CS536 Science of Programming - Assignment 3

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Problem 1

Solution:

a) Let $S \equiv$ **if** $x > y \to x := x-1 \;\square\; x > y \to y := y+1 \;\square\; x+y = 4 \to x := y/x \;\square\; x+y = 4 \to x := x/y$ **fi** , and let $\sigma = \{x = 3, y = 1\}$. Calculate $M(S, \sigma)$.

$$\begin{aligned}
\langle S, \sigma \rangle &= \langle \textbf{ if } x > y \to x := x - 1 \;\square\; x > y \to y := y + 1 \;\square\; x + y = 4 \to x := y/x \;\square\; x + y = 4 \to x := x/y \textbf{ fi } , \sigma \rangle \\
&= \langle \textbf{ if } x > y \to x := x - 1 \;\square\; x > y \to y := y + 1 \;\square\; x + y = 4 \to x := y/x \;\square\; x + y = 4 \to x := x/y \textbf{ fi } , \{x = 3, y = 1\} \rangle \\
&\to^* \langle \textbf{ if } T \to x := x - 1 \;\square\; T \to y := y + 1 \;\square\; T \to x := y/x \;\square\; T \to x := x/y \textbf{ fi } , \{x = 3, y = 1\} \rangle \\
\langle S, \sigma \rangle &\to \langle x := x - 1, \{x = 3, y = 1\} \rangle \to^* \langle E, \{x = 2, y = 1\} \rangle \\
\langle S, \sigma \rangle &\to \langle y := y + 1, \{x = 3, y = 1\} \rangle \to^* \langle E, \{x = 3, y = 2\} \rangle \\
\langle S, \sigma \rangle &\to \langle x := y/x, \{x = 3, y = 1\} \rangle \to^* \langle E, \{x = 0.33, y = 1\} \rangle \\
\langle S, \sigma \rangle &\to \langle x := x/y, \{x = 3, y = 1\} \rangle \to^* \langle E, \{x = 3, y = 1\} \rangle \\
M(S, \sigma) &= \{\{x = 2, y = 1\}, \{x = 3, y = 2\}, \{x = 0.33, y = 1\}, \{x = 3, y = 1\}\}
\end{aligned}$$

b) Let $W \equiv$ **do** $x > y \to x := x - 1 \;\square\; x > y \to y := y + 1 \;\square\; x + y = 4 \to x := y/x \;\square\; x + y = 4 \to x := x/y$ **od** , and let $\sigma = \{x = 3, y = 1\}$. Calculate $M(W, \sigma)$.

- After the first iteration, we have the following states as we calculated above: $\{\{x = 2, y = 1\}, \{x = 3, y = 2\}, \{x = 0.33, y = 1\}, \{x = 3, y = 1\}\}$
- After the second iteration, from state $\{x = 2, y = 1\}$ we have: $\{\{x = 1, y = 1\}, \{x = 2, y = 2\}\}$, from state $\{x = 3, y = 2\}$ we have: $\{\{x = 2, y = 2\}, \{x = 3, y = 3\}\}$, state $\{x = 0.33, y = 1\}$ doesn't satisfy any conditions, and state $\{x = 3, y = 1\}$ appears again so it diverges $\perp_d$.
- After the third iteration, state $\{x = 1, y = 1\}$ and state $\{x = 3, y = 3\}$ doesn't satisfy any conditions, from state $\{x = 2, y = 2\}$ we have: $\{x = 1, y = 2\}$.
- Thus, $M(S, \sigma) = \{\{x = 0.33, y = 1\}, \perp_d, \{x = 1, y = 1\}, \{x = 3, y = 3\}, \{x = 1, y = 2\}\}$

Problem 2

Solution: