



$$1) f(x, y) = x^2 + y^2 \quad \frac{\partial f}{\partial x} = 2x \quad \frac{\partial f}{\partial y} = 2y$$

$$2) f(x, y) = x^2 - y^2 \quad \frac{\partial f}{\partial x} = 2x \quad \frac{\partial f}{\partial y} = -2y$$

$$3) f(x, y) = e^{-(x^2 + y^2)} \quad \frac{\partial f}{\partial x} = e^{-(x^2 + y^2)} (-2x) = \frac{-2x}{e^{(x^2 + y^2)}}$$

$$\frac{\partial f}{\partial y} = e^{-(x^2 + y^2)} (-2y) = \frac{-2y}{e^{(x^2 + y^2)}}$$

$$4) f(x, y) = \sin(x) + \cos(y)$$

$$\frac{\partial f}{\partial x} = \cos(x)$$

$$\frac{\partial f}{\partial y} = -\sin(y)$$

$$5) f(x, y) = \ln(x^2 + y^2 + 1)$$

$$\frac{\partial f}{\partial x} = \frac{2x}{x^2 + y^2 + 1}$$

$$\frac{\partial f}{\partial y} = \frac{2y}{x^2 + y^2 + 1}$$

$$1) \nabla f \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$$

$$2) \nabla f \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x \\ -2y \end{bmatrix}$$

$$3) \nabla f \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{-2x}{e^{x^2+y^2}} \\ \frac{-2y}{e^{x^2+y^2}} \end{bmatrix}$$

$$4) \nabla f = \begin{bmatrix} \cos(x) \\ -\sin(y) \end{bmatrix}$$

$$5) \nabla f = \begin{bmatrix} \frac{2x}{x^2+y^2+1} \\ \frac{2y}{x^2+y^2+1} \end{bmatrix}$$

$$1) \nabla f(1,1) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \nabla f(0,0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$2) \nabla f(1,1) = \begin{bmatrix} 2 \\ -2 \end{bmatrix}, \nabla f(0,0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \nabla f(-1,1) = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$$

$$3) \nabla f(1,1) = \begin{bmatrix} \frac{-2}{7.3} \\ \frac{-2}{7.3} \end{bmatrix} \quad \nabla f(0,0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\nabla f(-1,1) = \begin{bmatrix} \frac{2}{7.3} \\ \frac{-2}{7.3} \end{bmatrix}$$

$$4) \nabla f(1,1) = \begin{bmatrix} \cos(1) \\ -\sin(1) \end{bmatrix}$$

$$\nabla f(0,0) = \begin{bmatrix} \cos(0) \\ -\sin(0) \end{bmatrix}$$

$$\nabla f(-1,0) = \begin{bmatrix} \cos(-1) \\ -\sin(-1) \end{bmatrix}$$

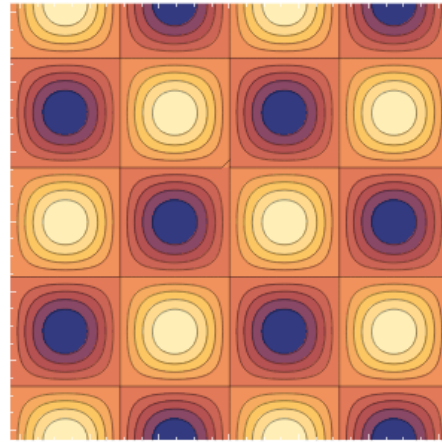
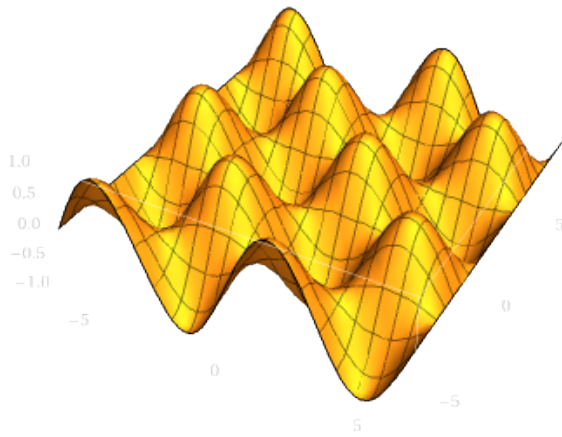
$$5) \nabla f(1,1) = \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \end{bmatrix} \quad \nabla f(0,0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\nabla f(-1,1) = \begin{bmatrix} \frac{-2}{3} \\ \frac{2}{3} \end{bmatrix}$$

$$f(x,y) = \sin(x) + \cos(y) \quad \nabla f = \begin{bmatrix} \cos(x) \\ -\sin(y) \end{bmatrix}$$

$$f\left(\frac{\pi}{2}, 0\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad f(0, \pi) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Visualization of $f(x, y) = \sin(x) \cdot \cos(y)$



$$\text{grad}(\sin(x) \cos(y)) = (\cos(x) \cos(y), -\sin(x) \sin(y))$$

