MCH 3008 PROJECT 1.2

Question 5.1:

a) Sketch the root locus with respect to K for the equation: $1+KL(s)=0$. Be sure to give the asymptotes and the frequency of any imaginary axis crossings.

$$L(s)=\frac{s+5}{s(s+20)(s^2+2s+4)}$$

b) Write a MATLAB program which will plot the closed loop poles on the s-plane for values of K (Use the rlocus function of MATLAB.) Compare the plot with the sketch in (a).

Question 5.2:

a) Sketch the root locus with respect to K for the equation: $1+KL(s)=0$. Be sure to give the asymptotes and the frequency of any imaginary axis crossings.

$$L(s)=\frac{s}{(s-1)(s+4)(s^2+6s+9)}$$

b) Write a MATLAB program which will plot the closed loop poles on the s-plane for values of K (Use the rlocus function of MATLAB.) Compare the plot with the sketch in (a).

Question 6.1: Consider the following second order plant

$$G(s)=\frac{1}{(s+4)(s+6)}$$

a) Design a controller $C(s)$ (lead, lag or lead-lag) with root locus design which will have a rise time less than 0.18 seconds, M_p less than 20% and can track a unit reference signal with a steady state erros less than 0.01.
b) Simulate the plant under control with a unit step input in Matlab and Simulink. Check is the design specifications are met or not.
c) Draw a bode diagram of the $C(s)G(s)$ without using Matlab.
d) Draw a bode diagram of the $C(s)G(s)$ with using Matlab.

Question 6.2: The transfer function of a plant is $G(s)=\frac{100}{(s+1)(s+7)}$.

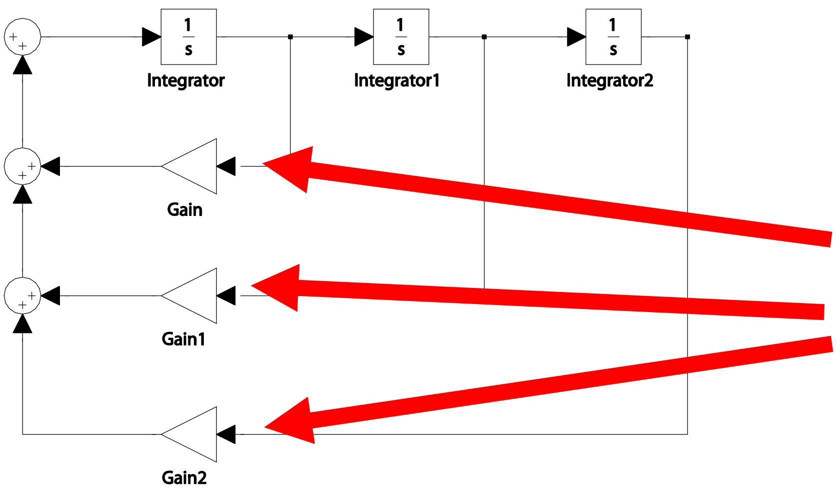
a) Our task is to design a controller $C(s)$ to meet the following specifications in closed loop with unit feedback: $M_p \leq 50\%,\ \omega_n = 20,$ and steady state error for unit step input ≤ 0.001. Use the root locus approach and a lag compensator.
b) Simulate the plant under control with a unit step input in Matlab and Simulink. Check is the design specifications are met or not.
c) Draw a bode diagram of the $C(s)G(s)$ without using Matlab.
d) Draw a bode diagram of the $C(s)G(s)$ with using Matlab.

Question 7.1:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}0\\0\\1\end{bmatrix}u$$

