Basics of Trees

EECS 233

Hierarchies Everywhere

- Your local directory structure
- Governmental and organizational structures
- biological taxonomy
- General approach to large-scale computer systems
  - Hostnames
  - IP addresses
  - Internet
- Trees are natural representation of hierarchies

Mundane Example: Maintaining a Sorted Collection of Data

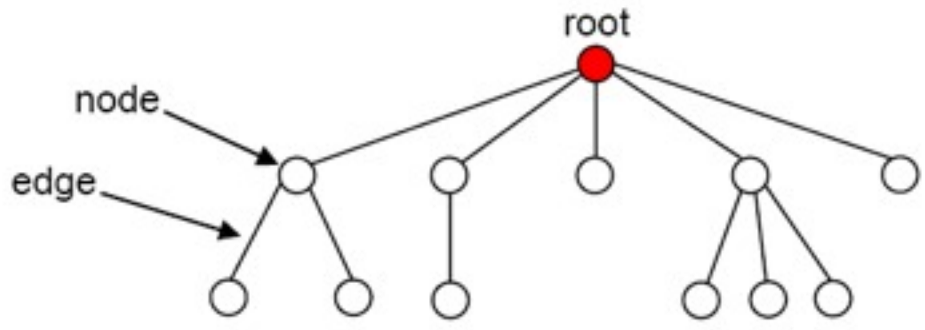
- A *data dictionary* is a sorted collection of data with the following key operations:
  - *search* for an item (and possibly delete it)
  - *insert* a new item
- If we use a list (in previous lectures) to implement a data dictionary, efficiency = *O*(n).

data structure	***searching for an item***	***inserting an item***
a list implemented using an array (sorted)	*O*(log n) using binary search	*O*(n) because we need to shift items over
a list implemented using a linked list	*O*(n) using linear search	*O*(n), (O(1) to do the actual insertion, but *O*(n) to find the right spot)

- Various tree structures allow for a more efficient data dictionary.

What Is A Tree?

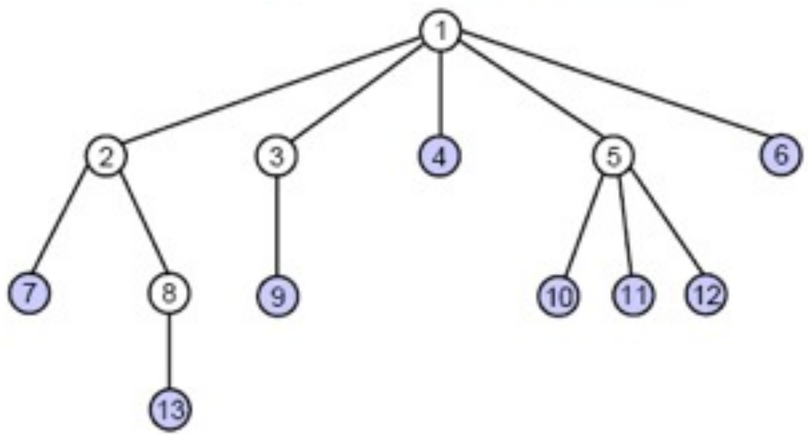
- A tree consists of:
  - a set of *nodes,* with one of them distinguished as a *root*
  - a set of *edges*, each of which connects a pair of nodes
  - no cycles



- Each node may have an associated *data item* (“payload”).
  - consists of one or more fields
  - *key field* = the field used when searching for a data item
- The node at the “top” of the tree is called the *root* of the tree.

