DoNRS: Homework #2

Due on 27 September

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1 Problem Description

In this assignment, we were asked to develop kinematic model of manipulator, solve forward kinematics problem and solve inverse kinematics problem. The manipulator chosen for the assignment is KUKA KR 10 R1100-2. The robot has 6 revolute joints.

2 Robot Kinematic Scheme

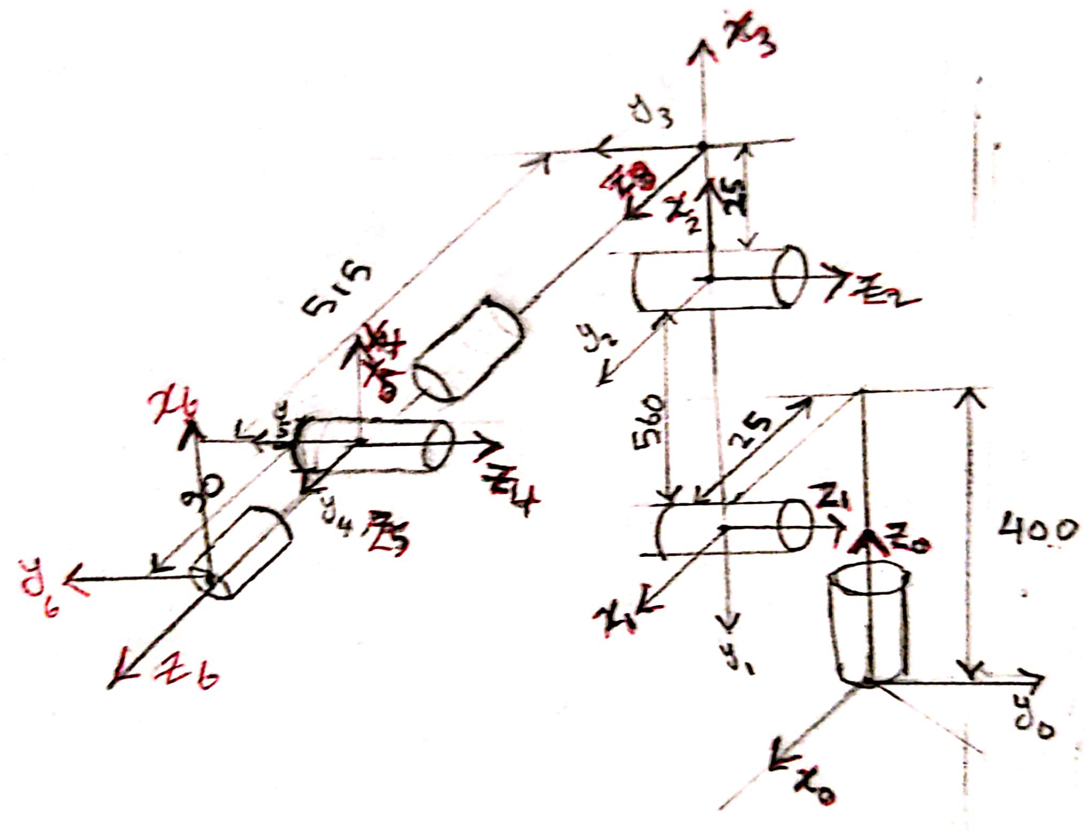

Figure 1: Robot Kinematic Scheme

3 Forward Kinematics

Forward kinematics are can be described with the following equation :

$$H = T_z(400)R_z(q_1)T_x(25)R_y(q_2)T_x(560)R_y(q_3)T_z(25)T_x(515)R_x(q_4)R_y(q_5)R_x(q_6)T_x(90)$$

To bulid the forward kinematics, we used DH parameters described in the table (1) and MATLAB 2019 to derive the equations.

	θ	d	a	α
1	θ_1	400	25	-90
2	θ_2	0	560	0
3	θ_3	0	25	-90
4	θ_4	515	0	90
5	θ_5	0	0	-90
6	θ_6	90	0	0

Table 1: DH Parameters

$$FK = \begin{pmatrix} n_x & s_x & r_x & p_x \\ n_y & s_y & r_y & p_y \\ n_z & s_z & r_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1}$$

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Px =cos(q1)/40 + (14*cos(q1)*cos(q2))/25 - (9*sin(q5)*(sin(q1)*sin(q4)+
    cos(q4)*(cos(q1)*cos(q2)*cos(q3)- cos(q1)*sin(q2)*sin(q3))))/100-
    (9*cos(q5)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))/100+
    (cos(q1)*cos(q2)*cos(q3))/40 - (103*cos(q1)*cos(q2)*sin(q3))/200-
    (103*cos(q1)*cos(q3)*sin(q2))/200 - (cos(q1)*sin(q2)*sin(q3))/40