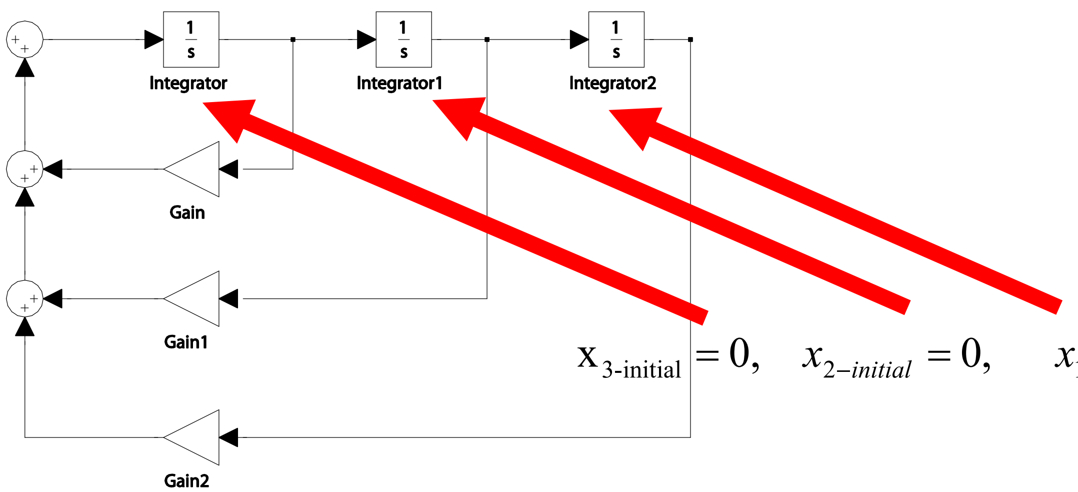
$$y=\begin{bmatrix}1&0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

a) Discuss whether the system should be controllable or not. Check your intuition by forming the controllability matrix and checking its rank.
b) Assume that all of the state variables can be measured directly; that is, x is measured. Find a state feedback law that places the dominant closed-loop poles of the system so that $t_r \leq 0.1$ sec and $M_p \leq 5\%$. Place the remaining pole of the system 4-5 times farther to the left so that the design specifications are more likely to hold.
c) Find the state feedback gains by first matching the coefficients of the desired polynomial and closed-loop characteristic equation. Then check your result by using the Matlab function “acker.m” or “place.m”.
d) Simulate the closed loop system with a Simullink model.
   - Model the plant with Simulink integrators and Simulink gains



Find the values of these gains from the plant transfer function.

   - Enter the following initial conditions into the simulink integrators



$x_{3\text{-initial}} = 0, \quad x_{2-initial} = 0, \quad x_{1-initial} = 3.$

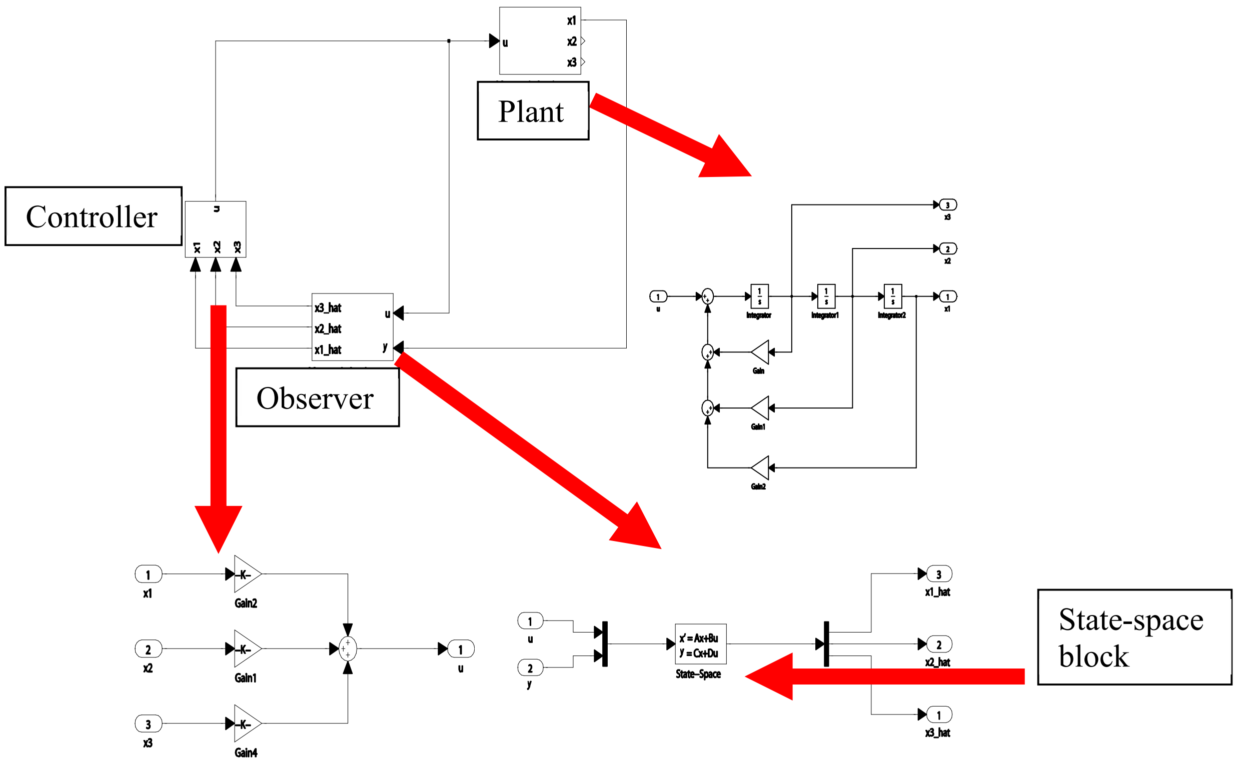
- Model the the state feedback controller with a Simulink gain.
- Add “time” and “to workspace” blocks.
- Simulate the system and plot x_1, x_2, x_3, y and u versus time.
- Discuss simulation results.

