MIT OPENCOURSEWARE ÇÖZÜMLÜ SORULARI -2

https://ocw.mit.edu/resources/res-6-007-signals-and-systems-spring-2011/

http://web.mit.edu/6.003/F11/www/handouts/hw8-solutions.pdf

***Soru – 1: Verilen* x(t) işaretlerinin Fourier seri katsayılarını bulunuz.**

P7.3

Find the Fourier series coefficients for each of the following signals:

(a) $x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right)$

(b) $x(t) = 1 + \cos(2\pi t)$

(c) $x(t) = [1 + \cos(2\pi t)]\left[\sin\left(10\pi t + \frac{\pi}{6}\right)\right]$

Hint: You may want to first multiply the terms and then use appropriate trigonometric identities.

Çözüm -1:

S7.3

(a)
$$x(t) = \sin\left(10\pi t + \frac{\pi}{6}\right) = \frac{e^{j\pi/6}}{2j}e^{j2\pi t5} - \frac{e^{-j\pi/6}}{2j}e^{-j2\pi t5}$$

We choose ω_0, the fundamental frequency, to be 2π.

$$x(t) = \sum_k a_k e^{jk\omega_0 t},$$

where

$$a_5 = \frac{e^{j\pi/6}}{2j}, \qquad a_{-5} = \frac{-e^{-j\pi/6}}{2j}$$

Otherwise $a_k = 0$.

(b)
$$x(t) = 1 + \cos(2\pi t) = 1 + \frac{e^{j2\pi t}}{2} + \frac{e^{-j2\pi t}}{2}$$

For $\omega_0 = 2\pi$, $a_{-1} = a_1 = \frac{1}{2}$, and $a_0 = 1$. All other a_k's $= 0$.

(c)
$$x(t) = [1 + \cos(2\pi t)]\left[\sin\left(10\pi t + \frac{\pi}{6}\right)\right]$$

$$= \sin\left(10\pi t + \frac{\pi}{6}\right) + \cos(2\pi t)\sin\left(10\pi t + \frac{\pi}{6}\right)$$

$$= \left(\frac{e^{j\pi/6}}{2j}e^{j2\pi t5} - \frac{e^{-j\pi/6}}{2j}e^{-j2\pi t5}\right) + \left(\tfrac{1}{2}e^{j2\pi t} + \tfrac{1}{2}e^{-j2\pi t}\right)\left(\frac{e^{j\pi/6}}{2j}e^{j2\pi t5} - \frac{e^{-j\pi/6}}{2j}e^{-j2\pi t5}\right)$$

$$= \frac{e^{j\pi/6}}{2j}e^{j2\pi t5} - \frac{e^{-j\pi/6}}{2j}e^{-j2\pi t5} + \frac{e^{j\pi/6}}{4j}e^{j2\pi t6} - \frac{e^{-j\pi/6}}{4j}e^{-j2\pi t4}$$
$$+ \frac{e^{j\pi/6}}{4j}e^{j2\pi t4} - \frac{e^{-j\pi/6}}{4j}e^{-j2\pi t6}$$

Therefore,

$$x(t) = \sum_k a_k e^{jk\omega_0 t},$$

where $\omega_0 = 2\pi$.

$$a_4 = \frac{e^{j\pi/6}}{4j}, \qquad a_{-4} = \frac{-e^{-j\pi/6}}{4j},$$
$$a_5 = \frac{e^{j\pi/6}}{2j}, \qquad a_{-5} = \frac{-e^{-j\pi/6}}{2j},$$
$$a_6 = \frac{e^{j\pi/6}}{4j}, \qquad a_{-6} = \frac{-e^{-j\pi/6}}{4j}$$

All other a_k's = 0.

Soru – 2: Fourier seri analiz denklemini kullanarak aşağıdaki işaretlerin seri katsayılarını bulunuz.

(a)

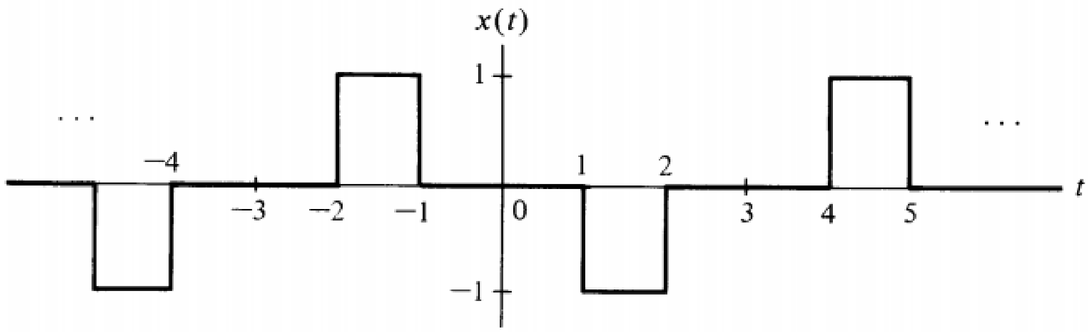


Figure P7.4-1

(b)

