Realistic Simulation of Free Falling Objects of Different Weights and in Different Environments

Batuhan Berk Şahin
dept. of Computer Engineering
Ankara University
batuberkshn@gmail.com

***Abstract*—In this report, we explore the realistic simulation of free-falling objects with different masses in both frictional and frictionless environments. We delve into the mathematical models behind these simulations, the fundamental equations of free fall motion, and the effects of air resistance on falling objects. Additionally, we discuss the concept of terminal velocity and its significance in understanding object behavior during free fall. The simulation results offer valuable insights into the behavior of objects in different conditions, which can be useful for various applications. Overall, this report provides a comprehensive overview of our modeling and simulation efforts in the context of free-falling objects.**

I. Introduction

This report was written for the second assignment of the Modeling and Simulation course. A realistic simulation of different objects of different masses in a frictional and frictionless environment was simulated.

II. Requirements

Unlike the previous assignment, the MATLAB version has been updated. Version has been updated as **R2023b**. **Symbolic Math Toolbox** and **Image Processing Toolbox** are installed as plugins to MATLAB, but these plugins are not needed to run the MATLAB file in the context of the assignment.

III. Preparing the Model

In the previous assignment we derived a mathematical model of an object in free fall in a frictionless environment. For this assignment we need to derive an extra mathematical model of an object in free fall in a frictional environment.

A. Determine the Fundamental Equation of Free Fall Motion

Based on Newton's second law [1], we have:

$$F = m \cdot a$$

And therefore:

$$a = \frac{F}{m}$$

Considering the forces acting on the object, we have:

$$F = F_{\text{gravity}} - F_{\text{drag}}$$

So, the acceleration of the object in a frictional environment is given by:

$$a = \frac{F_{\text{gravity}} - F_{\text{drag}}}{m}$$

The sum of acceleration over time gives velocity, and the sum of velocity over time gives displacement.

$$v = \int a\, dt$$

And:

$$x = \int v\, dt$$

In our simulation, we will discretize time and use these equations to update the object's velocity and position over time.

B. Equation of the Drag Force

The equation for the drag force [2] (F_{drag}) can be expressed as follows:

$$F_{\text{drag}} = \frac{1}{2}\rho A k v^2$$

Where:

$$\begin{aligned} F_{\text{drag}} &: \text{Drag force} \\ \rho &: \text{Air density} \\ A &: \text{Cross-sectional area of the object} \\ k &: \text{Drag coefficient} \\ v &: \text{Velocity of the object} \end{aligned}$$

This equation describes the force of air resistance acting on the object during its free fall in a frictional environment.

In order to keep the model simple, we take parameters such as air density and drag coefficient as constant, just like gravity was taken constant in the previous assignment.

$$\begin{aligned} g &: 9.81\text{m/s}^2 \\ \rho &: 1.2\text{kg/m}^3 \\ A &: 0.09\text{m}^2 \\ k &: 0.4 \end{aligned}$$

In this way, we keep the model simple and make it easier for ourselves to perform the simulation.

C. Terminal Velocity Equation

We need to understand the concept of terminal velocity, which is the maximum velocity a free-falling object reaches when the force of gravity pulling it downwards is equal to the air resistance pushing against it.

For this, we can use this equation [3]:

$$v_t = \sqrt{\frac{2mg}{\rho A k}}$$

Where:

v_t : Terminal velocity
m : Mass of the object
g : Acceleration due to gravity
ρ : Air density
A : Cross-sectional area of the object
k : Drag coefficient

The terminal velocity depends on the mass, air density, cross-sectional area, and drag coefficient of the object. When the object reaches terminal velocity, it will no longer accelerate and will fall at a constant speed.

