

A Book of Abstract Algebra | (2nd Edition)



Chapter 23, Problem 1EA

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Problem

For each of the following congruences, find m such that the congruence has a unique solution modulo m .

If there is no solution, write “none.”

(a) $60x \equiv 12 \pmod{24}$

(b) $42x \equiv 24 \pmod{30}$

(c) $49x \equiv 30 \pmod{25}$

(d) $39x \equiv 14 \pmod{52}$

(e) $147x \equiv 47 \pmod{98}$

(f) $39x \equiv 26 \pmod{52}$

Step-by-step solution

Step 1 of 8

Here, objective is to find m , where the given congruence has a unique solution modulo m .

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Step 2 of 8

Consider the congruent equation $ax = b(\text{mod } n)$, has solutions if and only if $\gcd(a, n)$ is divisible by b . If $\gcd(a, n) = 1$, then the congruence has unique solution

[Comment](#)

Step 3 of 8

(a)

Consider the congruence $60x = 12(\text{mod } 24)$

$$a = 60, b = 12, n = 24$$

$$\gcd(60, 24) = 6$$

$\gcd(a, n)$ is divisible by b . Since, the congruence has solutions.

$$60x = 12(\text{mod } 24)$$

$$60x = 12 + 24q$$

$$5x = 1 + 2q$$

$$5x = 1(\text{mod } 2)$$

$$a = 5, b = 1, n = 2$$

$$\gcd(5, 2) = 1$$

Hence, the value of m for which the congruence equation has a unique solution $= 2$

[Comment](#)

Step 4 of 8

(b)

Consider the congruence $42x = 24(\text{mod } 30)$

$$a = 42, b = 24, n = 30$$

$$\gcd(42, 30) = 6$$

$\gcd(a, n)$ is divisible by b . Since, the congruence has solutions.

$$42x = 24(\text{mod } 30)$$

$$42x = 24 + 30q$$

$$7x = 4 + 5q$$

$$7x = 4(\text{mod } 5)$$

$$a = 7, b = 4, m = 5$$

$$\gcd(7, 5) = 1$$

Hence, the value of m for which the congruence equation has a unique solution $= 5$

[Comment](#)

Step 5 of 8

(c)

Consider the congruence $49x = 30 \pmod{25}$

$$a = 49, b = 30, n = 25$$

$$\gcd(49, 25) = 1$$

Hence, the value of m for which the congruence equation has a unique solution $= 25$

[Comment](#)

Step 6 of 8

(d)

Consider the congruence $39x = 14 \pmod{52}$

$$a = 39, b = 14, n = 52$$

$$\gcd(39, 52) = 13$$

$\gcd(a, n)$ is not divisible by b

Hence, the congruence has no solutions.

[Comment](#)

Step 7 of 8

(e)

Consider the congruence $147x = 47 \pmod{98}$

$$a = 147, b = 47, n = 98$$

$$\gcd(147, 98) = 49$$

$\gcd(a, n)$ is not divisible by b

Hence, the congruence has no solutions.

[Comments \(1\)](#)

Step 8 of 8

(f)

Consider the congruence $39x = 26 \pmod{52}$

$$a = 39, b = 26, n = 52$$

$$\gcd(39, 52) = 13$$

$\gcd(a, n)$ is divisible by b . Since, the congruence has solutions.

$$39x = 26 \pmod{52}$$

$$3x = 2 + 4q$$

$$3x = 2 \pmod{4}$$

$$a = 3, b = 2, m = 4$$

$$\gcd(3, 4) = 1$$

Hence, the value of m for which the congruence equation has a unique solution $= 4$

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