Assignment-1

MDP Basics and Dynamic Programming Methods

Problem-1 : GridWorld

Environment Setting

- The environment consists of a 8×8 grid with the start state as $(0, 0)$ and terminal state as $(63, 63)$.
- The reward for reaching the terminal state is 0 else the reward $r(s, a, s') \in (0, -1)$.
- The possible actions in each stae are $\{Up = 0, Right = 1, Down = 2, Left = 3\}$.

In [1]:
```
import env
import agent
import numpy as np
import matplotlib
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from matplotlib import cm


Env = env.Grid_1()
agent_1 = agent.ValueIteration(Env)
agent_2 = agent.PolicyIteration(Env)
agent_3 = agent.ConfusedAgent(Env)
```

- We train 3 agents corresponding to Value Iteration, Policy Iteration and Random.

In [2]:
```
def plot(Env,policy,V):
    pp = np.reshape(np.argmax(policy, axis=1), Env.shape)
    print(pp)

    cmp = plt.matshow(np.reshape(V, Env.shape))
    # plt.arrow(0,0.5,0,-0.7,head_width = 0.1)

    plt.colorbar(cmp)

    for i in range(Env.shape[0]):
        for j in range(Env.shape[1]):
            if i == (Env.shape[0]-1) and j == (Env.shape[1]-1):
                continue
            if pp[i][j] == 0:
                plt.arrow(j,i+0.5,0,-0.7,head_width = 0.1)
            elif pp[i][j] == 2:
                plt.arrow(j,i-0.5,0,+0.7,head_width = 0.1)
            elif pp[i][j] == 1:
                plt.arrow(j-0.5,i,0.7,0,head_width = 0.1)
            elif pp[i][j] == 3:
                plt.arrow(j+0.5,i,-0.7,0,head_width = 0.1)

    plt.show()
```

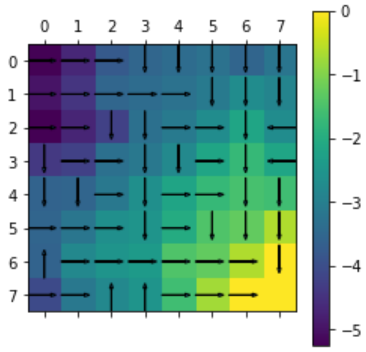
Function for plotting the optimal policies obtained by different agents

Part-a,b

Value-Iteration

In [3]:
```
### Value Iteration
# agent_1.set_gamma(0.5)
agent_1.clear()
itr = 0
while True:
    agent_1.reset()
    agent_1.update()

    if agent_1.get_delta() < agent_1.get_threshold():
        break
    itr += 1

print(itr)
policy = agent_1.get_policy()
print(np.reshape(agent_1.V,Env.shape))
plot(Env,policy,agent_1.V)
# agent_1.clear()
```

```
121
[[-5.22041551 -4.66374662 -3.76022869 -3.51898212 -3.39792422 -3.27240293
  -3.53854272 -3.20898484]
 [-4.9694722  -4.4278591  -3.51898212 -3.39792422 -3.27240293 -3.1021674
  -2.68712729 -2.8775704 ]
 [-5.24781006 -4.71161351 -4.22887404 -3.31763094 -3.1021674  -2.68712729
  -2.16822194 -2.68712729]
 [-4.3842771  -4.22887404 -3.31763094 -3.15465386 -2.78012179 -2.16822194
  -1.72471154 -2.16822194]
 [-3.58602673 -3.53169027 -3.15465386 -2.63622143 -2.17376983 -1.72471154
  -1.52491241 -1.60236044]
 [-3.53169027 -3.19126    -2.63622143 -2.53203929 -2.01816864 -1.44891952
  -1.25594018 -0.62961128]
 [-3.58602673 -2.80017455 -2.53203929 -2.38232128 -1.44891952 -1.25594018
  -0.62961128  0.        ]
 [-4.07129585 -3.14357618 -2.80017455 -2.53203929 -1.6153412  -0.73101098
   0.          0.        ]]
[[1 1 1 2 2 2 2 2]
 [1 1 1 1 1 2 2 2]
 [1 1 2 2 1 1 2 3]
 [2 1 1 2 2 1 2 3]
 [2 2 1 2 1 1 2 2]
 [1 1 1 2 1 2 2 2]
 [0 1 1 1 1 1 1 2]
 [1 1 0 0 1 1 1 0]]
```



1. In Value Iteration Algorithm, we initialize all the state values with 0 and apply the Bellman optimality operator until the point of constantcy (V^*) is reached.

$$V_{n+1}(s) = F(V_n(s)) = \max_{a \in A(s)} \left\{ r(s,a) + \gamma \times \sum_{s' \in S} P(s'|s,a) V_n(s') \right\} \forall s \in S$$

2. We get optimal policy using the following equation.

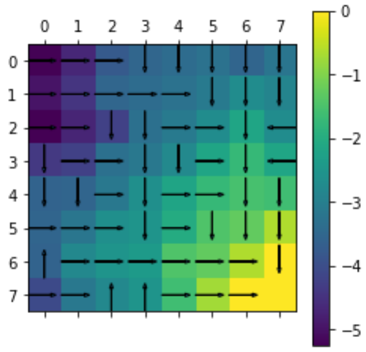
$$\pi(s) = \arg\max_{a \in A(s)} \left\{ r(s,a) + \gamma \times \sum_{s' \in S} P(s'|s,a) V_{n+1}(s') \right\} \forall s \in S$$

3. In the above part we show the final state value vector and the optimal policy for $\gamma = 1$.

Policy-Iteration

In [4]:
```
### Policy Iteration
# agent_2.set_gamma(0.5)
agent_2.clear()
while True:
    V = agent_2.evaluate_policy()
    # print(V)

    stable = agent_2.update()

    if stable == True:
        break

print(np.reshape(agent_2.V,Env.shape))
plot(Env,agent_2.policy,agent_2.V)
# agent_2.clear()
```

```
[[-5.22041551 -4.66374662 -3.76022869 -3.51898212 -3.39792422 -3.27240293
  -3.53854272 -3.20898484]
 [-4.9694722  -4.4278591  -3.51898212 -3.39792422 -3.27240293 -3.1021674
  -2.68712729 -2.8775704 ]
 [-5.24781006 -4.71161351 -4.22887404 -3.31763094 -3.1021674  -2.68712729
  -2.16822194 -2.68712729]
 [-4.3842771  -4.22887404 -3.31763094 -3.15465386 -2.78012179 -2.16822194
  -1.72471154 -2.16822194]
 [-3.58602673 -3.53169027 -3.15465386 -2.63622143 -2.17376983 -1.72471154
  -1.52491241 -1.60236044]
 [-3.53169027 -3.19126    -2.63622143 -2.53203929 -2.01816864 -1.44891952
  -1.25594018 -0.62961128]
 [-3.58602673 -2.80017455 -2.53203929 -2.38232128 -1.44891952 -1.25594018
  -0.62961128  0.        ]
 [-4.07129585 -3.14357618 -2.80017455 -2.53203929 -1.6153412  -0.73101098
   0.          0.        ]]
[[1 1 1 2 2 2 2 2]
 [1 1 1 1 1 2 2 2]
 [1 1 2 2 1 1 2 3]
 [2 1 1 2 2 1 2 3]
 [2 2 1 2 1 1 2 2]
 [1 1 1 2 1 2 2 2]
 [0 1 1 1 1 1 1 2]
 [1 1 0 0 1 1 1 0]]
```



1. In Policy Iteration, we start with a random policy. This algorithm involves 2 parts:
   A. Policy Evaluation
   B. Policy Improvement

Policy Evaluation

1. In this we evaluate the current policy and return the final state value vector.
2. Evaluation is done using the following equation:-

$$V_{n+1}(s) = \max_{a \in A(s)} \left\{ \pi(a, s)\left\{ r(s, a) + \gamma \times \sum_{s' \in S} P(s'|s, a)V_n(s') \right\} \right\} \forall s \in S$$

Policy updation

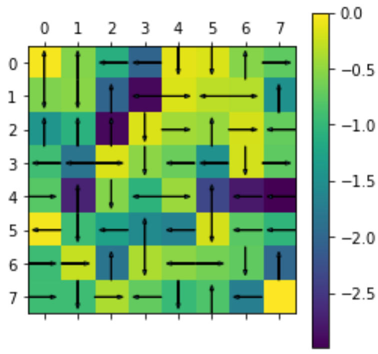
1. We update the our current policy using the final state vector obtained from the policy evaluation step.
2. Improvement is done as follows:-

$$\pi_{k+1}(s) = \arg\max_{a \in A(s)} \left\{ r(s, a) + \gamma \times \sum_{s' \in S} P(s'|s, a)V_{\pi_k}(s') \right\}$$

In the above part, we show the state value vector and optimal policy for Policy Iteration Algorithm.