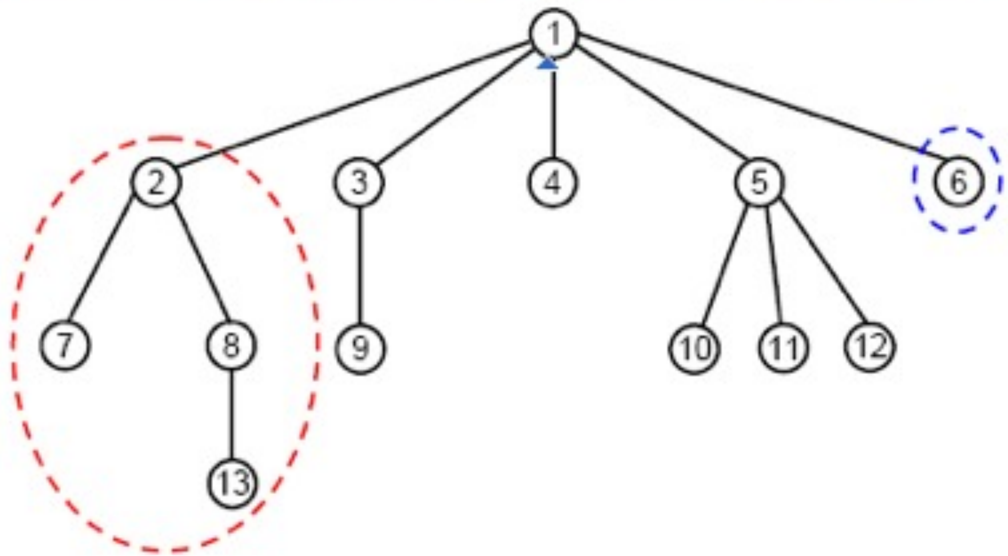
Relationships Between Nodes

- If a node N is connected to other nodes that are directly below it in the tree, N is referred to as their *parent* and the other nodes are referred to as its *children*.
  - example: node 5 is the parent of nodes 10, 11, and 12



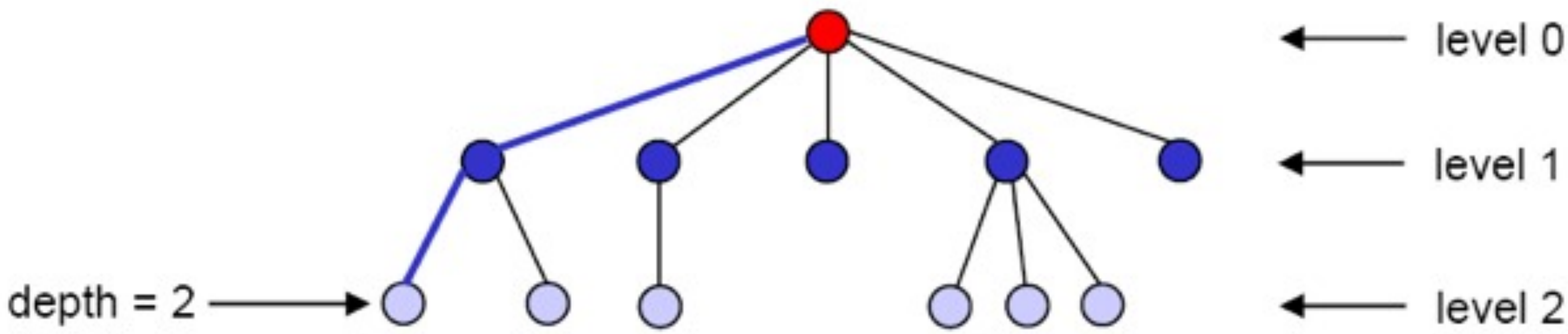
- Each node is the child of *at most one* parent.
- Other "family terms":
  - nodes with the same parent are *siblings*
  - a node's *ancestors* are its parent, its parent's parent, etc.
  - a node's *descendants* are its children, their children, etc.
- A *leaf node* is a node without children.
- An *interior node* is a non-leaf node (but sometimes meant as a non-leaf and non-root).

A Tree Is A Recursive Data Structure



- Each node in the tree is the root of a smaller tree!
  - refer to such trees as *subtrees* to distinguish them from the tree as a whole
  - example: node 2 is the root of the subtree circled above
  - example: node 6 is the root of a subtree with only one node
- We'll see that tree algorithms often lend themselves to recursive implementations.

