

DoNRS: Homework #5

Due on 25 October

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1 Task1

In this assignment, we were asked to Solve direct dynamic problem using Lagrange-Euler method for the robot in figure (1), supposing that the robot located in a vertical plane.

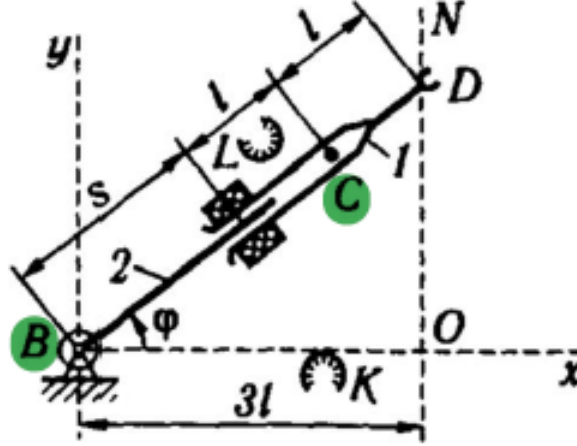


Figure 1: RP Robot

To solve this task, we start by finding the center of mass for each link, here it's B and C. But we notice that the center of mass for the first link is at the origin, so we find O_{c2}

$$O_{c2} = \begin{pmatrix} \cos(q_1) (L + q_2) \\ \sin(q_1) (L + q_2) \\ 0 \end{pmatrix}$$

Then we need to find the linear and rotational jacobian for each joint.

To get the dynamic equation in matrix form we need to find D , C and G matrices.

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau(t)$$

We compute the kinetic energy for each link and sum them to find the D matrix.

$$D_1 = m_1 J_{v1}^T J_{v1} + J_{w1}^T R_1 I_1 R_1^T J_{w1}$$

$$D_2 = m_2 J_{v2}^T J_{v2} + J_{w2}^T R_2 I_2 R_2^T J_{w2}$$

$$D =$$

$$\begin{pmatrix} m_2 L^2 + 2 m_2 L q_2 + m_2 q_2^2 + I_1 + I_2 & 0 \\ 0 & m_2 \end{pmatrix}$$

For the potential energy, we see that only the second link has a potential energy which is:

$$P_2 = m_2 g ((q_2 + L) \sin q_1)$$

For the G matrix we differentiate the potential energy with respect to q_1 and q_2 .

$$\mathbf{G} = \begin{pmatrix} g m_2 \cos(q_1) (L + q_2) \\ g m_2 \sin(q_1) \end{pmatrix}$$

To find the C matrix, we use the following formula :

$$\sum_{j=1}^n \sum_{i=1}^n c_{ijk}(q) \dot{q}_i \dot{q}_j$$

$$c_{ijk}(q) = \frac{1}{2} \left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right)$$

$$\mathbf{C} = \begin{pmatrix} \dot{q}_2 m_2 (L + q_2) & \dot{q}_1 m_2 (L + q_2) \\ \dot{q}_1 m_2 (L + q_2) & 0 \end{pmatrix}$$

The final torques are :

$$\mathbf{tor} = \begin{pmatrix} \ddot{q}_1 (m_2 L^2 + 2 m_2 L q_2 + m_2 q_2^2 + I_1 + I_2) + g m_2 \cos(q_1) (L + q_2) + 2 \dot{q}_1 \dot{q}_2 m_2 (L + q_2) \\ m_2 (L + q_2) \dot{q}_1^2 + \ddot{q}_2 m_2 + g m_2 \sin(q_1) \end{pmatrix}$$

Then we apply some input on our dynamic model to compute the changes of q & \dot{q} & \ddot{q} against time. We start with initial position $q = [\frac{\pi}{2} \ 0]$ and zero input. we notice in figure(2) that the first joint stays at its value and the second start decreasing because of gravity .