In [5]:
```
agent_3.get_policy()
print(np.reshape(agent_3.V,Env.shape))
plot(Env,agent_3.policy,agent_3.V)
```

```
[[-0.04345    -0.55666888 -1.11333777 -2.0168557  -0.1255213  -0.17023553
  -0.53937516 -0.74397335]
 [-0.57585387 -0.53619655 -2.0168557  -2.92573268 -0.17023553 -0.29575683
  -0.33141444 -1.4879467 ]
 [-1.42734639 -1.07780964 -2.92573268 -0.16297707 -0.41504011 -0.46599235
  -0.19044311 -0.70934846]
 [-0.91915017 -1.83830034 -0.16297707 -0.51843243 -0.6814095  -1.47286136
  -0.19979913 -0.74597145]
 [-0.77347435 -2.69456567 -0.55503856 -1.04926075 -0.44905829 -2.33561133
  -2.78466962 -2.98446874]
 [-0.05433646 -0.94084391 -1.28127419 -1.56769318 -1.67187533 -0.19297934
  -0.76222846 -1.0312007 ]
 [-0.94084391 -0.26813526 -1.83631275 -0.36901193 -0.51872993 -0.6263289
  -0.73101098 -2.00394986]
 [-0.92771967 -0.92771967 -0.36901193 -0.71241356 -0.96835084 -0.81930824
  -1.70363847  0.        ]]
[[0 0 3 3 2 2 0 1]
 [2 2 0 3 1 3 1 0]
 [0 0 0 2 1 0 1 3]
 [3 3 1 2 3 3 2 1]
 [1 0 2 3 1 0 3 3]
 [3 2 3 0 3 2 3 3]
 [1 1 0 2 3 1 2 0]
 [1 2 1 3 2 0 3 2]]
```



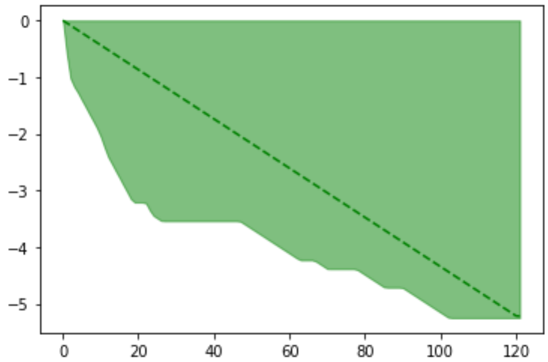
We observe that the final state value vectors and optimal policies is the same for both value iteration and policy iteration algorithm.

Part-c

In [6]:
```
def plot_mean_and_CI(mean, lb, ub, color_mean=None, color_shading=None):
    # plot the shaded range of the confidence intervals
    plt.fill_between(range(mean.shape[0]), ub, lb,
                     color=color_shading, alpha=.5)
    # plot the mean on top
    plt.plot(mean, color_mean)
```

In [29]:
```
agent_1.clear()

mu = []
low = []
high = []

while True:
    agent_1.reset()
    agent_1.update()

#     mu.append((np.matrix(np.reshape(agent_1.V,Env.shape)))[0][0])
#     mu.append(agent_1.V[0][0])
    mat = np.matrix(np.reshape(agent_1.V,Env.shape))
    mu.append(mat[0,0])

    low.append((np.matrix(np.reshape(agent_1.V,Env.shape))).min())
    high.append((np.matrix(np.reshape(agent_1.V,Env.shape))).max())

    if agent_1.get_delta() < agent_1.get_threshold():
        break

mu = np.array(mu)
high = np.array(high)
low = np.array(low)

print(mu)
plot_mean_and_CI(mu, high, low, color_mean='g--', color_shading='g')
```

```
[ 0.         -0.04345    -0.08689999 -0.13034999 -0.17379999 -0.21724999
 -0.26069998 -0.30414998 -0.34759998 -0.39104997 -0.43449997 -0.47794997
 -0.52139996 -0.56484996 -0.60829996 -0.65174996 -0.69519995 -0.73864995
 -0.78209995 -0.82554994 -0.86899994 -0.91244994 -0.95589993 -0.99934993
 -1.04279993 -1.08624993 -1.12969992 -1.17314992 -1.21659992 -1.26004991
 -1.30349991 -1.34694991 -1.3903999  -1.4338499  -1.4772999  -1.5207499
 -1.56419989 -1.60764989 -1.65109989 -1.69454988 -1.73799988 -1.78144988
 -1.82489988 -1.86834987 -1.91179987 -1.95524987 -1.99869986 -2.04214986
 -2.08559986 -2.12904985 -2.17249985 -2.21594985 -2.25939985 -2.30284984
 -2.34629984 -2.38974984 -2.43319983 -2.47664983 -2.52009983 -2.56354982
 -2.60699982 -2.65044982 -2.69389982 -2.73734981 -2.78079981 -2.82424981
 -2.8676998  -2.9111498  -2.9545998  -2.9980498  -3.04149979 -3.08494979
 -3.12839979 -3.17184978 -3.21529978 -3.25874978 -3.30219977 -3.34564977
 -3.38909977 -3.43254977 -3.47599976 -3.51944976 -3.56289976 -3.60634975
 -3.64979975 -3.69324975 -3.73669974 -3.78014974 -3.82359974 -3.86704974
 -3.91049973 -3.95394973 -3.99739973 -4.04084972 -4.08429972 -4.12774972
 -4.17119971 -4.21464971 -4.25809971 -4.30154971 -4.3449997  -4.3884497
 -4.4318997  -4.47534969 -4.51879969 -4.56224969 -4.60569969 -4.64914968
 -4.69259968 -4.73604968 -4.77949967 -4.82294967 -4.86639967 -4.90984966
 -4.95329966 -4.99674966 -5.04019966 -5.08364965 -5.12709965 -5.17054965
 -5.21399964 -5.22041551]
```

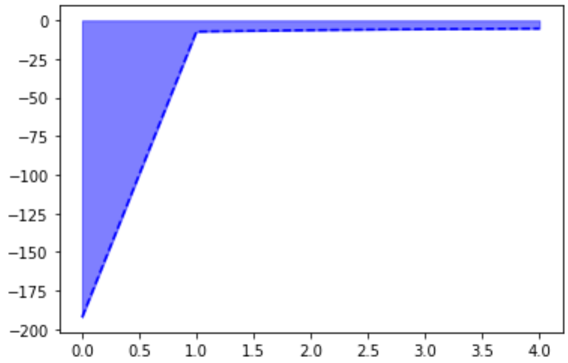


Above is the plot for the average value of the optimal state value function v/s number of iterations for value iteration

1. We observe that as the number of iterations increases, the optimal reward value (average state value) converges to a constant value.

In [28]:
```
agent_2.clear()
mu = []
low = []
high = []

while True:
    V = agent_2.evaluate_policy()

#     mu.append((np.matrix(np.reshape(V,Env.shape))).mean())
    mat = np.matrix(np.reshape(agent_2.V,Env.shape))
    mu.append(mat[0,0])
    low.append((np.matrix(np.reshape(V,Env.shape))).min())
    high.append((np.matrix(np.reshape(V,Env.shape))).max())

    stable = agent_2.update()

    if stable == True:
        break

mu = np.array(mu)
high = np.array(high)
low = np.array(low)

print(mu)
plot_mean_and_CI(mu, high, low, color_mean='b--', color_shading='b')
```

```
[-192.41222849   -7.22184676   -6.25734004   -5.62044015   -5.22041551]
```

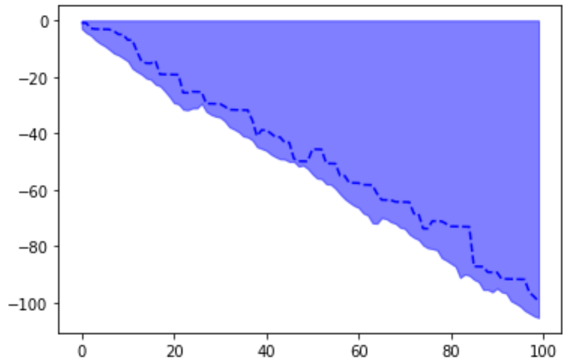


Above is the plot for the average value of the optimal state value function v/s number of iterations for policy iteration

1. We observe that as the number of iterations increases, the optimal reward value (average state value) converges to a constant value.

In [27]:
```
agent_3.clear()
mu = []
low = []
high = []

for i in range(100):
    agent_3.get_policy()

#     mu.append((np.matrix(np.reshape(agent_3.V,Env.shape))).mean())
    mat = np.matrix(np.reshape(agent_3.V,Env.shape))
    mu.append(mat[0,0])
    low.append((np.matrix(np.reshape(agent_3.V,Env.shape))).min())
    high.append((np.matrix(np.reshape(agent_3.V,Env.shape))).max())

mu = np.array(mu)
high = np.array(high)
low = np.array(low)

print(mu)
plot_mean_and_CI(mu, high, low, color_mean='b--', color_shading='b')
```

```
[ -0.85149252  -0.89494252  -2.82045201  -3.04805649  -3.09150649
  -3.13495648  -3.17840648  -3.77852536  -4.92484152  -4.96829152
  -6.91806603  -6.96151602 -10.97207975 -14.5501409  -15.12599477
 -15.16944477 -14.34906268 -19.06797577 -19.11142576 -19.15487576
 -19.19832576 -19.24177576 -25.58355328 -25.62700328 -25.22381249
 -25.26726249 -25.31071249 -29.43737012 -29.48082012 -29.52427012
 -29.56772011 -30.46266263 -31.61107069 -31.65452069 -31.69797068
 -31.74142068 -31.78487068 -35.36977725 -40.98164786 -38.837261
 -38.88071099 -39.99404876 -41.26846264 -41.31191264 -43.21823714
 -43.26168714 -48.93040912 -49.83392705 -49.87737705 -49.92082704
 -45.59886027 -45.64231027 -45.68576027 -50.64489701 -50.68834701
 -50.731797   -54.98156821 -55.02501821 -57.51033007 -57.55378007
 -57.59723007 -58.19734895 -58.24079895 -58.28424894 -61.28885544
 -63.54599585 -63.58944585 -63.63289584 -64.23301473 -64.27646472
 -64.31991472 -64.36336472 -68.40375532 -68.44720532 -73.68853667
 -73.73198667 -71.04513289 -71.08858288 -71.13203288 -72.0269754
 -72.92191792 -72.96536792 -73.00881791 -73.05226791 -73.09571791
 -87.10745609 -87.15090609 -87.19435609 -89.14413059 -89.18758059
 -89.23103059 -91.5098695  -91.5533195  -91.59676949 -91.64021949
 -91.68366949 -91.72711949 -96.30590684 -97.76609365 -99.46831686]
```



- By seeing the above plots we conclude that that the rewards obtained by confused agent may be higher but on an average the confused agent performs poorly than the learned agents.
- We also observe that the final average state value is same for both value iteration and policy iteration.

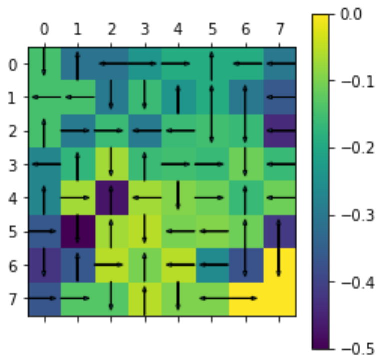
Comparison of different policies with different values of γ

Value Iteration

$\gamma = 0$

In [10]:
```
agent_1.clear()

# policies_value = []
# V_value = []

agent_1.set_gamma(0)
while True:
    agent_1.reset()
    agent_1.update()

    if agent_1.get_delta() < agent_1.get_threshold():
        break

policy = agent_1.get_policy()
print(np.reshape(agent_1.V,Env.shape))
plot(Env,policy,agent_1.V)
```

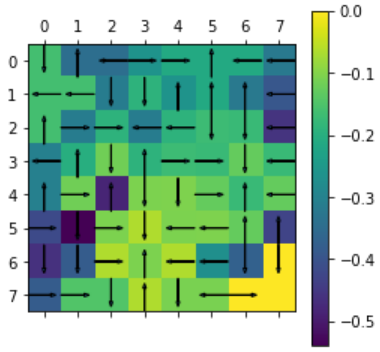
```
[[-0.14512055 -0.30882554 -0.30882554 -0.24017891 -0.19047494 -0.19047494
  -0.19047494 -0.30045085]
 [-0.14512055 -0.14512055 -0.29377688 -0.15884583 -0.24017891 -0.19047494
  -0.30045085 -0.36050373]
 [-0.14512055 -0.29377688 -0.15884583 -0.29377688 -0.15884583 -0.15183197
  -0.16053415 -0.44040548]
 [-0.27280255 -0.17011694 -0.06925676 -0.15884583 -0.15183197 -0.16053415
  -0.11034099 -0.16053415]
 [-0.27280255 -0.06925676 -0.47161264 -0.06925676 -0.09368626 -0.11034099
  -0.16053415 -0.11034099]
 [-0.36257063 -0.50018686 -0.06925676 -0.05339038 -0.10052064 -0.09368626
  -0.11034099 -0.41490262]
 [-0.4238164  -0.36257063 -0.05339038 -0.10052064 -0.05339038 -0.26303996
  -0.37154444  0.        ]
 [-0.36731126 -0.13121401 -0.13121401 -0.05339038 -0.09717174 -0.09717174
   0.          0.        ]]
[[2 0 3 1 1 0 3 3]
 [3 3 2 2 0 0 0 3]
 [0 1 1 3 3 2 2 3]
 [3 0 2 0 1 1 2 3]
 [0 1 0 3 2 1 0 3]
 [1 2 0 2 3 3 0 0]
 [2 0 1 0 3 3 2 2]
 [1 1 2 0 2 3 1 0]]
```



$\gamma = 0.1$

In [11]:
```
agent_1.clear()

# policies_value = []
# V_value = []

agent_1.set_gamma(0.1)
while True:
    agent_1.reset()
    agent_1.update()

    if agent_1.get_delta() < agent_1.get_threshold():
        break

policy = agent_1.get_policy()
print(np.reshape(agent_1.V,Env.shape))
plot(Env,policy,agent_1.V)
```

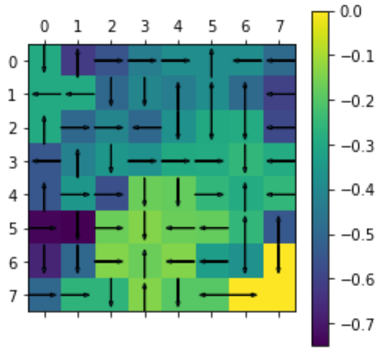
```
[[-0.16124344 -0.34313605 -0.34313605 -0.26134067 -0.21163671 -0.21163671
  -0.21163671 -0.32161262]
 [-0.16124344 -0.16124344 -0.31278745 -0.19012164 -0.26134067 -0.21163671
  -0.32161262 -0.39266309]
 [-0.16124344 -0.31278745 -0.19012164 -0.31278745 -0.19012164 -0.16916036
  -0.17329999 -0.45773387]
 [-0.30311091 -0.20139274 -0.11758913 -0.19012164 -0.16916036 -0.17329999
  -0.12766939 -0.17329999]
 [-0.30311091 -0.11758913 -0.48337086 -0.10692832 -0.10437809 -0.12766939
  -0.17329999 -0.12766939]
 [-0.41640676 -0.53837446 -0.10692832 -0.0640822  -0.10692832 -0.10437809
  -0.12766939 -0.42766846]
 [-0.462004   -0.38188914 -0.0640822  -0.10692832 -0.0640822  -0.26944764
  -0.37154444  0.        ]
 [-0.38188914 -0.14579189 -0.14579189 -0.0640822  -0.10796752 -0.10796752
   0.          0.        ]]
[[2 0 3 1 1 0 3 3]
 [3 3 2 2 0 0 0 3]
 [0 1 1 3 3 2 2 3]
 [3 0 2 0 1 1 2 3]
 [0 1 0 2 2 1 0 3]
 [1 2 1 2 3 3 0 0]
 [2 2 1 0 3 3 2 2]
 [1 1 2 0 2 3 1 0]]
```



$\gamma = 0.5$

In [12]:
```
agent_1.clear()

# policies_value = []
# V_value = []

agent_1.set_gamma(0.5)
while True:
    agent_1.reset()
    agent_1.update()

    if agent_1.get_delta() < agent_1.get_threshold():
        break

policy = agent_1.get_policy()
print(np.reshape(agent_1.V,Env.shape))
plot(Env,policy,agent_1.V)
```

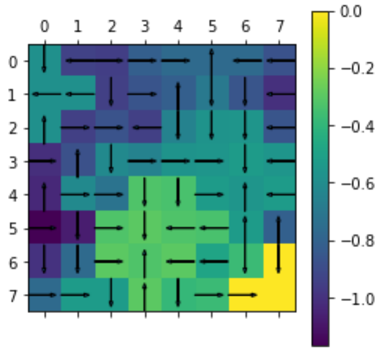
```
[[-0.29023224 -0.61763222 -0.55453705 -0.43064222 -0.38093826 -0.38093826
  -0.38093826 -0.49091417]
 [-0.29023224 -0.29023224 -0.49758728 -0.40763051 -0.43064222 -0.38093826
  -0.49091417 -0.60595501]
 [-0.29023224 -0.49758728 -0.40763051 -0.49758728 -0.36935294 -0.29562629
  -0.28759844 -0.5841998 ]
 [-0.54558845 -0.41890161 -0.35296401 -0.36935294 -0.29562629 -0.28759844
  -0.25413532 -0.28759844]
 [-0.54558845 -0.35296401 -0.56741776 -0.16961688 -0.17849163 -0.25413532
  -0.28759844 -0.25413532]
 [-0.73728736 -0.74944148 -0.16961688 -0.13819575 -0.16961688 -0.17849163
  -0.25413532 -0.54196692]
 [-0.67307102 -0.49851726 -0.13819575 -0.16961688 -0.13819575 -0.3321362
  -0.37154444  0.        ]
 [-0.49851726 -0.26242001 -0.26242001 -0.13819575 -0.19433754 -0.19433754
   0.          0.        ]]
[[2 0 1 1 1 0 3 3]
 [3 3 2 2 0 0 0 3]
 [0 1 1 3 2 2 2 3]
 [3 0 2 1 1 1 2 3]
 [0 1 1 2 2 1 0 3]
 [1 2 1 2 3 3 0 0]
 [2 2 1 0 3 3 2 2]
 [1 1 2 0 2 3 1 0]]
```



$\gamma = 0.75$

In [13]:
```
agent_1.clear()

# policies_value = []
# V_value = []

agent_1.set_gamma(0.75)
while True:
    agent_1.reset()
    agent_1.update()

    if agent_1.get_delta() < agent_1.get_threshold():
        break

policy = agent_1.get_policy()
print(np.reshape(agent_1.V,Env.shape))
plot(Env,policy,agent_1.V)
```

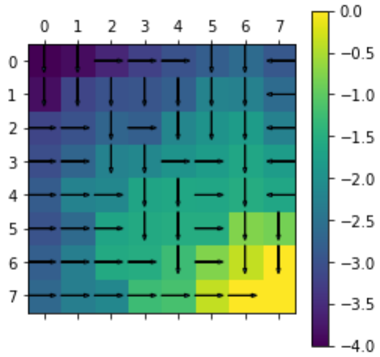
```
[[-0.58046375 -0.93655564 -0.94790034 -0.81157952 -0.76187555 -0.76187555
  -0.76187555 -0.87185147]
 [-0.58046375 -0.58046375 -0.94376715 -0.86320238 -0.81157952 -0.69392968
  -0.85745711 -1.00359146]
 [-0.58046375 -0.94376715 -0.86666614 -0.94376715 -0.64820076 -0.5688836
  -0.55607352 -0.85745711]
 [-1.00943013 -0.87793725 -0.61193023 -0.64820076 -0.5688836  -0.55607352
  -0.52739263 -0.55607352]
 [-1.0298701  -0.61193023 -0.7235689  -0.3212776  -0.33464276 -0.52739263
  -0.55607352 -0.52739263]
 [-1.16573032 -1.07088514 -0.3212776  -0.29434688 -0.3212776  -0.33464276
  -0.52739263 -0.81044199]
 [-0.99451468 -0.76093661 -0.29434688 -0.3212776  -0.29434688 -0.48379693
  -0.37154444  0.        ]
 [-0.76093661 -0.52483936 -0.52483936 -0.29434688 -0.38867459 -0.37154444
   0.          0.        ]]
[[2 3 1 1 1 0 3 3]
 [3 3 2 1 0 2 2 3]
 [0 1 1 3 2 2 2 3]
 [1 0 2 1 1 1 2 3]
 [0 1 1 2 2 1 0 3]
 [1 2 1 2 3 3 0 0]
 [2 2 1 0 3 3 2 2]
 [1 1 2 0 2 1 1 0]]
```



$\gamma = 1$

In [14]:
```
agent_1.clear()

# policies_value = []
# V_value = []

agent_1.set_gamma(1)
while True:

    agent_1.reset()
    agent_1.update()

    if agent_1.get_delta() < agent_1.get_threshold():
        break

policy = agent_1.get_policy()
print(np.reshape(agent_1.V,Env.shape))
plot(Env,policy,agent_1.V)
```

```
[[-4.00052043 -3.87213884 -3.5688979  -3.22967614 -2.98949723 -2.79902229
  -2.59654604 -2.8969969 ]
 [-3.85539988 -3.15875858 -2.98864165 -2.94775684 -2.78891101 -2.21474088
  -2.23604231 -2.59654604]
 [-3.15875858 -2.98864165 -2.69486477 -2.78891101 -2.16901196 -1.9474688
  -1.79563683 -2.23604231]
 [-3.04585131 -2.69486477 -2.22325213 -2.15399537 -1.9474688  -1.79563683
  -1.63510268 -1.79563683]
 [-2.84204343 -2.22325213 -2.15399537 -1.67138298 -1.61115823 -1.63510268
  -1.52476168 -1.63510268]
 [-2.97756514 -2.61499451 -1.67138298 -1.57086235 -1.51747197 -1.52476168
  -0.76130864 -0.83198647]
 [-2.79131282 -2.29112596 -1.57086235 -1.51747197 -1.25443201 -0.76130864
  -0.37154444  0.        ]
 [-2.65161468 -2.28430342 -2.15308941 -1.25443201 -1.15726028 -0.37154444
   0.          0.        ]]
[[2 2 1 1 1 2 2 3]
 [2 2 2 2 2 2 2 3]
 [1 1 2 1 2 2 2 3]
 [1 1 2 2 1 1 2 3]
 [1 1 1 2 2 1 2 3]
 [1 1 1 2 2 1 2 2]
 [1 1 1 1 2 1 2 2]
 [1 1 1 1 1 1 1 0]]
```

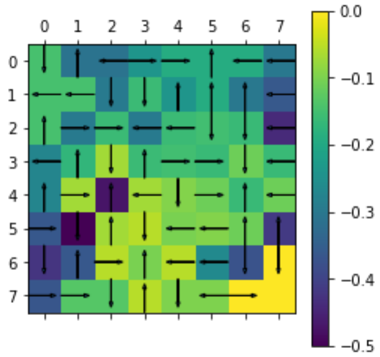


Policy Iteration

$\gamma = 0$

In [15]:
```
agent_2.clear()
agent_2.set_gamma(0)
while True:
    V = agent_2.evaluate_policy()
    # print(V)

    stable = agent_2.update()

    if stable == True:
        break

print(np.reshape(agent_2.V,Env.shape))
plot(Env,agent_2.policy,agent_2.V)
```

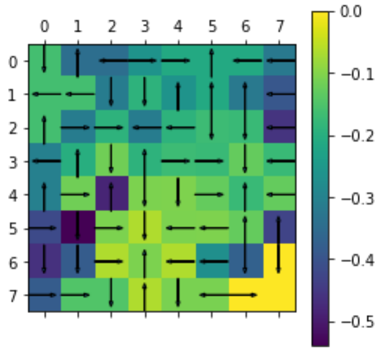
```
[[-0.14512055 -0.30882554 -0.30882554 -0.24017891 -0.19047494 -0.19047494
  -0.19047494 -0.30045085]
 [-0.14512055 -0.14512055 -0.29377688 -0.15884583 -0.24017891 -0.19047494
  -0.30045085 -0.36050373]
 [-0.14512055 -0.29377688 -0.15884583 -0.29377688 -0.15884583 -0.15183197
  -0.16053415 -0.44040548]
 [-0.27280255 -0.17011694 -0.06925676 -0.15884583 -0.15183197 -0.16053415
  -0.11034099 -0.16053415]
 [-0.27280255 -0.06925676 -0.47161264 -0.06925676 -0.09368626 -0.11034099
  -0.16053415 -0.11034099]
 [-0.36257063 -0.50018686 -0.06925676 -0.05339038 -0.10052064 -0.09368626
  -0.11034099 -0.41490262]
 [-0.4238164  -0.36257063 -0.05339038 -0.10052064 -0.05339038 -0.26303996
  -0.37154444  0.        ]
 [-0.36731126 -0.13121401 -0.13121401 -0.05339038 -0.09717174 -0.09717174
   0.          0.        ]]
[[2 0 3 1 1 0 3 3]
 [3 3 2 2 0 0 0 3]
 [0 1 1 3 3 2 2 3]
 [3 0 2 0 1 1 2 3]
 [0 1 0 3 2 1 0 3]
 [1 2 0 2 3 3 0 0]
 [2 0 1 0 3 3 2 2]
 [1 1 2 0 2 3 1 0]]
```



$\gamma = 0.1$

In [16]:
```
agent_2.clear()
agent_2.set_gamma(0.1)
while True:
    V = agent_2.evaluate_policy()
    # print(V)

    stable = agent_2.update()

    if stable == True:
        break

print(np.reshape(agent_2.V,Env.shape))
plot(Env,agent_2.policy,agent_2.V)
```

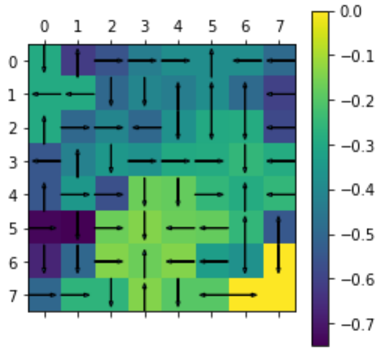
```
[[-0.16124344 -0.34313605 -0.34313605 -0.26134067 -0.21163671 -0.21163671
  -0.21163671 -0.32161262]
 [-0.16124344 -0.16124344 -0.31278745 -0.19012164 -0.26134067 -0.21163671
  -0.32161262 -0.39266309]
 [-0.16124344 -0.31278745 -0.19012164 -0.31278745 -0.19012164 -0.16916036
  -0.17329999 -0.45773387]
 [-0.30311091 -0.20139274 -0.11758913 -0.19012164 -0.16916036 -0.17329999
  -0.12766939 -0.17329999]
 [-0.30311091 -0.11758913 -0.48337086 -0.10692832 -0.10437809 -0.12766939
  -0.17329999 -0.12766939]
 [-0.41640676 -0.53837446 -0.10692832 -0.0640822  -0.10692832 -0.10437809
  -0.12766939 -0.42766846]
 [-0.462004   -0.38188914 -0.0640822  -0.10692832 -0.0640822  -0.26944764
  -0.37154444  0.        ]
 [-0.38188914 -0.14579189 -0.14579189 -0.0640822  -0.10796752 -0.10796752
   0.          0.        ]]
[[2 0 3 1 1 0 3 3]
 [3 3 2 2 0 0 0 3]
 [0 1 1 3 3 2 2 3]
 [3 0 2 0 1 1 2 3]
 [0 1 0 2 2 1 0 3]
 [1 2 1 2 3 3 0 0]
 [2 2 1 0 3 3 2 2]
 [1 1 2 0 2 3 1 0]]
```



$\gamma = 0.5$

In [17]:
```
agent_2.clear()
agent_2.set_gamma(0.5)
while True:
    V = agent_2.evaluate_policy()
    # print(V)

    stable = agent_2.update()

    if stable == True:
        break

print(np.reshape(agent_2.V,Env.shape))
plot(Env,agent_2.policy,agent_2.V)
```

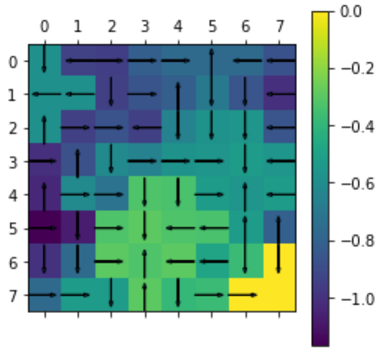
```
[[-0.29023224 -0.61763222 -0.55453705 -0.43064222 -0.38093826 -0.38093826
  -0.38093826 -0.49091417]
 [-0.29023224 -0.29023224 -0.49758728 -0.40763051 -0.43064222 -0.38093826
  -0.49091417 -0.60595501]
 [-0.29023224 -0.49758728 -0.40763051 -0.49758728 -0.36935294 -0.29562629
  -0.28759844 -0.5841998 ]
 [-0.54558845 -0.41890161 -0.35296401 -0.36935294 -0.29562629 -0.28759844
  -0.25413532 -0.28759844]
 [-0.54558845 -0.35296401 -0.56741776 -0.16961688 -0.17849163 -0.25413532
  -0.28759844 -0.25413532]
 [-0.73728736 -0.74944148 -0.16961688 -0.13819575 -0.16961688 -0.17849163
  -0.25413532 -0.54196692]
 [-0.67307102 -0.49851726 -0.13819575 -0.16961688 -0.13819575 -0.3321362
  -0.37154444  0.        ]
 [-0.49851726 -0.26242001 -0.26242001 -0.13819575 -0.19433754 -0.19433754
   0.          0.        ]]
[[2 0 1 1 1 0 3 3]
 [3 3 2 2 0 0 0 3]
 [0 1 1 3 2 2 2 3]
 [3 0 2 1 1 1 2 3]
 [0 1 1 2 2 1 0 3]
 [1 2 1 2 3 3 0 0]
 [2 2 1 0 3 3 2 2]
 [1 1 2 0 2 3 1 0]]
```



$\gamma = 0.75$

In [18]:
```
agent_2.clear()
agent_2.set_gamma(0.75)
while True:
    V = agent_2.evaluate_policy()
    # print(V)

    stable = agent_2.update()

    if stable == True:
        break

print(np.reshape(agent_2.V,Env.shape))
plot(Env,agent_2.policy,agent_2.V)
```

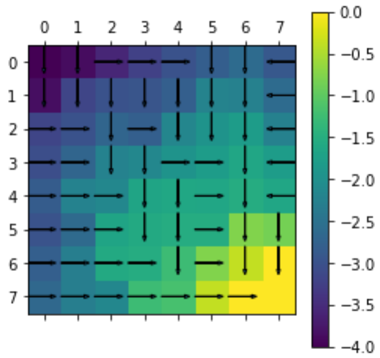
```
[[-0.58046375 -0.93655564 -0.94790034 -0.81157952 -0.76187555 -0.76187555
  -0.76187555 -0.87185147]
 [-0.58046375 -0.58046375 -0.94376715 -0.86320238 -0.81157952 -0.69392968
  -0.85745711 -1.00359146]
 [-0.58046375 -0.94376715 -0.86666614 -0.94376715 -0.64820076 -0.5688836
  -0.55607352 -0.85745711]
 [-1.00943013 -0.87793725 -0.61193023 -0.64820076 -0.5688836  -0.55607352
  -0.52739263 -0.55607352]
 [-1.0298701  -0.61193023 -0.7235689  -0.3212776  -0.33464276 -0.52739263
  -0.55607352 -0.52739263]
 [-1.16573032 -1.07088514 -0.3212776  -0.29434688 -0.3212776  -0.33464276
  -0.52739263 -0.81044199]
 [-0.99451468 -0.76093661 -0.29434688 -0.3212776  -0.29434688 -0.48379693
  -0.37154444  0.        ]
 [-0.76093661 -0.52483936 -0.52483936 -0.29434688 -0.38867459 -0.37154444
   0.          0.        ]]
[[2 3 1 1 1 0 3 3]
 [3 3 2 1 0 2 2 3]
 [0 1 1 3 2 2 2 3]
 [1 0 2 1 1 1 2 3]
 [0 1 1 2 2 1 0 3]
 [1 2 1 2 3 3 0 0]
 [2 2 1 0 3 3 2 2]
 [1 1 2 0 2 1 1 0]]
```



$\gamma = 1$

In [19]:
```
agent_2.clear()
agent_2.set_gamma(1)
while True:
    V = agent_2.evaluate_policy()
    # print(V)

    stable = agent_2.update()

    if stable == True:
        break

print(np.reshape(agent_2.V,Env.shape))
plot(Env,agent_2.policy,agent_2.V)
```

```
[[-4.00052043 -3.87213884 -3.5688979  -3.22967614 -2.98949723 -2.79902229
  -2.59654604 -2.8969969 ]
 [-3.85539988 -3.15875858 -2.98864165 -2.94775684 -2.78891101 -2.21474088
  -2.23604231 -2.59654604]
 [-3.15875858 -2.98864165 -2.69486477 -2.78891101 -2.16901196 -1.9474688
  -1.79563683 -2.23604231]
 [-3.04585131 -2.69486477 -2.22325213 -2.15399537 -1.9474688  -1.79563683
  -1.63510268 -1.79563683]
 [-2.84204343 -2.22325213 -2.15399537 -1.67138298 -1.61115823 -1.63510268
  -1.52476168 -1.63510268]
 [-2.97756514 -2.61499451 -1.67138298 -1.57086235 -1.51747197 -1.52476168
  -0.76130864 -0.83198647]
 [-2.79131282 -2.29112596 -1.57086235 -1.51747197 -1.25443201 -0.76130864
  -0.37154444  0.        ]
 [-2.65161468 -2.28430342 -2.15308941 -1.25443201 -1.15726028 -0.37154444
   0.          0.        ]]
[[2 2 1 1 1 2 2 3]
 [2 2 2 2 2 2 2 3]
 [1 1 2 1 2 2 2 3]
 [1 1 2 2 1 1 2 3]
 [1 1 1 2 2 1 2 3]
 [1 1 1 2 2 1 2 2]
 [1 1 1 1 2 1 2 2]
 [1 1 1 1 1 1 1 0]]
```



- We observe that different values of γ result in different policies.
- If value of $\gamma \rightarrow 0$, then the agent focusses on short term gains and acts greedily.
- If the value of $\gamma \rightarrow 1$, then the agent will put more weight on long term gains and will not act greedily.
- In our MDP, as $\gamma \rightarrow 1$, we obtain a better policy.

Question- 3

Part-a

The problem can be modelled as a MDP tuple $< S, A, P, R >$ where :

1. S is the set of all possible states of the gambler.

$$S = \{\$0 \ldots \$100\}$$

where $S = \$0$ and $S = \$100$ are terminal states.

2. A is the set of all possible bets he can place in a given state $s \in S$.

$$A(s) = \{1 \ldots \min(s, 100 - s)\}$$

3. $R(s, a, s^{'})$ is 0 if $s^{'} \in \{\$0 \ldots \$99\}$ else 1.
4. Given a state s, if the gambler makes a bet of $a \in \{1 \ldots \min(s, 100 - s)\}$, then $s^{'}$ can be:
   A. $s + a$ with $P(head)$
   B. $s - a$ with $1 - P(head)$

$$\therefore P(s, a, s + a) = P(head)$$
$$P(s, a, s - a) = 1 - P(head)$$

Bellman Equation for value iteration

1. The Bellman Update Equation for this problem will be as follows:

$$V_{n+1}(s) = \max_{a \in A(s)} \Big\{ P(head) \times \Big(r(s + a, a) + \gamma \times V_n(s + a) \Big) + \{1 - P(head)\} \times \Big(r(s - a, a) + \gamma$$

Part-b

In [24]:
```
Gambler_env = env.Gambler_env()
Gambler_env.set_p_h(0.3)
agent_1 = agent.Gambler_ValueIteration(Gambler_env)
Gambler_env.set_p_h(0.15)
agent_2 = agent.Gambler_ValueIteration(Gambler_env)
Gambler_env.set_p_h(0.65)
agent_3 = agent.Gambler_ValueIteration(Gambler_env)
```

In [25]:
```
def plot_fig1(y):
    x = range(100)

    for i in range(1,len(y)):
        plt.plot(x,y[i][:100])
    plt.show()
```

In [26]:
```
def plot_fig2(policy):
    x = range(100)

    plt.bar(x,policy)

    plt.show()
```

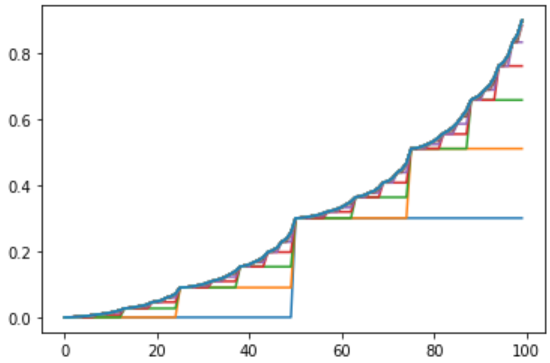
Plots showing the iterations of Value Iteration Algorithm and the optimal policy for different values of $P(head)$

