Simulation for Missile Tracking with Proportional Navigation and Pure Pursuit

Batuhan Berk Şahin
dept. of Computer Engineering
Ankara University
Ankara, Türkiye
batuberkshn@gmail.com

***Abstract*—Missile tracking systems play a crucial role in ensuring the success of missile missions. Developing, testing, and implementing these systems in real-world scenarios can be costly and resource-intensive. To address this challenge, simulations are employed to analyze the behavior of tracking methods. In this research, we implement and compare two different missile tracking methods: Proportional Navigation and Pure Pursuit.**

Keywords: Missile Tracking, Proportional Navigation, Pure Pursuit, Simulation, Kalman Filter.

I. Introduction

Various methods have been developed over time for missiles to track and reach their targets. The real-world implementation, development and testing of these methods impose high costs on organizations and individuals. In order to reduce costs and test the systems, simulations are carried out on these systems. In this research, two different tracking methods were implemented.

II. Problem Definition

Missile tracking systems face various problems. Some of these real-world problems are ignored or simplified in order to keep the focus of the research on the simulation of tracking systems and the analysis of the results of this simulation.

A. Tracking Systems

Two different methods will be used in this research to enable missiles to track their targets. The first one is the Proportional Navigation method and the other one is the Pure Pursuit method.

B. Target Observation

For the simulation set, it is necessary to reach the observed values of the target. In the real world, these values are reached by various methods. In this research, we will abstract the observer in the simulation set and reduce it to a simple form. It is assumed that the measured values of the target can be reached in any way. It can be assumed that there can be a radar observing the target from any distance or angle.

The radar's observation frequency and margin of error will be included in the model.

C. Filtering Problem

For the error in the observed values of the missile tracking system, the effect of filtering the values on the tracking system will be investigated. Kalman Filter will be applied as a filter.

III. Creating Models

A. Pure Pursuit Model

The Pure Pursuit method is simply based on a calculation that tracks the line of sight of the target per unit time.

The line of sight (L_s) can be calculated as:

$$L_s = \sqrt{(x_t - x)^2 + (y_t - y)^2}$$

The steering angle (δ) for Pure Pursuit can then be calculated as:

$$\delta = \text{atan2}(2L \cdot \sin(\alpha), L_s)$$

Where α is the angle between the vehicle's heading and the line of sight:

$$\alpha = \text{atan2}(y_t - y, x_t - x) - \theta$$

B. Proportional Navigation Model

$$\omega_{\text{PN}} = N \cdot V_c \cdot \dot{\lambda}$$

Where:

N : Unitless navigation gain (constant), between 3 to 5
V_c : Closing Velocity (or "range closing rate")
$\dot{\lambda}$: Line-of-sight (LOS) rotation rate

C. Blast-field Model

Since the missile does not know exactly where the target is when it is tracking it, it should detonate at the right time so that it has a better chance of hitting the target. The area affected by the explosion (W_r) and the distance at which the missile will explode are equal in this simulation scheme. We can say that the greater the impact of the missile's blast radius, the farther the missile will explode from the target.

The blast field model can be represented as:

$$W_r = \sqrt{(x - x_t)^2 + (y - y_t)^2}$$

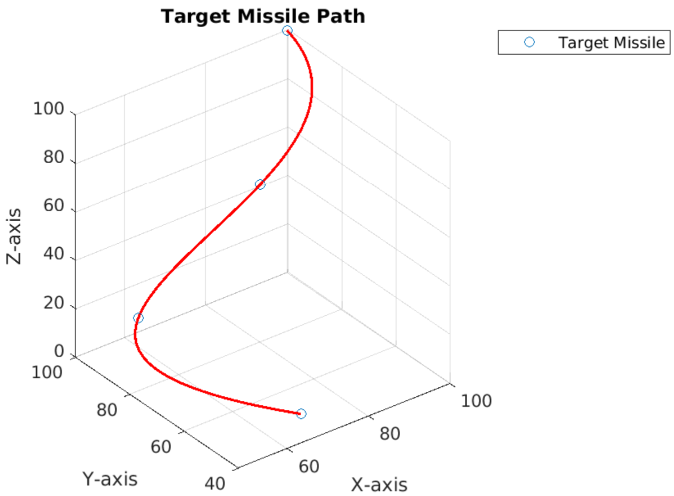

Fig. 1. Path followed by the target generated according to the Cubic Spline model

