A Book of Abstract Algebra | (2nd Edition)

Chapter AB, Problem 10E

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Problem

Prove that the following are true for any integers a, b, and c:

If $d|ab$ and $d|cb$ and gcd(a, c) = 1, then $d|b$.

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $if \;\; d \mid ab \;\; and \;\; d \mid cb \; and \;\; \gcd(a,c)=1, then \;\; d \mid b.$

Comment

Step 2 of 2

Proof:-

Let suppose $d \mid ab \;\; and \;\; d \mid cb.$

Then there exist number k and l such that:-

$$ab = kd \qquad .....(1)$$
$$cb = ld \qquad .....(2)$$

Let $\gcd(a,c)=1$, then integers a and c are relatively prime.

Let us consider the theorem.

Theorem:-Any two nonzero integers r and s have a unique positive greatest common divisor t, Moreover, t is equal to a “Linear combination” of r and s. That is,

$$t = kr + ls \quad for \;\; some \;\; \text{integer} \;\; k \; and \;\; l$$

Let us suppose $\gcd(a,c)=1.$ Then by above theorem:-

$$1 = ma + nc \quad for \ some \ \text{integer} \ m \ and \ n$$

$$1 = m\frac{kd}{b} + n\frac{kd}{b} \qquad \{from \ equation(1) \ and \ (2)\}$$

$$1 = \frac{kd}{b}(m+n)$$

$$b = kd(m+n)$$

Thus, d is a factor of $kd(m+n)$ that is a factor of *b*. Hence, *d* divides *b* that is $d \mid b$..

Proved

Comment