CS536 Science of Programming - Assignment 4

Batkhishig Dulamsurankhor - A20543498

March 21, 2024

Problem 1

Solution:

$p \equiv x > y \to \forall z.y \le z < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,z))$. Finish the syntactic substitutions.

a) $p[y+z/x] \equiv (x > y \to \forall z.y \le z < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,z)))[y+z/x]$
$\equiv (y+z) > y \to (\forall z.y \le z < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,z)))[y+z/x]$
$\equiv (y+z) > y \to \forall v.(y \le z < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,z)))[v/z][y+z/x]$
$\equiv (y+z) > y \to \forall v.(y \le v < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,v)))[y+z/x]$
$\equiv (y+z) > y \to \forall v.y \le v < y+z \to \exists w.(w > 0 \wedge (w * y < 0 \to f(w,v)))[y+z/x]$
$\equiv (y+z) > y \to \forall v.y \le v < y+z \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,v))$

b) $p[x+w/y] \equiv (x > y \to \forall z.y \le z < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,z)))[x+w/y]$
$\equiv x > (x+w) \to \forall z.(y \le z < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,z)))[x+w/y]$
$\equiv x > (x+w) \to \forall z.x+w \le z < x \to (\exists w.w > 0 \wedge (w * y < 0 \to f(w,z)))[x+w/y]$
$\equiv x > (x+w) \to \forall z.x+w \le z < x \to (\exists v.(w > 0 \wedge (w * y < 0 \to f(w,z)))[v/w])[x+w/y]$
$\equiv x > (x+w) \to \forall z.x+w \le z < x \to \exists v.(v > 0 \wedge (v * y < 0 \to f(v,z)))[x+w/y]$
$\equiv x > (x+w) \to \forall z.x+w \le z < x \to \exists v.v > 0 \wedge (v * (x+w) < 0 \to f(v,z))$

c) $p[w+z/y] \equiv (x > y \to \forall z.y \le z < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,z)))[w+z/y]$
$\equiv x > (w+z) \to (\forall z.y \le z < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,z)))[w+z/y]$
$\equiv x > (w+z) \to (\forall v.(y \le z < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,z)))[v/z])[w+z/y]$
$\equiv x > (w+z) \to \forall v.(y \le v < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,v)))[w+z/y]$
$\equiv x > (w+z) \to \forall v.y+z \le v < x \to (\exists w.w > 0 \wedge (w * y < 0 \to f(w,v)))[w+z/y]$
$\equiv x > (w+z) \to \forall v.y+z \le v < x \to (\exists u.(w > 0 \wedge (w * y < 0 \to f(w,v)))[u/w])[w+z/y]$
$\equiv x > (w+z) \to \forall v.y+z \le v < x \to \exists u.(u > 0 \wedge (u * y < 0 \to f(u,v)))[w+z/y]$
$\equiv x > (w+z) \to \forall v.y+z \le v < x \to \exists u.u > 0 \wedge (u * (w+z) < 0 \to f(u,v))$

d) $p[x+y/z] \equiv (x > y \to \forall z.y \le z < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,z)))[x+y/z]$
$\equiv x > y \to (\forall z.y \le z < x \to \exists w.w > 0 \wedge (w * y < 0 \to f(w,z)))[x+y/z]$

Problem 2

Solution:

a) True.

Problem 3

Solution:

a) True.

Problem 4

Calculate $sp(i \leq j \wedge j < n+1, i \coloneqq f(i+j); j \coloneqq g(i-j))$.

Solution:
$sp(i \leq j \wedge j < n+1, i \coloneqq f(i+j); j \coloneqq g(i-j))$
$\equiv sp(sp(i \leq j \wedge j < n+1, i \coloneqq f(i+j)), j \coloneqq g(i-j))$
$\equiv sp(i_0 \leq j \wedge j < n+1 \wedge i = (f(i+j))[i_0, i], j \coloneqq g(i-j))$
$\equiv sp(i_0 \leq j \wedge j < n+1 \wedge i = f(i_0+j), j \coloneqq g(i-j))$
$\equiv i_0 \leq j_0 \wedge j_0 < n+1 \wedge i = f(i_0+j) \wedge j = g(i-j_0)$

Problem 5

Calculate $sp(T, y \coloneqq x;\ \textbf{if } x < 0 \textbf{ then } y \coloneqq -y \textbf{ fi})$.

