# Phys 414 Final Project Report

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## Newton

### Part A

Starting with

$$\frac{dm(r)}{dr} = 4\pi r^2 \rho(r) \qquad (1) \qquad\qquad \frac{dp(r)}{dr} = -\frac{Gm(r)\rho(r)}{r^2} \qquad (2)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G \qquad (3)$$

$$\rho = K\rho^{1+\frac{1}{n}} \qquad (4) \qquad\qquad \rho = \rho_c\theta^n \qquad (5)$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 K\rho_c^{1/n}(n+1)\frac{d\theta}{dr}\right) = -4\pi G\rho_c\theta^n \qquad (6)$$

$$\alpha^2 = \left(\frac{K\rho_c^{1/n}(n+1)}{4\pi G\rho_c}\right) \qquad (7) \qquad\qquad r = \alpha\epsilon, \quad dr = \alpha d\epsilon \qquad (8)$$

$$\frac{1}{\alpha^2\epsilon^2}\frac{d}{\alpha d\epsilon}\left(\frac{\alpha^2}{\alpha}\frac{d\theta}{d\epsilon}\right) = -\theta^n \qquad (9)$$

Which gives Lane-Emden

$$\frac{1}{\epsilon^2}\frac{d}{d\epsilon}\left(\epsilon^2\frac{d\theta}{d\epsilon}\right)+\theta^n=0 \tag{10}$$

for n =1

$$\begin{aligned}\theta(0)&=1\\ \theta'(0)&=0\\ \theta(\epsilon)&=1-\frac{1}{6}\epsilon^2+\frac{n}{120}\epsilon^4+\dots\end{aligned} \tag{11}$$

$$\theta(\epsilon)=1-\frac{1}{6}\epsilon^2+\frac{1}{120}\epsilon^4+\dots \tag{12}$$

$$Mass=M=\int_0^R 4\pi r^2\rho\,dr \tag{13}$$

using (8) and (5)

$$M=4\pi\int_0^{\alpha\epsilon}\alpha^2\epsilon^2\rho_c\theta^n\alpha\,d\epsilon \tag{14}$$

$$M=4\pi a^3\rho_c\int_0^{a\epsilon}\theta^n\epsilon^2\,d\epsilon \tag{15}$$

Using (10)

$$\theta^n\epsilon^2\,d\epsilon=-d\left(\epsilon^2\frac{d\theta}{d\epsilon}\right) \tag{16}$$

Place it on the

$$M=4\pi a^3\rho_c\left(-\epsilon^2\frac{d\theta}{d\epsilon}\right)|_\epsilon \tag{17}$$

Using (8) to replace a

$$M=4\pi\rho_c R^3\left(-\frac{d\theta}{d\epsilon}\right)|_\epsilon\frac{1}{\epsilon} \tag{18}$$

And finally

$$M = 4\pi\rho_c R^3 \frac{-\theta'(\epsilon)}{\epsilon} \tag{19}$$

Using (17) and (7), we can write

$$M = -4\pi a^{\left(\frac{3-n}{1-n}\right)} \left(\frac{K}{G}\right) \left(\frac{n+1}{4\pi}\right)^{-\frac{n}{1-n}} \epsilon^{-\frac{2n}{1-n}} \epsilon^2 \theta'(\epsilon) \tag{20}$$

and

$$\frac{R}{\epsilon} = a \tag{21}$$

$$a^{\frac{3-n}{1-n}} = \left(\frac{R}{\epsilon}\right)^{\frac{3-n}{1-n}} \tag{22}$$

