CS536 Science of Programming - Assignment 1

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HW 1

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(1) (a) no, because semantic equality doesn't always have to imply syntactic equality. In other words, two expressions can have the same value, but the syntax can be different.

(b) yes, ~~syntactica equality~~ is always ~~have the same~~ syntactically equal expressions will always have the same value (meaning). If two expressions are not ~~syntacti~~ semantically equal, it will always be syntactically different.

(2) (a) If you wash the boss's car, then you might be promoted.

(b) If there are winds from south, then it means a spring thaw.

(c) If willy cheats, then he will get caught.

(d) If you begin your climb early, then you will reach the summit.

(e) If you don't pay a subscription fee, then you can't access the website.

(3) $p \wedge \neg(q \vee r) \rightarrow q \vee r \rightarrow \neg p$.

(3) $p \wedge \neg(q \vee r) \rightarrow q \vee r \rightarrow \neg p$

p	q	r	$\neg p$	$q \vee r$	$\neg(q \vee r)$	$p \wedge \neg(q \vee r)$	$q \vee r \rightarrow \neg p$	$p \wedge \neg(q \vee r) \rightarrow q \vee r \rightarrow \neg p$
F	F	F	T	F	T	F	T	T
F	F	T	T	T	F	F	T	T
F	T	F	T	T	F	F	T	T
F	T	T	T	T	F	F	T	T
T	F	F	F	F	T	T	T	T
T	F	T	F	T	F	F	F	T
T	T	F	F	T	F	F	F	T
T	T	T	F	T	F	F	F	T

This tautology, because the truth table is always True.

(4) (a) $p \wedge \neg r \vee s \rightarrow \neg q \wedge r \rightarrow \neg p \leftrightarrow \neg s \rightarrow t$

$p \wedge (\neg r) \vee s \rightarrow (\neg q) \wedge r \rightarrow (\neg p) \leftrightarrow (\neg s) \rightarrow t$

$(p \wedge (\neg r)) \vee s \rightarrow ((\neg q) \wedge r) \rightarrow (\neg p) \leftrightarrow ((\neg s) \rightarrow t)$

$((p \wedge (\neg r)) \vee s) \rightarrow ((\neg q) \wedge r) \rightarrow (\neg p) \leftrightarrow ((\neg s) \rightarrow t)$

$((p \wedge (\neg r)) \vee s) \rightarrow (((\neg q) \wedge r) \rightarrow (\neg p)) \leftrightarrow ((\neg s) \rightarrow t)$

$(((p \wedge (\neg r)) \vee s) \rightarrow (((\neg q) \wedge r) \rightarrow (\neg p))) \leftrightarrow ((\neg s) \rightarrow t)$

$((((p \wedge (\neg r)) \vee s) \rightarrow (((\neg q) \wedge r) \rightarrow (\neg p))) \leftrightarrow ((\neg s) \rightarrow t))$

(b) $\forall m.\ 0 < m < n \rightarrow \exists\, 0 \leq j < m.\ b[0] \leq b[j] \wedge b[j] \leq b[m]$

$(\forall m. ((0 < m < n) \rightarrow (\exists\, 0 \leq j < m. ((b[0] \leq b[j]) \wedge (b[j] \leq b[m])))))$

(5) (a) $\cancel{\neg p \to q \to r \Leftrightarrow (p \vee r)}$

$\neg p \to q \to r \Leftrightarrow q \to (p \vee r)$

$\neg p \to q \to r$

$\Leftrightarrow \neg p \to (\neg q \vee r)$ Definition of $\to$

$\Leftrightarrow \neg\neg p \vee (\neg q \vee r)$ Definition of $\to$

$\Leftrightarrow p \vee (\neg q \vee r)$ Double negation

$\Leftrightarrow (p \vee \neg q) \vee r$ Associativity of $\vee$

$\Leftrightarrow (\neg q \vee p) \vee r$ Commutativity of $\vee$

$\Leftrightarrow \neg q \vee (p \vee r)$ Associativity of $\vee$

$\Leftrightarrow q \to (p \vee r)$ Definition of $\to$

(b) $(p \to q) \wedge (p \to r) \Leftrightarrow p \to q \wedge r$

$(p \to q) \wedge (p \to r)$

$\Leftrightarrow (\neg p \vee q) \wedge (\neg p \vee r)$ Definition of $\to$

$\Leftrightarrow (q \vee \neg p) \wedge (r \vee \neg p)$ Commutativity of $\vee$

$\Leftrightarrow (q \wedge r) \vee \neg p$ Reverse Distributivity of $\vee$

$\Leftrightarrow \neg p \vee (q \wedge r)$ Commutativity of $\vee$

$\Leftrightarrow p \to (q \vee r)$ Definition of $\to$

$\Leftrightarrow p \to q \vee r$ removed parenthesis.

(6) (a) The sum of two negative integers will always be negative.

~~$\forall x. \exists y.$~~

$\forall x.\ x<0 \rightarrow \forall y.\ y<0 \rightarrow (x+y)<0$

$\forall x<0.\ \forall y<0.\ (x+y)<0$

(b) For each positive integer, there doesn't exist any negative integer that has the same value as it.

$\forall x.\ x>0 \rightarrow \neg\exists y.\ y<0 \wedge x=y$

$\forall x>0.\ \neg\exists y<0.\ x=y.$

(c) For any two integers, there is always another integer equals to their (the first two integers') product.