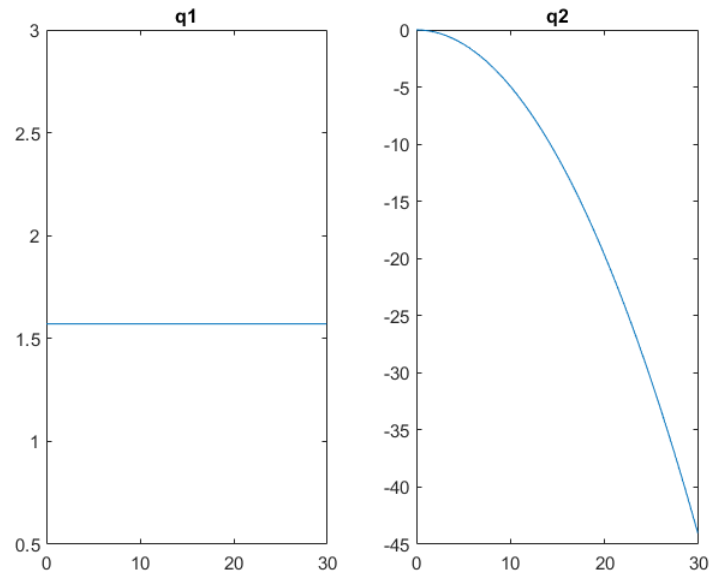


Figure 2: Zero input

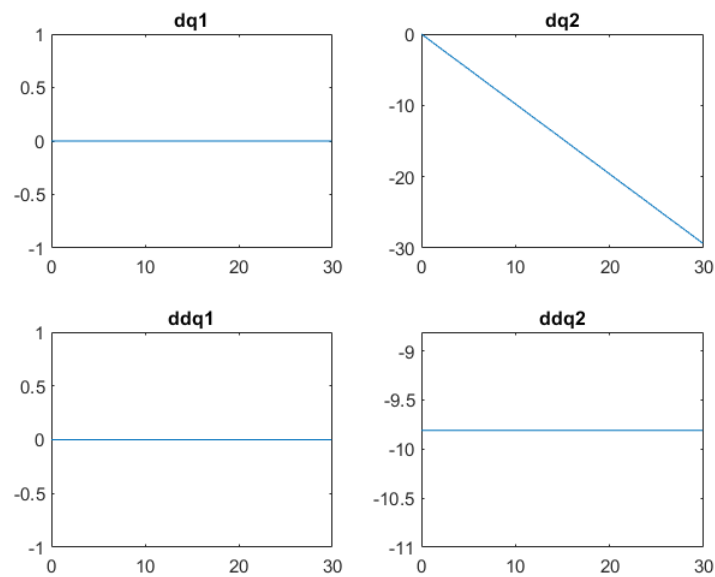


Figure 3: Zero input

For the initial position $q = [0 \ 0]$ and zero input in figure(4) we see that q_1 start decreasing and settles on $-\frac{\pi}{2}$ and q_2 start increasing because of gravity.