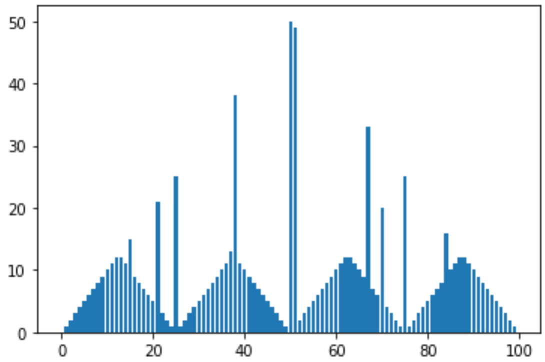
$P(head) = 0.3$

In [44]:
```
agent_1.clear()
y = []
while True:

    agent_1.reset()
    agent_1.update()

    y.append(agent_1.V)

    if agent_1.get_delta() < agent_1.get_threshold():
        break

policy = agent_1.get_policy()
print(agent_1.get_policy())
print(agent_1.V)
```

```
[ 0.  1.  2.  3.  4.  5.  6.  7.  8.  9. 10. 11. 12. 12. 11. 15.  9.  8.
  7.  6.  5. 21.  3.  2.  1. 25.  1.  2.  3.  4.  5.  6.  7.  8.  9. 10.
 11. 13. 38. 11. 10.  9.  8.  7.  6.  5.  4.  3.  2.  1. 50. 49.  2.  3.
  4.  5.  6.  7.  8.  9. 10. 11. 12. 12. 11. 10.  9. 33.  7.  6. 20.  4.
  3.  2.  1. 25.  1.  2.  3.  4.  5.  6.  7.  8. 16. 10. 11. 12. 12. 11.
 10.  9.  8.  7.  6.  5.  4.  3.  2.  1.]
[0.00000000e+00 2.66917018e-04 8.89723393e-04 1.92325355e-03
 2.96574464e-03 4.32158176e-03 6.41084517e-03 8.50388325e-03
 9.88581548e-03 1.18309578e-02 1.44052725e-02 1.77664658e-02
 2.13694839e-02 2.71868419e-02 2.83462775e-02 3.00251072e-02
 3.29527183e-02 3.52816705e-02 3.94365260e-02 4.60307893e-02
 4.80175751e-02 5.16971693e-02 5.92215525e-02 6.31880185e-02
 7.12316130e-02 9.00000000e-02 9.06228064e-02 9.20760213e-02
 9.44875916e-02 9.69200708e-02 1.00083691e-01 1.04958639e-01
 1.09842394e-01 1.13066903e-01 1.17605568e-01 1.23612303e-01
 1.31455087e-01 1.39862129e-01 1.53435964e-01 1.56141314e-01
 1.60058584e-01 1.66889676e-01 1.72323898e-01 1.82018561e-01
 1.97405175e-01 2.02041008e-01 2.10626728e-01 2.28183623e-01
 2.37438710e-01 2.56207097e-01 3.00000000e-01 3.00622806e-01
 3.02076021e-01 3.04487592e-01 3.06920071e-01 3.10083691e-01
 3.14958639e-01 3.19842394e-01 3.23066903e-01 3.27605568e-01
 3.33612303e-01 3.41455087e-01 3.49862129e-01 3.63435964e-01
 3.66141314e-01 3.70058584e-01 3.76889676e-01 3.82323898e-01
 3.92018561e-01 4.07405175e-01 4.12041008e-01 4.20626728e-01
 4.38183623e-01 4.47438710e-01 4.66207097e-01 5.10000000e-01
 5.11453215e-01 5.14844050e-01 5.20471047e-01 5.26146832e-01
 5.33528612e-01 5.44903490e-01 5.56298920e-01 5.63822773e-01
 5.74412993e-01 5.88428706e-01 6.06728536e-01 6.26344968e-01
 6.58017250e-01 6.64329733e-01 6.73470028e-01 6.89409244e-01
 7.02089095e-01 7.24709975e-01 7.60612075e-01 7.71429020e-01
 7.91462366e-01 8.32428453e-01 8.54023656e-01 8.97816560e-01
 0.00000000e+00]
```

In [45]:
```
plot_fig1(y)
plot_fig2(policy)
```



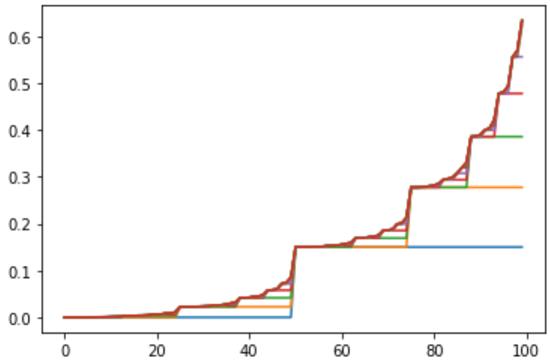


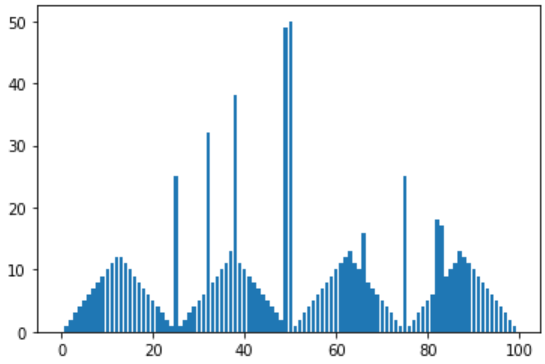
$P(head) = 0.15$

In [34]:
```
agent_2.clear()
y = []
while True:

    agent_2.reset()
    agent_2.update()

    y.append(agent_2.V)

    if agent_2.get_delta() < agent_2.get_threshold():
        break

policy = agent_2.get_policy()
print(policy)
print(agent_2.V)
```

```
[ 0.  1.  2.  3.  4.  5.  6.  7.  8.  9. 10. 11. 12. 12. 11. 10.  9.  8.
  7.  6.  5.  4.  3.  2.  1. 25.  1.  2.  3.  4.  5.  6. 32.  8.  9. 10.
 11. 13. 38. 11. 10.  9.  8.  7.  6.  5.  4.  3.  2. 49. 50.  1.  2.  3.
  4.  5.  6.  7.  8.  9. 10. 11. 12. 13. 11. 10. 16.  8.  7.  6.  5.  4.
  3.  2.  1. 25.  1.  2.  3.  4.  5.  6. 18. 17.  9. 10. 11. 13. 12. 11.
 10.  9.  8.  7.  6.  5.  4.  3.  2.  1.]
[0.00000000e+00 1.92759560e-06 1.28536475e-05 3.73999231e-05
 8.57080408e-05 1.42805249e-04 2.49880557e-04 5.10991667e-04
 5.71386939e-04 6.72316904e-04 9.52034995e-04 1.30250559e-03
 1.66587038e-03 3.37663800e-03 3.40670777e-03 3.49636996e-03
 3.80934292e-03 3.94646937e-03 4.48212975e-03 6.24514185e-03
 6.34689997e-03 6.72948446e-03 8.68337057e-03 9.09506179e-03
 1.11058025e-02 2.25000000e-02 2.25109230e-02 2.25728373e-02
 2.27119329e-02 2.29856789e-02 2.33092297e-02 2.39159898e-02
 2.53956194e-02 2.57378593e-02 2.63097958e-02 2.78948650e-02
 2.98808650e-02 3.19399321e-02 4.16342820e-02 4.18046774e-02
 4.23127631e-02 4.40862765e-02 4.48633264e-02 4.78987352e-02
 5.78891372e-02 5.84657665e-02 6.06337453e-02 7.17057666e-02
 7.40386835e-02 8.54328810e-02 1.50000000e-01 1.50010923e-01
 1.50072837e-01 1.50211933e-01 1.50485679e-01 1.50809230e-01
 1.51415990e-01 1.52895619e-01 1.53237859e-01 1.53809796e-01
 1.55394865e-01 1.57380865e-01 1.59439932e-01 1.69134282e-01
 1.69304677e-01 1.69812763e-01 1.71586277e-01 1.72363326e-01
 1.75398735e-01 1.85389137e-01 1.85965766e-01 1.88133745e-01
 1.99205767e-01 2.01538684e-01 2.12932881e-01 2.77500000e-01
 2.77561897e-01 2.77912745e-01 2.78700953e-01 2.80252180e-01
 2.82085635e-01 2.85523942e-01 2.93908510e-01 2.95847869e-01
 2.99088843e-01 3.08070902e-01 3.19324902e-01 3.30992949e-01
 3.85927598e-01 3.86893172e-01 3.89772324e-01 3.99822234e-01
 4.04225516e-01 4.21426166e-01 4.78038444e-01 4.81306010e-01
 4.93591223e-01 5.56332677e-01 5.69552540e-01 6.34119659e-01
 0.00000000e+00]
```

In [35]:
```
plot_fig1(y)
plot_fig2(policy)
```



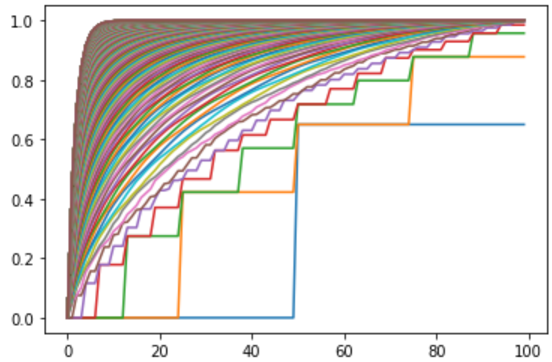


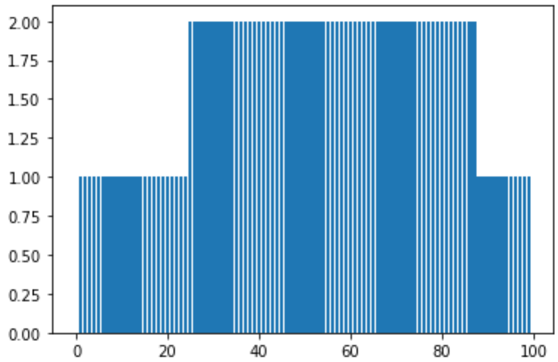
$P(head) = 0.65$

In [37]:
```
agent_3.clear()
y = []
while True:

    agent_3.reset()
    agent_3.update()

    y.append(agent_3.V)
    if agent_3.get_delta() < agent_3.get_threshold():
        break

policy = agent_3.get_policy()
print(policy)
print(agent_3.V)
```

```
[0. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
 1. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2.
 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2.
 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 2. 1. 1. 1. 1. 1. 1. 1. 1.
 1. 1. 1. 1.]
[0.         0.46137334 0.70981407 0.84360309 0.91565822 0.9544721
 0.97538609 0.98666051 0.99274307 0.99602866 0.99780691 0.99877234
 0.99929903 0.99958853 0.99974948 0.99984048 0.9998932  0.99992476
 0.99994446 0.99995737 0.99996628 0.99997274 0.99997763 0.99998147
 0.99998454 0.99998706 0.99998911 0.99999081 0.9999922  0.99999336
 0.99999432 0.99999513 0.9999958  0.99999638 0.99999686 0.99999728
 0.99999763 0.99999794 0.9999982  0.99999843 0.99999862 0.9999988
 0.99999894 0.99999907 0.99999919 0.99999929 0.99999938 0.99999945
 0.99999952 0.99999958 0.99999963 0.99999968 0.99999972 0.99999975
 0.99999978 0.99999981 0.99999984 0.99999986 0.99999987 0.99999989
 0.99999991 0.99999992 0.99999993 0.99999994 0.99999995 0.99999995
 0.99999996 0.99999997 0.99999997 0.99999998 0.99999998 0.99999998
 0.99999998 0.99999999 0.99999999 0.99999999 0.99999999 0.99999999
 0.99999999 1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         1.         1.
 1.         1.         1.         1.         0.        ]
```

