A Book of Abstract Algebra | (2nd Edition)

Problem

Let p be a prime number, and ω a primitive pth root of unity in the field F.

Explain why $x^p - a$ factors in $F[x]$ as $x^p - a = p(x)f(x)$, where both factors have degree ≤ 2.

Step-by-step solution

Step 1 of 3

Here, objective is to explain why $x^p - a$ factors in $F(x)$

Comment

Step 2 of 3

Consider the polynomial $x^p - a \in F(x)$

Where, P is a prime and $x^p - a$ is reducible in $F(x)$

And the factors $p(x)$ and $f(x)$ have degree ≤ 2.

Comment

Step 3 of 3

Let

$$x^p - a = p(x)f(x)$$

$$0 < \deg p(x), \deg f(x) \leq \deg x^p - a$$

We know that,

$$\deg x^p - a = \deg p(x) + \deg f(x)$$

Then, one of the factors are taking of the form $x-a; a \in F$

Thus $x^p - a$ has zero *a* in $F(x)$.

That is,

$p(x)$ and $f(x)$ both the factors are in $F(x)$

Hence,

$x^p - a$ Factors in $F(x)$, where both the factors $p(x)$ and $f(x)$ have degree ≤ 2.

Comment