How accurate are the computed timings for sunrise and sunset?

Akbar Ali S.F.A. Saifee^

akberali.saifee@gmail.com

^Formerly Associate Professor of Physics, Maharashtra College of Arts, Science and Comm., Mumbai-400008.

Abstract: Although noon at any place on the Earth can be predicted with an accuracy of 0.1 second, sunrise and sunset have an uncertainty of minutes because of unpredictable refraction of rays of light near horizon. In this article, the theoretical and the experimental findings concerning the effect are discussed. Also the effect of the height of the observer above local horizon level on observed sunrise and sunset timings is discussed and the correction required for the same in the computed timings are suggested.

Knowledge of sunrise and sunset timings for a place and their day to day variations is important for people from various fields like farming, cattle grazing, outdoor photography, street lighting, tourism, sports, solar panels and airplane flights etc. But over and above all these, knowledge of either sunrise time or sunset time or both with 'precision' is very important in many religions. Not long ago, sunrise and sunset could be observed from villages and even from outskirts of cities. For cities, sunrise, noon and sunset timings were calculated from the knowledge of astronomy and monthly or yearly charts were prepared. The modern versions of these charts are the result-sheets of the specific computer programs. In these charts, the sunrise and the sunset timings are based on certain norms. Since the real conditions differ from these norms, the calculated timings generally do not tally precisely with the observed timings of sunsets and sunrises.

Uncertainty due to variation in refraction. The sunrays have to pass through different layers of atmosphere before reaching to the observer on the Earth. At noon (known as transit time in astronomy), since the rays are passing normally through the atmospheric layers, there is no refraction of light and hence the accuracy of the computed time depends only on the accuracy of the knowledge of the geometrical position of the Sun. But, at sunrise and sunset the situation is extremely different. The observed sunrise is advanced and the observed sunset is delayed due to the refraction of light. As accepted by the astronomers, all the computer programs assume a nominal horizontal refraction of 0.567^o for estimating sunrise and sunset times. The real value of refraction might be quite different from this. The astronomical refraction increases with decrease in temperature and increase in pressure. [1]. But even after using proper algorithms for the temperature and pressure, the timings for the sunset and sunrise cannot be predicted accurately because there is another factor which affects the refraction most. Andrew T. Young, who is a pioneer in the field of sunset science, has discussed the theoretical reasons behind the variation in the refraction. He has shown that at and below the astronomical horizon, the refraction depends primarily on atmospheric structure below the observer i.e. on the temperatures and pressures of different layers of the atmosphere through which the rays are passing. The refraction varies so much (tens of minutes of arc or even several degrees) that only very crude predictions can be made for the sunset or sunrise timings. [2].

Schaefer and Liller [3] studied the variation of refraction at sunset from the experimental measurements at different places. At Vina del Mar (Latitude 32.950^oS), where 77 observations were made, mean refraction at sunset was found to be 0.510^o with standard deviation of 0.048^o, minimum of 0.234^o and maximum of 0.667^o. Vina del Mar is a coastal city of Chile having a warm-summer Mediterranean climate. The measurements were done there in the months of Mar, Apr, Aug, Sep, Oct and Dec, the average temperature during which months are approximately 16, 14, 12, 13, 14 and 17^o C respectively. From the details given in their research paper, I have found that in Vina del Mar the refraction is greater than the nominal value of 0.567^o in only 8% cases.

Effect of seasonal variation in refraction was specifically found in another study. Sampson et al studied the variation in the refraction at both sunrise and sunset at Edmonton Canada (Latitude 53.526^{o}N), which has a continental sub-arctic climate. They analyzed the refraction values of 234 sunrises and 124 sunsets. They found that at sunset, the astronomical refraction had a mean of 0.579^o and standard deviation of 0.108^o. From one of the charts in their research paper, I have found that in Edmonton the refraction at sunset is greater than the nominal value of 0.567^o in about 35% cases. They found the seasonal variation in the refraction as shown in the table below. I have also added the values of the approximate monthly average temperature of Edmonton in their table.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number of observations	10	8	10	20	8	9	9	7	22	9	3	9
Mean (deg)	0.777	0.647	0.590	0.564	0.509	0.531	0.534	0.496	0.507	0.550	0.658	0.672
SD (deg)	0.143	0.051	0.046	0.102	0.020	0.029	0.019	0.036	0.029	0.034	0.143	0.091
Min. (deg)	0.568	0.593	0.528	0.443	0.482	0.502	0.507	0.442	0.449	0.512	0.571	0.578
Max. (deg)	1.108	0.755	0.661	0.894	0.538	0.586	0.560	0.543	0.563	0.613	0.823	0.801
Average Temp. (^{o}C)	-13	-9	-4	5	12	16	18	17	11	6	-4	-11

