CS536 Science of Programming - Assignment 3

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Problem 1

Solution:

a) Let $S \equiv$ **if** $x > y \to x \coloneqq x{-}1 \,\square\, x > y \to y \coloneqq y{+}1 \,\square\, x{+}y = 4 \to x \coloneqq y/x \,\square\, x{+}y = 4 \to x \coloneqq x/y$ **fi** , and let $\sigma = \{x = 3, y = 1\}$. Calculate $M(S, \sigma)$.

$\langle S, \sigma\rangle \quad = \langle$ **if** $x > y \to x \coloneqq x - 1 \,\square\, x > y \to y \coloneqq y + 1 \,\square\, x + y = 4 \to x \coloneqq y/x \,\square\, x + y = 4 \to x \coloneqq x/y$ **fi** $, \sigma\rangle$

$= \langle$ **if** $x > y \to x \coloneqq x - 1 \,\square\, x > y \to y \coloneqq y + 1 \,\square\, x + y = 4 \to x \coloneqq y/x \,\square\, x + y = 4 \to x \coloneqq x/y$ **fi** $, \{x = 3, y = 1\}\rangle$

$\to^* \langle$ **if** $T \to x \coloneqq x - 1 \,\square\, T \to y \coloneqq y + 1 \,\square\, T \to x \coloneqq y/x \,\square\, T \to x \coloneqq x/y$ **fi** $, \{x = 3, y = 1\}\rangle$

$\langle S, \sigma\rangle \quad \to \langle x \coloneqq x - 1, \{x = 3, y = 1\}\rangle \to^* \langle E, \{x = 2, y = 1\}\rangle$
$\langle S, \sigma\rangle \quad \to \langle y \coloneqq y + 1, \{x = 3, y = 1\}\rangle \to^* \langle E, \{x = 3, y = 2\}\rangle$
$\langle S, \sigma\rangle \quad \to \langle x \coloneqq y/x, \{x = 3, y = 1\}\rangle \to^* \langle E, \{x = 0.33, y = 1\}\rangle$
$\langle S, \sigma\rangle \quad \to \langle x \coloneqq x/y, \{x = 3, y = 1\}\rangle \to^* \langle E, \{x = 3, y = 1\}\rangle$
$M(S, \sigma) = \{\{x = 2, y = 1\}, \{x = 3, y = 2\}, \{x = 0.33, y = 1\}, \{x = 3, y = 1\}\}$

b) Let $W \equiv$ **do** $x > y \to x \coloneqq x - 1 \,\square\, x > y \to y \coloneqq y + 1 \,\square\, x + y = 4 \to x \coloneqq y/x \,\square\, x + y = 4 \to x \coloneqq x/y$ **od** , and let $\sigma = \{x = 3, y = 1\}$. Calculate $M(W, \sigma)$.

- After the first iteration, we have the following states as we calculated above: $\{\{x = 2, y = 1\}, \{x = 3, y = 2\}, \{x = 0.33, y = 1\}, \{x = 3, y = 1\}\}$
- After the second iteration, from state $\{x = 2, y = 1\}$ we have: $\{\{x = 1, y = 1\}, \{x = 2, y = 2\}\}$, from state $\{x = 3, y = 2\}$ we have: $\{\{x = 2, y = 2\}, \{x = 3, y = 3\}\}$, state $\{x = 0.33, y = 1\}$ doesn't satisfy any conditions, and state $\{x = 3, y = 1\}$ appears again so it diverges $\perp_d$.
- After the third iteration, state $\{x = 1, y = 1\}$ and state $\{x = 3, y = 3\}$ doesn't satisfy any conditions, from state $\{x = 2, y = 2\}$ we have: $\{x = 1, y = 2\}$.
- Thus, $M(W, \sigma) = \{\{x = 0.33, y = 1\}, \perp_d, \{x = 1, y = 1\}, \{x = 3, y = 3\}, \{x = 1, y = 2\}\}$

Problem 2

Solution:

If we look at Jason's program, it terminates when $b[k_1] = 1$ and $b[k_2] = 2$ because there is no guard that handles this case. In this case, we should increase both pointers meaning we have paired a 1 with a 2.