Py =sin(q1)/40 + (14*cos(q2)*sin(q1))/25 + (9*sin(q5)*(cos(q1)*sin(q4) +
   cos(q4)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1))))/100 -
   (9*cos(q5)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))/100 -
   (sin(q1)*sin(q2)*sin(q3))/40 + (cos(q2)*cos(q3)*sin(q1))/40 -
   (103*cos(q2)*sin(q1)*sin(q3))/200 - (103*cos(q3)*sin(q1)*sin(q2))/200

Pz =(103*sin(q2)*sin(q3))/200 - (103*cos(q2)*cos(q3))/200 - (cos(q2)*sin(q3))/40 -
    (cos(q3)*sin(q2))/40 - (14*sin(q2))/25 - (9*cos(q5)*(cos(q2)*cos(q3)-
    sin(q2)*sin(q3)))/100 + (9*cos(q4)*sin(q5)*(cos(q2)*sin(q3) + cos(q3)*sin(q2)))/100 + 2/5

nx =sin(q6)*(cos(q4)*sin(q1) - sin(q4)*(cos(q1)*cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3))) +
    cos(q6)*(cos(q5)*(sin(q1)*sin(q4) + cos(q4)*(cos(q1)*cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3))) -
    sin(q5)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))

ny = - sin(q6)*(cos(q1)*cos(q4) - sin(q4)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1))) -
    cos(q6)*(cos(q5)*(cos(q1)*sin(q4) + cos(q4)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1))) +
    sin(q5)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2)))

nz = sin(q4)*sin(q6)*(cos(q2)*sin(q3) + cos(q3)*sin(q2)) - cos(q6)*(sin(q5)*(cos(q2)*cos(q3) -
     sin(q2)*sin(q3)) + cos(q4)*cos(q5)*(cos(q2)*sin(q3) + cos(q3)*sin(q2)))
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sx = cos(q6)*(cos(q4)*sin(q1) - sin(q4)*(cos(q1)*cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3))) -
     sin(q6)*(cos(q5)*(sin(q1)*sin(q4) + cos(q4)*(cos(q1)*cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3)))-
     sin(q5)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2)))

sy = sin(q6)*(cos(q5)*(cos(q1)*sin(q4) + cos(q4)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1)))+
     sin(q5)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2))) -
     cos(q6)*(cos(q1)*cos(q4) - sin(q4)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1)))

sz =sin(q6)*(sin(q5)*(cos(q2)*cos(q3) - sin(q2)*sin(q3)) + cos(q4)*cos(q5)*(cos(q2)*sin(q3) +
     cos(q3)*sin(q2))) + cos(q6)*sin(q4)*(cos(q2)*sin(q3) + cos(q3)*sin(q2))

rx = - sin(q5)*(sin(q1)*sin(q4) + cos(q4)*(cos(q1)*cos(q2)*cos(q3) - cos(q1)*sin(q2)*sin(q3)))-
    cos(q5)*(cos(q1)*cos(q2)*sin(q3) + cos(q1)*cos(q3)*sin(q2))

ry = sin(q5)*(cos(q1)*sin(q4) + cos(q4)*(sin(q1)*sin(q2)*sin(q3) - cos(q2)*cos(q3)*sin(q1))) -
    cos(q5)*(cos(q2)*sin(q1)*sin(q3) + cos(q3)*sin(q1)*sin(q2))

rz = cos(q4)*sin(q5)*(cos(q2)*sin(q3) + cos(q3)*sin(q2)) - cos(q5)*(cos(q2)*cos(q3) - sin(q2)*sin(q3))
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4 Inverse Kinematics

4.1 Position

In solving the IK problem, we know the matrix T_0^6. by writing :

$$T_0^6 = T_0^5 T_5^6$$
$$T_0^6 (T_5^6)^{-1} = T_0^5$$

and we know that :

$$T_5^6 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.09 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

because the end effector frame has the same orientation with frame 5 with only translation on z (along the link). then we can find T_0^5 and after that we write :

$$P_0^5 = T_0^5 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

we know that $P_0^5 = P_0^4$, therefore :

$$P_0^4 = T_0^3 T_3^4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = T_0^3 P_3^4$$

from our robot we know that P_3^4 doesn't depend on q_4 therefore we find P_0^4 from our model then we solve:

$$P_0^4 = f(q_1, q_2, q_3)$$

4.2 Orientation

After finding q_1, q_2, q_3, we're able to find the matrix R_0^3 and we already know R_0^6 therefore, we write :

$$R_0^6 = R_0^3 R_3^6$$
$$R_3^6 = (R_0^3)^{-1} R_0^6$$

then we solve: $R_3^6 = f(q_4, q_5, q_6)$ to find q_4, q_5, q_6 using Euler angles equation as stated in code.

5 GitHub

Link to assignment : HW2