In their observations for sunrise they found a mean value of 0.714^o with standard deviation of 0.184^o, minimum value of 0.420^o and maximum of 2.081^o. Sunrise refraction was found by them to be, in general, greater than that at sunset and exhibiting more variability. The seasonal variation in the refraction at sunrise was similar to that at sunset but with all the values higher than those for the sunset. [4].

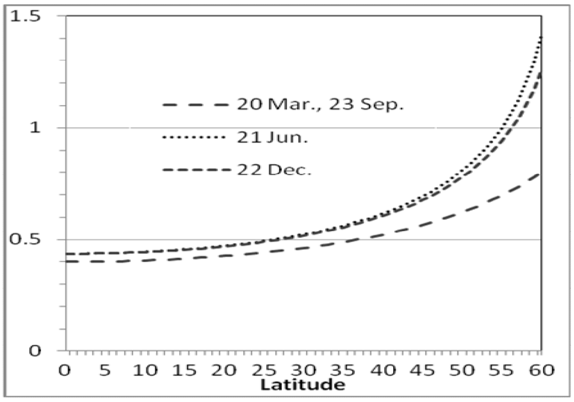

Fig.1. Advancement in sunset in min. if refraction is 0.467^o.

From the above observations at Vina del Mar and Edmonton, we can reach to the following inferences. (1) The refraction is high at low temperatures and vice versa. (2) In warmer climates e.g. tropical climates, the refraction would be less than the nominal value of 0.567^o on most of the days i.e. the sunset would be earlier than the expected. Of course this advancement would not be very large because the distribution of astronomical refraction values is positively skewed and the minimum refraction is very close to the mean. If the refraction is 0.1^o less than the nominal i.e. the refraction is 0.467, the amount of advancement in the sunset, worked out by me, for equinoxes on 20th Mar. and 23rd Sep., summer solstice on 21st Jun. and winter solstice on 22 Dec. (in the northern hemisphere) are shown in Fig.1. The advancement would not be noticeable for latitudes up to 45^o. (3) In colder climates, the mean value of refraction itself would be high and variation on the higher side would also be large i.e. at such places, on many

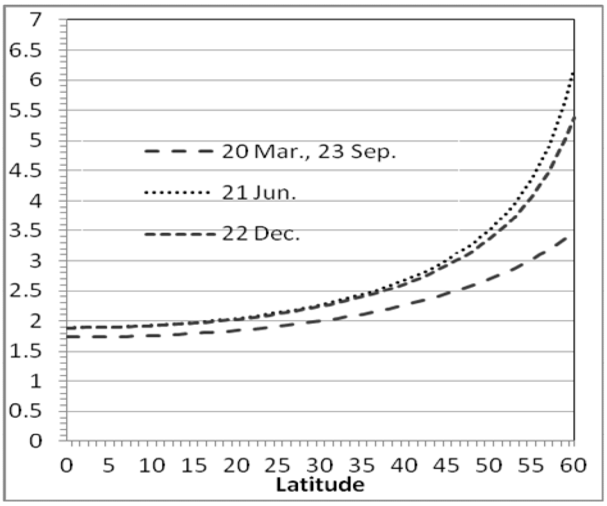

Fig.2. Delay in sunset in min. if refraction is 1.0^o.

days, the sunset would be delayed in comparison to the expected. If refraction is 1^o i.e. 0.433^o greater than the nominal value, the delay in the sunset would be as shown in Fig.2. This delay cannot be ignored. It must be remembered that on a few days of a year, the refraction might be greater than even 1^o in colder climates. (4) In the case of sunrise, the effects are opposite of and greater those for sunset i.e. sunrise is earlier due to larger refraction and delayed due to smaller refraction by larger amounts.