D. Target Trajectory Model

The model of the target's path is based on a cubic spline model with 4 defined positions.

$$f(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

where:

$$x(t) = \begin{cases} a_{x,1}t^3 + b_{x,1}t^2 + c_{x,1}t + d_{x,1}, & t_1 \leq t \leq t_2 \\ a_{x,2}t^3 + b_{x,2}t^2 + c_{x,2}t + d_{x,2}, & t_2 \leq t \leq t_3 \\ a_{x,3}t^3 + b_{x,3}t^2 + c_{x,3}t + d_{x,3}, & t_3 \leq t \leq t_4 \end{cases}$$

and

$$y(t) = \begin{cases} a_{y,1}t^3 + b_{y,1}t^2 + c_{y,1}t + d_{y,1}, & t_1 \leq t \leq t_2 \\ a_{y,2}t^3 + b_{y,2}t^2 + c_{y,2}t + d_{y,2}, & t_2 \leq t \leq t_3 \\ a_{y,3}t^3 + b_{y,3}t^2 + c_{y,3}t + d_{y,3}, & t_3 \leq t \leq t_4 \end{cases}$$

Fig. 1, this generated path can be observed.

E. Radar Measurements Model

In our simulation setup, the radar that takes measurements and sends position information to the missile is inaccurate. The model of these erroneous measurements has a Gaussian error distribution. In the simulation setup, the standard deviation of the distribution is a variable. The cases where the radar is more or less inaccurate can be observed separately.

The probability density function (PDF) of the Gaussian distribution for measurement errors is given by:

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Denoting the true position as (x_t, y_t) and the radar-measured position as (x_r, y_r), the errors in the x and y directions (e_x and e_y) can be modeled as:

$$e_x \sim \mathcal{N}(0, \sigma)$$

$$e_y \sim \mathcal{N}(0, \sigma)$$

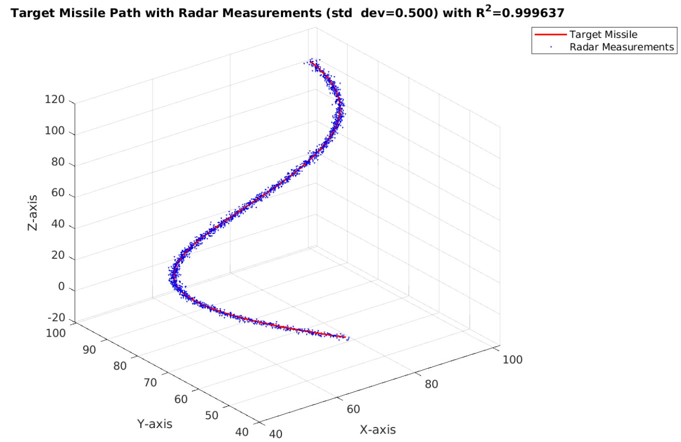

Fig. 2. Representation of erroneous measurements in the same space as the true target position

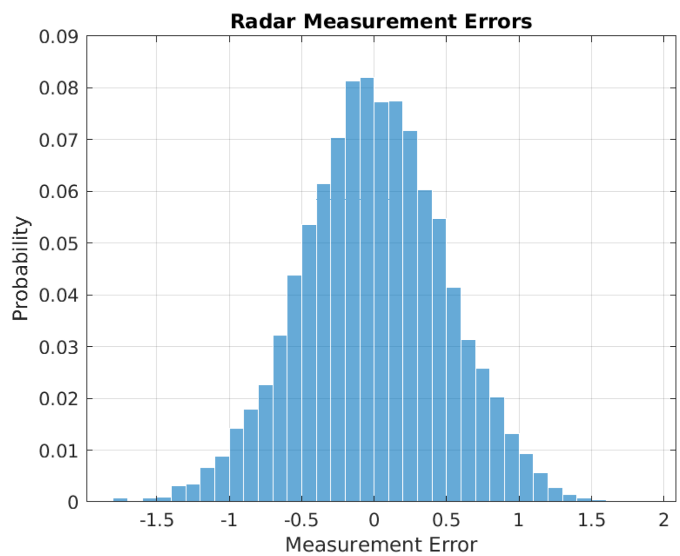

Fig. 3. Gaussian error distribution

The radar-measured position is then obtained by adding these errors to the true position:

$$x_r = x_t + e_x$$

$$y_r = y_t + e_y$$

Comparison of these erroneous measurements with the actual target position is shown in Fig. 2 and the error distribution is shown in Fig. 3.

