DoNRS: Homework #3

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In this task, forward and inverse kinematics equations were derived for RRP robot. Jacobian matrix was computed in three different approaches and was analyzed for singularities. Finally, the velocity vector of the tool frame was computed and plotted.

1 Forward Kinematics

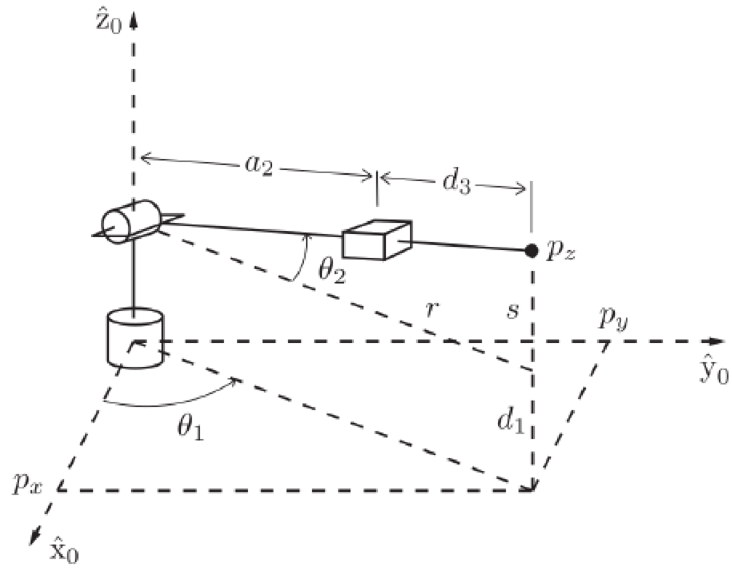

Figure 1: RRP Robot

From figure(1) we see that p_x and p_y can be described by :

$$p_x = rcos\theta_1 \tag{1}$$

$$p_y = rsin\theta_1 \tag{2}$$

where : $r = (a_2 + d_3)cos\theta_2$. To compute p_z we use :

$$p_z = d_1 - s \tag{3}$$

where : $s = (a_2 + d_3)sin\theta_2$.
By using all the previous equations together we get the vector X that describes the position of the end effector in the workspace.

$$X = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} (a_2 + d_3)cos\theta_2 cos\theta_1 \\ (a_2 + d_3)cos\theta_2 sin\theta_1 \\ d_1 - (a_2 + d_3)sin\theta_2 \end{pmatrix} \tag{4}$$

2 Inverse Kinematics

Knowing p_x, p_y and p_z we try to find the q vector.
From the top-view of the robot we can find θ_1. There are two solutions :

$$\theta_1 = atan2(p_y, p_x) \tag{5}$$
$$\theta_1 = \pi + atan2(p_y, p_x) \tag{6}$$

θ_2 could be found from the side-view of the robot.

$$\theta_2 = atan2(s, r) \tag{7}$$
$$\theta_2 = \pi - atan2(s, r) \tag{8}$$

where the first solution corresponds to the first solution of θ_1 and the second corresponds to the second solution of θ_1. From the side-view we can also find d_3.

$$d_3 = \sqrt{r^2 + s^2} - a_2 \tag{9}$$

The 1st solution:

$$\begin{aligned} \theta_1 &= atan2(p_y, p_x) \\ \theta_2 &= atan2(s, r) \\ d_3 &= \sqrt{r^2 + s^2} - a_2 \end{aligned}$$

The 2nd solution:

$$\begin{aligned} \theta_1 &= \pi + atan2(p_y, p_x) \\ \theta_2 &= \pi - atan2(s, r) \\ d_3 &= \sqrt{r^2 + s^2} - a_2 \end{aligned}$$

if we allow d_3 to be negative we will have two extra solutions for θ_2.
The 3rd solution:

$$\begin{aligned} \theta_1 &= atan2(p_y, p_x) \\ \theta_2 &= \pi + atan2(s, r) \\ d_3 &= -\sqrt{r^2 + s^2} - a_2 \end{aligned}$$

The 4th solution:

$$\begin{aligned} \theta_1 &= \pi + atan2(p_y, p_x) \\ \theta_2 &= -atan2(s, r) \\ d_3 &= -\sqrt{r^2 + s^2} - a_2 \end{aligned}$$

3 Jacobian Matrix

The Jacobian matrix consists of two parts, Linear velocity Jacobian J_v and the Angular velocity Jacobian J_ω.

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

The Jacobian Matrix is derived using three different approaches, Analitycal, Geometric and numerical approach by using MATLAB.
In the Analitycal approach we find J_v using partial derivatives of the vector X derived from the forward kinematics.

$$J_v = \begin{bmatrix} \frac{\partial X_1}{\partial \theta_1} & \frac{\partial X_1}{\partial \theta_2} & \frac{\partial X_1}{\partial d_3} \\ \frac{\partial X_2}{\partial \theta_1} & \frac{\partial X_2}{\partial \theta_2} & \frac{\partial X_2}{\partial d_3} \\ \frac{\partial X_3}{\partial \theta_1} & \frac{\partial X_3}{\partial \theta_2} & \frac{\partial X_3}{\partial d_3} \end{bmatrix} = \begin{bmatrix} -(a_2+d_3)cos\theta_2 sin\theta_1 & -(a_2+d_3)sin\theta_2 cos\theta_1 & cos\theta_2 cos\theta_1 \\ (a_2+d_3)cos\theta_2 cos\theta_1 & -(a_2+d_3)sin\theta_2 sin\theta_1 & cos\theta_2 sin\theta_1 \\ 0 & -(a_2+d_3)cos\theta_2 & -sin\theta_2 \end{bmatrix}$$

Results of the three methods are shown in code.

