A Book of Abstract Algebra | (2nd Edition)

Chapter 23, Problem 6EA

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Problem

Solve the following Diophantine equations. (If there is no solution, write “none.”)

(a) $14x + 15y = 11$

(b) $4x + 5y = 1$

(c) $21x + 10y = 9$

(d) $30x^2 + 24y = 18$

Step-by-step solution

Step 1 of 6

Here, objective is to solve the given Diophantine equations.

Comment

Step 2 of 6

Diophantine equation is in one or more unknowns, with the integer coefficients.

The Diophantine equation is of the form $ax + by = c; a, b, c$ are integers..

Now Euclidian algorithm is used to solve the Diophantine equation.

Comment

(a)

Consider the congruence $14x + 15y = 11$

$\gcd(14,15) = 1$

$\gcd(14,15) = 1$ is divisible by 11. So there is an integer pair solutions.

Apply Euclidian algorithm:

$$14x + 15y = 11 \ldots\ldots\ldots\ldots\ldots\ldots(1)$$
$$15 = 1 \times 14 + 1$$
$$14 = 14 \times 1 + 0$$

By applying extended Euclidian algorithm,

$$1 = (1 \times 15) + (-1 \times 14)$$
$$11 = (11 \times 15) + (-11 \times 14).$$
$$(14 \times -11) + (15 \times 11) = 11 \ldots\ldots\ldots\ldots\ldots(2)$$

By comparing equations (1) and (2)

$$x = -11, y = 11$$

Hence, the solution of $14x + 15y = 11$ Diophantine equations is $x = -11, y = 11$

Comment

(b)

Consider the congruence $4x+5y=1$

$\gcd(4,5)=1$

$\gcd(4,5)=1$ is divisible by 1. So there is an integer pair solutions.

Apply Euclidian algorithm:

$$4x+5y=1.................(1)$$
$$5=1\times4+1$$
$$4=4\times1+0$$

By applying extended Euclidian algorithm,

$$1=(1\times5)+(-1\times4)$$
$$1=(1\times5)+(-1\times14).$$
$$(4\times-1)+(5\times1)=1................(2)$$

By comparing equations (1) and (2)

$x=-1, y=1$

Hence, the solution of $4x+5y=1$ Diophantine equations is $x=-1, y=1$

Comment

Step 5 of 6

(c)

Consider the congruence $21x+10y=9$

$\gcd(21,10)=1$

$\gcd(21,10)=1$ is divisible by 9.So there is an integer pair solutions.

Apply Euclidian algorithm:

$$21x+10y=9................(1)$$
$$10=0\times21+10$$
$$21=2\times10+1$$
$$10=10\times1+0$$

By applying extended Euclidian algorithm,

$$1=(1\times21)+(-2\times0)$$
$$1=(-2\times10)+(1\times21).$$
$$9=(-18\times10)+(9\times21).$$
$$(21\times9)+(10\times-18)=9................(2)$$

By comparing equations (1) and (2)

$$x=9, y=-18$$

Hence, the solution of $21x+10y=9$ Diophantine equations is $x=9, y=-18$

Comment

Step 6 of 6

(d)

Consider the congruence $30x^2 + 24y = 18$

$30z + 24y = 18;\ \text{where}\ z = x^2$

$\gcd(30, 24) = 6$

$\gcd(30, 24) = 6$ is divisible by 18.

Apply Euclidian algorithm:

$$30z + 24y = 18 \ldots\ldots\ldots\ldots\ldots\ldots(1)$$
$$24 = 0 \times 30 + 24$$
$$30 = 1 \times 24 + 6$$
$$24 = 4 \times 6 + 0$$

By applying extended Euclidian algorithm,

$$6 = (1 \times 30) + (-1 \times 24)$$
$$6 = (-1 \times 24) + (1 \times 30).$$
$$18 = (-3 \times 24) + (3 \times 30).$$
$$(30 \times 3) + (24 \times -3) = 18 \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

By comparing equations (1) and (2)

$z = 3, y = -3, x^2 \neq 3.$ x must be an integer.

Hence, the Diophantine equations have no solution.

Comment