DoNRS: Homework #5

Due on 25 October

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1 Task1

In this assignment, we were asked to Solve direct dynamic problem using Lagrange-Euler method for the robot in figure (1), supposing that the robot located in a vertical plane.

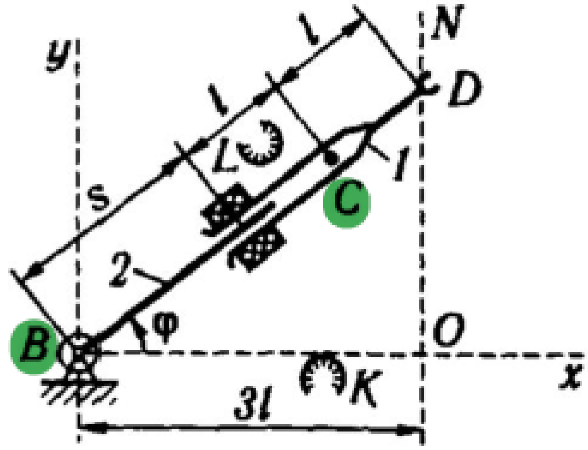

Figure 1: RP Robot

To solve this task, we start by finding the center of mass for each link, here it's B and C. But we notice that the center of mass for the first link is at the origin, so we find O_{c2}

Oc2 =

$$\begin{pmatrix} \cos(q_1)\ (L+q_2) \\ \sin(q_1)\ (L+q_2) \\ 0 \end{pmatrix}$$

Then we need to find the linear and rotational jacobian for each joint.
To get the dynamic equation in matrix form we need to find D , C and G matrices.

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau(t)$$

We compute the kinetic energy for each link and sum them to find the D matrix.

$$D_1 = m_1 J_{v1}^T J_{v1} + J_{w1}^T R_1 I_1 R_1^T J_{w1}$$

$$D_2 = m_2 J_{v2}^T J_{v2} + J_{w2}^T R_2 I_2 R_2^T J_{w2}$$

D =

$$\begin{pmatrix} m_2 L^2 + 2\, m_2\, L\, q_2 + m_2\, q_2{}^2 + I_1 + I_2 & 0 \\ 0 & m_2 \end{pmatrix}$$

For the potential energy, we see that only the second link has a potential energy which is:

$$P_2 = m_2 g((q_2 + L) sin q_1)$$

For the G matrix we differentiate the potential energy with respect to q_1 and q_2.

G =

$$\begin{pmatrix} g\, m_2 \cos(q_1)\ (L + q_2) \\ g\, m_2 \sin(q_1) \end{pmatrix}$$

To find the C matrix, we use the following formula :

$$\sum_{j=1}^{n} \sum_{i=1}^{n} c_{ijk}(q) \dot{q}_i \dot{q}_j$$

$$c_{ijk}(q) = \frac{1}{2} \left(\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right)$$

C =

$$\begin{pmatrix} \mathrm{dq}_2\, m_2\ (L + q_2) & \mathrm{dq}_1\, m_2\ (L + q_2) \\ \mathrm{dq}_1\, m_2\ (L + q_2) & 0 \end{pmatrix}$$

The final torques are :

tor =

$$\begin{pmatrix} \mathrm{ddq}_1\ \left(m_2 L^2 + 2\, m_2\, L\, q_2 + m_2\, {q_2}^2 + I_1 + I_2\right) + g\, m_2 \cos(q_1)\ (L + q_2) + 2\, \mathrm{dq}_1\, \mathrm{dq}_2\, m_2\ (L + q_2) \\ m_2\ (L + q_2)\, {\mathrm{dq}_1}^2 + \mathrm{ddq}_2\, m_2 + g\, m_2 \sin(q_1) \end{pmatrix}$$

Then we apply some input on our dynamic model to compute the changes of q & $\dot{q}$ & $\ddot{q}$ against time.
We start with initial position $q = [\frac{\pi}{2}\ 0]$ and zero input. we notice in figure(2) that the first joint stays at its value and the second start decreasing because of gravity .

