A Book of Abstract Algebra | (2nd Edition)

Chapter 24, Problem 3EH

Bookmark

Show all steps: ON

Problem

Let us denote an arbitrary polynomial $p(x, y)$ in $A[x, y]$ by $\Sigma\, a_{ij}x^iy^j$ where Σ ranges over *some* pairs i, j of nonnegative integers.

Imitating the definitions of sum and product of polynomials in $A[x]$, give a definition of sum and product of polynomials in $A[x, y]$.

Step-by-step solution

Step 1 of 1

Let $a(x,y)$ and $b(x,y)$ be polynomials in $A[x,y]$ of degree n and m respectively.

$a(x,y) = a_{00} + (a_{01}y + a_{10}x) + \ldots. + \sum_{i_1+j_1=k_1} a_{i_1j_1}x^{i_1}y^{j_1} = \sum_{k_1=0}^{n} \sum_{i_1+j_1=k_1} a_{i_1j_1}x^{i_1}y^{j_1}$, i_1, j_1 are

Non-negative positive integers, similarly

$b(x,y) = b_{00} + (b_{01}y + b_{10}x) + \ldots. + \sum_{i_2+j_2=k_2} b_{i_2j_2}x^{i_2}y^{j_2} = \sum_{k_2=0}^{m} \sum_{i_2+j_2=k_2} b_{i_2j_2}x^{i_2}y^{j_2}$, i_2, j_2 are

Non-negative positive integers

$$\Rightarrow \begin{aligned} a(x,y)+b(x,y) &= \sum_{k_1=0}^{n} \sum_{i_1+j_1=k_1} a_{i_1j_1}x^{i_1}y^{j_1} + \sum_{k_2=0}^{m} \sum_{i_2+j_2=k_2} b_{i_2j_2}x^{i_2}y^{j_2} \\ &= \sum_{k=0}^{\min(n,m)} \sum_{i+j=k} (a_{ij}+b_{ij})x^iy^j + \sum_{k=0}^{\max(m,n)} \sum_{i+j=k} c_{ij}x^iy^j \end{aligned}$$

if $m < n$ then $c_{ij} = a_{ij}$ else $c_{ij} = b_{ij}$

$$\begin{aligned} a(x,y)b(x,y) &= \Big(\sum_{k_1=0}^{n} \sum_{i_1+j_1=k_1} a_{i_1j_1}x^{i_1}y^{j_1}\Big)\times\Big(\sum_{k_2=0}^{m} \sum_{i_2+j_2=k_2} b_{i_2j_2}x^{i_2}y^{j_2}\Big) \\ &= \sum_{k=0}^{m+n} \sum_{i_1+j_1+i_2+j_2=k} (a_{i_1j_1}b_{i_2j_2})x^{i_1+i_2}y^{j_1+j_2} \end{aligned}$$

i_1, i_2, j_1, j_2 are Non-negative

positive integers

Comment