e) Check if this system is observable or not. (Form the observability matrix and check if it is invertable or not.)

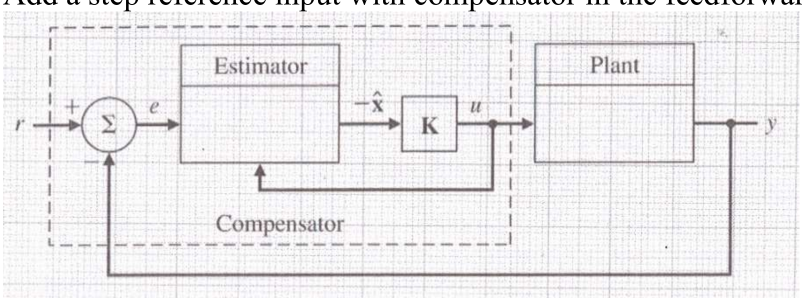
f) Suppose now that the state variables are not all available for measurement. Design an observer which has poles with magnitudes about 4 times larger than the real parts of the closed-loop system. Determine the observer poles.

g) Do coefficient matching to find observer gains.

h) Simulate the closed loop system with the observer.
- Model the observer with a Simulink “State-Space” block.
- Simulate the system and plot $x_1, x_2, x_3, \hat{x}_1, \hat{x}_2, \hat{x}_3, y$ and u versus time.
- Discuss simulation results.

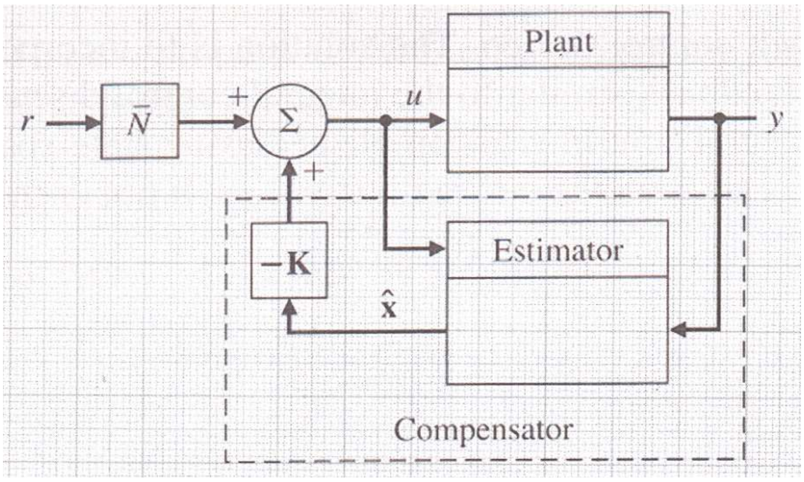


i) Add a step reference input with compensator in the feedforward path.



- Use a step of 5 units.
- Simulate the system and plot $x_1, x_2, x_3, \hat{x}_1, \hat{x}_2, \hat{x}_3, y$ and u versus time.
- Discuss simulation results. Is there a steady state error?

j) Now we want to avoid the steady state error. For this purpose, use the compensator in the feedback path structure.



