseteq K$.

Since $a \notin F$, $F \neq F(a)$, so $[F(a):F] > 1$.

But also，$\left[F(a):F\right] \mid \left[K:F\right]=p$.

Now，$\left[F(a):F\right]\neq 1$，implies that $\left[F(a):F\right]=p$.

$$\left[K:F\right]=\left[K:F(a)\right]\left[F(a):F\right]$$
$$p=\left[K:F(a)\right]\cdot p$$

Therefore，$\left[K:F(a)\right]=1$.Hence，$K=F(a)$.

Comment