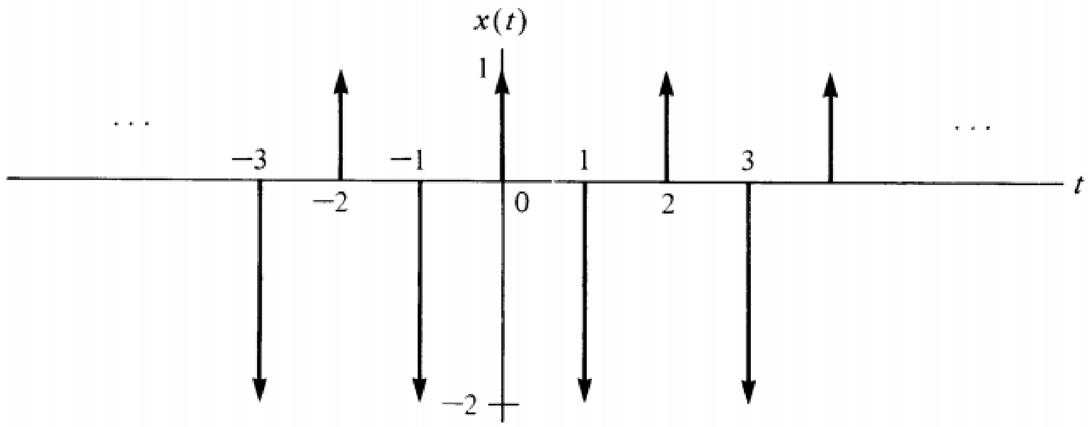


Figure P7.4-2

Çözüm -2:

S7.4

(a)

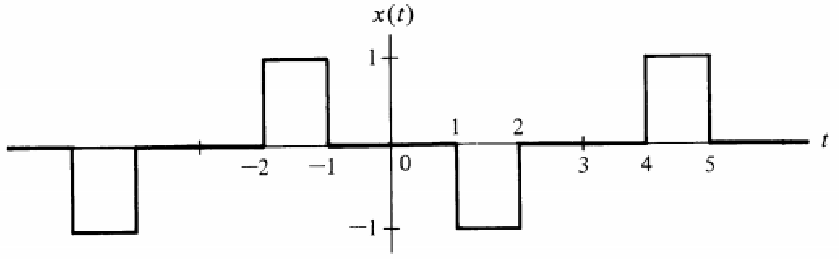


Figure S7.4-1

Note that the period is $T_0 = 6$. Fourier coefficients are given by

$$a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t}\, dt$$

We take $\omega_0 = 2\pi/T_0 = \pi/3$. Choosing the period of integration as -3 to 3, we have

$$\begin{aligned}
a_k &= \frac{1}{6} \int_{-2}^{-1} e^{-jk(\pi/3)t}\, dt - \frac{1}{6} \int_{1}^{2} e^{-jk(\pi/3)t}\, dt \\
&= \frac{1}{6} \frac{1}{-jk(\pi/3)} e^{-jk(\pi/3)t} \Big|_{-2}^{-1} - \frac{1}{6} \frac{1}{-jk(\pi/3)} e^{-jk(\pi/3)t} \Big|_{1}^{2} \\
&= \frac{1}{-j2\pi k} [e^{+j(\pi/3)k} - e^{+j(2\pi/3)k} - e^{-j(2\pi/3)k} + e^{-j(\pi/3)k}] \\
&= \frac{\cos(2\pi/3)k}{j\pi k} - \frac{\cos(\pi/3)k}{j\pi k}
\end{aligned}$$

Therefore,

$$x(t) = \sum_k a_k e^{jk\omega_0 t}, \qquad \omega_0 = \frac{\pi}{3}$$

and

$$a_k = \frac{\cos(2\pi/3)k - \cos(\pi/3)k}{j\pi k}$$

(b)

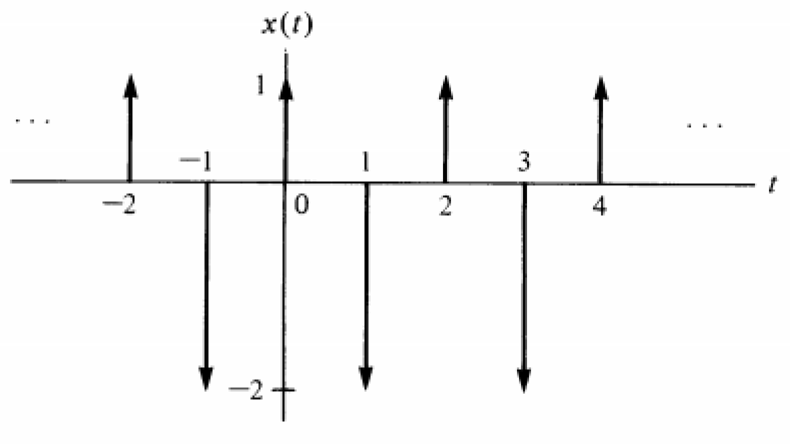


Figure S7.4-2

The period is $T_0 = 2$, with $\omega_0 = 2\pi/2 = \pi$. The Fourier coefficients are