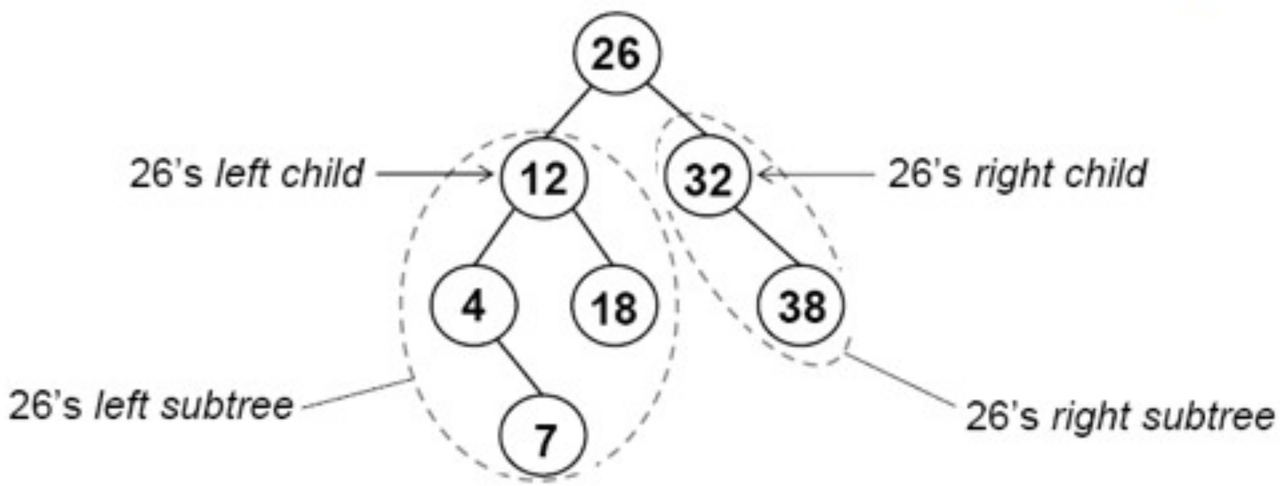
Path, Depth, Level, and Height



- There is exactly one *path* (one sequence of edges) connecting each node to the root.
- *depth* of a node = # of edges on the path from it to the root
- Nodes with the same depth form a *level* of the tree.
- The *height* of a tree is the maximum depth of its nodes.
  - example: the tree above has a height of 2

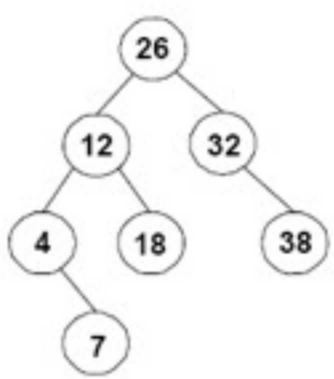
Binary Trees

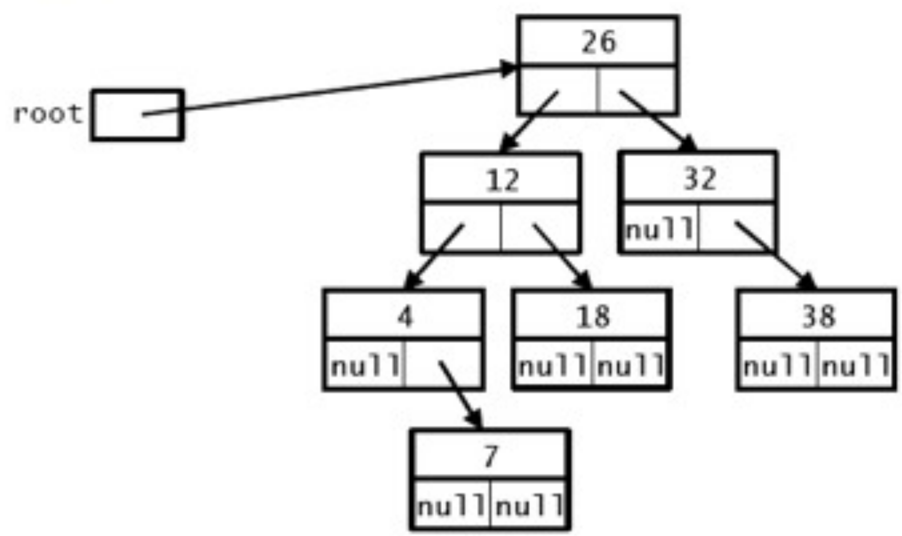
- In a *binary tree*, nodes have *at most two* children.
- Recursive definition: a binary tree is a collection of nodes that is either:
  - 1) empty, or
  - 2) contains a node R (the root of the tree) that has
    - ✓ a binary *left subtree*, whose root (if any) connects to R
    - ✓ a binary *right subtree*, whose root (if any) connects to R