Error due to rounding off of the calculated time. All the computer programs can do calculations for timings in hours, minutes and seconds but when the results are to be shown in only hours and minutes then they round off the seconds to the nearest minute. For example, the sunset timings 18:11:31 and 18:12:29 of two different days will both be shown in hours and minutes as 18:12. In the former case sunset would be earlier than the calculated time while in the later case sunset would be later than the calculated time.

Minimum error in the watch/clock used. If a watch/clock is set to give right time in hours and minutes, an error of $\pm$ 30 seconds must always be considered.

Effect of the dip of the horizon: Let us first understand the outline of how a computer program is written in the case of sunrise and sunset timings. The following explanation is given only for the sunset. The reader can easily imagine the situations for the sunrise.
Sunset occurs when the 'apparent position' of the Sun's upper limb reaches the 'apparent horizon' of the observer. The apparent horizon is the apparent boundary between the Earth and sky. But the astronomers do calculations for astronomical horizon, which is defined as the imaginary horizontal plane always at a 90-degree angle from the observer's zenith. Zenith is the point in the celestial sphere overhead the observer.
The Sun's geometric position can of course be calculated with high accuracy by a computer. Hence, noon known as transit of the sun can be predicted with high precision. But in the case of sunset, Sun's apparent upper limb is affected by the refraction while the visible horizon is affected by the position of the observer and also by the refraction. Another difficult part has to do with just where the apparent horizon is. If it's a sea horizon, the apparent horizon depends also on the height of the waves. If it's a land horizon, irregularities in the topography determine the geometric position where the Sun disappears. The angle between astronomical horizon of the observer and the apparent horizon is called dip.
The computer programs written for finding the sunset time at a place first calculate the transit time at that place (i.e. when the centre of the sun's disc is in the prime meridian which is a vertical plane containing zenith). Then the time is calculated for the moment when the upper limb of the Sun will be observed at the astronomical horizon considering the nominal refraction. The calculations require a large number of rigorous steps. Even the results given by the US Naval Observatory's website [5], which are considered to be the most accurate, do calculations for the astronomical horizon and not the apparent horizon.[6]. Hence if we use these results for a practical situation, these will have discrepancies with the actually observed values due to the dip of the horizon. Further, since the horizon depends on refraction, an uncertainty has also to be considered in the observed values.

Correction for the dip of the horizon: There are four simple possibilities for the apparent horizon. The real situation might be quite complex.
(A). Observer at a height on sea shore. Refer to Fig.3. SGH is the surface representing the mean sea level and the observer's eye O is a height 'h' above the point S on the surface. I would prefer it to call height above local horizon level for a consistency with the other cases described below. C is the

centre and R is the radius of the Earth. The observer's astronomical horizon is the dashed line through O, perpendicular to the vertical line OC. (Of course, the height of the eye, and consequently the distance to the horizon, is greatly exaggerated in this diagram.) If we ignore refraction then the line of sight to the apparent horizon would be the straight line OG and the corresponding dip is d_g. Due to dip of the horizon, the sun will be visible on a longer diurnal path to the observer at O than the observer at S i.e. the sunset will be delayed at O than at S while the sunrise will be advanced.

