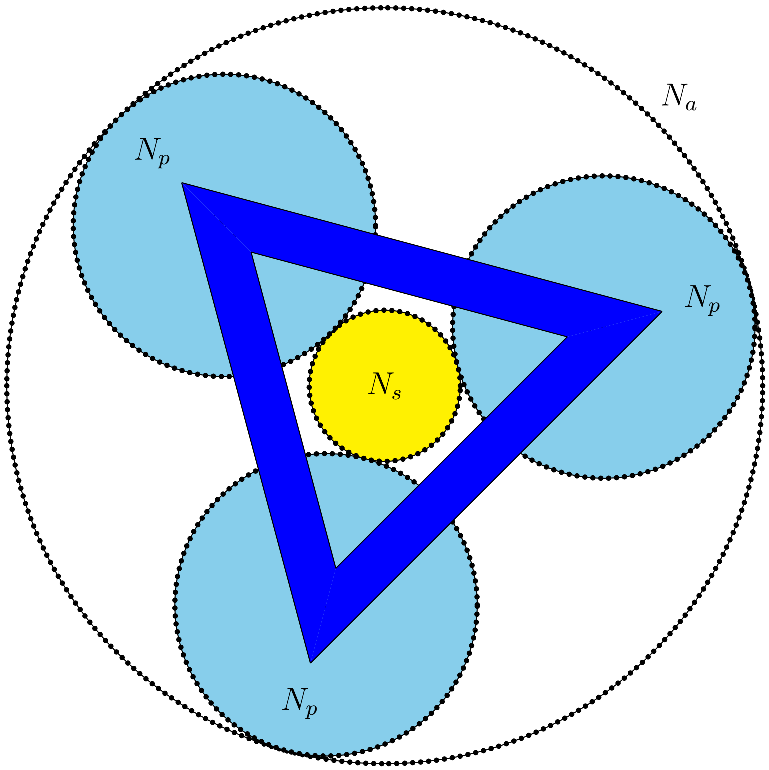
The system

The system consists of the following components:

- "Sun gear" in centre, with N_s teeth and angular velocity ω_s.
- "Planet gears" surrounding sun gear, with N_p teeth and angular velocity ω_p.
- "Annular gear" surrounding planet gears, with N_a teeth and angular velocity ω_a.
- "Planet carrier" which is fixed to each planet's axle with angular velocity ω_c.



Input and output can be connected to any of the gears, or to the carrier. If a planet gear is directly used for input or output, the system reduces to a simple 2-gear system.

In terms of reference frames, we have:

- The system frame
  - The sun frame
  - The carrier and planets frame
    - Planet 1 frame
    - ⋮ ⋮ ⋮
    - Planet n frame
  - The annular frame

Solving

Fixed carrier

In the case where the carrier is fixed, gear ratios and thus rotation rates are trivial to obtain:

$$
\begin{aligned}
\omega_s : \omega_p &= N_p : -N_s \\
\omega_p : \omega_a &= N_a : N_p \\
\omega_s : \omega_a &= N_a : -N_s \\
\therefore\ \omega_s : \omega_p : \omega_a : \omega_c &= 1 : -\frac{N_s}{N_p} : -\frac{N_s}{N_a} : 0
\end{aligned}
$$

Note: all rotation rates are relative to orientation of parent frame.

Fixed annular gear

To fix the annular gear instead of the carrier, we require that ω_c become free and that $\omega_a = 0$. This is achieved by applying a rotation to the top-level frames to cancel out the rotation of the annular gear's frame. We calculate the new rotation rates ω_x for all child frames of the system frame:

$$
\omega_x \to \omega_x' - \left(-\frac{N_s}{N_a}\right) \quad \text{only for } \omega_x \text{ that are relative to the system frame}
$$

Note that ω_p is not changed since it is measured relative to the carrier frame, not the system frame.

$$
\begin{aligned}
\omega_s : \omega_p : \omega_a : \omega_c &= 1 + \frac{N_s}{N_a} : -\frac{N_s}{N_p} : 0 : \frac{N_s}{N_a} \\
&= \frac{N_a + N_s}{N_a} : -\frac{N_s}{N_p} : 0 : \frac{N_s}{N_a}
\end{aligned}
$$

For convenience, normalize the sun gear:

$$
\omega_s : \omega_p : \omega_a : \omega_c = 1 : -\frac{N_s N_a}{N_p\left(N_a + N_s\right)} : 0 : \frac{N_s}{N_a + N_s}
$$