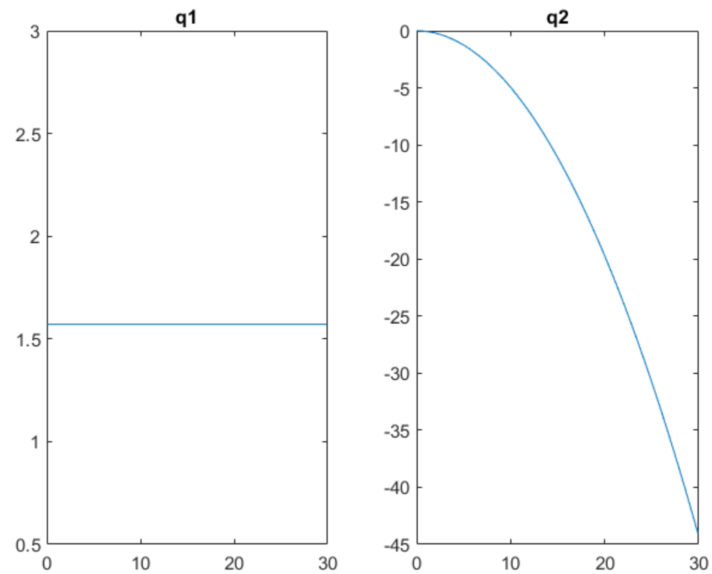

Figure 2: Zero input

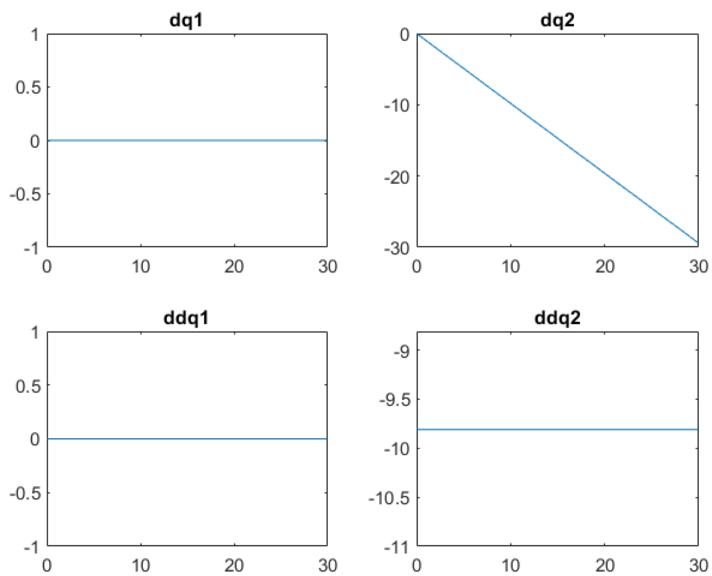

Figure 3: Zero input

For the initial position $q = [0\ 0]$ and zero input in figure(4) we see that q_1 start decreasing and it seems that it settles on $\frac{-\pi}{2}$ and q_1 start increasing.

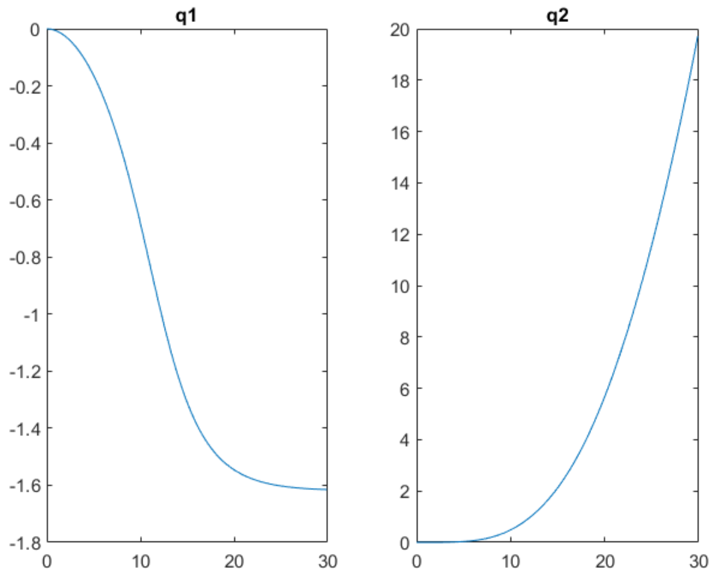

Figure 4: Zero input

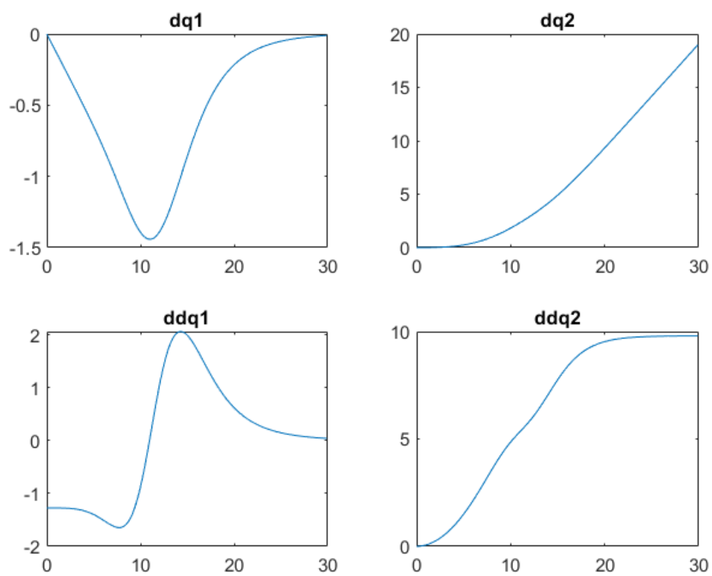

Figure 5: Zero input

For the initial position $q = [0\ 0]$ and input U=[10 10] in figure (6) we see that q_1 start increasing at first then start again decreasing and q_2 start increasing because of the force on the link.

