A Book of Abstract Algebra | (2nd Edition)

Chapter 28, Problem 2EB

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Problem

Prove that the set of all $(x, y, z) \in \mathbb{R}^3$ which satisfy the pair of equations $ax + by + c = 0$, $dx + ey + f = 0$ is a subspace of $\mathbb{R}^3$.

Step-by-step solution

Step 1 of 2

(a_1, a_2, a_3) represents a vector space in 3 dimension or $\mathbb{R}^3$ as it satisfies all conditions for vector space.

For 3 dimension, any subspace must be a plane or line or a point passing through origin. The reason for it lies in the fact that any linear combination of 2 vectors lying on plane and line also lies on that vector space.

Given condition for subspace is

$$ax + by + cz = 0$$

$$dx + ey + fz = 0$$

This represents an equation of plane or line depending on *a, b, c, d, e, f* in $\mathbb{R}^3$ passing through origin. Hence it represents a vector space.

Comment

Step 2 of 2

Above mentioned method is useful in simple geometrical vector spaces but is not much useful in complex spaces. Here 2 step check is needed to determine if given subset is a subspace. First determine if any linear combination of vector in subspace is closed under given operation.

Second, determine if 0 satisfies given conditions.

STEP 1: For any 2 vectors (p,q,r) and (k,l,m),

$$ap+bq+cr=0 \qquad (1)$$
$$ak+bl+cm=0 \qquad (2)$$
$$dp+eq+fr=0 \qquad (3)$$
$$dk+el+fm=0 \qquad (4)$$

Combining above 4 equations, $s(1)+t(2)$ and $s(3)+t(4)$ gives

$$a(sp+tk)+b(sq+tl)+c(sr+tm)=0$$
$$d(sp+tk)+e(sq+tl)+f(sr+tm)=0$$

Thus linear combination of 2 vectors in subspace lies in subspace.

STEP 2: Check if $(0,0,0)$ satisfies given condition,

$$a\cdot 0+b\cdot 0+c\cdot 0=0$$
$$d\cdot 0+e\cdot 0+f\cdot 0=0$$

$$\boxed{\text{Hence given set represents a subspace}}$$

Comment