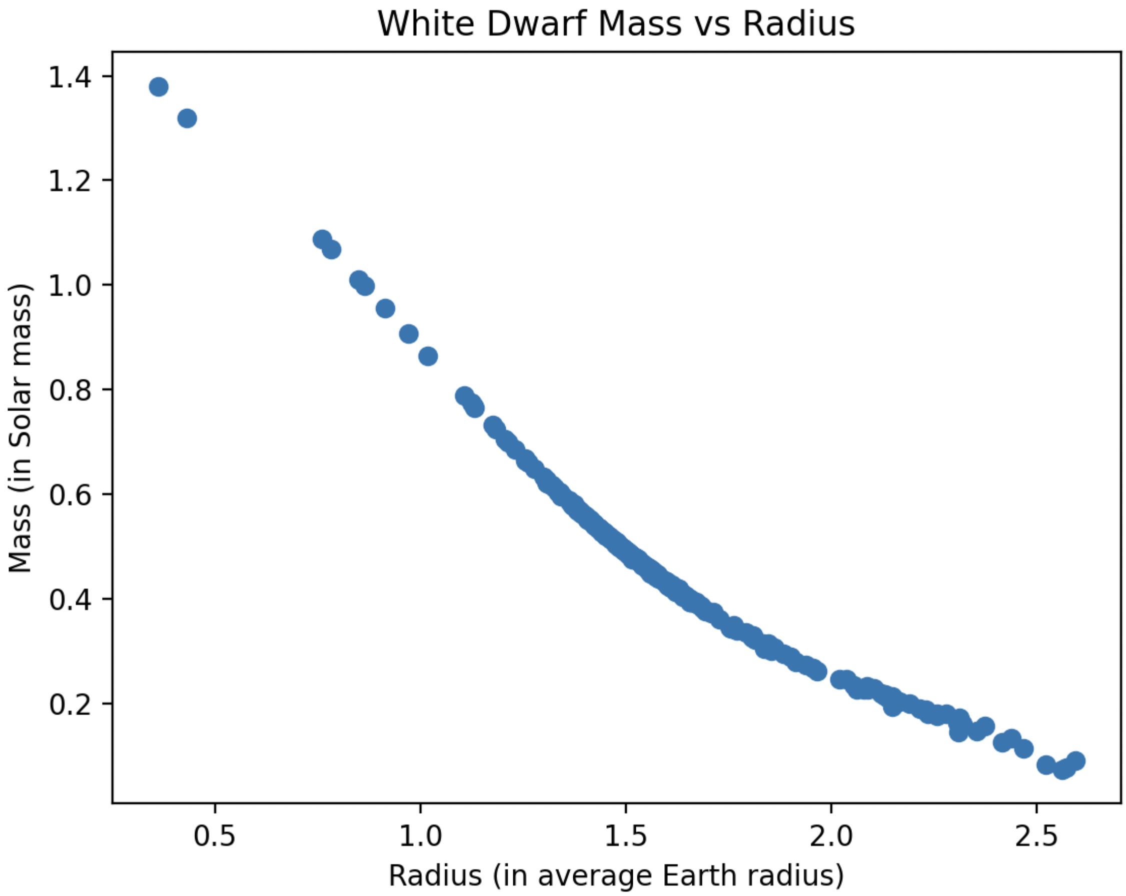
$$M = CR^{\frac{3-n}{1-n}} \tag{23}$$

## Part B)

Code is at the repository

https://github.com/batuhanarat/Phys414_Project

This is the plot that is requested.




## Part C)

Given

$$P = C\left[x(2x^2-3)(x^2+1)^{\frac{1}{2}} + 2\sinh^{-1}x\right] \tag{24}$$

$$x = \left(\frac{\rho}{D}\right)^{\frac{1}{q}}$$

When we write it's Taylor expansion

$$C(0)+\frac{8}{5}x^5C'(0)-\frac{4}{7}x^7C'(0)+O(x^9) \tag{25}$$

We can see that it has a leading term of

$$\frac{8}{5}Cx^5=\frac{8}{5}\left(\frac{\rho}{D}\right)^{\frac{5}{q}} \tag{26}$$

From there, we can define

$$\frac{8}{5}\left(\frac{\rho}{D}\right)^{\frac{5}{q}}=K_*\rho^{1+\frac{1}{n_*}} \tag{27}$$

from there we can define

$$K_*=\frac{8C}{5D^{5/q}} \tag{28}$$

$$n_*=\frac{q}{5-q} \tag{29}$$

In the code , we find corresponding n* and q values

$$n_*=1.5441641728490145 \tag{30}$$

$$q=3.03471802120345 \tag{31}$$

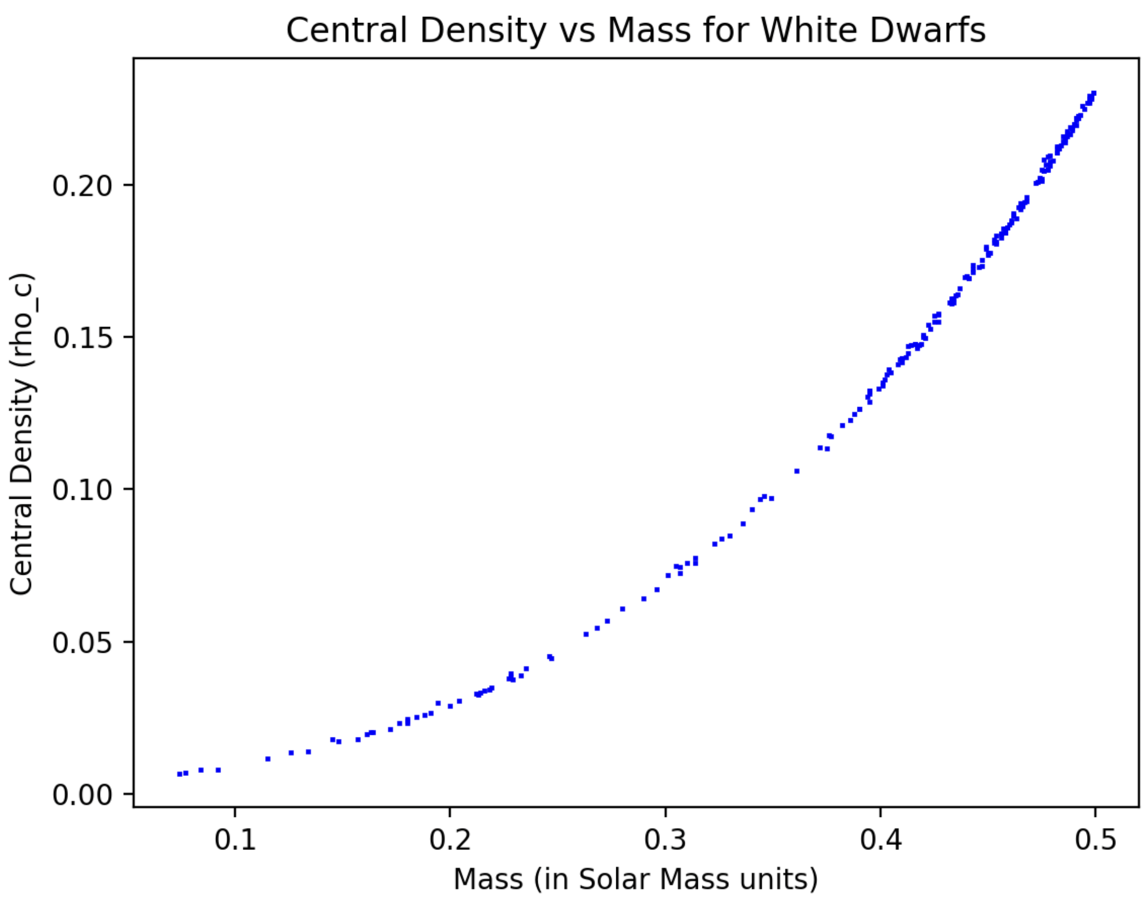
In the pdf it is explicitly said we should take q as an integer, therefore we can take closest integer and our numbers become

