A Book of Abstract Algebra (2nd Edition)

Chapter 30, Problem 6EF

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Problem

By de Moivre's theorem,

$$\omega = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$$

is a complex seventh root of unity. Since

$$x^7 - 1 = (x - 1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$$

 ω is a root of $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$.

Prove that the regular polygon of seven sides is not constructible.

Step-by-step solution

Step 1 of 4

Here, objective is to prove that regular polygon of seven sides is not constructible.

Comment

Step 2 of 4

Regular *n*-gon has *n* equal length of sides and all angles are equal.

Regular *n* sided polygon is constructible if and only if the angle $\frac{2\pi}{n}$ is constructible

An angle $\frac{2\pi}{N}$ is constructible if and only if N is either a power of two or power of two and a set of distinct Fermat points

Comment

Step 3 of 4

Consider regular seven sided Polygon or 7 – gon

Number of sides n = 7

Comment

Step 4 of 4

To verify $\frac{2\pi}{7}$ is constructible or not:

$$\frac{2\pi}{N} = \frac{2\pi}{7}$$

$$N = 7$$

Here,

7 cannot be written in the form of $2^{2^n} + 1$. So 7 is not a Fermat prime

Therefore, $\frac{2\pi}{7}$ is not a constructible angle.

Regular 7 - gon is constructible if and only if the angle $\frac{2\pi}{7}$ is constructible

| Hence, |
|------------------------------------------------------|
| Regular polygon of seven sides is not constructible. |
| |
| Comment |
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