N_u: Feedforward gain to avoid steady state error

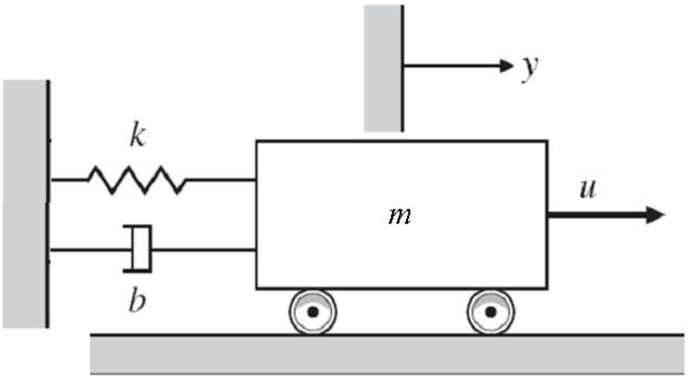
N_x: State reference gain to convert the reference for y into a reference for x.

$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} F & G \\ H & J \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\bar{N} = N_u + KN_x$$

- Use a step of 5 units
- Simulate the system and plot $x_1, x_2, x_3, \hat{x}_1, \hat{x}_2, \hat{x}_3, y$ and u versus time.
- Discuss simulation results. Is there a steady state error?

Question 7.2: In the following figure the force u is applied to the mass m. The spring with the stiffness coefficient k and the damper with damping coefficient b model the coupling between the mass and the wall (Take m=10, k= 1000 and b=0.01).



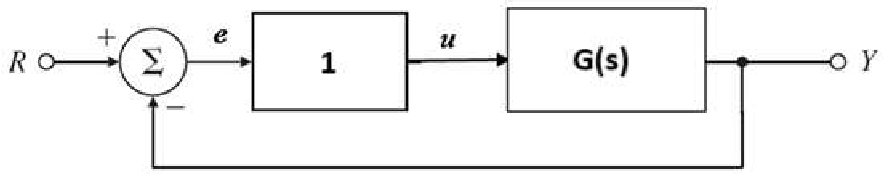
a) Write the equations of motion of this system with input u and output y.
b) Find the state space representation of this system. (Find F, G, H, J matrices).
c) Is this system controllable? Why?
d) Design a linear state feedback regulator for this system such that the closed loop poles are at -2 and -5. (Find the gain K.)
e) Simulate the closed loop system with a Simulink model.
   - Model the plant with Simulink integrators and Simulink gains
   - Model the the state feedback controller with a Simulink gain.
   - Add “time” and “to workspace” blocks.
   - Simulate the system and plot x_1, x_2, y and u versus time.
   - Discuss simulation results.

f) Is this system observable? Why?
g) Design an observer such that the observer poles are at -10 and -15.
h) Simulate the closed loop system with the observer.
   - Model the observer with a Simulink “State-Space” block.
   - Simulate the system and plot $x_1, x_2, \hat{x}_1, \hat{x}_2, y$ and u versus time.
   - Discuss simulation results.
i) Add a step reference input with compensator in the feedforward path.
   - Use a step of 10 units.
   - Simulate the system and plot $x_1, x_2, \hat{x}_1, \hat{x}_2, y$ and u versus time.
   - Discuss simulation results. Is there a steady state error?
j) Now we want to avoid the steady state error. For this purpose, use the compensator in the feedback path structure.
   N_u: Feedforward gain to avoid steady state error
   N_x: State reference gain to convert the reference for y into a reference for x.

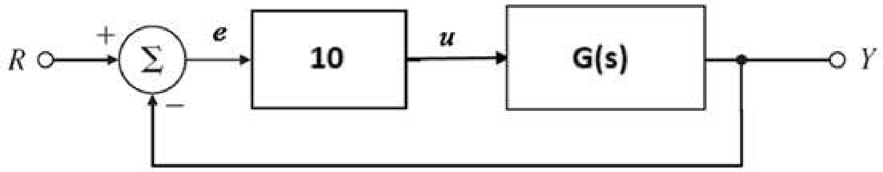
$$\begin{bmatrix} N_x \\ N_u \end{bmatrix} = \begin{bmatrix} F & G \\ H & J \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\bar{N} = N_u + K N_x$$

   - Use a step of 10 units
   - Simulate the system and plot $x_1, x_2, \hat{x}_1, \hat{x}_2, y$ and u versus time.
   - Discuss simulation results. Is there a steady state error?