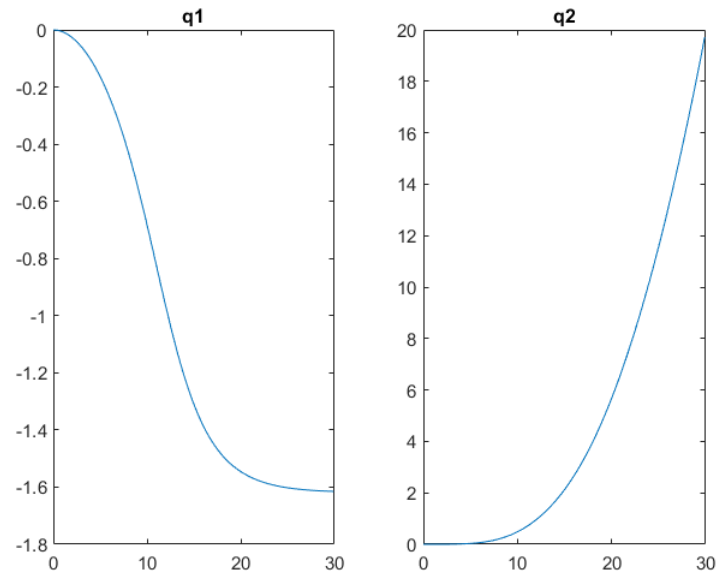


Figure 4: Zero input

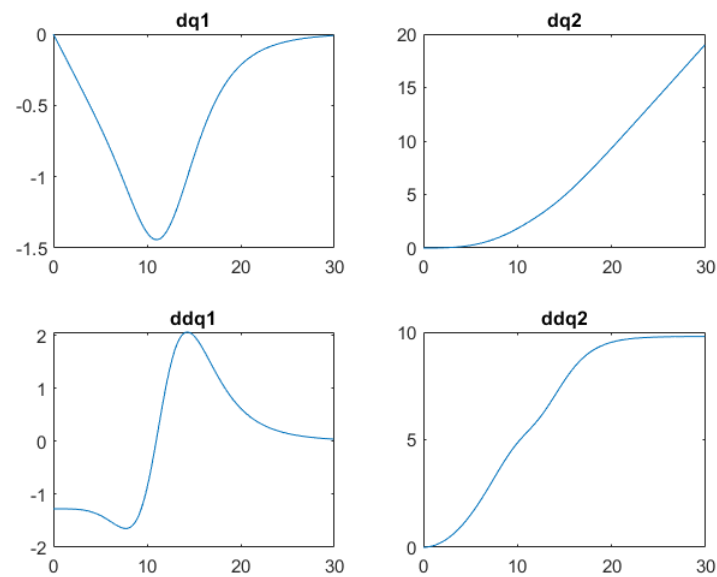
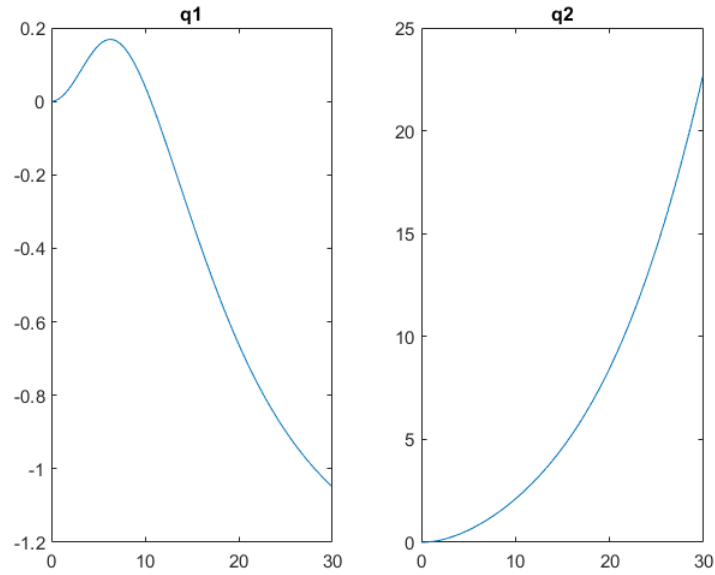
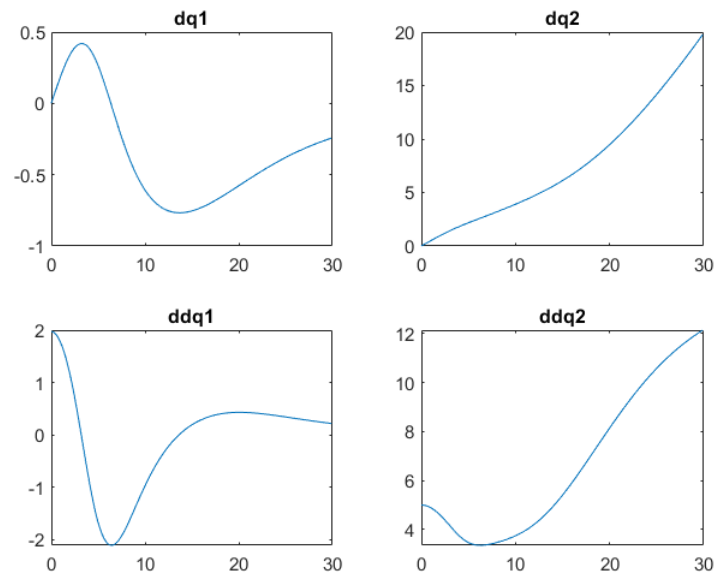
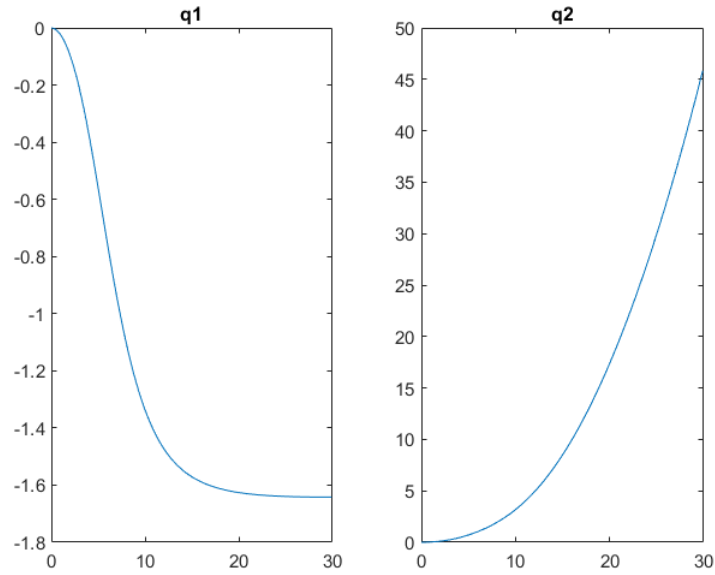
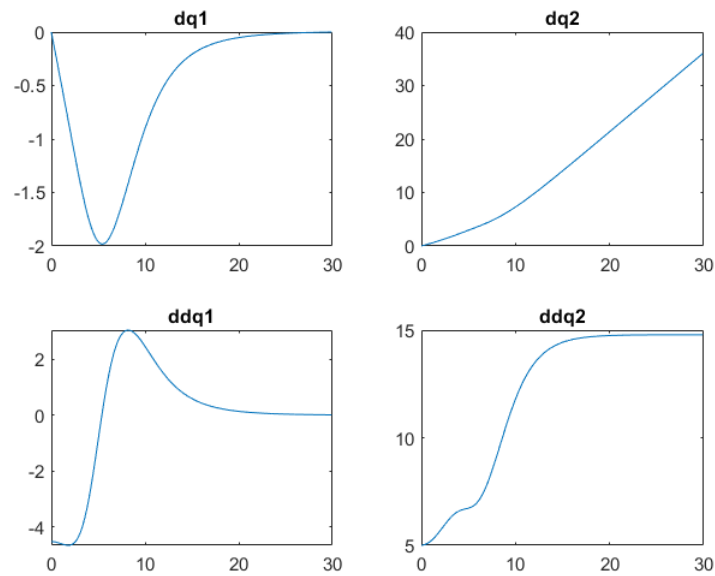


