

Hashing 1

EECS 233

Hashing

- We have learned several data structures that allow us to store and search for data items using their keys fields.
 - Lists and arrays - linear search complexity
 - Sorted arrays - log complexity for search; linear for inserts
 - Various trees
 - ✓ Generally logarithmic time complexity
 - ✓ Rich operations:
 - Insert, remove, search, find max/min, top-k
- Can we do better than logarithmic complexity?

Taking Stock...

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- The efficiency of the different data structures and operations:

Data structure	Search for an item	Insert an item
List (unsorted array)	$O(N)$	$O(N)$
List (sorted array)	$O(\log N)$	$O(N)$
List (linked list)	$O(N)$	$O(1)$
Binary search tree	$O(h)$ ($h = \log N$ if balanced)	$O(h)$
Balanced search tree	$O(\log N)$	$O(\log N)$

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- Hash tables and hashing techniques may allow us to search, insert, and delete an item in sub-logarithmic time, e.g., $O(1)$

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- Sorted arrays:
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- Example: storing grades about students in this class
 - Give each student a unique key (integer from 1-80).
 - Store the student records in an array, in the position determined by the key
 - We can perform both search/update and insertion in $O(1)$ time (for up to 80 students).

Hash Functions

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 - Cannot be arbitrarily assigned
 - Phone book: key is person's name, not a unique number
 - Web cache: key is a URL

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- Problem with student grades:
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- In many real-world problems, the key attribute has semantic meaning
 - Cannot be arbitrarily assigned
 - Phone book: key is person's name, not a unique number
 - Web cache: key is a URL
- To handle these problems, we use a *hash function*
 - Convert ("map") keys into array indices
 - Domain: the keys; range: integers in [0, size-of-array)
 - The word "hash": to chop into small pieces (Merriam-Webster)
 - ✓ Chopping large domain space into small number of array cells

Hash Tables

- Example: student list
 - Index = SSN mod 80
- Problem: multiple keys mapping to the same index
 - Two students with SSN 511-00-0001 and 511-00-0161
 - ✓ $511000001 \text{ mod } 80 = 511000161 \text{ mod } 80 = 1$
 - We need techniques to handle such cases called *collisions*
- The resulting data structure is known as a *hash table*
 - Hash function
 - Array
 - Procedures and data structures to handle collisions
- Operations:
 - Insert
 - Remove
 - Contains (search)
 - isEmpty, makeEmpty, etc.
 - No findMax!

Hash Functions Desiderata

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■ Example 1: Salary as key

- 10-workers shop
- Keep employees record in hash table with $(\text{salary mod } 10)$ hash function
- What if all salaries are multiple of 10K? - Bad function!
- What if all salaries are multiple of 2K? - Better function!
- Random salaries - Good function!

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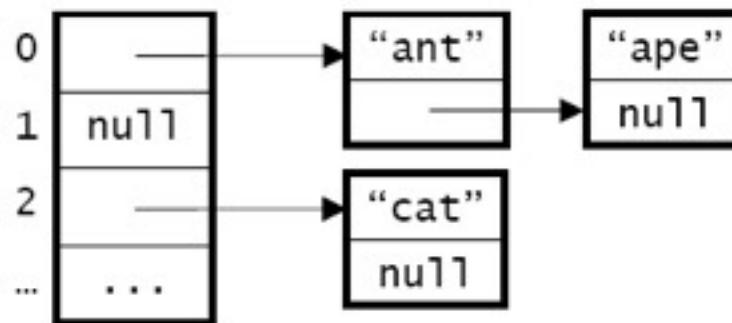
- Example 2: String as key (e.g., Webster)
 - keys = character strings composed of lower-case letters
 - hash function:
 - ✓ $h(\text{key}) = (\text{the byte sum of all characters}) \bmod \text{table-size}$
 - ✓ example: $h(\text{"cat"}) = ('c' + 'a' + 't') \bmod 100000$
 - ✓ But: assume words are mostly up to 20 characters-long
 - ✓ Max sum is $127 * 20 = 2540$; any larger table is of no use!

