A Book of Abstract Algebra | (2nd Edition)

Chapter 32, Problem 2ED

Bookmark

Show all steps: ON

Problem

If $\alpha = \sqrt[4]{2}$ is a real fourth root of 2, then the four fourth roots of 2 are $\pm\alpha$ and $\pm i\alpha$. Explain parts 1–6, briefly but carefully:

$[\mathbb{Q}(\alpha) : \mathbb{Q}] = 4.$

Step-by-step solution

Step 1 of 2

The objective is to show that $\left[\mathbb{Q}\left(\sqrt[4]{2}\right):\mathbb{Q}\right]=4$.

Comment

Step 2 of 2

Clearly , $\sqrt[4]{2}$ is the root of x^4-2.

Also , x^4-2 is irreducible polynomial of lowest degree 4 over $\mathbb{Q}$ by Eisenstein $(p=2)$.

Therefore , $\left[\mathbb{Q}\left(\sqrt[4]{2}\right):\mathbb{Q}\right]=\deg\left(x^4-2\right)=4$.

Comment