Figure 5: Zero input

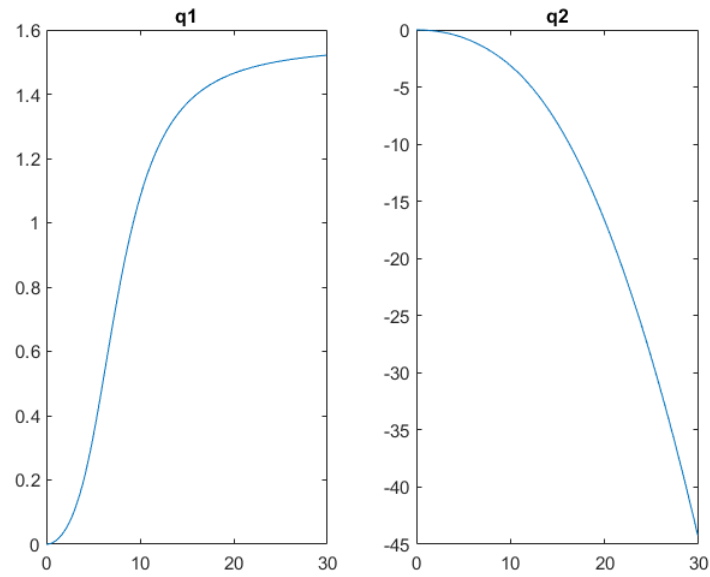
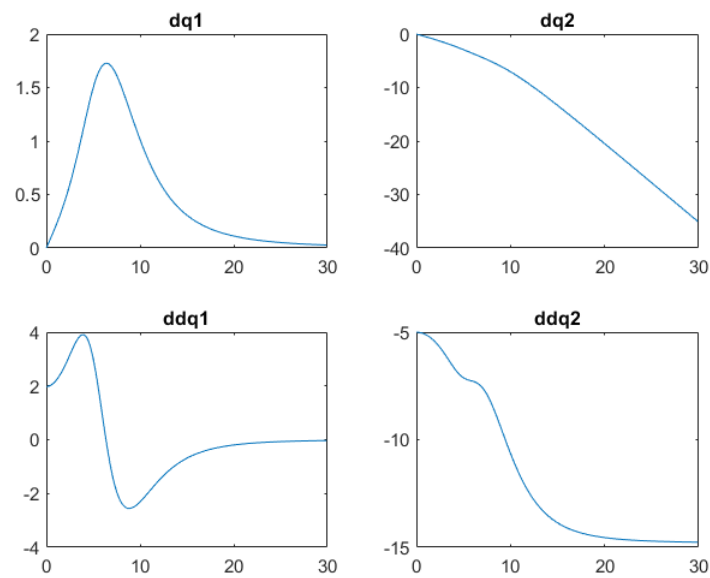
For the initial position $q = [0 \ 0]$ and input $U=[10 \ 10]$ in figure (6) we see that q_1 start increasing at first then start again decreasing because of gravity and q_2 start increasing.

Figure 6: $U=[10 \ 10]$ Figure 7: $U=[10 \ 10]$

For the initial position $q = [0 \ 0]$ and input $U=[-10 \ 10]$ in figure (8) we see that q_1 start decreasing until it seems to settle on $-\frac{\pi}{2}$ and it's faster than when it was zero torque , q_2 start increasing because of the force on the link. q_2 changes the center of mass for whole robot until the torque won't able to rotate the joint anymore.

Figure 8: $U = [-10 \ 10]$ Figure 9: $U = [-10 \ 10]$

For the initial position $q = [0 \ 0]$ and input $U = [10 \ -10]$ in figure (10) we see that q_1 start increasing because here the force and the torque together help to rotate the link. q_2 start decreasing because of the force on the link and keep decreasing .

Figure 10: $U=[10 \ -10]$ Figure 11: $U=[10 \ -10]$

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Link to assignment : HW5