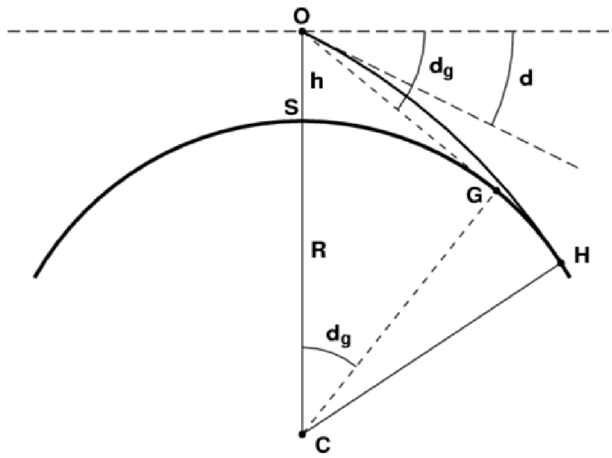

Fig.3. Dip of the Horizon

Due to refraction, the line of sight to the apparent horizon is not straight, but curved. The arc OH represents the curved line of sight; H is the (refracted) apparent horizon. Notice that the refracted dip ‘d’ between the horizontal (dashed) plane and the tangent to the curved line of sight is now less than the geometric dip d_g but the depression of the Sun corresponding to H would be greater than that for G i.e. sunset would be more delayed due to refraction and the sunrise would be more advanced. Also the range of horizon, represented by SH, is increased due to refraction.

It is very easy to calculate d_g and the linear distance OG to horizon by using geometry. But to calculate OH is difficult because the curvature of OH depends on the temperature variation in the atmospheric layers. Typical values used in practice for the horizon range is 3.86 km times the square root of the height in meters and the corresponding solar depression is 2.076′ times the square root of the height in meters. [6]. For a man having his eyes at 1.5 m from the ground, the horizon range would be 3.86 km multiplied by square root of 1.5 i.e. 4.72 km. For an observer at a height of 100m from the ground, the horizon range would be 38.6 km!

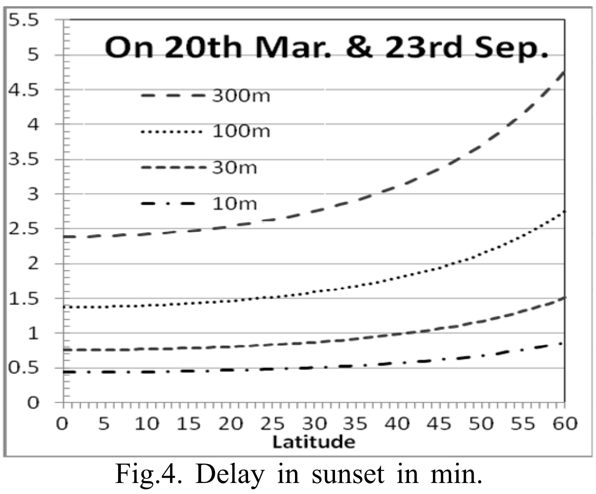

Fig.4. Delay in sunset in min. due to height of the observer.

Using the above mentioned formula for solar depression, the delay in the sunset for different latitudes from 0^o to 60^o as well as for different values of h on 20th Mar. / 23rd Sep., 21st Jun. and 22nd Dec. would be as shown in Fig.4, 5 and 6 respectively. These graphs are for the Northern hemisphere of the Earth. The effect of the height of the observer is the minimum on 21st Mar. /

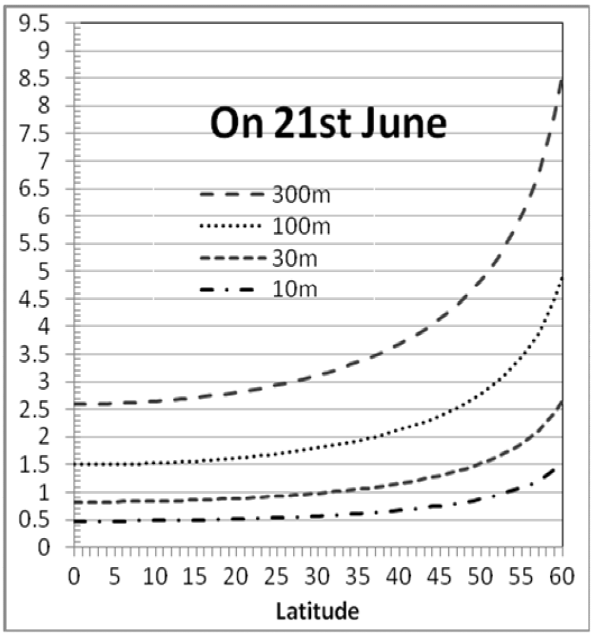

Fig.5. Delay in sunset in min. due to height of the observer.