Binary Tree Representation in Java

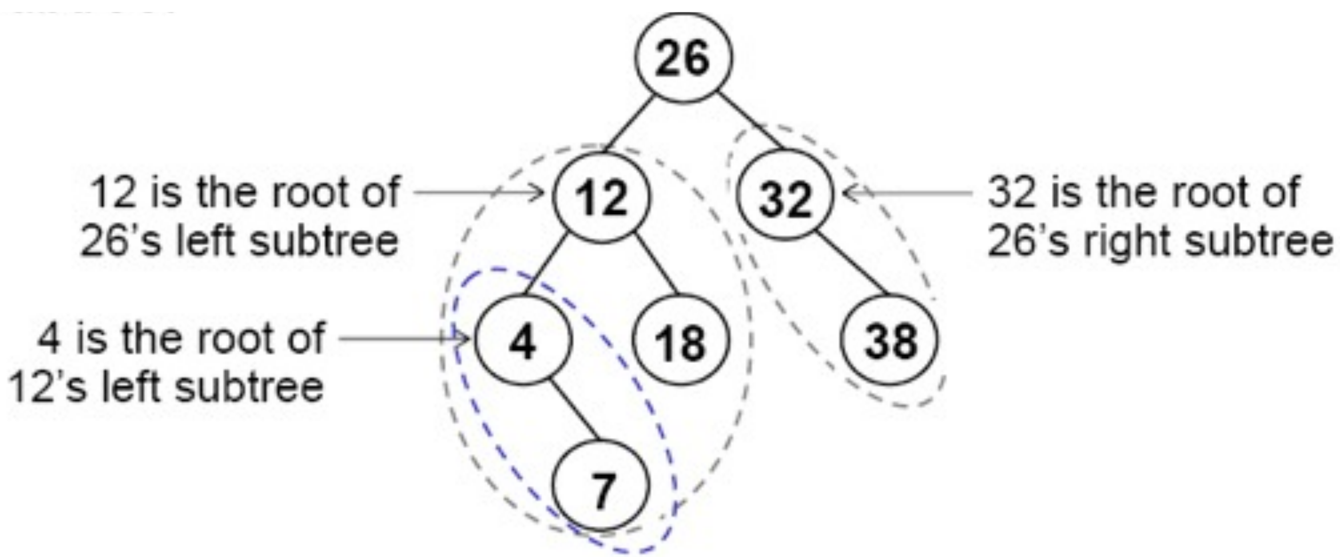
```
public class LinkedTree {
    private class Node {
        private int key;
        private String data;
        private Node left;      // reference to left child
        private Node right;     // reference to right child
        …
    }
    private Node root;
    …
}
```





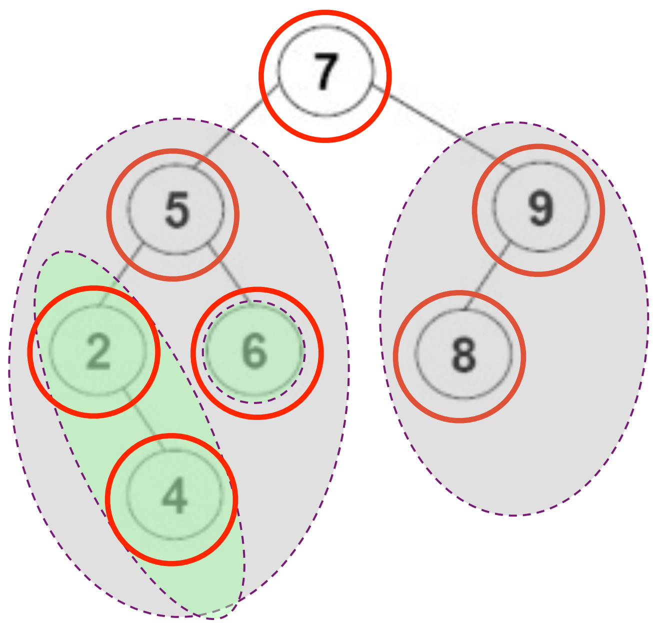
Traversing A Binary Tree

- Traversing a tree involves *visiting* all of the nodes in the tree.
  - visiting a node = processing its data in some way, e.g., print it
- We will look at four types of traversals. Each of them visits the nodes in a different order.
- To understand traversals, keep in mind the recursive definition of a binary tree: every node is the root of a subtree.



