# A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 4EH

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## Problem

An integer $a$ is called a *quadratic residue* modulo $m$ if there is an integer $x$ such that $x^2 \equiv a \pmod{m}$. This is the same as saying that $\bar{a}$ is a square in $\mathbb{Z}_m$. If $a$ is not a quadratic residue modulo $m$, then $a$ is called a *quadratic nonresidue* modulo $m$. Quadratic residues are important for solving quadratic congruences, for studying sums of squares, etc. Here, we will examine quadratic residues modulo an arbitrary prime $p > 2$.

Let $h : \mathbb{Z}_p^* \to \mathbb{Z}_p^*$ be defined by $h(\bar{a}) = \bar{a}^2$.

Evaluate $\left(\frac{17}{23}\right); \left(\frac{3}{29}\right); \left(\frac{5}{11}\right); \left(\frac{8}{13}\right); \left(\frac{2}{23}\right)$.

## Step-by-step solution

**Step 1** of 7

Here, objective is to evaluate the given Legendre symbols.

Comment

**Step 2** of 7

Consider the congruence $x^2 = a \pmod{p}$ where $p$ is odd prime, is solvable, if and only if the Legendre symbol $\left(\frac{a}{P}\right) = 1$ .Where, $\left(\frac{a}{P}\right) = a^{(p-1)/2} \pmod{p}$

Comment

**Step 3** of 7

Consider the Legendre symbol $\frac{17}{23}$

$$\begin{aligned}\frac{17}{23} &= \frac{6}{17}\\ &= \frac{3}{17}\\ &= \frac{2}{3}\\ &= -\frac{1}{3}\\ &= -1\end{aligned}$$

Hence, $\frac{17}{23} = -1$

Comment

**Step 4** of 7

Consider the Legendre symbol $\frac{3}{29}$

$$\begin{aligned}\frac{3}{29} &= \frac{2}{3}\\ &= -\frac{1}{3}\\ &= -1\end{aligned}$$

Hence, $\frac{3}{29} = -1$

Comment

**Step 5** of 7

Consider the Legendre symbol $\frac{5}{11}$

$$\begin{aligned}\frac{5}{11} &= \frac{1}{5}\\ &= 1\\ &= 1\end{aligned}$$

Hence, $\frac{5}{11} = 1$

Comment

**Step 6** of 7

Consider the Legendre symbol $\frac{8}{13}$

$$\begin{aligned}\frac{8}{13} &= \frac{6}{17}\\ &= -\frac{4}{13}\\ &= \frac{2}{13}\\ &= -\frac{1}{13}\\ &= -1\end{aligned}$$

Hence, $\frac{8}{13} = -1$

Comment

**Step 7** of 7

Consider the Legendre symbol $\frac{2}{23}$

$$\begin{aligned}\frac{2}{23} &= \frac{1}{23}\\ &= 1\end{aligned}$$

Hence, $\frac{2}{23} = 1$

Comment