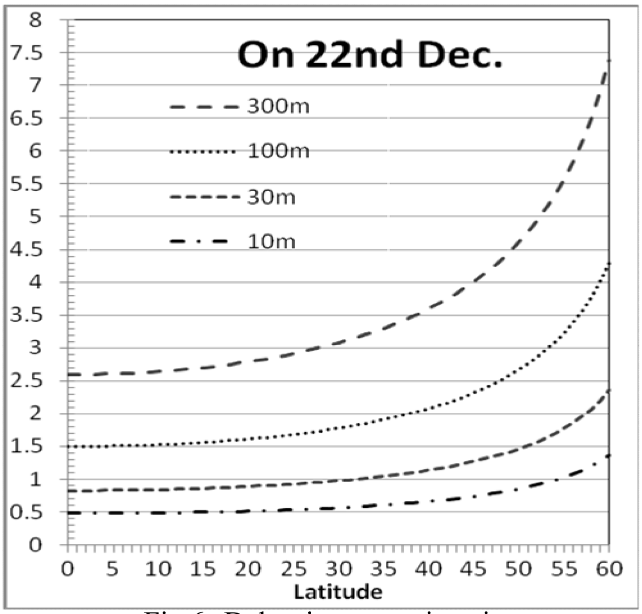

Fig.6. Delay in sunset in min. due to height of the observer.

23rd Sep. while it is a maximum on 23rd Jun. and is another maximum on 23rd Dec. For other days of the year it lies between the minimum and the relevant maximum. In the case of the Southern hemisphere, the graphs of 21st Mar. / 23rd Sep. will remain the same but the graphs of 23rd Jun. and 23rd Dec. would have to be swapped. These graphs can be used for any value of height by simply multiplying a value observed from the graph for the particular latitude with the square root of the ratio of the required height with the height shown on the relevant graph e.g. at latitude of 38^o, the delay in sunset for 'h' = 100m on 23rd Dec. is 2 min. Hence for h = 700m, the delay would be square root of 7 times 2 min i.e. 5.3 min.

This description is applicable to all the places on sea shore. Taking the example of Mumbai (latitude approximately 19^o), the sunset will be delayed by about 10 seconds for a person standing on a beach, by about half a minute for a person on the third floor of a building near the beach, by about one minute on the 10th storey of a tower near Mumbai central, by about one a half minutes for a person on Malabar Hills, by about two minutes on the 40th storey of a tower and by about three minutes on the 90th storey of an under construction tower .

The effects for similar heights in London (latitude approximately 51^o) would be much more than that in Mumbai. The tallest bldg. in London is about 300m high and on the topmost floor of that building, the delay in sunset would be by about 5 minutes than the ground floor of that building!

(B). Observer at a height over a plateau. Refer to Fig.7. In this case the ground level of the place and its apparent horizon are raised above mean sea level by the same amount i.e. SP = HQ. Hence the sunset for the ground level, which here we can call local horizon level, would be as predicted by the computer program. Therefore the delay in the sunset for the height of the observer in comparison to the ground level would have the same pattern as is described above for the case of places near seashore i.e. only h = OS has to be considered in determining the effect of the height on the delay in the sunset timing.

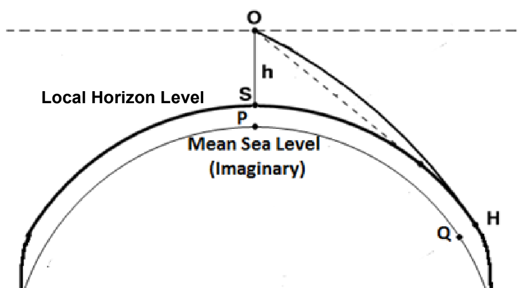

Fig.7. Observer at a height over a plateau

(C). Observer on a hill or a plain having uniform slope i.e. the observer is over a place the ground of which is at a bigger height above mean sea level than that of the horizon. Refer to Fig.8. The dashed curve represents the local horizon level of the place. If h' = SP-HQ is the height of the ground level of the place above

