A Book of Abstract Algebra | (2nd Edition)

Chapter 24, Problem 2EA | 1 Bookmark | Show all steps: ON

Problem

REMARK ON NOTATION: In some of the problems which follow, we consider polynomials with coefficients in $\mathbb{Z}_n$ for various *n*. To simplify notation, we denote the elements of $\mathbb{Z}_n$ by 1, 2, …, *n* – 1 rather than the more correct $\bar{1}, \bar{2}, \ldots, \overline{n-1}$.

Find the quotient and remainder when $x^3 + x^2 + x + 1$ is divided by $x^2 + 3x + 2$ in $\mathbb{Z}[x]$ and in $\mathbb{Z}_5[x]$.

Step-by-step solution

Step 1 of 2

Consider the polynomial $p(x) = x^3 + x^2 + x + 1$ and $q(x) = x^2 + 3x + 2$.

Objective of this question is to find quotient and remainder when $p(x) = x^3 + x^2 + x + 1$ divided by $q(x) = x^2 + 3x + 2$ in $\mathbb{Z}[x]$ and $\mathbb{Z}_5[x]$.

First consider the ring $\mathbb{Z}[x]$.

Given polynomials $p(x)$ and $q(x)$ are the elements of $\mathbb{Z}[x]$.

Now do long division.

$$
\begin{array}{r|l}
 & x-2 \\
\hline
 & x^3+x^2+x+1 \\
 & \underline{x^3+3x^2+2x} \\
x^2+3x+2 & \quad -2x^2-x+1 \\
 & \quad \underline{-2x^2-6x-4} \\
 & \qquad\qquad 5x+5
\end{array}
$$

Then, quotient and remainder when $p(x) = x^3 + x^2 + x + 1$ divided by $q(x) = x^2 + 3x + 2$ in

$\mathbb{Z}[x]$ are $\boxed{x-2}$ and $\boxed{5x+5}$ respectively.

Comment

Step 2 of 2

Consider the ring $\mathbb{Z}_5[x]$.

Change polynomials $p(x)$ and $q(x)$ as the element of $\mathbb{Z}_5[x]$.

$$\begin{aligned} p(x) &= 1(\bmod 5)x^3 + 1(\bmod 5)x^2 + 1(\bmod 5)x + 1(\bmod 5) \\ &= x^3 + x^2 + x + 1 \end{aligned}$$

$$\begin{aligned} q(x) &= 1(\bmod 5)x^2 + 3(\bmod 5)x + 2(\bmod 5) \\ &= x^2 + 3x + 2 \end{aligned}$$

Now do long division. Here the operations multiplication and addition are multiplication modulo 5 and addition modulo 5.

$$\begin{array}{r|l} & x-2 \\ \hline & x^3+x^2+x+1 \\ & \underline{x^3+3x^2+2x} \\ x^2+3x+2 & \quad -2x^2-x+1 \\ & \quad \underline{-2x^2-x-4} \\ & \qquad\qquad 0 \end{array}$$

Then, quotient and remainder when $p(x)=x^3+x^2+x+1$ divided by $q(x)=x^2+3x+2$ in $\mathbb{Z}_5[x]$ are $\boxed{x-2}$ and $\boxed{0}$ respectively.

Comment