A Book of Abstract Algebra | (2nd Edition)

Chapter 30, Problem 3EC

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Problem

An angle α is called *constructible* iff there exist constructible points *A, B*, and *C* such that $\angle LABC = \alpha$.

Prove the following:

If $\cos\alpha$, $\cos\beta \in \mathbb{D}$, then $\cos(\alpha+\beta)$, $\cos(\alpha-\beta) \in \mathbb{D}$.

Step-by-step solution

Step 1 of 3

Here, objective is to prove that $\cos\alpha, \cos\beta \in D$, then $\cos(\alpha+\beta), \cos(\alpha-\beta) \in D$

Comment

Step 2 of 3

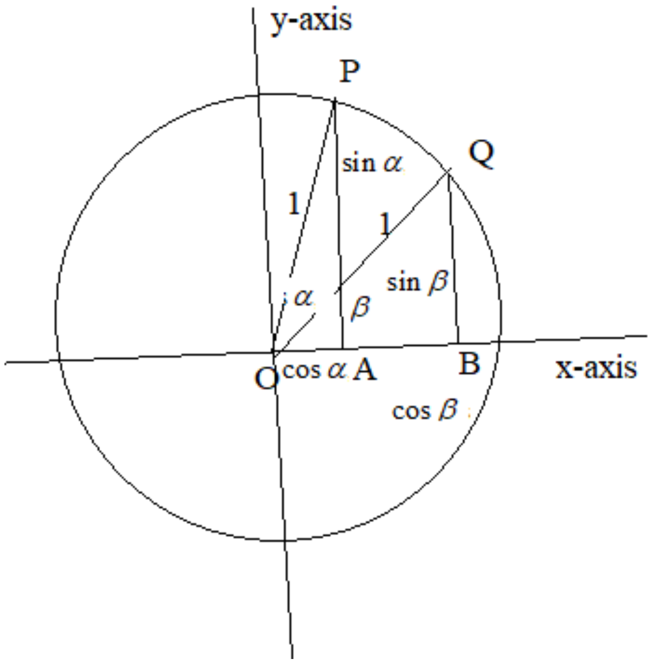
Constructible point is the end point of given unit segment or it is intersection of two lines determined by constructional points.

Consider $\cos\alpha, \cos\beta \in D$, then the points $(\cos\alpha, 0), (\cos\beta, 0)$ are constructible from $\{O, I\}$

Comment

Step 3 of 3

Consider the below figure:



If $(\cos\alpha, 0), (\cos\beta, 0)$ are constructible, then $\sin\alpha, \sin\beta$ are constructible, by using the identities

$$\sin\alpha = 1 - \cos^2\alpha$$
$$\sin\beta = 1 - \cos^2\beta$$

Consequently,

$\cos(\alpha+\beta), \cos(\alpha-\beta)$ are also constructible. Since

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

if $\cos(\alpha+\beta), \cos(\alpha-\beta)$ are constructible then $\cos(\alpha+\beta), \cos(\alpha-\beta) \in D.$

Therefore, $\cos(\alpha+\beta), \cos(\alpha-\beta) \in D$, if $\cos\alpha, \cos\beta \in D$.

Hence, proved

Comment