# A Book of Abstract Algebra | (2nd Edition)

Chapter 33, Problem 1EB

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## Problem

Let $G$ be a group. The symbol $H \lhd G$ is commonly used as an abbreviation of "$H$ is a *normal* subgroup of $G$." A *normal series* of $G$ is a finite sequence $H_0$, $H_1$, …,$H_n$ of subgroups of $G$ such that

$$\{e\} = H_0 \lhd H_1 \lhd \cdots \lhd H_n = G$$

Such a series is called a *solvable series* if each quotient group $H_{i+1}/H_i$ is abelian. $G$ is called a *solvable group* if it has a solvable series.

Explain why every abelian group is, trivially, a solvable group.

## Step-by-step solution

**Step 1** of 4

Here, objective is to explain why every Abelian group is trivially a solvable group.

Comment

**Step 2** of 4

A group $G$ is solvable, if there exist a finite chain of successive subgroups $1 = G_0 \le G_1 \le G_2 \le ............ \le G_n$ having the following properties.

$G_i$ is the normal subgroup of $G_{i+1}; \forall \quad 0 \le i \le n-1$

$\dfrac{G_{i+1}}{G_i}$ is an abelian group $\forall \quad 0 \le i \le n-1$

Comment

**Step 3** of 4

A group is a solvable group, if that group is constructed from abelian groups using extensions. That means solvable group has Abelian series.

A group is said to be Abelian group, if it satisfies the condition $xy = yx$ for all group elements *x* and *y*.

Comment

**Step 4** of 4

Consider *G* is an Abelian group.

By the definition of group,

$$\begin{aligned} 1 &= G_0 \le G_1 \\ &= G \end{aligned}$$

Where, *G* is a chain of successive subgroups.

We know that,

$G_0 = e$ is a normal subgroup of $G_1 = G$ trivially.

Then,

$$\begin{aligned} \frac{G_1}{G_0} &= \frac{G}{e} \\ &\cong G \end{aligned}$$

*G* is an Abelian group.

Therefore, *G* is a solvable group.

Hence, every Abelian group is trivially a solvable group.

Comment