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

Choosing the period of integration as $-\frac{1}{2}$ to $\frac{3}{2}$, we have

$$\begin{aligned} a_k &= \frac{1}{2}\int_{-1/2}^{3/2} x(t)e^{-jk\omega_0 t}\,dt \\ &= \frac{1}{2}\int_{-1/2}^{3/2} [\delta(t) - 2\delta(t-1)]e^{-jk\omega_0 t}\,dt \\ &= \frac{1}{2} - e^{-jk\omega_0} = \frac{1}{2} - (e^{-j\pi})^k \end{aligned}$$

Therefore,

$$a_0 = -\tfrac{1}{2}, \qquad a_k = \tfrac{1}{2} - (-1)^k$$

It is instructive to plot a_k, which we have done in Figure S7.4-3.

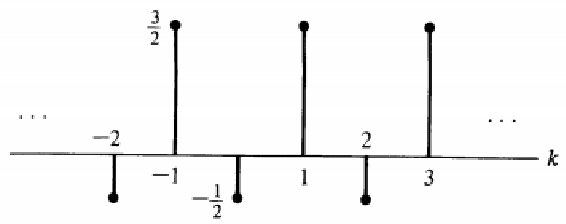
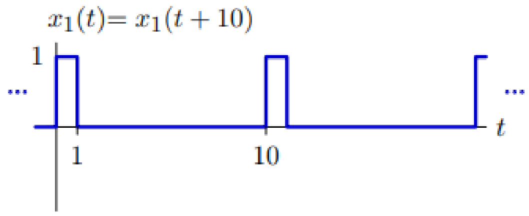


Figure S7.4-3

Soru – 3: Aşağıdaki **x1(t)** ***sinyalinin Fourier Seri katsayılarını bulunuz.***

1. Fourier Series

Determine the Fourier series coefficients a_k for $x_1(t)$ shown below.



Çözüm – 3:

$$a_k = \frac{1}{T}\int_T x(t)e^{-j\frac{2\pi}{T}kt}dt = \frac{1}{10}\int_0^1 1e^{-j\frac{2\pi}{10}kt}dt = \frac{1}{10}\left.\frac{e^{-j\frac{\pi}{5}kt}}{-j\frac{\pi}{5}k}\right|_0^1 = \frac{1}{j2\pi k}\left(1 - e^{-j\pi k/5}\right)$$

Notice that this expression is badly formed at $k = 0$. We could use l'Hôpital's rule to evaluate this expression, but an easier method (which is also more robust against errors) is to simply evaluate the average value of $x_1(t)$ to find that $a_0 = 1/10$.

This solution could also be written in terms of sinusoids as

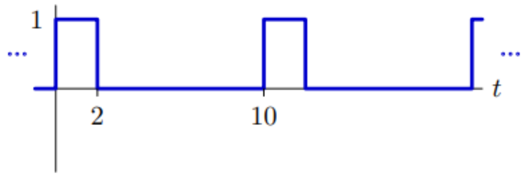
$$a_k = \begin{cases} \frac{1}{10} & k = 0 \\ \frac{1}{\pi k}e^{-j\pi k/10}\sin(\pi k/10) & k \neq 0 \end{cases}.$$

$a_0 =$	$\frac{1}{10}$	
$a_k =$	$\frac{1}{\pi k} e^{-j\pi k/10} \sin(\pi k/10)$	for $k \neq 0$

Soru – 4: Aşağıdaki x2(t) sinyalinin Fourier Seri katsayılarını bulunuz.

Determine the Fourier series coefficients b_k for $x_2(t)$ shown below.

$x_2(t) = x_2(t+10)$



Çözüm – 4:

$$b_k = \frac{1}{T}\int_T x(t)e^{-j\frac{2\pi}{T}kt}dt = \frac{1}{10}\int_0^2 1e^{-j\frac{2\pi}{10}kt}dt = \frac{1}{10}\left.\frac{e^{-j\frac{\pi}{5}kt}}{-j\frac{\pi}{5}k}\right|_0^2 = \frac{1}{j2\pi k}\left(1 - e^{-j2\pi k/5}\right)$$

As with the previous part, this expression is badly formed for $k = 0$. We therefore obtain $b_0 = 1/5$ by calculating the average value of $x_2(t)$.

This solution could also be written in terms of sinusoids as

$$b_k = \begin{cases} \frac{1}{5} & k = 0 \\ \frac{1}{\pi k}e^{-j\pi k/5}\sin(\pi k/5) & k \neq 0 \end{cases}.$$

$b_0 =$	$\frac{1}{5}$	
$b_k =$	$\frac{1}{\pi k} e^{-j\pi k/5} \sin(\pi k/5)$	for $k \neq 0$