

A Book of Abstract Algebra | (2nd Edition)

Chapter 31, Problem 5EA

Bookmark

Show all steps: ☒ ON

Problem

Find irreducible polynomials $a(x)$ over \mathbb{Q} , and $b(x)$ over $\mathbb{Q}(i)$, such that $\mathbb{Q}(i, \sqrt{3})$ is the root field of $a(x)$ over \mathbb{Q} , and is the root field of $b(x)$ over $\mathbb{Q}(i)$. Then do the same for $\mathbb{Q}(\sqrt{2}, \sqrt{3})$.

Step-by-step solution

Step 1 of 3

The objective is to find the irreducible polynomials $a(x)$ over \mathbb{Q} , and $b(x)$ over $\mathbb{Q}(i)$, such that $\mathbb{Q}(i, \sqrt{3})$ is the root field of $a(x)$ over \mathbb{Q} , and is the root field of $b(x)$ over $\mathbb{Q}(i)$. Then do the same for $\mathbb{Q}(\sqrt{2}, \sqrt{3})$.

[Comment](#)

Step 2 of 3

Take $x = \sqrt{3} + i$.

Then

$$x^2 = 2 + 2\sqrt{3}i$$

$$x^2 - 2 = 2\sqrt{3}i$$

$$(x^2 - 2)^2 = (2\sqrt{3}i)^2$$

$$x^4 - 4x^2 + 4 = -12$$

$$x^4 - 4x^2 + 16 = 0$$

The polynomial $a(x) = x^4 - 4x^2 + 16$ is irreducible over \mathbb{Q} and $\mathbb{Q}(i, \sqrt{3})$ is the root field of $a(x)$ over \mathbb{Q} .

The polynomial $b(x) = x^2 - 3$ is irreducible over $\mathbb{Q}(i)$ and $\mathbb{Q}(i, \sqrt{3})$ is the root field of $b(x)$ over $\mathbb{Q}(i)$.

[Comment](#)

Step 3 of 3

Take $x = \sqrt{2} + \sqrt{3}$.

Then

$$x^2 = 5 + 2\sqrt{6}$$

$$x^2 - 5 = 2\sqrt{6}$$

$$(x^2 - 5)^2 = (2\sqrt{6})^2$$

$$x^4 - 10x^2 + 25 = 24$$

$$x^4 - 10x^2 + 1 = 0$$

The polynomial $a(x) = x^4 - 10x^2 + 1$ is irreducible over \mathbb{Q} and $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ is the root field of $a(x)$ over \mathbb{Q} .

The polynomial $b(x) = x^2 - 3$ is irreducible over $\mathbb{Q}(i)$ and $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ is the root field of $b(x)$ over $\mathbb{Q}(\sqrt{2})$.

[Comment](#)

COMPANY

About Chegg
Chegg For Good
College Marketing
Corporate Development
Investor Relations
Jobs
Join Our Affiliate Program
Media Center
Site Map

LEGAL & POLICIES

Advertising Choices
Cookie Notice
General Policies
Intellectual Property Rights
Terms of Use
Global Privacy Policy
Honor Code
Honor Shield

CHEGG PRODUCTS AND SERVICES

Cheap Textbooks
Chegg Coupon
Chegg Play
Chegg Study Help
College Textbooks
eTextbooks
Flashcards
Learn
Chegg Math Solver
Mobile Apps
Sell Textbooks
Solutions Manual
Study 101
Textbook Rental
Used Textbooks
Digital Access Codes
Chegg Money

CHEGG NETWORK

EasyBib
Internships.com
Thinkful

CUSTOMER SERVICE

Customer Service
Give Us Feedback
Help with eTextbooks
Help to use EasyBib Plus
Manage Chegg Study Subscription
Return Your Books
Textbook Return Policy