In [38]:
```
plot_fig1(y)
plot_fig2(policy)
```





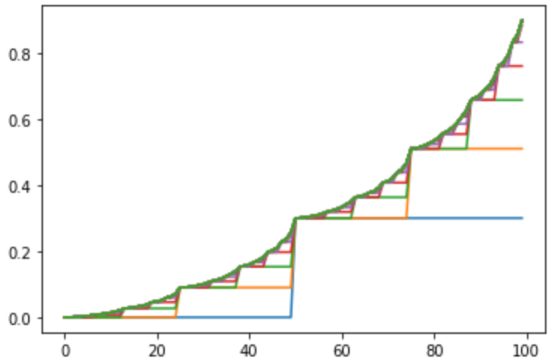
Part-c

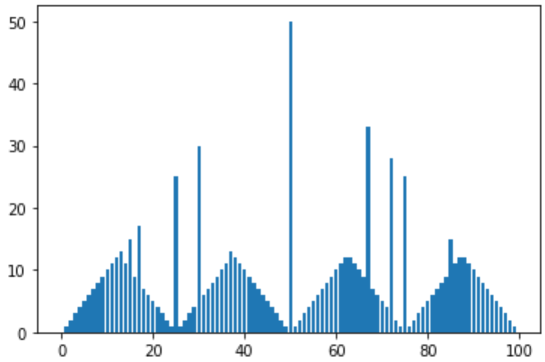
$\theta = 0.00000000000000001$

In [39]:
```
agent_1.set_threshold(0.00000000000000001)
agent_1.clear()
y = []
while True:

    agent_1.reset()
    agent_1.update()

    y.append(agent_1.V)
    if agent_1.get_delta() < agent_1.get_threshold():
        break

policy = agent_1.get_policy()
print(policy)
print(agent_1.V)
```

```
[ 0.  1.  2.  3.  4.  5.  6.  7.  8.  9. 10. 11. 12. 13. 11. 15.  9. 17.
  7.  6.  5.  4.  3.  2.  1. 25.  1.  2.  3.  4. 30.  6.  7.  8.  9. 10.
 11. 13. 12. 11. 10.  9.  8.  7.  6.  5.  4.  3.  2.  1. 50.  1.  2.  3.
  4.  5.  6.  7.  8.  9. 10. 11. 12. 12. 11. 10.  9. 33.  7.  6.  5.  4.
 28.  2.  1. 25.  1.  2.  3.  4.  5.  6.  7.  8.  9. 15. 11. 12. 12. 11.
 10.  9.  8.  7.  6.  5.  4.  3.  2.  1.]
[0.00000000e+00 2.66917018e-04 8.89723393e-04 1.92325355e-03
 2.96574464e-03 4.32158176e-03 6.41084517e-03 8.50388325e-03
 9.88581548e-03 1.18309578e-02 1.44052725e-02 1.77664658e-02
 2.13694839e-02 2.71868419e-02 2.83462775e-02 3.00251072e-02
 3.29527183e-02 3.52816705e-02 3.94365260e-02 4.60307893e-02
 4.80175751e-02 5.16971693e-02 5.92215525e-02 6.31880185e-02
 7.12316130e-02 9.00000000e-02 9.06228064e-02 9.20760213e-02
 9.44875916e-02 9.69200708e-02 1.00083691e-01 1.04958639e-01
 1.09842394e-01 1.13066903e-01 1.17605568e-01 1.23612303e-01
 1.31455087e-01 1.39862129e-01 1.53435964e-01 1.56141314e-01
 1.60058584e-01 1.66889676e-01 1.72323898e-01 1.82018561e-01
 1.97405175e-01 2.02041008e-01 2.10626728e-01 2.28183623e-01
 2.37438710e-01 2.56207097e-01 3.00000000e-01 3.00622806e-01
 3.02076021e-01 3.04487592e-01 3.06920071e-01 3.10083691e-01
 3.14958639e-01 3.19842394e-01 3.23066903e-01 3.27605568e-01
 3.33612303e-01 3.41455087e-01 3.49862129e-01 3.63435964e-01
 3.66141314e-01 3.70058584e-01 3.76889676e-01 3.82323898e-01
 3.92018561e-01 4.07405175e-01 4.12041008e-01 4.20626728e-01
 4.38183623e-01 4.47438710e-01 4.66207097e-01 5.10000000e-01
 5.11453215e-01 5.14844050e-01 5.20471047e-01 5.26146832e-01
 5.33528612e-01 5.44903490e-01 5.56298920e-01 5.63822773e-01
 5.74412993e-01 5.88428706e-01 6.06728536e-01 6.26344968e-01
 6.58017250e-01 6.64329733e-01 6.73470028e-01 6.89409244e-01
 7.02089095e-01 7.24709975e-01 7.60612075e-01 7.71429020e-01
 7.91462366e-01 8.32428453e-01 8.54023656e-01 8.97816560e-01
 0.00000000e+00]
```

In [40]:
```
plot_fig1(y)
plot_fig2(policy)
```



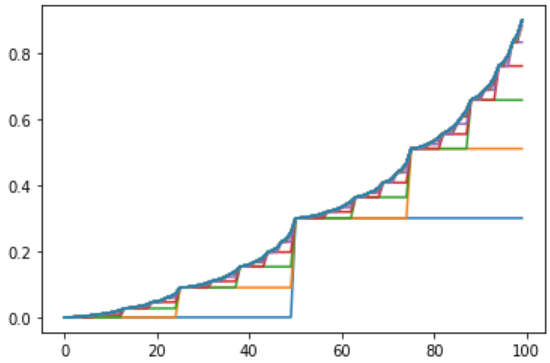


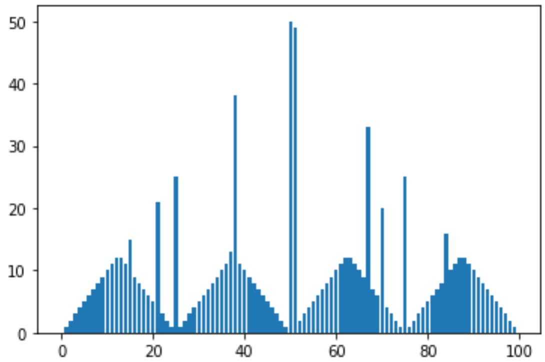
$\theta = 0.00000000000001$

In [41]:
```
agent_1.set_threshold(0.00000000000001)
agent_1.clear()
y = []
while True:

    agent_1.reset()
    agent_1.update()

    y.append(agent_1.V)
    if agent_1.get_delta() < agent_1.get_threshold():
        break

policy = agent_1.get_policy()
print(policy)
print(agent_1.V)
```

```
[ 0.  1.  2.  3.  4.  5.  6.  7.  8.  9. 10. 11. 12. 12. 11. 15.  9.  8.
  7.  6.  5. 21.  3.  2.  1. 25.  1.  2.  3.  4.  5.  6.  7.  8.  9. 10.
 11. 13. 38. 11. 10.  9.  8.  7.  6.  5.  4.  3.  2.  1. 50. 49.  2.  3.
  4.  5.  6.  7.  8.  9. 10. 11. 12. 12. 11. 10.  9. 33.  7.  6. 20.  4.
  3.  2.  1. 25.  1.  2.  3.  4.  5.  6.  7.  8. 16. 10. 11. 12. 12. 11.
 10.  9.  8.  7.  6.  5.  4.  3.  2.  1.]
[0.00000000e+00 2.66917018e-04 8.89723393e-04 1.92325355e-03
 2.96574464e-03 4.32158176e-03 6.41084517e-03 8.50388325e-03
 9.88581548e-03 1.18309578e-02 1.44052725e-02 1.77664658e-02
 2.13694839e-02 2.71868419e-02 2.83462775e-02 3.00251072e-02
 3.29527183e-02 3.52816705e-02 3.94365260e-02 4.60307893e-02
 4.80175751e-02 5.16971693e-02 5.92215525e-02 6.31880185e-02
 7.12316130e-02 9.00000000e-02 9.06228064e-02 9.20760213e-02
 9.44875916e-02 9.69200708e-02 1.00083691e-01 1.04958639e-01
 1.09842394e-01 1.13066903e-01 1.17605568e-01 1.23612303e-01
 1.31455087e-01 1.39862129e-01 1.53435964e-01 1.56141314e-01
 1.60058584e-01 1.66889676e-01 1.72323898e-01 1.82018561e-01
 1.97405175e-01 2.02041008e-01 2.10626728e-01 2.28183623e-01
 2.37438710e-01 2.56207097e-01 3.00000000e-01 3.00622806e-01
 3.02076021e-01 3.04487592e-01 3.06920071e-01 3.10083691e-01
 3.14958639e-01 3.19842394e-01 3.23066903e-01 3.27605568e-01
 3.33612303e-01 3.41455087e-01 3.49862129e-01 3.63435964e-01
 3.66141314e-01 3.70058584e-01 3.76889676e-01 3.82323898e-01
 3.92018561e-01 4.07405175e-01 4.12041008e-01 4.20626728e-01
 4.38183623e-01 4.47438710e-01 4.66207097e-01 5.10000000e-01
 5.11453215e-01 5.14844050e-01 5.20471047e-01 5.26146832e-01
 5.33528612e-01 5.44903490e-01 5.56298920e-01 5.63822773e-01
 5.74412993e-01 5.88428706e-01 6.06728536e-01 6.26344968e-01
 6.58017250e-01 6.64329733e-01 6.73470028e-01 6.89409244e-01
 7.02089095e-01 7.24709975e-01 7.60612075e-01 7.71429020e-01
 7.91462366e-01 8.32428453e-01 8.54023656e-01 8.97816560e-01
 0.00000000e+00]
```

