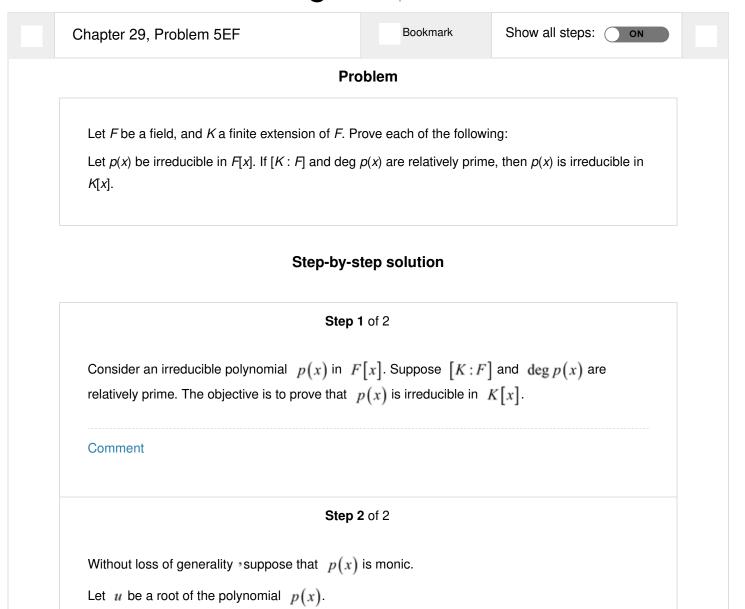
A Book of Abstract Algebra (2nd Edition)



Then $p(x) = p_F(x)$ is the minimal polynomial of u over F.

Compute $\lceil K(u) : F \rceil$ in wo ways:

$$[K(u):K]\cdot [K:F] = [K(u):F] = [K(u):F(u)]\cdot [F(u):F]$$

It follows that $[F(u):F]|[K(u):K]\cdot [K:F]$.

By hypothesis $\cdot \deg(p_F(x)) = [F(u):F]$ is relatively prime to [K:F].

Therefore $\cdot [F(u):F] | [K(u):K]$; note that $[F(u):F] \leq [K(u):K]$.

But $F \subseteq K$ implies that $p_K(x) | p_F(x)$ in K[x] so

$$\deg p_F = \lceil F(u) : F \rceil \ge \lceil K(u) : K \rceil = \deg p_K$$

Hence $\rightarrow \deg p_F = \deg p_K$.

Since $p_K(x) | p_F(x)$ and both polynomials are monic $p_K(x) = p_F(x)$.

That is as the minimal polynomial of u over K, $p(x) = p_F$ is irreducible over K.

Comment