Preorder Traversal

- Preorder traversal of the tree whose root is N:
  - visit the root, N
  - recursively perform a preorder traversal of N's left subtree
  - recursively perform a preorder traversal of N's right subtree



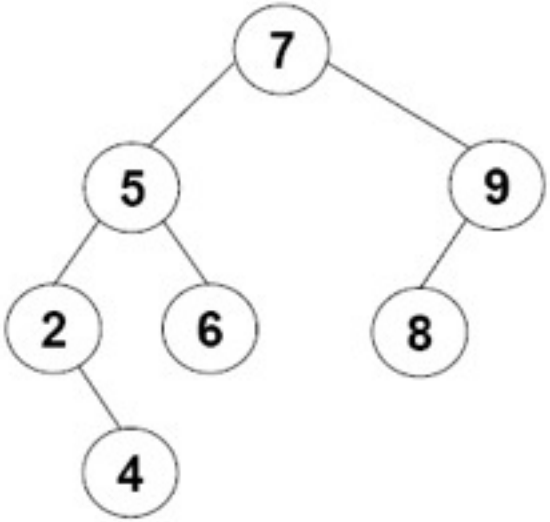
Result: 7 5 2 4 6 9 8

Implementing Preorder Traversal in Java

```
public class LinkedTree {
    …
    private Node root;
    public void preorderPrint() {
        if (root != null)
            preorderPrintTree(root);
    }
    private void preorderPrintTree(Node root) {
        System.out.print(root.key + “ ”);
        if (root.left != null)
            ?
        if (root.right != null)
            ?
    }
}
```

Postorder Traversal

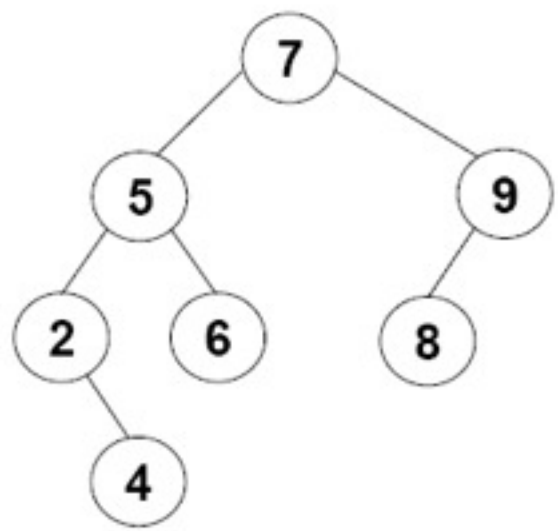
- postorder traversal of the tree whose root is N:
  - recursively perform a postorder traversal of N's left subtree
  - recursively perform a postorder traversal of N's right subtree
  - visit the root, N



Result: 4 2 6 5 8 9 7

Inorder Traversal

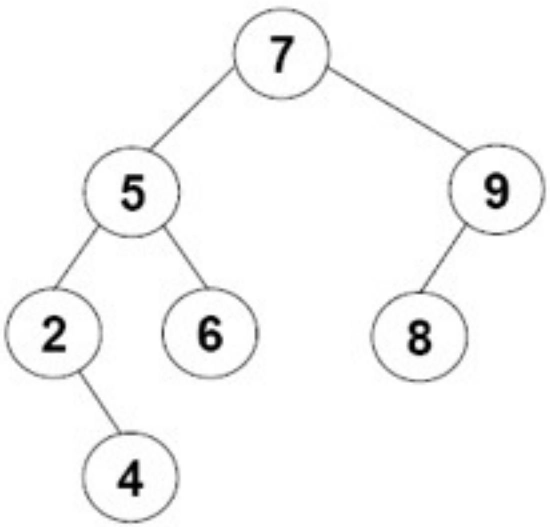
- Inorder traversal of the tree whose root is N:
  - recursively perform an inorder traversal of N’s left subtree
  - visit the root, N
  - recursively perform an inorder traversal of N’s right subtree



Result: 2 4 5 6 7 8 9

Level-Order Traversal

- Visit the nodes one level at a time, from top to bottom and left to right
- AKA, Breadth (or width) first traversal