IV. Simulation

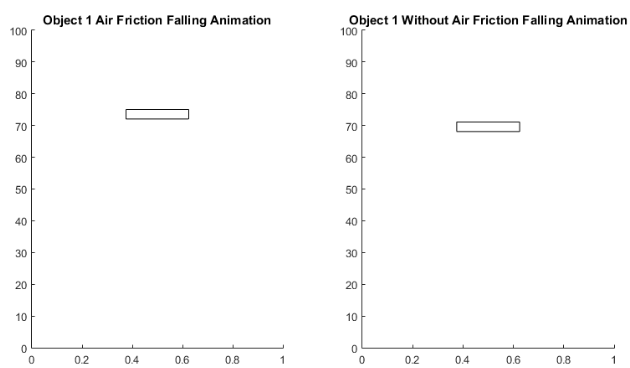

Fig. 1. Non-real-time 2D simulation

After the model was created, the simulation setup was prepared. The basic loop logic that provides the equations and the plot layouts where the results of this logic will be observed were set.

There are 3 different plots for 4 devices in the simulation. The first and largest of these is the visual output plot, which can be observed in Figure 1. The second one is the position-time plot of the object and the last one is the velocity-time plot. In the velocity-time plot, the limit velocity can also be observed.

In the simulation there are 4 different setups for 2 objects of 2 different weights and 2 different environments (with and without friction). At the beginning of the application, the initial height and mass information for the 1st and 2nd objects are obtained.

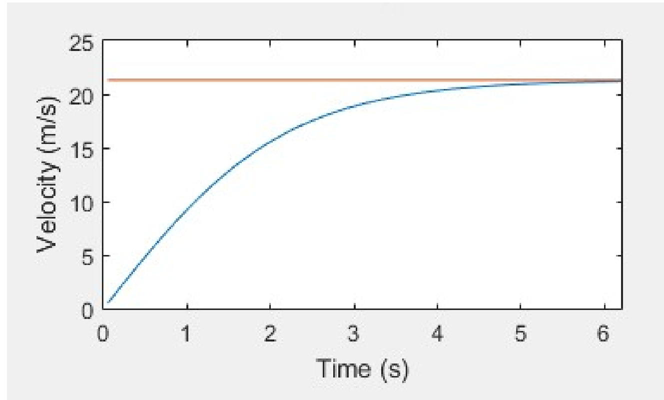

Fig. 2. Velocity-time plot of a 1 kg object falling freely from a height of 100 meters in a frictional environment

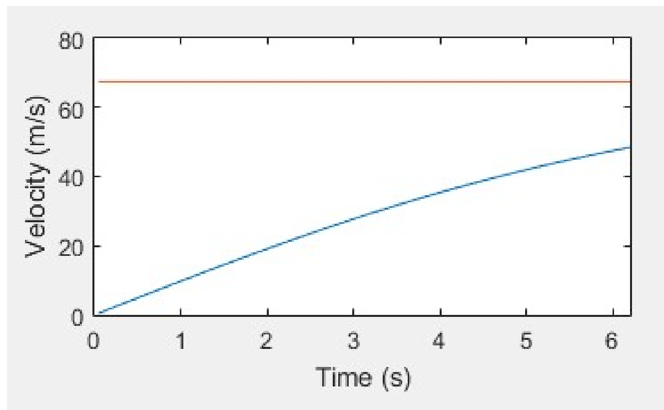

Fig. 3. Velocity-time plot of a 10 kg object falling freely from a height of 100 meters in a frictional environment

V. Conclusion

The simulation provides valuable insights into the behavior of free-falling objects of different masses in both frictional and frictionless environments. Based on this simplified model, we can use parameters such as air density and drag coefficient to make sense of the natural phenomenon we are studying.

On the other hand, we can optimize the internal parameters (drag coefficient being the biggest example) by keeping the input and output constant as we expect. This would be very beneficial for us in terms of cost.

Finally, by looking at the output of the simulation, we can more easily understand and comment on the free fall motion in frictional and frictionless environments.

References

[1] Newton's Laws of Motion article on Wikipedia, 'Laws' section, heading 'Second law' https://en.wikipedia.org/wiki/Newton%27s_laws_of_motion

[2] Air Friction Part, http://hyperphysics.phy-astr.gsu.edu/hbase/airfri.html

[3] Terminal Velocity article on Wikipedia, under 'Physics' section, https://en.wikipedia.org/wiki/Terminal_velocity