4 Singularities

To analyze Jacobian, we compute the determinant of the J_v matrix and find the values that make the determinant equals zero.

$$det(J_v) = -cos\theta_2(d_3+a_2)^2 = 0$$

we see that the determinant equals zero when:
1) $cos\theta_2 = 0$ which means that $\theta_2 = \pm\frac{\pi}{2} + \pi k$. In this case both p_x and p_y are equal to zero. This means that the end-effector point intersects z_0 axis, and any rotation of the first joint will not affect the end-effector position. This singular case is represented by the group of points $(0, 0, p_z)$.
2)$d_3 = -a_2$, we know that $d_3 = \sqrt{r^2+s^2} - a_2$ which means that both r and s is 0. Since $r = \sqrt{p_x^2+p_y^2} = 0$ and $s = d_1 - p_z = 0$, p_x and p_y are equal to zero and $p_z = d_1$. we see that this case is similar to the previous case. In fact it's a point from the group of points described in the first case.

5 Tool Frame Velocities

To compute the Tool Frame Velocities, we use :

$$\dot{X} = J\dot{q} \tag{10}$$

Joints variables are changing with time and are described by :

$$\theta_1(t) = sint$$
$$\theta_2(t) = cos2t$$
$$d_3(t) = sin3t$$

we differentiate the equations and substitute them in equation(10) to compute $\dot{X}$ vector.

$$\dot{\theta_1}(t) = cost$$
$$\dot{\theta_2}(t) = -2sin2t$$
$$\dot{d_3}(t) = 3cos3t$$

by using MATLAB, we plot the linear and angular velocities of the tool frame with respect to time.

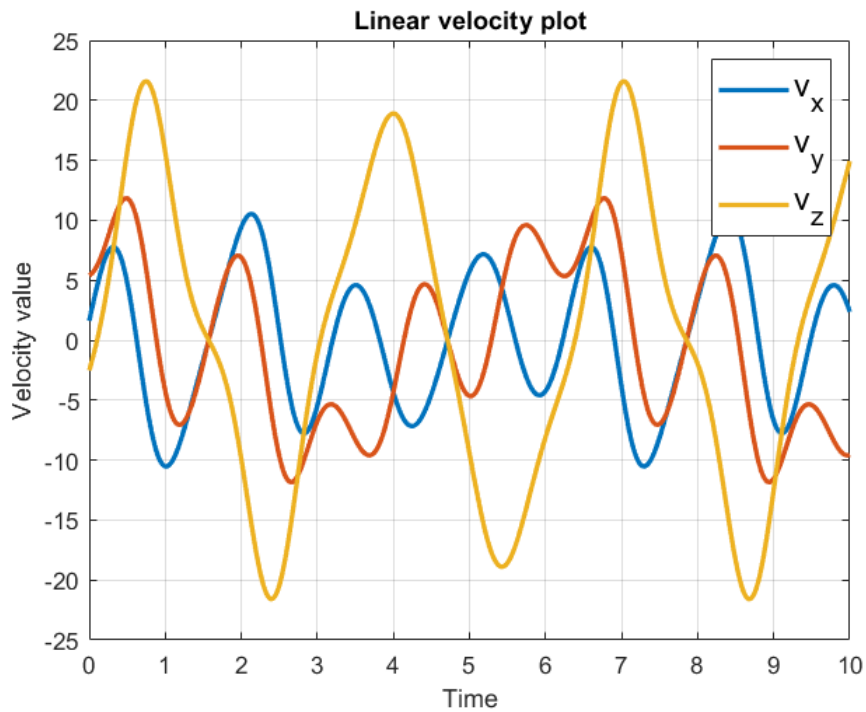

Figure 2: Linear Velocities

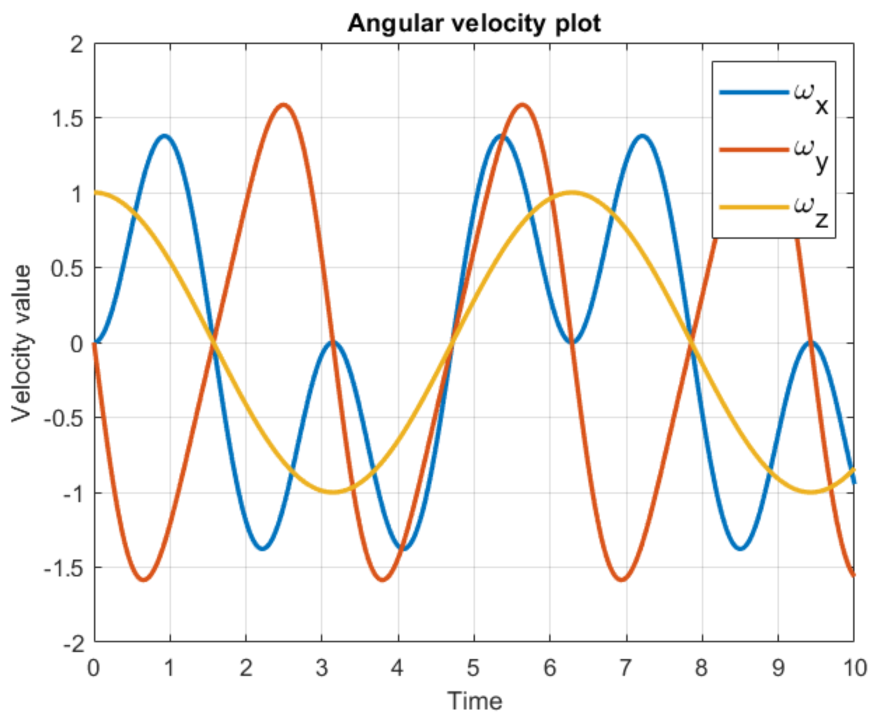

Figure 3: Angular Velocities