In [42]:
```
plot_fig1(y)
plot_fig2(policy)
```





1. We see from the above plots that as $\theta \to 0$, the state value matrix and the optimal policy becomes stable for $P(head) = 0.3$

Question-2

Part-a

The problem can be modelled as a MDP tuple $< S, A, P, R >$ where :

1. S is the set of all possible states (s_1, s_2) where s_i is the number of cars at location i.
$$S = \{(0,0) \dots (20,20)\}$$
2. A is the set of all possible movement of cars that can be done given a state $s = (s_1, s_2)$.
$$A(s) = \{-min(5, s_2), \dots, +min(5, s_1)\}$$
3. Let number of cars rented at location 1 and location 2 be r_1 and r_2 respectively and cars returned be c_1 and c_2.That is, given $s = (s_1, s_2)$, $s' = (s_1 - r_1 + r_2 + c_1, s_2 - r_2 + r_1 + c_2)$.
4. $\therefore R(s, a, s') = (r_1 + r_2) \times \$10 - (c_1 + c_2) \times \$2$. where r_1, r_2, c_1, c_2 are sampled from poisson's distribution with $\lambda_{rent1} = 3$, $\lambda_{rent2} = 4$,$\lambda_{return1} = 3$, $\lambda_{return2} = 4$.
5. Given a state s, if number of cars moved are (c_1, c_2) and final state is s', then:
$$P(s, a, s') = \frac{\lambda_{return1}^{c_1}}{c_1!} \exp^{-\lambda_{return1}} \times \frac{\lambda_{return2}^{c_2}}{c_2!} \exp^{-\lambda_{return2}}$$

Assumption: I have assumed an upper bound of 8 on the number of cars that can be rented or returned as $P(E) \to 0$ for $E \geq 8$.

Bellman Equation for policy iteration

1. The Bellman Update Equation for this problem will be as follows:
   A. Policy evaluation: Let a be the action determined by π.

$$V_{n+1}(s) = \pi(a|s) \times \left\{ \sum_{rent_1} \sum_{rent_2} \sum_{return_1} \sum_{return_2} P(rent) \times P(return) \times \{R(s_1, a, s_2) + \gamma \times V_n(s_1, s_2)\} \right\}$$

```
2. Policy Improvement:
```

$$\pi_{k+1}(s) = \arg\max_{a \in A} V_\pi(s)$$

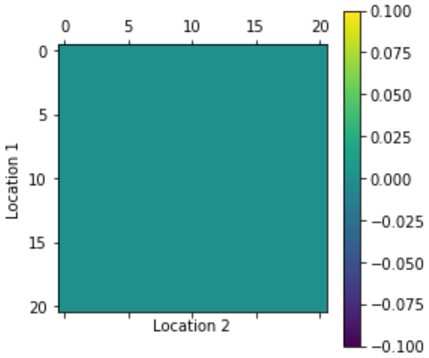
Part-b

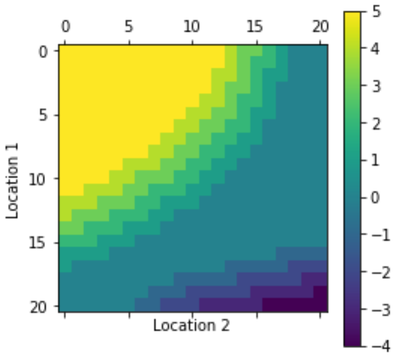
In [5]:
```
def three_dimentional_plot(V):
    fig = plt.figure()
    ax = fig.gca(projection = '3d')
    X = np.arange(0,V.shape[0],1)
    Y = np.arange(0,V.shape[1],1)
    X,Y = np.meshgrid(X,Y)
    surf = ax.plot_surface(X,Y,V,rstride = 1,cstride = 1,cmap = cm.coolwarm,linewi
    ax.set_xlabel('Location 2')
    ax.set_ylabel('Location 1')
    plt.show()
```

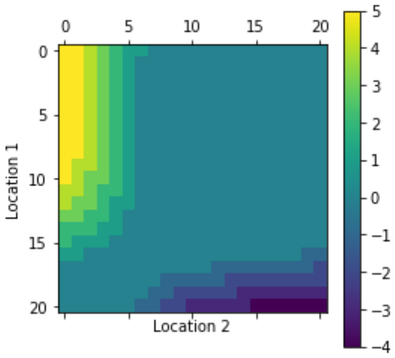
In [4]:
```
def plot_policy(all_P):
    itr = 0
    for pi in all_P:
        cmp = plt.matshow(pi)
        plt.xlabel('Location 2')
        plt.ylabel('Location 1')
        plt.colorbar(cmp)
    plt.show()
```

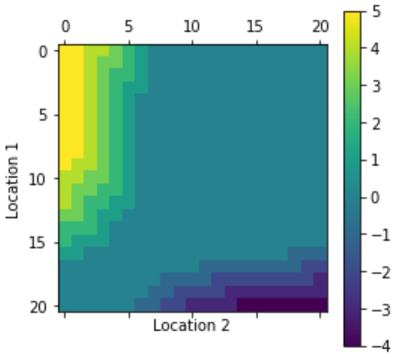
In [4]:
```
Jack_env = env.Jack_env()
agent_1 = agent.Jack_PolicyIteration(Jack_env)


all_V = []
all_P = []

all_P.append(agent_1.policy.copy())

while True:
    agent_1.evaluate_policy()
    stable = agent_1.update()

    all_V.append(agent_1.V.copy())
    all_P.append(np.flip(agent_1.policy.copy(),0))
    if stable == True:
        break
```

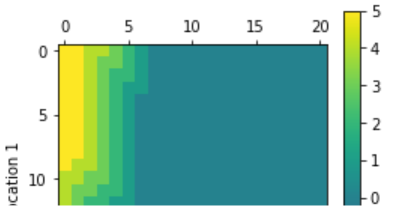
In [16]:
```
plot_policy(all_P)
```



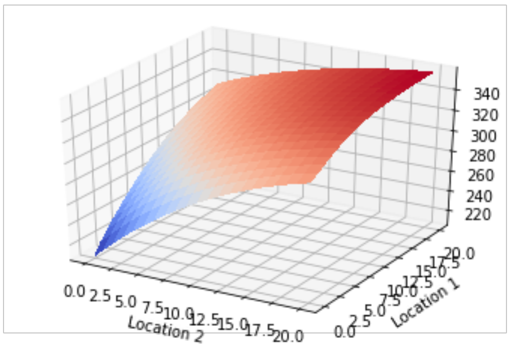








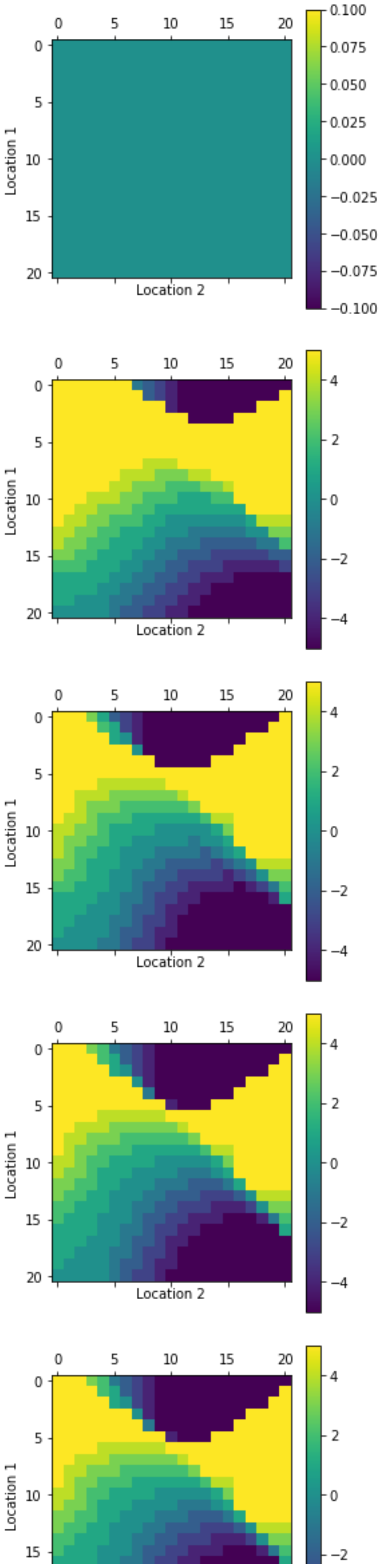
In [6]:
```
three_dimentional_plot(agent_1.V)
```



Part-c

In [2]:
```
Jack_env = env.Jack_env()
agent_2 = agent.Jack_PolicyIteration_2(Jack_env)


all_V = []
all_P = []

all_P.append(agent_2.policy.copy())

while True:
    agent_2.evaluate_policy()
    stable = agent_2.update()

    all_V.append(agent_2.V.copy())
    all_P.append(np.flip(agent_2.policy.copy(),0))
    if stable == True:
        break
```

In [6]:
```
plot_policy(all_P)
```



In [7]:
```
three_dimentional_plot(agent_2.V)
```