IV. Filtering

Even if the measurements are too few or too many, the measurements should be filtered to increase the consistency of the missile's tracking system. Although there are many ways to do this, in this simulation setup it was decided to use a Kalman filter.

For 3D Constant Velocity, a Kalman filter was applied over the radar measurements. As a result, it was observed that the R^2 score increased.

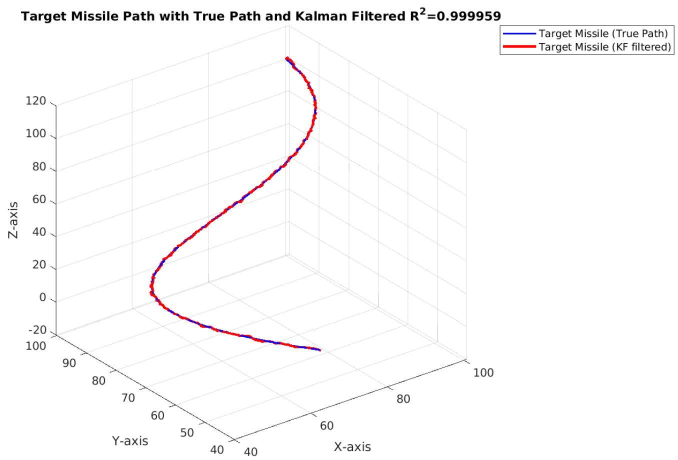

Fig. 4. Kalman filtered estimated path

The path followed by the target and the estimated path resulting from the Kalman filter are shown in Fig. 4 can be observed.

A. R^2 Score

The R^2 method, also known as the coefficient of determination, helps us understand how well our model predicts the target variable. The R^2 value ranges from 0 to 1, where 1 indicates a perfect prediction, and 0 means the model doesn't provide any predictive power.

The formula for R^2 is as follows:

$$R^2 = 1 - \frac{\text{SSR}}{\text{SST}} \tag{1}$$

Where:

SSR : Sum of squared residuals

SST : Total sum of squares

V. Simulation

MATLAB version 2023b was used as the software for all simulation setups.

A. Visualization

For both the Pure Pursuit method and the Proportional Navigation method, MATLAB was used to visually simulate the target and missile movements and results in 3D space. The missiles always started moving from the point (0, 0, 0) and tried to capture the target. If they hit the target, the text **IMPACT** appears on the screen, if not, **NOT IMPACT** appears on the screen.

The variables for Pure Pursuit and Proportional Navigation are the speed (s), the diameter of the blast-field (W_r) and the standard deviation (σ) of the radar measurements. In addition, for Proportional Navigation there is a variable Navigation Constant (N) that can be changed in the simulation.

