A Book of Abstract Algebra (2nd Edition)

Chapter 16, Problem 2EI

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Problem

Let *H* and *K* be normal subgroups of a group *G*, with $H \subseteq k$ Define ϕ: $G/H \to G/K$ by $\phi(Ha) = Ka$. Prove part:

ϕ is a homomorphism.

Step-by-step solution

Step 1 of 3

Suppose that *G* is any group and let *H, K* are normal subgroups of *G* with $H \subseteq K$.

Consider a mapping $\phi : G/H \to G/K$ defined by

$\phi(Ha) = Ka$, for all $a \in G$.

Objective is to prove that function ϕ is homomorphism.

If *G* and *H* are two groups, a homomorphism from *G* to *H* is a function $f : G \to H$ such that for any two elements *a*, *b* in *G*,

$f(ab) = f(a)f(b)$.

Comment

Step 2 of 3

Assume that $Ha, Hb \in G/H$, for some $a, b \in G$. Then use the definition of mapping in the following manner:

$$\begin{aligned}\phi(Ha \cdot Hb) &= \phi(Hab) \\ &= Kab \\ &= (Ka)\cdot(Kb) \\ &= \phi(Ha)\cdot\phi(Hb).\end{aligned}$$

Comment

Step 3 of 3

Since the condition $\phi(Ha \cdot Hb) = \phi(Ha) \cdot \phi(Hb)$ holds, therefore ϕ is homomorphism mapping.

Comment