IE452/IE552: Algebraic and Geometric Methods in Data Analysis

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Due Date: March,20 2020

1. (3+4+4 pts) Find the SVD of the matrix

   (a) U= $\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$ (b) V= $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 1 & 3 & 1 \\ 1 & 3 & 2 \end{bmatrix}$ (c) T= $\begin{bmatrix} 2 & 0 & 8 & 6 \\ 1 & 6 & 0 & 1 \\ 5 & 0 & 7 & 4 \end{bmatrix}$

2. (14 pts) Using Cramer-Chernoff bounds, solve the following;

   (a) (8 pts) Consider a random variable X$\sim N(0,\sigma^2)$, obtain an upper bound for P[X$\geq$t].

   (b) (6 pts) Consider X is a geometric random variable with probability of success p. Given $0 < \delta < 1$, find $P\big[|X - E[X]| \leq \delta E[X]\big]$.

3. (10 pts) Let R be a random matrix of order k×d such that $R_{ij} \overset{\text{i.i.d}}{\sim} N(0,1)$ and u be any fixed vector $\in R^d$. Define $v = \frac{1}{\sqrt{k}} R \cdot u$. Given $P[|\,\|v\|_2^2 - \|u\|_2^2\,| \geq \varepsilon\,\|u\|_2^2] \leq f(n^2)$. Compute $f(\cdot)$.

4. (15 pts) Raquel, a graduate student wants to increase her happiness and she either meets with her friends or studies on her research. If she meets with her friends, she increases her happiness levels 2 points with probability $\frac{3}{4}$ and decreases her happiness levels 1 point with probability $\frac{1}{4}$. If she chooses to study on her research, she wins 1002 happiness points with probability $\frac{2}{3}$ and lose 2001 happiness points with probability $\frac{1}{3}$. Considering her situation, what are the expected happiness points and variances in both options? Using Chebyshev's inequality, compute an upper bound on the probability that after 1000 rounds of

each option, the winnings deviate by more than ± 100 from the expected value. Write a simulation in software (MATLAB or R) to estimate the probability that make a loss after 1000 rounds of each option. Please display your code,the results and submit it with the rest of the homework.