

# A Book of Abstract Algebra | (2nd Edition)



Chapter 30, Problem 1EF



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## Problem

By de Moivre's theorem,

$$\omega = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$$

is a complex seventh root of unity. Since

$$x^7 - 1 = (x - 1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$$

$\omega$  is a root of  $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ .

Prove that  $\omega^3 + \omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2} + \omega^{-3} = 0$ .

## Step-by-step solution

### Step 1 of 3

Here, objective is to prove that  $\omega^3 + \omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2} + \omega^{-3} = 0$ .

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### Step 2 of 3

De Moivre's theorem:

$\omega = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$  is a complex seventh root of unity.

Since  $x^7 - 1 = (x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$

$\omega$  is a root of  $P(x) = (x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$

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### Step 3 of 3

Consider  $\omega$  is a root of  $P(x) = (x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$

Then,

$$P(\omega) = 0$$

$$\omega^6 + \omega^5 + \omega^4 + \omega^3 + \omega^2 + \omega + 1 = 0$$

$$\omega^3 \cdot \omega^3 + \omega^3 \cdot \omega^2 + \omega^3 \cdot \omega + \omega^3 \cdot 1 + \omega^3 \cdot \omega^{-1} + \omega^3 \cdot \omega^{-2} + \omega^3 \cdot \omega^{-3} = 0$$

$$\omega^3 (\omega^3 + \omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2} + \omega^{-3}) = 0$$

$$\omega^3 = 0 \text{ or}$$

$$(\omega^3 + \omega^2 + \omega + 1 + \omega^{-1} + \omega^{-2} + \omega^{-3}) = 0$$

Hence, proved

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