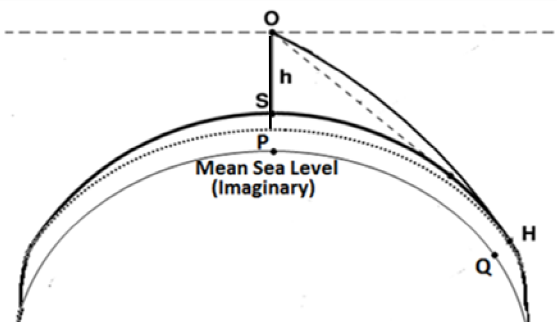

Fig.8. Observer on a hill

the local horizon level and if the eyes of the observer are at height OS = h” above the ground level then the effective height of the eyes of the observer above the horizon level is h = h’+ h” and the calculation for the delay in the sunset time must be done for the effective height h using the formula given in the case (A). It must be remembered that the direction of sunset goes on changing day to day through a year and therefore the value of h may also go on changing accordingly.

(D). Observer in a valley i.e. observer at a place where there is a hill at the horizon in the direction of the setting sun. Refer to Fig.9. In this case the sun goes behind the hill before reaching to the astronomical horizon. As per the previous discussions, the upper limb of the sun going behind the hill can be considered as sunset. But psychologically and for that reason for religious purposes, sunset is estimated for the imaginary situation in which the ground level is plane up to the horizon. If this is the consideration, h = OS must be used in the formula for calculating delay in the sunset timing and the graphs shown in the case (A) can be used for estimating the correction.

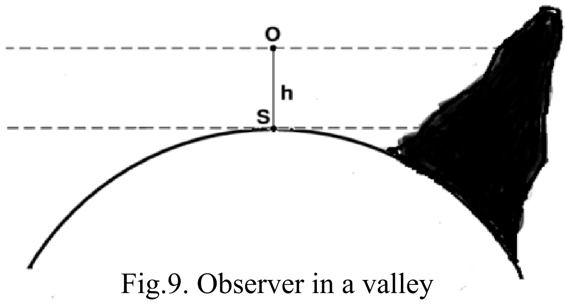

Fig.9. Observer in a valley

Conclusion: Although transit of the Sun (i.e. noon) can be predicted with high accuracy, the prediction of sunrise and sunset time has uncertainty of few minutes of clock. This is because the refraction of sunrays at sunrise and sunset depend on temperature, pressure and variation in these two across the layers of the atmosphere. The uncertainty in the sunrise time is much greater than that in the sunset time because of the lower temperature before sunrise than at the sunset and the greater temperature gradient in the atmosphere before sunrise. The uncertainty is much smaller on warmer days than on colder days at a particular place and uncertainty is much greater in arctic and subarctic regions than in the tropical regions. In fact, in the tropical climate the sunset, on most of the days of a year, is a few seconds before the time predicted by the computer programs. If in a tropical climate, we want to be sure of sunset for religious or any other reason, we must allow a gap of at least two minute after the predicted time—one min. of this for the errors of rounding off of the computed time and the minimum uncertainty in the reading of the watch/clock and the other min. for the approximate correction for ordinary height of our eyes (up to 10 m) above the local horizon level and an increase in refraction up to 0.1^{o} above the nominal value during colder days. If the height of our eyes above the local horizon level is large enough, then a calculated correction for the height must also be added. The graphs given in this article can be used for the calculation of the minimum and maximum values of the correction. In other types of climates the computed values of the sunset and sunrise timings must be used with caution because the uncertainty and the correction are quite large. The correction can be estimated by using the graphs given in this article but for estimating the uncertainty, each place requires to be considered separately for each season.

Refrences:

1. Jean Meeus, *Astronomical Algorithms*, Willmann-Bell Inc., 1991.

2. Andrew T. Young, *Sunset Science. IV. Low-Altitude Refraction*, The Astronomical Journal, 127:3622–3637, June 2004.

3. B.E. Schaefer and W. Liller, *Refraction Near the Horizon,* Publications of the Astronomical Society of the Pacific, 102:796-805, 1990 July.

4. Russell D. Sampson et al., *Variability in the Astronomical Refraction of the Rising and Setting Sun*, Publications of the Astronomical Society of the Pacific, 115:1256–1261, 2003 October.

5. http://www.usno.navy.mil/USNO/astronomical-applications

6. A.T. Young, *Atmospheric Refraction,* http://mintaka.sdsu.edu/GF/explain/atmos_refr/astr_refr.html