Solution:
Let $p \equiv T$, $S \equiv y \coloneqq x;\ \textbf{if } x < 0 \textbf{ then } y \coloneqq -y \textbf{ fi}$

- $lhs(S) = \{y\}$
- $rhs(S) = \{x, y\}$
- $free(p) = \varnothing$
- $aged(p, S) = \{y\}$

$sp(T, y \coloneqq x;\ \textbf{if } x < 0 \textbf{ then } y \coloneqq -y \textbf{ fi})$
$\equiv sp(T \wedge y = y_0 \wedge x < 0, y \coloneqq -y) \vee sp(T \wedge y = y_0 \wedge x \geq 0, skip)$
$\equiv (T \wedge y_0 = y_0 \wedge x < 0 \wedge y = -y_0) \vee (T \wedge y = y_0 \wedge x \geq 0)$
$\equiv (x < 0 \wedge y = -y_0) \vee (y = y_0 \wedge x \geq 0)$

Problem 6

Calculate $sp(x = y,\ \textbf{if } x \geq 0 \rightarrow x \coloneqq y+1; z \coloneqq x \ \square\ x \leq 0 \rightarrow y \coloneqq x-1; z \coloneqq y \textbf{ fi})$.

Let $p \equiv x = y$, $S \equiv\ \textbf{if } x \geq 0 \rightarrow x \coloneqq y+1; z \coloneqq x \ \square\ x \leq 0 \rightarrow y \coloneqq x-1; z \coloneqq y \textbf{ fi}$

- $lhs(S) = \{x, y, z\}$
- $rhs(S) = \{x, y\}$
- $rhs(p) \vee free(p, S) = \{x, y\}$
- $aged(p, S) = \{x, y\}$

$sp(x = y,\ \textbf{if } x \geq 0 \rightarrow x \coloneqq y+1; z \coloneqq x \ \square\ x \leq 0 \rightarrow y \coloneqq x-1; z \coloneqq y \textbf{ fi})$

- $sp(x = y \wedge x = x_0 \wedge y = y_0 \wedge x \geq 0, x \coloneqq y+1; z \coloneqq x)$
  $\equiv sp(sp(x = y \wedge x = x_0 \wedge y = y_0 \wedge x \geq 0, x \coloneqq y+1), z \coloneqq x)$
  $\equiv sp(x_0 = y \wedge x_0 = x_0 \wedge y = y_0 \wedge x_0 \geq 0 \wedge x = y+1, z \coloneqq x)$
  $\equiv x_0 = y \wedge x_0 = x_0 \wedge y = y_0 \wedge x_0 \geq 0 \wedge x = y+1 \wedge z = x$
- $sp(x = y \wedge x = x_0 \wedge y = y_0 \wedge x \leq 0, y \coloneqq x-1; z \coloneqq y)$
  $\equiv sp(sp(x = y \wedge x = x_0 \wedge y = y_0 \wedge x \leq 0, y \coloneqq x-1), z \coloneqq y)$
  $\equiv sp(x = y_0 \wedge x = x_0 \wedge y_0 = y_0 \wedge x \leq 0 \wedge y = x-1, z \coloneqq y)$
  $\equiv x = y_0 \wedge x = x_0 \wedge y_0 = y_0 \wedge x \leq 0 \wedge y = x-1 \wedge z = y$

- $sp(x = y,\ \textbf{if}\ x \geq 0 \rightarrow x \coloneqq y + 1; z \coloneqq x \ \Box\ x \leq 0 \rightarrow y \coloneqq x - 1; z \coloneqq y\ \textbf{fi})$
  $\equiv (x_0 = y \wedge x_0 = x_0 \wedge y = y_0 \wedge x_0 \geq 0 \wedge x = y + 1 \wedge z = x) \vee (x = y_0 \wedge x = x_0 \wedge y_0 = y_0 \wedge x \leq 0 \wedge y = x - 1 \wedge z = y)$