Yıldız Teknik Üniversitesi

Elektrik-Elektronik Fakültes

Bilgisayar Mühendisliği Bölümü

BLM 2642

Bilgisayar Mühendisleri için Diferasniyel Denklemler

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Homework 2

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Answer 1)

$P(t) = 2$
$Q(t) = e^t$

$\frac{dy}{dt} + 2y = e^t \qquad y' + P(t)\,y = Q(t)$

$\mu = e^{\int P(t)} = e^{2t}$

$\mu \cdot y = \int \mu Q(t)$

$e^{2t} \cdot y = \int e^{2t} \cdot e^t$

$e^{2t} \cdot y = \frac{e^{3t}}{3} + C$

$$\boxed{y = e^t/3 + C/e^{2t}}$$

Answer 2)

$\frac{dy}{dx} = xy^2 \qquad \int y^{-2}\,dy = \int x\,dx$

$-y^{-1} = x^2/2$

$$\boxed{y = -2/x^2}$$

Answer 3)

$(x^2 + y)dx + (y^2 + x)dy = 0$

$M dx + N dy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \rightarrow \frac{\partial (x^2 + y)}{\partial y} = \frac{\partial (y^2 + x)}{\partial y}$

$\Rightarrow \quad 1 = 1$ ifade exact diff

$$\int (x^2+y)dx = \int (y^2+x)dy$$

$$\frac{x^3}{3} + xy + c(y) = y^3/3 + xy + c(x)$$

$$c(y) = y^3/3$$

$$\boxed{x^3/3 + xy + y^3/3 = K}$$

Answer 4)

$$\frac{dy}{dx} + y = xy^2$$

$$y' + p(x)\cdot y = Q(x)\cdot y^n$$

$$p(x) = 1 \quad Q(x) = x \quad n = 2$$

$$y = xy^2 + \frac{dv}{dx}\cdot y^2$$

$$y = y^2\left(x + \frac{dv}{dx}\right)$$

$$\frac{1}{y} = x + \frac{dv}{dx}$$

$$v = x + \frac{dv}{dx}$$

$$v = y^{1-n}$$

$$v = y^{-1}$$

$$\frac{dv}{dx} = -y^{-2}\frac{dy}{dx}$$

$$-\frac{dv}{dx} \; y^2 = \frac{dv}{dx}$$

$$e^x \cdot v = e^{-x}(x+1) + C \quad \cdot e^x$$

$$v = x + 1 + C\cdot e^x$$

$$\boxed{y = \frac{1}{x+1+Ce^x}}$$

$$p(x) = -1 \qquad \mu = e^{\int -1} = e^{-x}$$

$$Q(x) = -x$$

$$y \quad \mu\cdot v = \int -1\cdot\mu \Rightarrow e^x\cdot v = \int -x\cdot e^{-x}$$

$$e^{-x}(x+1) + C$$

Answer 5)

$$\frac{dy}{dt} = 2t \qquad \int dy = \int 2t\,dt$$

$$y = t^2 + C$$

$$y(0) = 0 + C = 3$$

$$C = 3$$

$$\boxed{y(t) = t^2 + 3}$$

Answer 6)

$$\frac{dT}{dt} = -k(T - T_a)$$

$$T' + kT = kT_a$$

$$y' + P(x)\,y = Q(x)$$

$$T' + P(k).T = Q(k)$$

$$P(k) = k$$

$$Q(k) = kT_a$$

$$\mu = e^{\int P(x)} = e^{\int k} = e^{kt}$$

$$\mu.T = \int \mu.Q(k)$$

$$e^{kt}.T = \int e^{kt}.k.T_a$$

$$e^{kt}.T = k.T_a.\frac{e^{kt}}{k} + C$$

$$e^{kt}.T = T_a.e^{kt} + C$$

$$T = T_a + \frac{C}{e^{kt}}$$

$$T(0) = T_a + C$$

$$C = T_0 - T_a$$

$$\boxed{T = T_a + \frac{T_0 - T_a}{e^{kt}}}$$

Answer 7)

$$\frac{dP}{dt} + P = e^{-2t}$$

$$y' + p(x)\,y = Q(x)$$

$$P' + A(x)\cdot P = B(t)$$

$$A(x) = 1 \qquad B(t) = e^{-2t}$$

$$M = e^{\int A(x)} = e^{\int 1} = e^{t}$$

$$MA(x) = \int B(x)\cdot M$$

$$e^{t}\cdot P = \int e^{-2t}\cdot e^{t}$$

$$e^{t}\cdot P = -e^{-t} + C$$

$$P = -e^{-2t} + C/e^{t}$$

$$P_0 = -1 + C$$

$$C = P_0 + 1$$

$$\boxed{P = e^{-2t} + \frac{(P_0+1)}{e^{t}}}$$

Answer 8)

$$f(w) = w^2 + 2w + 5$$

$$df/dw = 2w + 2$$

$$a = 0.1$$

$$w_{n+1} = w_n + a\cdot df/dw$$

$$w_{n+1} = w_n + 0.1(2w+2)$$

$$w_{n+1} = 0.8w_n \quad 0.2$$

$$w_1 = 0.8\cdot 3 - 0,2$$

$$w_1 = 2,2$$

$$w_2 = 0.8\cdot 2,2 - 0,2$$

$$w_2 = 1,56$$

$$w_3 = 0,8\cdot 1,56 - 0,2$$

$$w_3 = 1,048$$

$$\boxed{\begin{aligned} w_1 &= 2,2 \\ w_2 &= 1,56 \\ w_3 &= 1,048 \end{aligned}}$$