A Book of Abstract Algebra | (2nd Edition)

Problem

Prove each of the following:

A point P is constructible iff both its coordinates are constructible numbers.

Step-by-step solution

Step 1 of 3

Here, objective is to prove that a point is constructible if and only if, both of its coordinates are constructible.

Comment

Step 2 of 3

Constructible point:

A point is said to be constructible, if it is either the end point of a given line segment or intersection of lines which are determined by previously obtained constructible points.

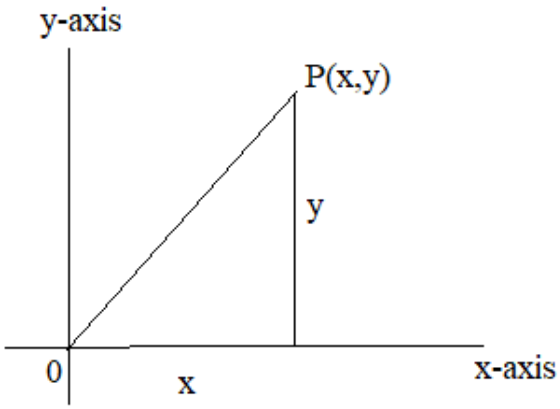
Constructible number:

It is a coordinate of a constructible point.

Comment

Step 3 of 3

Consider below figure:



If, the lengths *x, y* are constructible which are also called constructible numbers.

Draw the line along x-axis with the length *x* and then, from the end point draw the perpendicular line along y-axis with the length *y.*

Then the end point is the point $P(x, y)$.

If the point $P(x, y)$ is constructible,

Then draw the perpendicular lines from point *p* to x-axis and y-axis which results the lengths *x, y.*

Therefore,

A point is constructible if and only if both its coordinates are constructible

Hence, proved

Comment