

# A Book of Abstract Algebra | (2nd Edition)

Chapter AB, Problem 7E

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## Problem

*Prove that the following are true for any integers  $a$ ,  $b$ , and  $c$ :*

If  $\gcd(ab, c) = 1$ , then  $\gcd(a, c) = 1$  and  $\gcd(b, c) = 1$ .

## Step-by-step solution

### Step 1 of 3

#### Objective:-

The objective is to prove *if  $\gcd(ab, c) = 1$ , then  $\gcd(a, c) = 1$  and  $\gcd(b, c) = 1$ .*

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### Step 2 of 3

Proof:-

Let us consider the theorem.

**Theorem:-**Any two nonzero integers  $r$  and  $s$  have a unique positive greatest common divisor  $t$ . Moreover,  $t$  is equal to a "Linear combination" of  $r$  and  $s$ . That is,

$$t = kr + ls \text{ for some integer } k \text{ and } l$$

Let us suppose  $\gcd(ab, c) = 1$ . Then by above theorem:-

$$1 = k(ab) + lc \text{ for some integer } k \text{ and } l$$

This can be written as:-

$$1 = (kb)a + lc \text{ for some integer } k \text{ and } l$$

The integer 1 can be written as linear combination of the integer  $a$  and  $c$ .

Hence, by theorem:-

$$\gcd(a, c) = 1$$

Proved

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### Step 3 of 3

Let us suppose  $\gcd(ab, c) = 1$ . Then by above theorem:-

$$1 = k(ab) + lc \text{ for some integer } k \text{ and } l$$

This can be written as:-

$$1 = (ka)b + lc \text{ for some integer } k \text{ and } l$$

The integer 1 can be written as linear combination of the integer  $b$  and  $c$ .

Hence, by theorem:-

$$\gcd(b, c) = 1$$

Proved

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