$MAJORITY \equiv$
$k_1 \coloneqq 0; k_2 \coloneqq 0;$
do $b[k_1] = 2 \to k_1 \coloneqq k_1 + 1$
$\square\ b[k_2] = 1 \to k_2 \coloneqq k_2 + 1$
$\square\ b[k_1] = 1 \wedge b[k_2] = 2 \to k_1 \coloneqq k_1 + 1; k_2 \coloneqq k_2 + 1$ **od**

It is possible that $\perp_e \in M(MAJORITY, \sigma)$. Let's handle the out of bound exception:

$MAJORITY \equiv$

$k_1 := 0; k_2 := 0;$

do $k_1 < n \wedge b[k_1] = 2 \rightarrow k_1 := k_1 + 1$

$\Box\ k_2 < n \wedge b[k_2] = 1 \rightarrow k_2 := k_2 + 1$

$\Box\ k_1 < n \wedge k_2 < n \wedge b[k_1] = 1 \wedge b[k_2] = 2 \rightarrow k_1 := k_1 + 1; k_2 := k_2 + 1$ **od**

Problem 3

Solution:

a) True. Because the nature of nondeterminism, we can end up with more than one state.

b) False. Because a state σ doesn't have to satisfy the precondition p and the correctness triple can still be true since we are not running S with σ.

c) False. Because σ satisfies the precondition and there exist some τ that is either terminated in error or doesn't satisfy the postcondition.

d) False. Partial correctness can end with error.

e) True. Without pseudo states, the resulting state after S should satisfy the inverse of the postcondition.

Problem 4

Solution:

a) Valid since it's according to Backward Assignment Rule.

b) Not valid. Because postcondition is stronger.

c) Valid. No state can satisfy the precondition, thus the precondition is always F and the triple is correct.

d) Valid because the precondition and the postcondition are semantically equal, and running $s := s + 1$ doesn't affect the postcondition. Therefore, as long as a state satisfies the precondition, the resulting state will satisfy the postcondition.

e) Vaild since it's according to Backward Assignment Rule.

Problem 5

Solution:

a) All integers except even numbers. If x is even number including 0, loop terminates eventually $x := 0$ and it doesn't satisfy the post condition, $x = 0 \not\models x < 0$. For odd numbers and negative numbers, the program will diverge because x never becomes 0. Thus the triple is partially correct.

a) No possible values. There doesn't exist value of $\sigma(x)$ that satisfies the triple.

Problem 6

Solution:

a) True. Strengthening precondition will still satisfy the postcondition. So q_1 and q_2 both will be satisfied.

b) False. Weakining precondition will not guarantee that the postcondition is satisfied. So q_1 and q_2 may or may not be satisfied.

Problem 7

Solution:

a) False.

b) False.

Problem 8

Solution:
a) True. According to the weakest precondition.
b) True. Since w is the weakest precondition, making it stronger will satisfy the postcondition.
c) False.
d) True.
e) False. Pseudo state can exist.

Problem 9

Solution:
a) $wlp(S,q) \equiv wlp(y := y/x, sqrt(y) > x)$
$\equiv sqrt(y/x) > x$ // backward assignment rule

b) $wp(S,q) \equiv D(S) \land wlp(S,q) \land D(wlp(S,q))$

- $D(S) \equiv D(y/x) \equiv x \neq 0$
- $wlp(S,q) \equiv sqrt(y/x) > x$
- $D(wlp(S,q)) \equiv D(sqrt(y/x) > x) \equiv y/x \geq 0 \land x \neq 0$
- $wp(S,q) \equiv x \neq 0 \land sqrt(y/x) > x \land y/x \geq 0 \land x \neq 0$
  $\Leftrightarrow x \neq 0 \land sqrt(y/x) > x \land y/x \geq 0$

Problem 10

Solution:
a) $wlp(S,q) \equiv wlp(\textbf{if } y \geq 0 \textbf{ then } x := -y/x \textbf{ else } x := x/y \textbf{ fi}, r < x \leq y)$
$\equiv (y \geq 0 \to wlp(x := -y/x, r < x \leq y)) \land (\neg(y \geq 0) \to wlp(x := x/y, r < x \leq y))$
$\equiv (y \geq 0 \to r < -y/x \leq y) \land (y < 0 \to r < x/y \leq y)$

b) $wp(S,q) \equiv D(S) \land wlp(S,q) \land D(wlp(S,q))$

- $D(S) \equiv \textbf{if } y \geq 0 \textbf{ then } x := -y/x \textbf{ else } x := x/y \textbf{ fi} \equiv x \neq 0 \land y \neq 0$
  $\equiv D(y \geq 0) \land (y \geq 0 \to D(x := -y/x)) \land (y < 0 \to D(x := x/y))$
  $\equiv T \land (y \geq 0 \to x \neq 0) \land (y < 0 \to y \neq 0)$
  $\equiv (y < 0 \lor x \neq 0) \land T$
  $\equiv y < 0 \lor x \neq 0$
- $wlp(S,q) \equiv (y \geq 0 \to r < -y/x \leq y) \land (y < 0 \to r < x/y \leq y)$
- $D(wlp(S,q)) \equiv D((y \geq 0 \to r < -y/x \leq y) \land (y < 0 \to r < x/y \leq y))$
- $wp(S,q) \equiv y < 0 \lor x \neq 0 \land (y \geq 0 \to r < -y/x \leq y) \land (y < 0 \to r < x/y \leq y) \land D((y \geq 0 \to r < -y/x \leq y) \land (y < 0 \to r < x/y \leq y))$