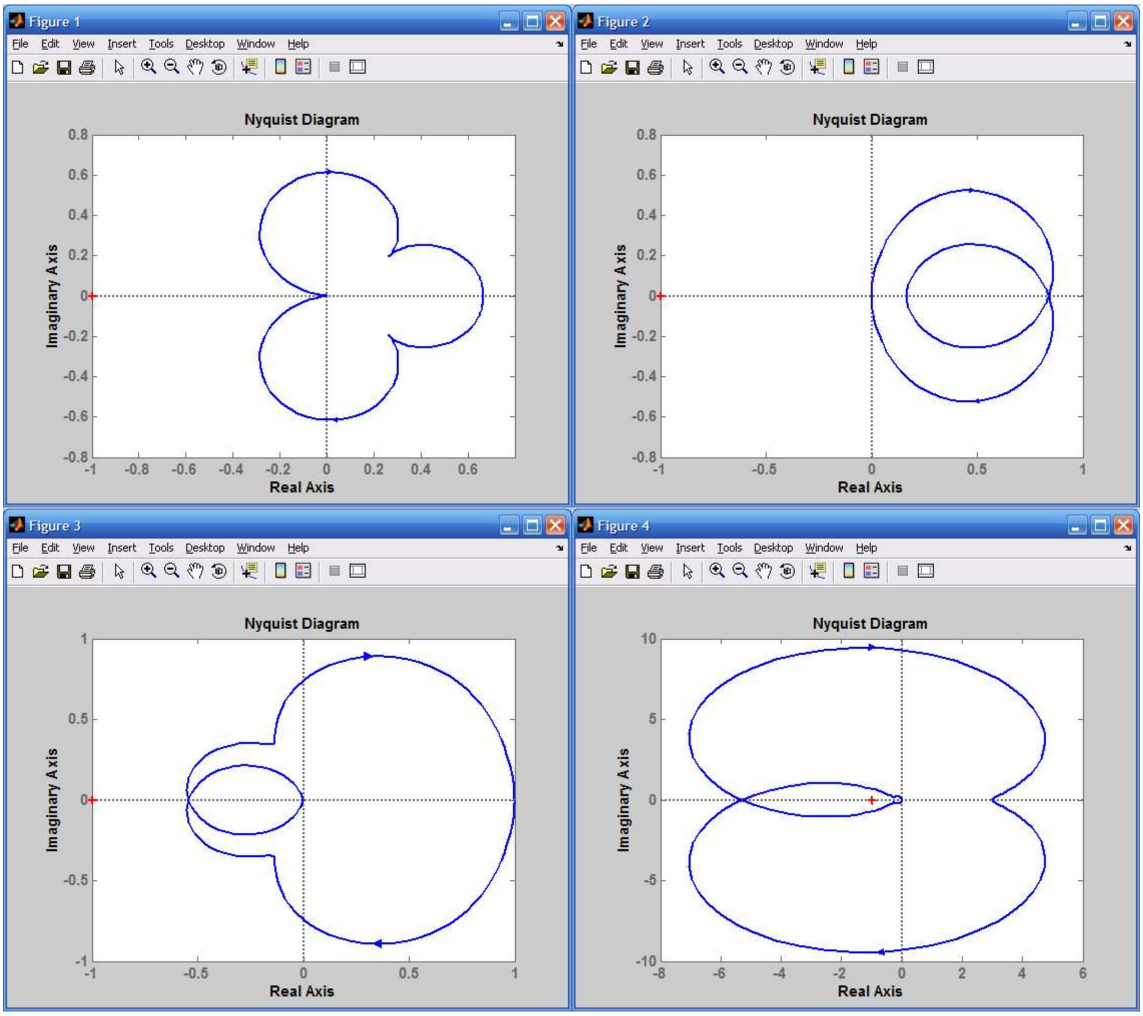
Question 8.1: Consider the feedback loop below. Where, $G(s) = \frac{s^2+1}{s(s^2+2s+3)}$. Write a MATLAB program which will plot the Nyquist diagram of the open loop system on the *s*-plane. Is the closed loop system stable?



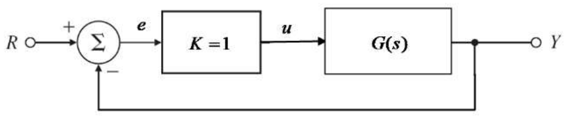
Question 8.2: Consider the feedback loop below. Where, $G(s) = \frac{1}{s^4+5}$. Write a MATLAB program which will plot the Nyquist diagram of the open loop system on the *s*-plane. Is the closed loop system stable?



Question 9.1: Consider the Nyquist plots below. They belong to four open loop systems $G(s)$ which are stable.

