

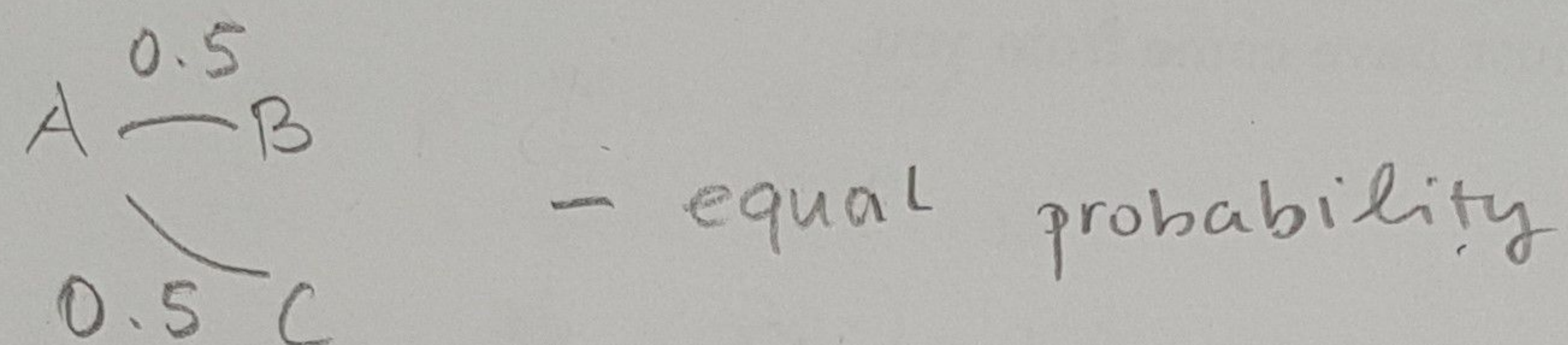
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Frequency-based approach

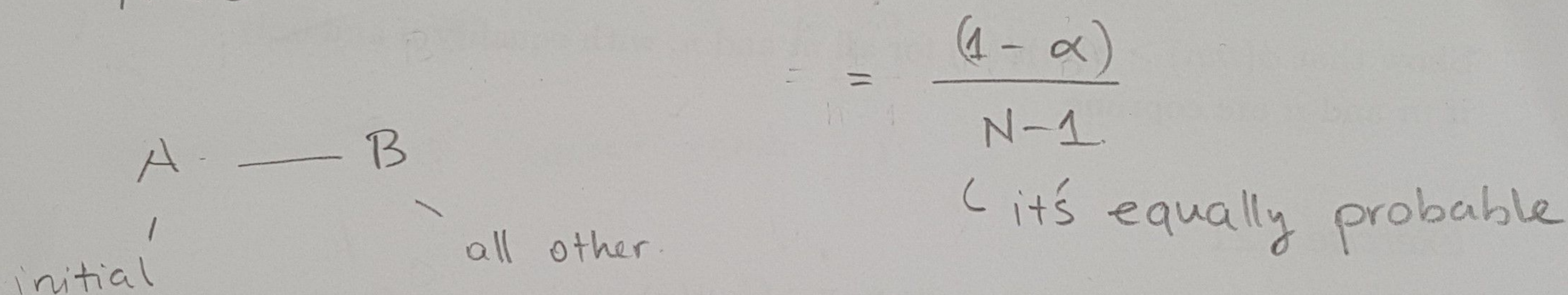
Fictitious play.

$$N = 3$$

Prob of Mutation = 0.3



$$N = 5$$



$$= \frac{1}{N} + \left(1 - \frac{1}{N}\right) \left(1 - \frac{\alpha N}{(N-1)}\right)^n$$

Spectral Theory

Operator: takes a function and gives another vector. Ex, derivative, inverse, fourier transform, matrix

two element set: a straight line $(x_1, x_2; x_1 + x_2 = 1)$

three " set: triangle. $(x_1, x_2, x_3; x_1 + x_2 + x_3 = 1)$

$$x_1, x_2, x_3 \geq 0$$

For n probability, we have $(n-1)$ simplex

$$x_i \geq 0, \quad x_1 + \dots + x_i + \dots + x_n = 1$$

- I take a point in simplex, apply P and get another point.
 - P should guarantee that point should be in simplex & it's coming from spectral theory.
- which should be in simplex.

$$P = \begin{bmatrix} 0.7 & 0.3 \\ 0.6 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} 0.7 - \lambda & 0.3 \\ 0.6 & 0.4 - \lambda \end{bmatrix}$$

- if you take any transition matrix, largest eigen value will be 1.

$$(0.7 - \lambda)(0.4 - \lambda) - 0.18 = 0$$

$$0.28 - 1.1\lambda + \lambda^2 - 0.18 = 0$$

$$0.1 - 1.1\lambda + \lambda^2 = 0$$

$$\begin{array}{rcl} 0.1 & -\lambda & \lambda = 0.1 \\ 1 & -\lambda & \lambda = 1 \end{array}$$

- The norm of the vector cannot increase. (if it is a probability vector) and it cannot decrease as well (should be 1).

Since

- Sum of the rows is 1, I'm operating vector to its convex comb.
- Summation of convex comb. is 1.

- Point \xrightarrow{P} Point \xrightarrow{P} Point $\dots \rightarrow$ Point

n-step Probability

sequence of points in Simplex.

P has at most eigenvalue 1 (can't be increased)

$\det(P) < 1$, meaning you decrease the distance with the operator

So, it's not possible to move out of the simplex.

Moreover, if you keep applying P , you reached a point. (if exists)

Unique

invariant distribution.

$$\pi_i * P = \pi_i \quad / \quad (1 \ 0) \xrightarrow{P} \pi_i$$

Last part

$$Pv = \lambda v \quad \text{mod}(\lambda) < 1$$

$$P^2 v = \lambda^2 v \quad v = a_1 v_1 + \dots + a_k v_k$$

$$Pv = a_1^2 v_1 + \dots + a_k^2 v_k$$

$$P^t v = a_1^t v_1 + \dots + a_k^t v_k$$

P^t can be done t-th power of the eigenvalue of P as above

- 2nd eigenvalue very much less than 1, it will depend on eigengap.

- Eigen structure, gives how fast it converges (if it converges)

- You can reach π_i as fast as eigen-gap.

- To compute eigen values: all of them is different

repeated/eigen values are non-real/imaginary. (comes in conjugate pairs)

n - #repetition times