

A Book of Abstract Algebra | (2nd Edition)

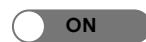


Chapter AA, Problem 22E



Bookmark

Show all steps:



ON

Problem

Prove the following identities involving cartesian products:

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

Step-by-step solution

Step 1 of 2

Objective:-

The objective is to prove $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

[Comment](#)

Step 2 of 2

Proof:-

Let A and B are two sets.

The Cartesian product of two sets A and B is:-

$$A \times B = \{(x, y) : x \in A, y \in B\}$$

The intersection of two sets A and B is:-

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Let $(x, y) \in A \times (B \cup C)$.

$$(x, y) \in A \times (B \cup C)$$

$$\Rightarrow x \in A, y \in B \cup C$$

$$\Rightarrow x \in A, y \in B \text{ or } y \in C$$

$$\Rightarrow (x \in A, y \in B) \text{ or } (x \in A, y \in C)$$

$$\Rightarrow (x, y) \in (A \times B) \text{ or } (x, y) \in (A \times C)$$

$$\Rightarrow (x, y) \in (A \times B) \cup (A \times C)$$

So,

$$A \times (B \cup C) \subseteq (A \times B) \cup (A \times C) \quad \text{.....(1)}$$

Let $(x, y) \in (A \times B) \cup (A \times C)$.

$$(x, y) \in (A \times B) \cup (A \times C)$$

$$\Rightarrow (x, y) \in (A \times B) \cup (A \times C)$$

$$\Rightarrow (x, y) \in (A \times B) \text{ or } (x, y) \in (A \times C)$$

$$\Rightarrow (x \in A, y \in B) \text{ or } (x \in A, y \in C)$$

$$\Rightarrow x \in A, y \in B \text{ or } y \in C$$

$$\Rightarrow x \in A, y \in B \cup C$$

$$\Rightarrow (x, y) \in A \times (B \cup C)$$

So,

$$(A \times B) \cup (A \times C) \subseteq A \times (B \cup C) \quad \text{.....(2)}$$

Let us consider the equation (1) and (2).

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

[Comment](#)