A Book of Abstract Algebra | (2nd Edition)

Chapter 30, Problem 6EG

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Problem

Prove each of the following:

The circle $x^2 + y^2 + ax + by + c = 0$ is constructible if $a, b, c \in \mathbb{D}$.

Step-by-step solution

Step 1 of 4

Here, objective is to prove that the circle $x^2 + y^2 + ax + by + c = 0$ is constructible, if $a,b,c \in D$.

Consider $a \in D$ if and only if the point $(a,0)$ is constructible from $\{O,I\}$

Comment

Step 2 of 4

A circle is constructible if its centre and radius are constructible.

The standard form of circle equation is

$(x-A)^2+(y-B)^2=R^2$

centre-(A,B)

radius-R

Comment

Step 3 of 4

Consider the equation of the circle $x^2+y^2+ax+by+c=0$

Transform in to standard form of circle,

$x^2+y^2+ax+by+c=0$

$x^2+y^2+ax+by+\frac{a^2}{4}+\frac{b^2}{4}=\frac{a^2}{4}+\frac{b^2}{4}-c$

$\left(x+\frac{a}{2}\right)^2+\left(y+\frac{b}{2}\right)^2=\frac{a^2}{4}+\frac{b^2}{4}-c$

$\text{centre}=\left(-\frac{a}{2},-\frac{b}{2}\right)$

$\text{radius}=\frac{a^2}{4}+\frac{b^2}{4}-c$

Comment

Step 4 of 4

The centre and radius are constructible if and only if $a,b,c \in D$

If the centre and radius are constructible, then circle $x^2 + y^2 + ax + by + c = 0$ is constructible

Therefore, the circle $x^2 + y^2 + ax + by + c = 0$ is constructible, if $a,b,c \in D$.

Hence, proved

Comment