# A Book of Abstract Algebra | (2nd Edition)

Chapter AC, Problem 5E

Bookmark

Show all steps: ON

## Problem

*Use mathematical induction to prove the following:*

$$\frac{1}{2!}+\frac{2}{3!}+\cdots+\frac{n}{(n+1)!}=\frac{n!-1}{n!}$$

## Step-by-step solution

**Step 1** of 3

The objective is to prove that $\frac{1}{2!}+\frac{2}{3!}+\cdots+\frac{n}{(n+1)!}=\frac{(n+1)!-1}{(n+1)!}$ using mathematical induction.

Comment

**Step 2** of 3

Step 1: For $n=1$,

$$\frac{1}{2!}+\frac{2}{3!}+\cdots+\frac{n}{(n+1)!}=\frac{1}{2!}$$

$$=\frac{1}{2}$$

$$\frac{(n+1)!-1}{(n+1)!}=\frac{(1+1)!-1}{(1+1)!}$$

$$=\frac{1}{2}$$

The theorem holds for $n=1$.

Assume that the theorem is true for $n=k$.

Then, $\frac{1}{2!}+\frac{2}{3!}+\cdots+\frac{k}{(k+1)!}=\frac{(k+1)!-1}{(k+1)!}$ … (1)

Comment

**Step 3** of 3

For $n=k+1$,

$$\frac{1}{2!}+\frac{2}{3!}+\cdots+\frac{k+1}{((k+1)+1)!}=\left(\frac{1}{2!}+\frac{2}{3!}+\cdots+\frac{k}{(k+1)!}\right)+\frac{k+1}{(k+2)!}$$

$$=\frac{(k+1)!-1}{(k+1)!}+\frac{k+1}{(k+2)!} \qquad [\text{using } (1)]$$

$$=\frac{(k+2)((k+1)!-1)+k+1}{(k+2)!}$$

$$=\frac{(k+2)!-1}{(k+2)!}$$

The theorem is true for $n=k+1$.

Therefore, it is proved that $\frac{1}{2!}+\frac{2}{3!}+\cdots+\frac{n}{(n+1)!}=\frac{(n+1)!-1}{(n+1)!}$ using mathematical induction.

Comment