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 - ✓ But: assume words are mostly up to 20 characters-long
 - ✓ Max sum is $127 * 20 = 2540$; any larger table is of no use!
- Requirements for good hash functions:
 - Full table size utilization
 - Even (“uniform”) key mapping throughout the table
 - ✓ Utilizing known key distribution in the objects
 - ✓ Making hash function “random” - the distribution of keys is independent of distribution of indexes they map to

Handling Collisions with Chaining

- If multiple items are assigned the same hash code, we “chain” them together. Each position in the hash table serves as a *bucket* that is able to store multiple data items.
- Two implementations:
 - each bucket is itself an array (or points to an array)
 - ✓ disadvantages: (1) large buckets can waste memory, (2) a bucket may become full; *overflow* occurs when we try to add an item to a full bucket
 - a linked list
 - ✓ disadvantage: memory overhead for the references



Operations with Chaining

■ Search for an item

- Need to traverse the corresponding linked list or array
- To guarantee short lists, the number of hash table slots should be of the same order as the total number of items
- *Load factor*: ratio of the number of items to the number of hash table slots

■ Inserting an item

- Insertion in a linked list or array
- New list node must be allocated
- Array size need to be dynamically adjusted

■ Removing an item

- What if a large array is almost empty (wasting storage)?
- List node must be garbage-collected

Handling Collisions with Open Addressing

- When the position assigned by the hash function is occupied, find another open position - a process called *probing*.
 - Example: $h(\text{key}) = \langle\text{character encoding of first char}\rangle - \langle\text{encoding of 'a'}\rangle$
 - “wasp” has a hash code of 22, but it ends up in position 23, because position 22 is occupied.

- The hash table also performs probing to search for an item.
 - example: when searching for “wasp”, we look in position 22 and then look in position 23
 - we can only stop a search when we reach an empty position

0	“ant”
1	
2	“cat”
3	
4	“emu”
5	
6	
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...	...
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Linear Probing for Open Addressing

- Probe sequence: $h(\text{key})$, $h(\text{key}) + 1$, $h(\text{key}) + 2$, ..., wrapping around as necessary.
 - Example:
 - ✓ “ape” ($h = 0$) would be placed in position 1, because position 0 is already full.
 - ✓ “bear” ($h = 1$): try 1, $1 + 1$, $1 + 2$ – open!
 - ✓ where would “whale” end up?
- Advantage: if there is an open position, linear probing will eventually find it.
- Disadvantage: “clusters” of occupied positions develop
 - Increases the lengths of subsequent probes.
 - As load factor increases, both search and insert times look increasingly linear!

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Quadratic Probing for Open Addressing

- Probe sequence: $h(key)$, $h(key) + 1$, $h(key) + 4$, $h(key) + 9$, ..., wrapping around as necessary.

- the offsets are perfect squares: $h + 1^2, h + 2^2, h + 3^2, \dots$

- ## ➤ Example:

- ✓ “ape” ($h = 0$): try 0, $0 + 1$ – open!
 - ✓ “bear” ($h = 1$): try 1, $1 + 1$, $1 + 4$ – open!
 - ✓ “whale”?

- Advantage: reduces clustering

- Disadvantage: it may fail to find an existing open position.

Example:

- table size = 10
 - x = occupied
 - trying to insert a key with $h(key) = 0$

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Double Hashing for Open Addressing

- Use two hash functions:
 - h_1 computes the hash code
 - h_2 computes the increment for probing
 - Probe sequence: $h_1, h_1 + h_2, h_1 + 2 \cdot h_2, \dots$
- Example:
 - $h_1 =$ our previous function h
 - $h_2 =$ number of characters in the string
 - “ape” ($h_1 = 0, h_2 = 3$): try 0, $0 + 3$ – open!
 - “bear” ($h_1 = 1, h_2 = 4$): try 1 – open!
 - “whale”?
- Combines the good features of linear and quadratic probing:
 - reduces clustering
 - Theorem: will find an open position if there is one, provided the table size is a prime number.
 - Disadvantage: the need for two hash functions

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- remove “ape”

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- search for “ape”: try 0, $0 + 1$ – no item

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- but “bear” is further down in the table!