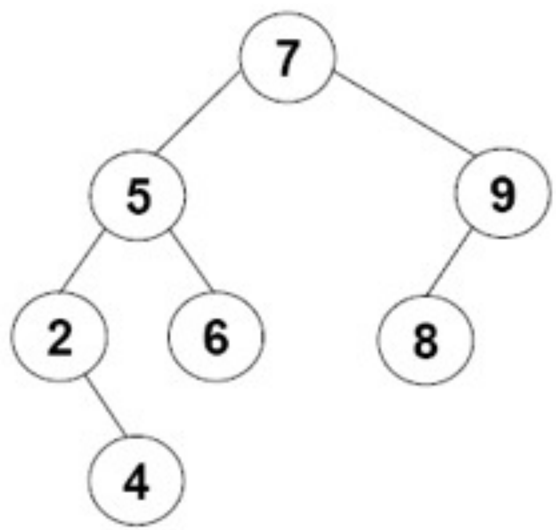


- Result: 7 5 9 2 6 8 4

Level-Order Traversal using Queue

- We can implement this type of traversal using a queue.

Initialization: insert the root in the queue
while queue is not empty
    Remove from a node from queue
    Print the node
    For each child, insert it into the queue

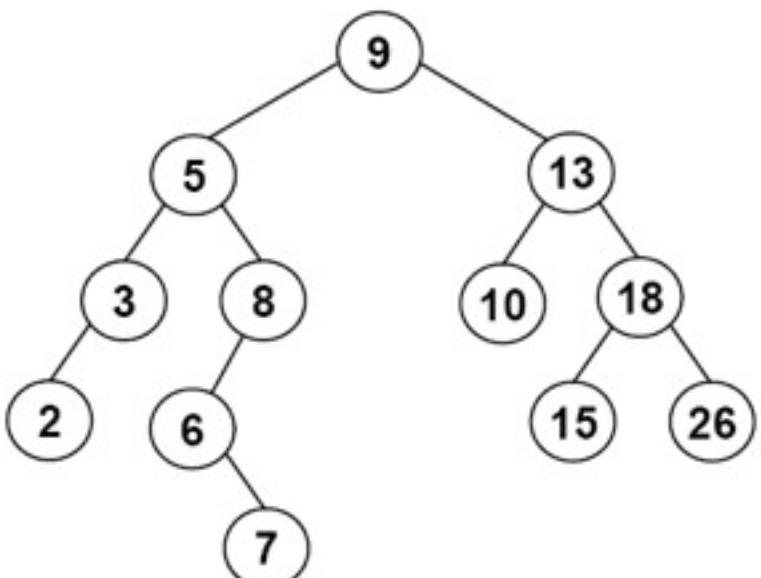


- Result: 7 5 9 2 6 8 4

Tree-Traversal Summary

- preorder: root, left subtree, right subtree
- postorder: left subtree, right subtree, root
- inorder: left subtree, root, right subtree
- level-order: top to bottom, left to right

- Perform each type of traversal on the tree below:



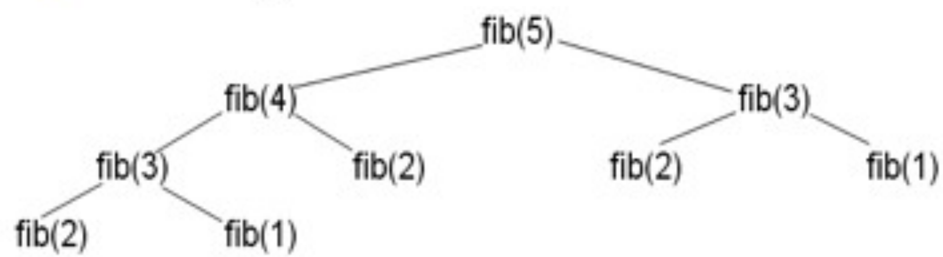
Back to Fibonacci: Order-of-Traversal versus Recursive Calls

- An execution of a recursive method forms a call tree (though not an actual tree data structure)

preorder: root, left subtree, right subtree
postorder: left subtree, right subtree, root
inorder: left subtree, root, right subtree
level-order: top to bottom, left to right

  - Example fib() and the call tree for fib(5)

```
public static long fib( int n )
{
  if (n <= 1)
    return 1;
  else
    return fib(n - 1) + fib(n - 2);
}
```



  - Analogy
    - ✓ Recursive method invocation ≡ visiting a tree node

- Q1: Recursive calls are pre-order, post-order, in-order, or level order?
- Q2: How much stack space is needed for the execution?
  - Or, would fib(1000) cause stack overflow?