

A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 1EB

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Problem

Show that, in any extension of \mathbb{Z}_3 which contains a root u of

$$a(x) = x^3 + 2x + 1 \in \mathbb{Z}_3[x]$$

it happens that $u + 1$ and $u + 2$ are the remaining two roots of $a(x)$. Use this fact to find the root field of $x^3 + 2x + 1$ over \mathbb{Z}_3 . List the elements of the root field.

Step-by-step solution

Step 1 of 3

The objective is to show that in any extension of \mathbb{Z}_3 which contains a root u of

$$a(x) = x^3 + 2x + 1 \in \mathbb{Z}_3[x], \text{ it happens that } u + 1 \text{ and } u + 2 \text{ are the remaining roots of } a(x).$$

Also find the root field of $x^3 + 2x + 1$ over \mathbb{Z}_3 and list its elements.

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Step 2 of 3

Let u be a root of $a(x) = x^3 + 2x + 1 \in \mathbb{Z}_3[x]$.

Then $a(u) = u^3 + 2u + 1 = 0$ (1)

$$\begin{aligned} a(u+1) &= (u+1)^3 + 2(u+1) + 1 \\ &= u^3 + 3u^2 + 3u + 1 + 2u + 2 + 1 \\ &= u^3 + 3u^2 + 5u + 4 \\ &= u^3 + 2u + 1 \end{aligned}$$

By (1), $a(u+1) = 0$.

$$\begin{aligned} a(u+2) &= (u+2)^3 + 2(u+2) + 1 \\ &= u^3 + 6u^2 + 12u + 8 + 2u + 4 + 1 \\ &= u^3 + 6u^2 + 14u + 13 \\ &= u^3 + 2u + 1 \end{aligned}$$

By (1), $a(u+2) = 0$.

Therefore, it is shown that $u+1$ and $u+2$ are the remaining roots of $a(x)$.

[Comment](#)

Step 3 of 3

In \mathbb{Z}_3 , u , $u+1$, and $u+2$ are the roots of $x^3 + 2x + 1$.

Also, \mathbb{Z}_3 contains u , $u+1$, and $u+2$.

Therefore, $\mathbb{Z}_3(u, u+1, u+2)$ is the root field of $x^3 + 2x + 1$ over \mathbb{Z}_3 and u , $u+1$, and $u+2$ are its elements.

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