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Chapter AB, Problem 13E

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Problem

Prove that the following are true for any integers a, b, and c:

If gcd(*a*, *b*) = *c*, then lcm(*a*, *b*) = *ab*/*c*.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $if \ \ \gcd(a,b)=c, then \ \ lcm(a,b)=\frac{ab}{c}.$

Comment

Step 2 of 2

Proof:-

Let us consider the theorem.

Theorem:-If *p* and *q* are two integers with greatest common divisor $\gcd(p,q)$ and least common multiple $lcm(p,q)$, then

$$p\times q=\gcd(p,q)\times lcm(p,q)$$

Let us suppose $\gcd(a,c)=c.$ Then by above theorem:-

$$a\times b=c\times lcm(a,b)$$

$$ab=c\cdot lcm(a,b)$$

$$lcm(a,b)=\frac{ab}{c}$$

Proved

Comment