A Book of Abstract Algebra | (2nd Edition)

Chapter AA, Problem 13E

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Problem

Prove the following:

If $A \subseteq B$, then $A \cup B = B$. Conversely, if $A \cup B = B$, then $A \subseteq B$.

Step-by-step solution

Step 1 of 3

Objective:-

The objective is to prove that if $A \subseteq B, \text{then } A \cup B = B$. Conversely, if $A \cup B = B, \text{then } A \subseteq B$.

Comment

Step 2 of 3

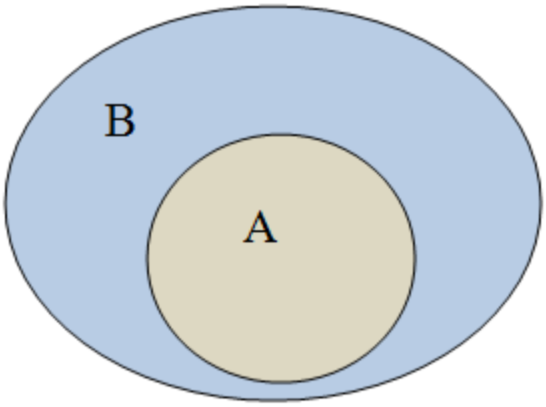
Proof:-

Let *A* and *B* are two sets. Let $x \in A \subseteq B$.

Subsets:-If sets *A* and *B* are such that every elements of *A* are also elements of *B*, then *A* is said to be subset of *B*.

$$A \subseteq B \Leftrightarrow \{x \in A \Rightarrow x \in B\}$$

So the set *B* contains the set *A* and set *A* completely lies within set *B*.



The union of two sets *A* and *B* is:-

$$A \cup B = \{x : x \in A \ \ or \ \ x \in B\}$$

Since set *B* contains the set *A*, the union of set *A* and *B* is same as the set *B*.

Hence,

$$If \ \ A \subseteq B, then \ \ A \cup B = B.$$

Proved

Comment

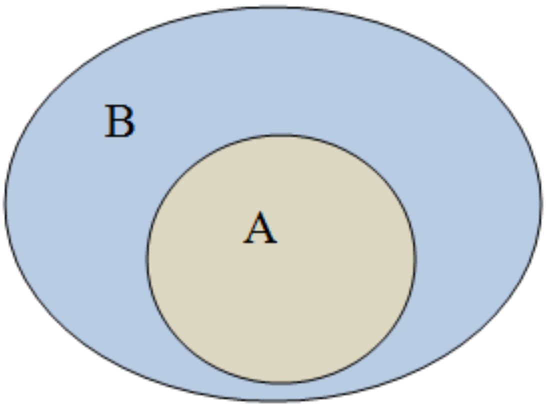
Step 3 of 3

Conversely:-

The union of two sets *A* and *B* is:-

$$A \cup B = \{x : x \in A \ \ or \ \ x \in B\}$$

If $A \cup B = B$, then set *B* completely contains the set *A*.



Subsets:-If sets *A* and *B* are such that every elements of *A* are also elements of *B*, then *A* is said to be subset of *B*.

$$A \subseteq B \Leftrightarrow \{x \in A \Rightarrow x \in B\}$$

According to this definition *A* is subset of *B*.

Hence,

$$A \cup B = B, then \ \ A \subseteq B.$$

Proved

Comment