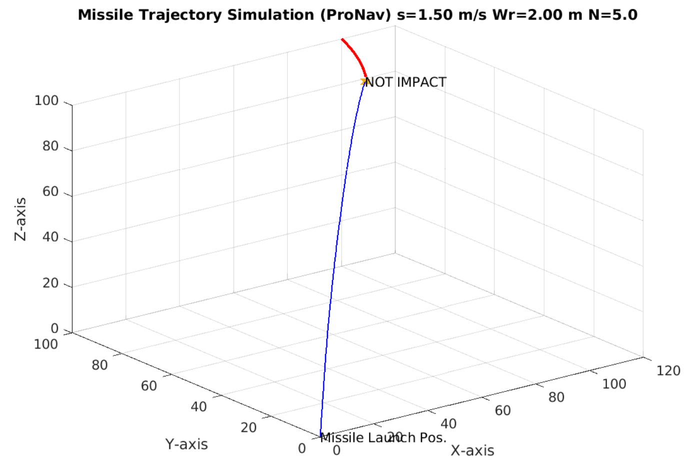

Fig. 5. Missile Trajectory Simulation for Proportional Navigation

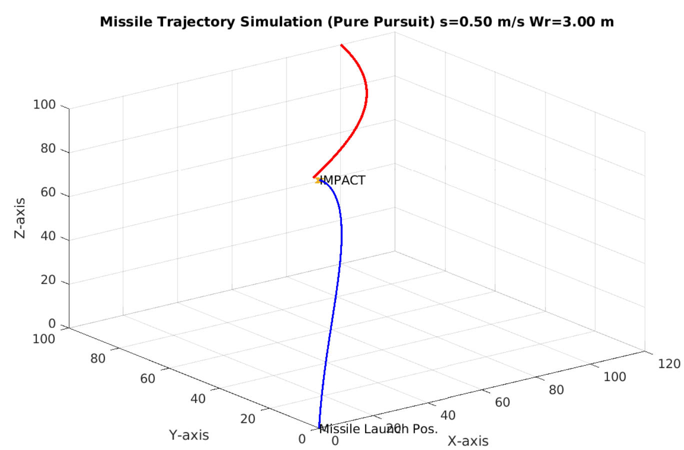

Fig. 6. Missile Trajectory Simulation for Pure Pursuit Navigation

B. Monte Carlo Simulation

The Monte Carlo simulation was performed with different sets of parameters to observe the system's behavior under varying conditions. The key parameters considered were the radar error standard deviation (σ), missile speed (s), blast-field radius (W_r) and Navigation Constant (N).

The following parameter values were used in the Monte Carlo simulation:

- Number of Monte Carlo runs: 40
- Radar error standard deviation (σ): [0.75, 1.5, 2]
- Missile pursuit speed: 0.5:0.25:2
- Missile pursuit warhead radius: [0.5, 1, 1.5, 2]
- Proportional Navigation Navigation Constant (N): 3:1:5
- Proportional Navigation speed: 0.5:0.25:2
- Proportional Navigation warhead radius: [0.5, 1, 1.5, 2]

The simulation resulted in a total of **10040** trials for Proportional Navigation and **3320** trials for Pure Pursuit.

VI. Metrics

A. Tables

TABLE I
IMPACT OF RADAR ERROR STANDARD DEVIATION ON SIMULATION RESULTS FOR PURSUIT MISSILES

Std Deviation (σ)	Mean Impact Rate (%)
0.75	0.749107
1.50	0.544643
2.00	0.485714

TABLE II
IMPACT OF RADAR ERROR STANDARD DEVIATION AND NAVIGATION CONSTANT ON SIMULATION RESULTS FOR PROPORTIONAL NAVIGATION MISSILES

Std Deviation (σ)	Navigation Constant (N)	Mean Impact Rate (%)
0.75	3	0.750000
	4	0.798214
	5	0.808929
1.50	3	0.565179
	4	0.567857
	5	0.575000
2.00	3	0.486607
	4	0.480357
	5	0.480357

TABLE III
IMPACT OF MISSILE PURSUIT SPEED ON SIMULATION RESULTS FOR PURE PURSUIT MISSILES

Speed	Mean Impact Rate (%)
0.50	0.493750
0.75	0.583333
1.0	0.597917
1.25	0.654167
1.50	0.643750
1.75	0.564583
2.0	0.614583

TABLE IV
IMPACT OF MISSILE WARHEAD RADIUS ON SIMULATION RESULTS FOR PURE PURSUIT MISSILES

Warhead Radius	Mean Impact Rate (%)
0.5	0.322619
1.0	0.641667
1.5	0.698810
2.0	0.709524

TABLE V
IMPACT OF MISSILE PURSUIT SPEED ON SIMULATION RESULTS FOR PROPORTIONAL NAVIGATION MISSILES (N=3)

Speed	Mean Impact Rate (%)
0.50	0.566667
0.75	0.600000
1.0	0.616667
1.25	0.575000
1.50	0.612500
1.75	0.602083
2.0	0.631250

TABLE VI
IMPACT OF MISSILE WARHEAD RADIUS ON SIMULATION RESULTS FOR PROPORTIONAL NAVIGATION MISSILES (N=3)

Warhead Radius	Mean Impact Rate (%)
0.5	0.332143
1.0	0.638095
1.5	0.698810
2.0	0.733333

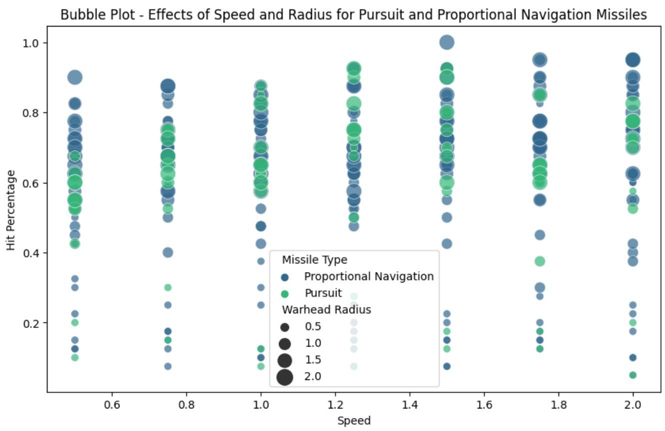

Fig. 7. Effects of Speed and Radius for Pursuit and Proportional Navigation Missiles

