

A Book of Abstract Algebra | (2nd Edition)

Chapter 30, Problem 1EA

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Problem

If O and I are any two points in the plane, consider a coordinate system such that



the interval OI coincides with the unit interval on the x axis. Let \mathbb{D} be the set of real numbers such that $a \in \mathbb{D}$ iff the point $(a, 0)$ is constructible from $\{O, I\}$. Prove the following:

If $a, b \in \mathbb{D}$, then $a + b \in \mathbb{D}$ and $a - b \in \mathbb{D}$.

Step-by-step solution

Step 1 of 4

Here, objective is to prove that, $a + b \in \mathbb{D}$ and $a - b \in \mathbb{D}$.

Consider $a \in \mathbb{D}$ if and only if the point $(a, 0)$ is constructible from $\{O, I\}$

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Step 2 of 4

Constructible point:

This point is either the end point of given unit segment or it is the intersection of two lines determined by previous constructible points.

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Step 3 of 4

D is a set of real numbers. Therefore, we can add, subtract, multiply any two points of them.

Let $a, b \in D$, then the points $(a, 0)$ and $(b, 0)$ are constructible from $\{O, I\}$

Consider the point $(a, 0)$ is constructible from $\{O, I\}$

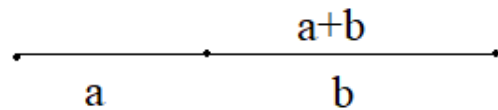


fig: addition

Draw a line through the two points $\{O, I\}$ with a distance $|a|$, then a circle of length $|b|$ around I

This circle intersects the line at a distance of $|a+b|$ from O

Then, the point $(a+b, 0)$ is constructible from $\{O, I\}$

Hence, $a+b \in D$

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Step 4 of 4

Consider the points $(a, 0), (b, 0)$ are constructible from $\{O, I\}$

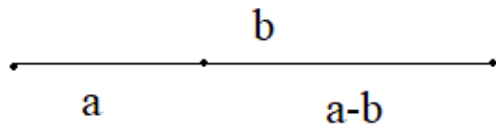


fig 2:subtraction

Draw a line through the two points $\{O, I\}$ with a distance $|a|$, then a circle of length $|a - b|$ around I . This circle intersects the line at a distance of $|b|$ from O

Then, the point $(a - b, 0)$ is constructible from $\{O, I\}$

Hence, $a - b \in D$

Therefore, if $a, b \in D$, then $a + b \in D$ and

Hence, proved

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