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- search for “bear”: try 1 – no item,
- but “bear” is further down in the table!
- Cannot tell if it is not in the table

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- Any difference with quadratic probing or double hashing?

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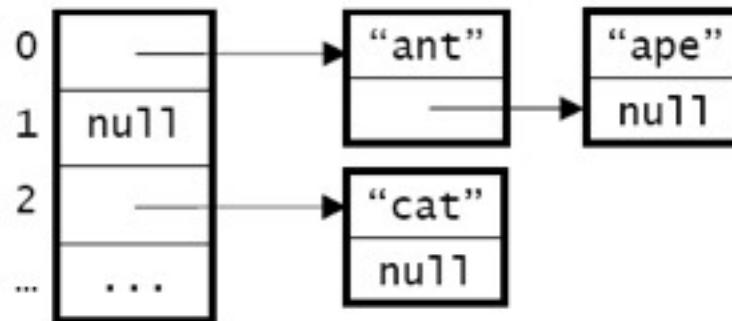
- When we remove an item from a position, we need to leave a special value in that position to indicate that an item was removed.
- Three types of positions: occupied, empty, “removed”.
- When we search for a key, we stop probing when we encounter an empty position, but not when we encounter a removed position.
 - How to search for “ape”?
 - How to search for “bear”?
- We can insert items in either empty or removed positions.

Previous Lecture

- Hash functions desiderata
- Handling collisions with open addressing
 - Linear probing
 - Quadratic probing
 - Double hashing
 - Need to distinguish between “removed” and “empty” positions

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- search for “bear”: try 1 – no item,

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1	
2	“cat”
3	“bear”
4	“emu”
5	
...	...
22	“wolf”
23	“wasp”
24	“yak”
25	“zebra”

Removing Items with Open Addressing

- Consider the following scenario:

- using linear probing
- insert “ape” ($h = 0$): try 0, $0 + 1$ – open!
- insert “bear” ($h = 1$): try 1, $1 + 1$, $1 + 2$ – open!
- remove “ape”
- search for “ape”: try 0, $0 + 1$ – no item
- search for “bear”: try 1 – no item,
- but “bear” is further down in the table!

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1	
2	“cat”
3	“bear”
4	“emu”
5	
...	...
22	“wolf”
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- search for “bear”: try 1 – no item,
- but “bear” is further down in the table!
- Cannot tell if it is not in the table

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...	...
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Removing Items with Open Addressing

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- Any difference with quadratic probing or double hashing?

Implementation of Hash Tables

- A simple hash table with open addressing.

```
public class HashTable {  
    private class Entry {  
        private String key;  
        private boolean removed;  
        private String etymology;  
        ...  
    }  
    private Entry[] table;  
    private int tableSize;  
    ...  
}
```

- for an empty position, `table[i]` will equal null
- for a removed position, `table[i]` will refer to an `Entry` object whose *removed* field equals true
- for an occupied position, `table[i]` will refer to an `Entry` object whose *removed* field equals false

Constructors

- Initializing the hash table

```
public HashTable(int size) {  
    table = new Entry[size];  
    tableSize = size;  
}
```

- Initializing an entry (before insertion)

```
private Entry(String key, String etymology) {  
    this.key = key;  
    this.etymology = etymology;  
    removed = false;  
    ...  
}
```

Finding An Open Position

- Using double hashing

```
private int probe(String key) {  
    int i = h1(key); // first hash function  
    int j = h2(key); // second hash function  
  
    // keep probing while the current position is occupied (non-empty and non-  
    // removed)  
    while ( ? )  
        ?  
  
    return i;  
}
```

- Does it always terminate?