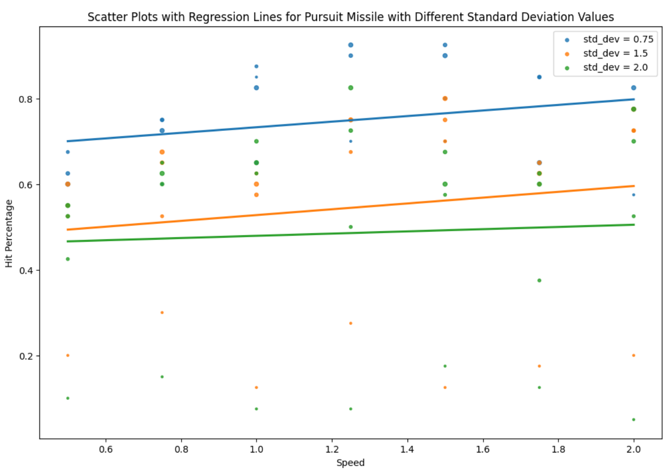

Fig. 8. Regression Lines on Std Dev for Pursuit Missile

B. Graphs

Results of speed and blast-field radius for pure pursuit and proportional navigation shown on the bubble plot Fig. 7 for pure pursuit and proportional navigation.

The effect of speed at different standard deviation values on both the missile using Pure Pursuit and the missile using Proportional Navigation is shown in Fig. 8 and Fig. 9 for different standard deviation values.

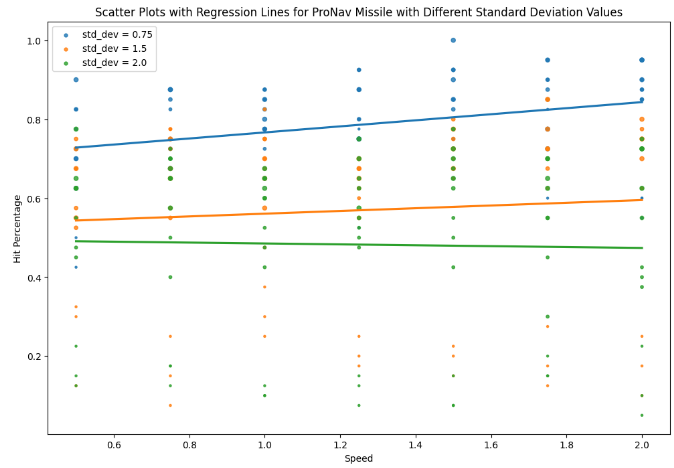

Fig. 9. Regression Lines on Std Dev for ProNav Missile

VII. Conclusion

This research delves into the simulation of missile tracking using Proportional Navigation and Pure Pursuit methods. We abstract real-world complexities to create models that capture essential aspects of missile-target interactions. Our investigation includes the impact of radar errors, the application of Kalman filtering, and the evaluation of tracking performance under various conditions.

The Monte Carlo simulation provides a comprehensive view of the system's behavior across different parameter sets. Notably, the impact of radar error standard deviation, missile speed, blast-field radius, and Navigation Constant on tracking performance is thoroughly examined.

The findings suggest that Proportional Navigation demonstrates robustness in tracking targets under varying conditions. The results also highlight the importance of filtering techniques, such as the Kalman filter, in enhancing the accuracy of missile tracking systems.

In conclusion, the simulation results contribute valuable insights into the performance of missile tracking methods. This research serves as a foundation for further studies and optimizations in the field of missile guidance and control.

References

[1] THREE PLANE APPROACH FOR 3D TRUE PROPORTIONAL NAVIGATION, INANÇ MORAN, 2005

[2] BIASED PROPORTIONAL NAVIGATION GUIDANCE FOR IMPACT ANGLE CONTROL WITH EXTENSION TO THREE-DIMENSIONAL ENGAGEMENTS, KORAY SAVAŞ ERER, 2015

[3] Introduction to Proportional Navigation, blahdy, 2015, https://www.moddb.com/members/blahdy/blogs/gamedev-introduction-to-proportional-navigation-part-i