A Book of Abstract Algebra | (2nd Edition)

Problem

A polygon is called *constructible* iff its vertices are constructible points. Prove the following:

The regular hexagon is constructible.

Step-by-step solution

Step 1 of 4

Here, objective is to prove that the regular hexagon is constructible.

Comment

Step 2 of 4

Regular *n*-gon has *n* equal length of sides and all angles are equal.

Regular *n*-gon is constructible if and only if the angle $\frac{2\pi}{n}$ is constructible

An angle $\frac{2\pi}{N}$ is constructible if and only if *N* is either a power of two or power of two and a set of

Fermat points

Comment

Step 3 of 4

Consider regular Hexagon.

Regular Hexagon is a six-sided Polygon or $6-\text{gon}$

Number of sides $n=6$

Comment

Step 4 of 4

To verify $\frac{2\pi}{n}$ is constructible or not:

$$\frac{2\pi}{n}=\frac{2\pi}{6}$$

$$6=2\times 3$$

6 is a product of power of two and 3 is a Fermat prime.

Therefore, $\frac{2\pi}{6}$ is constructible angle, which Implies Regular $6-\text{gon}$ is constructible.

Hence,

Regular hexagon is constructible.

Comment