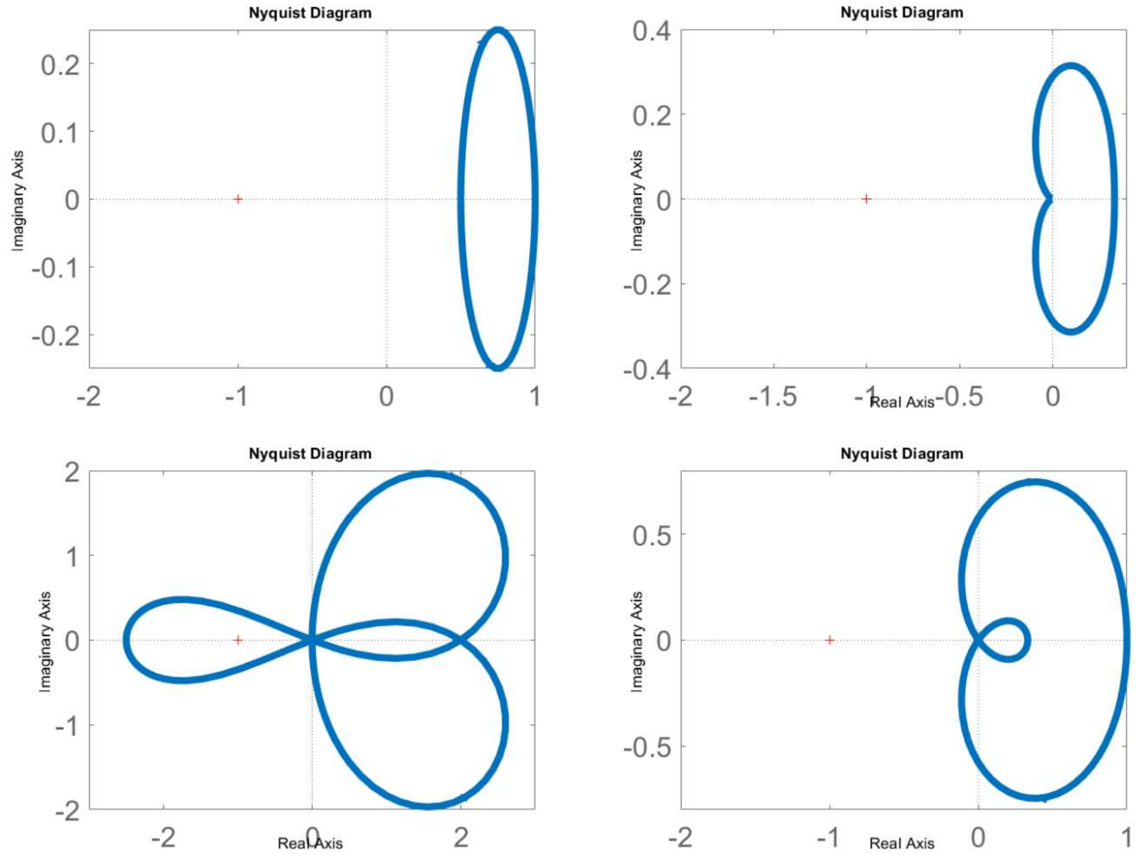


a) Which of them will be closed loop stable when the feedback loop is connected as below? Why?

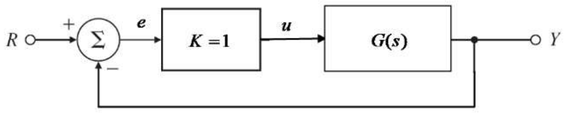


b) Which of them are closed loop stable for any value of the gain K? Why?

Question 9.2: Consider the Nyquist plots below. They belong to four open loop systems $G(s)$ which are stable.



a) Which of them will be closed loop stable when the feedback loop is connected as below? Why?



b) Which of them are closed loop stable for any value of the gain K? Why?

Question 10.1: Consider the following state space system

$$\dot{x} = Fx + Gu$$

$$y = Hx + Ju$$

with

$$F = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 \end{bmatrix}, J = 0.$$

a) Can you place the poles of this system to -1 and -1. Why?
b) Can you place the poles of this system to -2 and -2. Why?

Question 10.2: Consider the following state space system

$$\dot{x} = Fx + Gu$$

$$y = Hx + Ju$$

with

$$F = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}, G = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 \end{bmatrix}, J = 0.$$

a) Can you place the poles of this system to -5 and -5. Why?
b) Can you place the poles of this system to -10 and -10. Why?