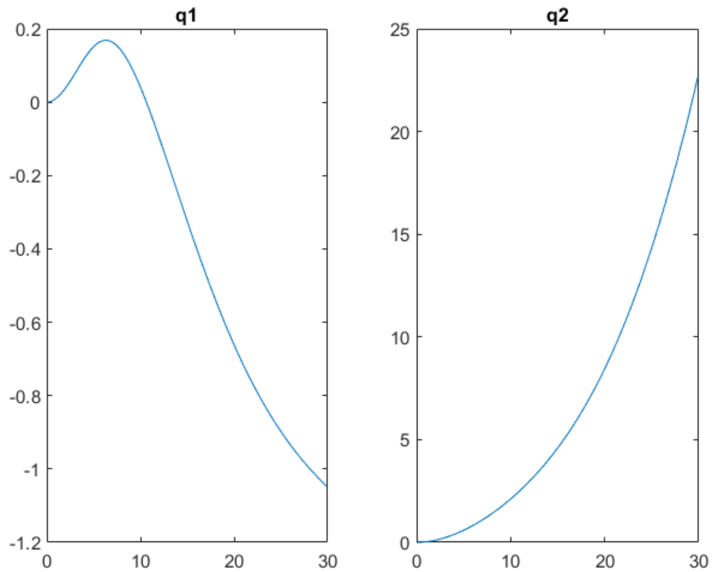

Figure 6: U=[10 10]

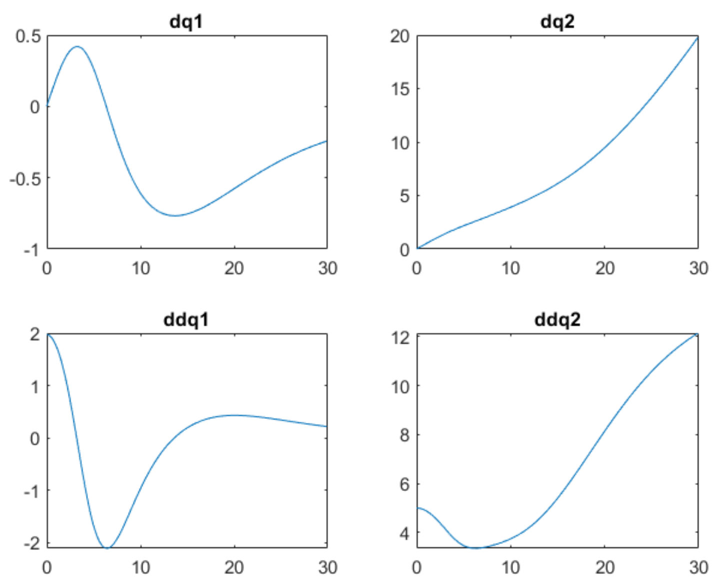

Figure 7: U=[10 10]

For the initial position $q = [0\ 0]$ and input U=[-10 10] in figure (8) we see that q_1 start decreasing until it seems to settle on $\frac{-\pi}{2}$ and it's faster than when it was zero torque , q_2 start increasing because of the force on the link.

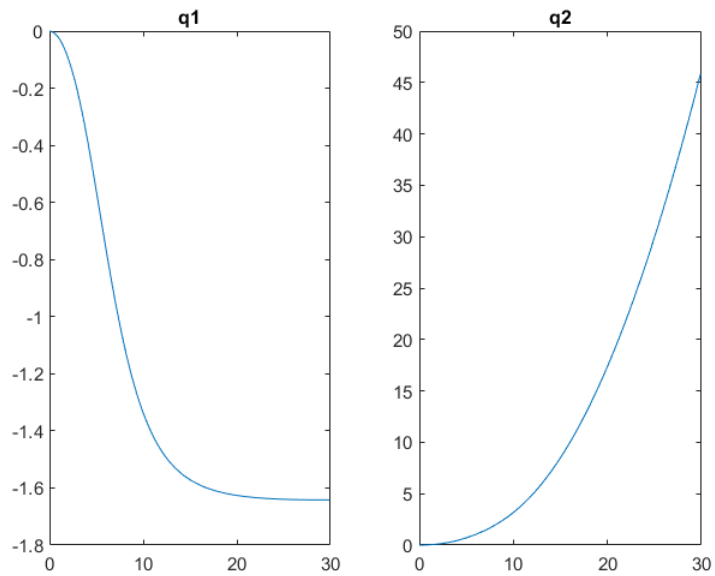

Figure 8: U=[-10 10]