$\forall x.\ \forall y.\ \exists z.\ x*y=z$

(7) (a) $\neg(\forall x.\ x<0 \rightarrow \forall y.\ y<0 \rightarrow (x+y)<0)$

$\Leftrightarrow \exists x.\ \neg(x<0 \rightarrow \forall y.\ y<0 \rightarrow (x+y)<0)$ DeMorgan's law in predicate logic

$\Leftrightarrow \exists x.\ x<0 \wedge \neg(\forall y.\ y<0 \rightarrow (x+y)<0)$ Negation of implication

$\Leftrightarrow \exists x.\ x<0 \wedge \exists y.\ \neg(y<0 \rightarrow (x+y)<0)$ DeMorgan's law in predicate logic

$\Leftrightarrow \exists x.\ x<0 \wedge \exists y.\ y<0 \wedge \neg((x+y)<0)$ Negation of implication

$\Leftrightarrow \exists x.\ x<0 \wedge \exists y.\ y<0 \wedge x+y \geqslant 0$

(b) $\neg(\forall x.\ x>0 \rightarrow \neg\exists y.\ y<0 \wedge x=y)$

$\Leftrightarrow \exists x.\ \neg(x>0 \rightarrow \neg\exists y.\ y<0 \wedge x=y)$ DeMorgan's law in predicate logic

$\Leftrightarrow \exists x.\ x>0 \wedge \neg(\neg\exists y.\ y<0 \wedge x=y)$ Negation of implication

$\Leftrightarrow \exists x.\ x>0 \wedge \exists y.\ y<0 \wedge x=y$ Double negation

(c) $\neg(\forall x.\ \forall y.\ \exists z.\ x*y=z)$

$\exists x.\ \neg(\forall y.\ \exists z.\ x*y=z)$ DeMorgan's law in predicate logic

$\exists x.\ \exists y.\ \neg(\exists z.\ x*y=z)$ DeMorgan's law in predicate logic

$\exists x.\ \exists y.\ \forall y.\ \neg(x*y=z)$ DeMorgan's law in predicate logic

$\exists x.\ \exists y.\ \forall y.\ x*y \neq z$

⑧ (a) $\sigma = \{p = T\}$, $e \equiv (r \to p) \wedge (\neg r \to p)$

proper:

$(r \to p) \wedge (\neg r \to p)$

$\Leftrightarrow (\neg r \vee p) \wedge (\neg\neg r \vee p)$ Definition of $\to$

$\Leftrightarrow (\neg r \vee p) \wedge (r \vee p)$ Double negation

$\Leftrightarrow (\neg r \wedge r) \vee p$ Distributivity of $\vee$

$\Leftrightarrow F \vee p$ Contradiction

$\Leftrightarrow p$ Identity

$\sigma(e) = \sigma(p) = T$

$\sigma \models e$. (satisfies).

(b) $\tau = \{p = F, q = T\}$, $\sigma = \tau \cup \{r = F\}$, $e \equiv p \wedge q$

proper:

$\sigma(e) = \sigma(p \wedge q)$

$= \sigma(p) \wedge \sigma(q)$

$= F \wedge T$

$= F$

$\sigma \not\models e$ (doesn't satisfy).

(8) (c) $\sigma = \{b=5, i=0, x=6\}$, $e \equiv x > b[i]$

improoper, b has to be an array

(d) $\sigma = \{x=5, b=(5,8)\}$, $e \equiv x+1 = b[0]$

proper.

$$\begin{aligned}
\sigma(e) &= (\sigma(x+1 = b[0])) \\
&= (\sigma(x+1) = \sigma(b[0])) \\
&= (\sigma(x) + \sigma(1) = \sigma(b)(\sigma[0])) \\
&= (5+1 = \sigma(b)(0)) \\
&= (6 = 5) = F
\end{aligned}$$

$\sigma \nvDash e$ (doesn't satisfy)

(e) $\sigma = \{p=T, b=\alpha\}$, where $\alpha = (2,0,4)$, $e \equiv p \leftrightarrow b[b[1]]=2$

proper.

$$\begin{aligned}
\sigma(e) &= (\sigma(p \leftrightarrow b[b[1]] = 2) \\
&= (\sigma(p) \leftrightarrow \sigma(b[b[1]] = 2)) \\
&= (T \leftrightarrow \sigma(b[b[1]]) = \sigma(2)) \\
&= (T \leftrightarrow \sigma(b)(\sigma(b[1])) = 2) \\
&= (T \leftrightarrow \sigma(b)(\sigma(b)(\sigma(1))) = 2) \\
&= (T \leftrightarrow \sigma(b)(\alpha(1)) = 2) \\
&= (T \leftrightarrow \sigma(b)(0) = 2) \\
&= (T \leftrightarrow \alpha(0) = 2) \\
&= (T \leftrightarrow 2 = 2) \\
&= (T \leftrightarrow T) = T
\end{aligned}$$

$\sigma \vDash e$ (satisfies).

(g) $\sigma \models p \leftrightarrow q \leftrightarrow r$.

p	q	r	$q \leftrightarrow r$	$p \leftrightarrow q \leftrightarrow r$
F	F	F	T	F
F	F	T	F	T
F	T	F	F	T
F	T	T	T	F
T	F	F	T	T
T	F	T	F	F
T	T	F	F	F
T	T	T	T	T

$\sigma = \{p=F, q=F, r=T\}$
$\sigma = \{p=F, q=T, r=F\}$ } either one of them is T.
$\sigma = \{p=T, q=F, r=F\}$
$\sigma = \{p=T, q=T, r=T\}$ → or all of them is T.

(10) (a) $isGreater(b, m, x) \equiv$

$$\equiv \forall\, 0 \leq i < m.\ m \leq size(b) \wedge b[i] < x$$

(b) $hasGreater(a, b) \equiv$

$$\equiv \forall\, 0 \leq i < size(a).\ \exists\, 0 \leq j < size(b).\ a[i] > b[j]$$