CS536 Science of Programming - Assignment 5

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Problem 1

Full proof outline under partial correctness for q10 Assignment 4.

$\{n > 0\}$
$k \coloneqq n - 1; \{n > 0 \wedge k = n - 1\} x \coloneqq n; \{n > 0 \wedge k = n - 1 \wedge x = n\} \{1 \leq k \leq n \wedge x = n!/k!\}$
$\{\textbf{inv}\ p \equiv 1 \leq k \leq n \wedge x = n!/k!\}$
while $k > 1$ **do**
$\quad \{p \wedge k > 1\} \{p[x * k/x][k - 1/k]\} k \coloneqq k - 1; \{p[x * k/x]\} x \coloneqq x * k; \{p\}$
od
$\{p \wedge k \leq 1\} \{x = n!\}$

Problem 2

Minimal proof outline under partial correctness for q10 Assignment 4.

$\{n > 0\}$
$k \coloneqq n - 1; x \coloneqq n;$
$\{\textbf{inv}\ p \equiv 1 \leq k \leq n \wedge x = n!/k!\}$
while $k > 1$ **do**
$\quad k \coloneqq k - 1; x \coloneqq x * k;$
od
$\{x = n!\}$

Problem 3

Full proof outline with backward assignment.

$\{y \geq 1\}$
$\{1 \leq 1 = 2^0 \leq y\} x \coloneqq 0; \{1 \leq 1 = 2^x \leq y\} r \coloneqq 1;$
$\{\textbf{inv}\ p \equiv 1 \leq r = 2^x \leq y\}$
while $2 * r \leq y$ **do**
$\quad \{p \wedge 2 * r \leq y\}$
$\quad \{1 \leq 2 * r = 2^{x+1} \leq y\}$
$\quad r \coloneqq 2 * r; \{1 \leq r = 2^{x+1} \leq y\}$
$\quad x \coloneqq x + 1; \{1 \leq r = 2^x \leq y\}$
od
$\{p \wedge 2 * r > y\}$
$\{r = 2^x \leq 2^{x+1}\}$

Problem 4

Full proof outline with forward assignment.

$\{y \geq 1\}$
$x \coloneqq 0; \{y \geq 1 \wedge x = 0\} r \coloneqq 1; \{y \geq 1 \wedge x = 0 \wedge r = 1\}$
$\{\textbf{inv } p \equiv 1 \leq r = 2^x \leq y\}$
while $2 * r \leq y$ **do**
$\{p \wedge 2 * r \leq y\}$
$r \coloneqq 2 * r; \{1 \leq r_0 = 2^x \leq y \wedge 2 * r_0 \leq y \wedge r = 2 * r_0\}$
$x \coloneqq x + 1; \{1 \leq r_0 = 2^{x_0} \leq y \wedge 2 * r_0 \leq y \wedge r = 2 * r_0 \wedge x = x_0 + 1\}$
od
$\{p \wedge 2 * r > y\}$
$\{r = 2^x \leq 2^{x+1}\}$