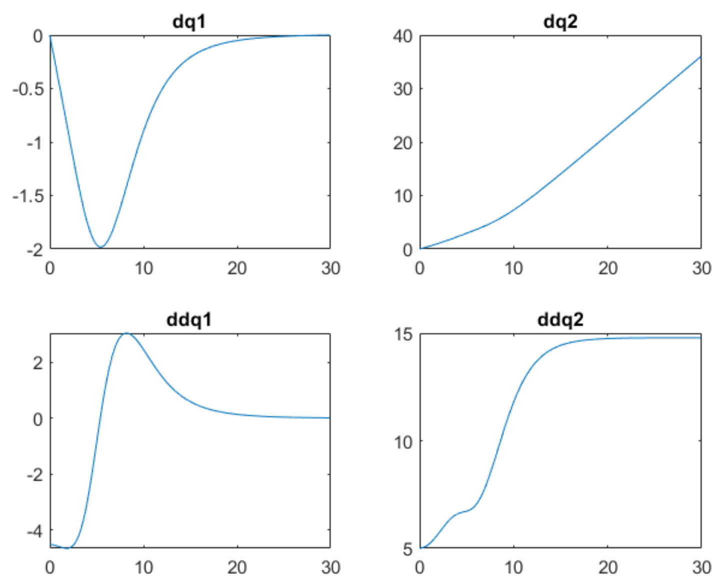

Figure 9: U=[-10 10]

For the initial position $q = [0\ 0]$ and input U=[10 -10] in figure (10) we see that q_1 start increasing because here the force and the torque together help to rotate the link. q_2 start decreasing because of the force on the link and keep decreasing .

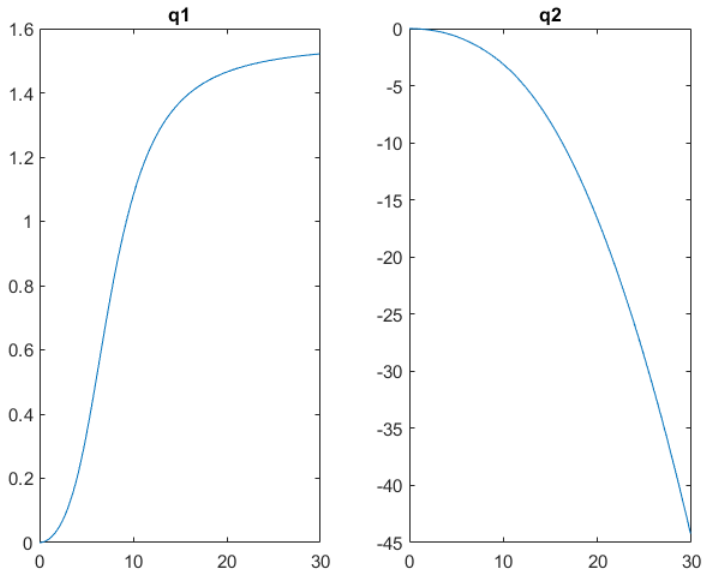

Figure 10: U=[10 -10]

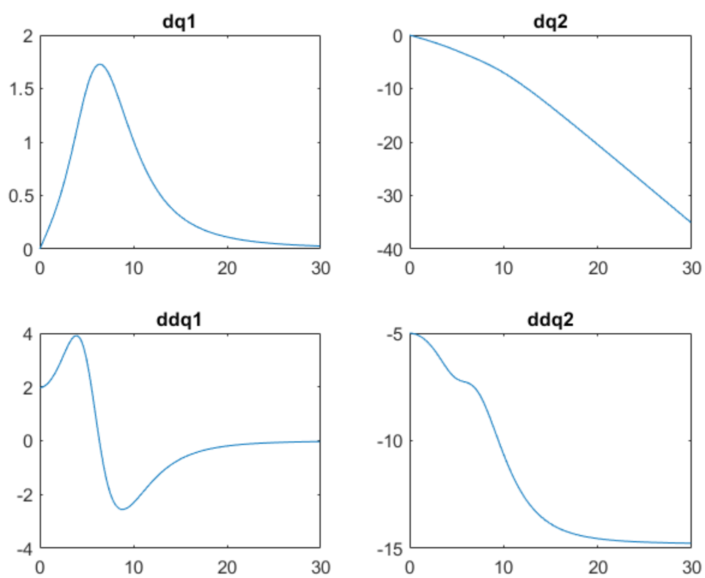

Figure 11: U=[10 -10]

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Link to assignment : HW5