A Book of Abstract Algebra | (2nd Edition)

Chapter 17, Problem 1EJ

Bookmark

Show all steps: ON

Problem

Prove that each of the following is true in a nontrivial ring.

If $a \neq \pm 1$ and $a^2 = 1$, then $a + 1$ and $a - 1$ are divisors of zero.

Step-by-step solution

Step 1 of 3

Consider an arbitrary nontrivial ring *R*. Suppose that $a \in R$ along with the certain conditions:

$a \neq \pm 1$ and $a^2 = 1$.

Objective is to show that $a+1$ and $a-1$ both are divisors of zero.

A nonzero element $x \in R$ is said to be a divisors of zero if there exists a nonzero $y \in R$ such that the product

$xy = 0$,

where zero stands for the zero element of the ring.

Comment

Step 2 of 3

The condition $a^2 = 1$ implies that

$$a^2 - 1 = 0$$
$$(a-1)(a+1) = 0.$$

Note that neither $a-1=0$ nor $a+1=0$ because if so then $a = \pm 1$, a contradiction of the hypothesis that $a \neq \pm 1$.

So, from the condition $(a-1)(a+1) = 0$, it conclude that 0 is the product of 2 nonzero elements. This fulfils the definition of divisors of zero.

Comment

Step 3 of 3

Hence, both $a+1$ and $a-1$ are divisors of zero.

Comment