Finding An Open Position

- Using double hashing

```
private int probe(String key) {  
    int i = h1(key); // first hash function  
    int j = h2(key); // second hash function  
  
    // keep probing while the current position is occupied (non-empty and non-  
    // removed)  
    while ( table[i] != null && table[i].removed==false )  
        ?  
  
    return i;  
}
```

- Does it always terminate?

Finding An Open Position

- Using double hashing

```
private int probe(String key) {  
    int i = h1(key); // first hash function  
    int j = h2(key); // second hash function  
  
    // keep probing while the current position is occupied (non-empty and non-  
    // removed)  
    while ( table[i] != null && table[i].removed==false )  
  
        i = (i + j) % tableSize;  
  
    return i;  
}
```

- Does it always terminate?

Finding An Open Position: Infinite Loops

- The loop in our probe method could become infinite.
 - When would this happen?
- To avoid infinite loops, we can stop probing after checking n positions ($n = \text{table size}$) because the probe sequence will just repeat after that point. Why?
 - for double hashing:
 - ✓ $(h1 + n*h2) \% n = h1 \% n$
 - ✓ $(h1 + (n+1)*h2) \% n = (h1 + n*h2 + h2) \% n = (h1 + h2)\%n$
 - ✓ $(h1 + (n+2)*h2) \% n = (h1 + n*h2 + 2*h2) \% n = (h1 + 2*h2)\%n$
 - ✓ ...
 - for quadratic probing:
 - ✓ $(h1 + n^2) \% n = h1 \% n$
 - ✓ $(h1 + (n+1)^2) \% n = (h1 + n^2 + 2n + 1) \% n = (h1 + 1)\%n$
 - ✓ $(h1 + (n+2)^2) \% n = (h1 + n^2 + 4n + 4) \% n = (h1 + 4)\%n$
 - ✓ ...

Finding An Open Position: Infinite Loop Protection

```
private int probe(String key) {  
    int i = h1(key); // first hash function  
    int j = h2(key); // second hash function  
    int iterations = 0;  
  
    // keep probing until we get an empty or removed position  
    while (table[i] != null && table[i].removed==false) {  
        i = (i + j) % tableSize;  
        iterations++;  
        if (iterations > tableSize) return -1;  
    }  
  
    return i;  
}
```

Finding the Position of A Key

- Different from probe()
 - Returns position of the key, not an available position.

```
private int findKey(String key) {  
    int i = h1(key); // first hash function  
    int j = h2(key); // second hash function  
    int iterations = 0;  
  
    // keep probing while the entry is not empty  
  
    while ( ? ) {  
        // return if key is found, otherwise continue  
        ?  
  
        i = (i + j) % tableSize;  
        iterations++;  
        if (iterations > tableSize) return -1;  
    }  
  
    return -1;  
}
```

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    int iterations = 0;  
  
    // keep probing while the entry is not empty  
  
    while ( table[i] != null ) {  
        // return if key is found, otherwise continue  
        if (table[i].key == key)  
            return i;  
  
        i = (i + j) % tableSize;  
        iterations++;  
        if (iterations > tableSize) return -1;  
    }  
  
    return -1;  
}
```

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    int j = h2(key); // second hash function  
    int iterations = 0;  
  
    // keep probing while the entry is not empty  
  
    while ( table[i] != null ) {  
        // return if key is found, otherwise continue  
        if (table[i].removed==false && table[i].key.equals(key))  
            return i;  
  
        i = (i + j) % tableSize;  
        iterations++;  
        if (iterations > tableSize) return -1;  
    }  
  
    return -1;  
}
```

Search() Method

- Search for the entry with the key, and return the Etymology (in other implementation, we may return other fields instead)

```
public String search(String key) {  
    int i = findKey(key);  
    if (i == -1)  
        return null;  
    else  
        return table[i].etymology ;  
}
```

- It calls the helper method `findKey()` to locate the position of the key.

Remove() Method

- Search the hash table and delete the key if found

```
public void remove(String key) {  
    int i = findKey(key);  
    if (i == -1)  
        return;  
    table[i].removed = true;  
}
```

- It also uses findKey().

Etc.

- “Hashing” to be continued!