A Book of Abstract Algebra (2nd Edition)

Problem	
Let $p(x)$ be an irreducible polynomial of degree n over F . Let c denote a root of $p(x)$ in some extension of F (as in the basic theorem on field extensions). Prove: Every element in $F(c)$ can be written as $r(c)$, for some $r(x)$ of degree $< n$ in $F[x]$. [HINT: Given any element $t(c) \in F(c)$, use the division algorithm to divide $t(x)$ by $p(x)$.]	>
Step-by-step solution	
Step 1 of 3 A	
Let $p(x)$ be an irreducible polynomial of degree n over F . Let c denote a root of $p(x)$ in some extension of F .	
Prove: Every element in $F(c)$ can be written as $r(c)$, for some $r(x)$ of degree $< n$ in $F[x]$.	
Comment	
Step 2 of 3 ▲	
Here we use "Division Algorithm".	
Let $t(c) \in F(c)$ be any element. Consider $t(x)$ be any polynomial over $F(c)$.	
Also, it is given that $p(x)$ is irreducible over F and c denote a root of $p(x)$ in some extension of F .	
So, by division algorithm, there exists two polynomials $q(x)$ and $r(x)$ such that	
$t(x) = q(x)p(x) + r(x)$, where $\deg r(x) < n$.	
Now, put $x = c$ in above expression and use the fact that c is root of $p(x)$.	
Hence, $t(c) = q(c)p(c)+r(c)=r(c)$ [: $p(c)=0$]	
That is, $t(c) = r(c)$.	
Hence, the result.	
Comment	
Step 3 of 3 A	
Comment	
	extension of F (as in the basic theorem on field extensions). Prove: Every element in $F(c)$ can be written as $r(c)$, for some $r(x)$ of degree $< n$ in $F(x)$. [HINT: Given any element $f(c) \in F(c)$, use the division algorithm to divide $f(x)$ by $p(x)$.] Step-by-step solution Step 1 of 3 ^* Let $p(x)$ be an irreducible polynomial of degree n over F . Let c denote a root of $p(x)$ in some extension of F . Prove: Every element in $F(c)$ can be written as $F(c)$, for some $F(c)$ of degree $< n$ in $F(x)$. Comment Step 2 of 3 ^* Here we use "Division Algorithm". Let $F(c) \in F(c)$ be any element. Consider $F(c)$ be any polynomial over $F(c)$. Also, it is given that $F(c)$ is irreducible over $F(c)$ and $F(c)$ and $F(c)$ in some extension of $F(c)$. So, by division algorithm, there exists two polynomials $F(c)$ and $F(c)$ such that $F(c) = f(c) = f(c)$ where $F(c) = f(c) = f(c)$ is root of $F(c) = f(c)$. Hence, $F(c) = f(c) = f(c) = f(c)$. Hence, $F(c) = f(c) = f(c)$. Hence, the result.

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