$$q=3$$

$$n=1.5$$

After finding those values, we can use Lane-Emden equation to find theta’(epsilon) and epsilon

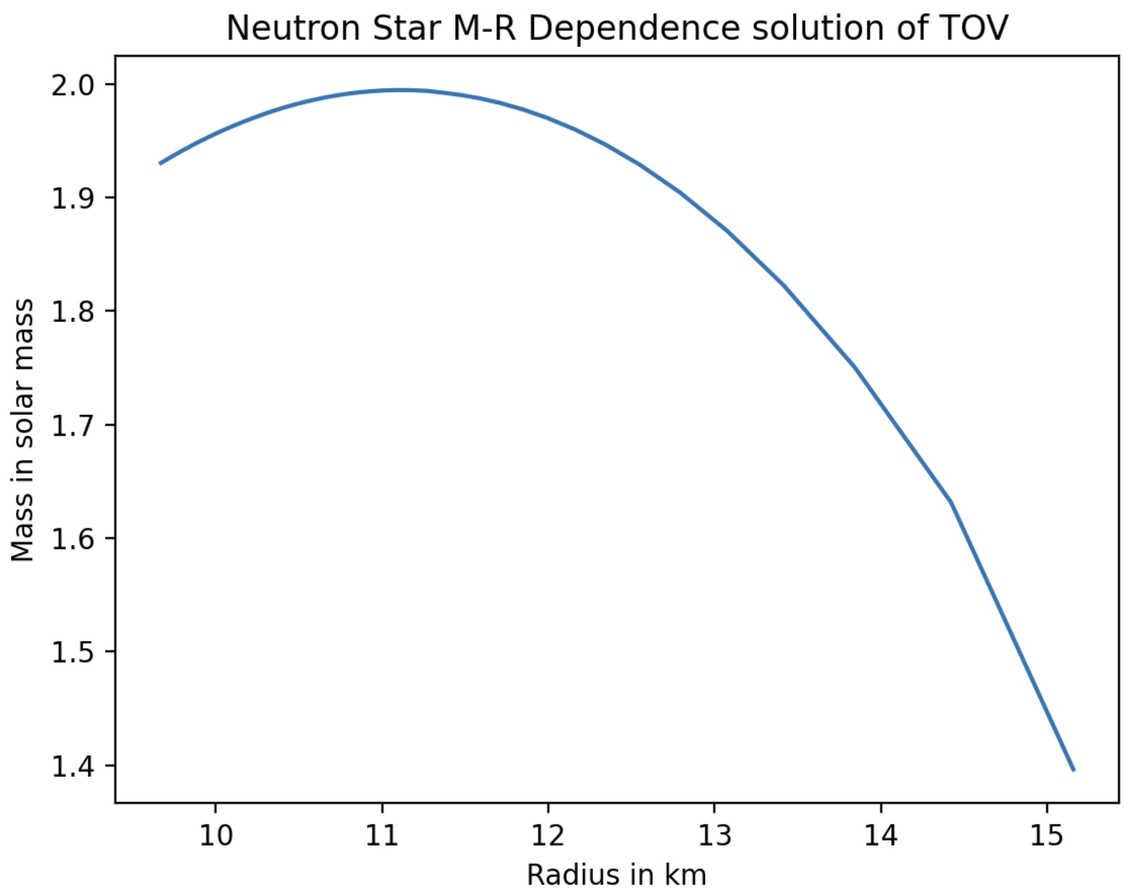
This is the plot that is requested.




# Einstein

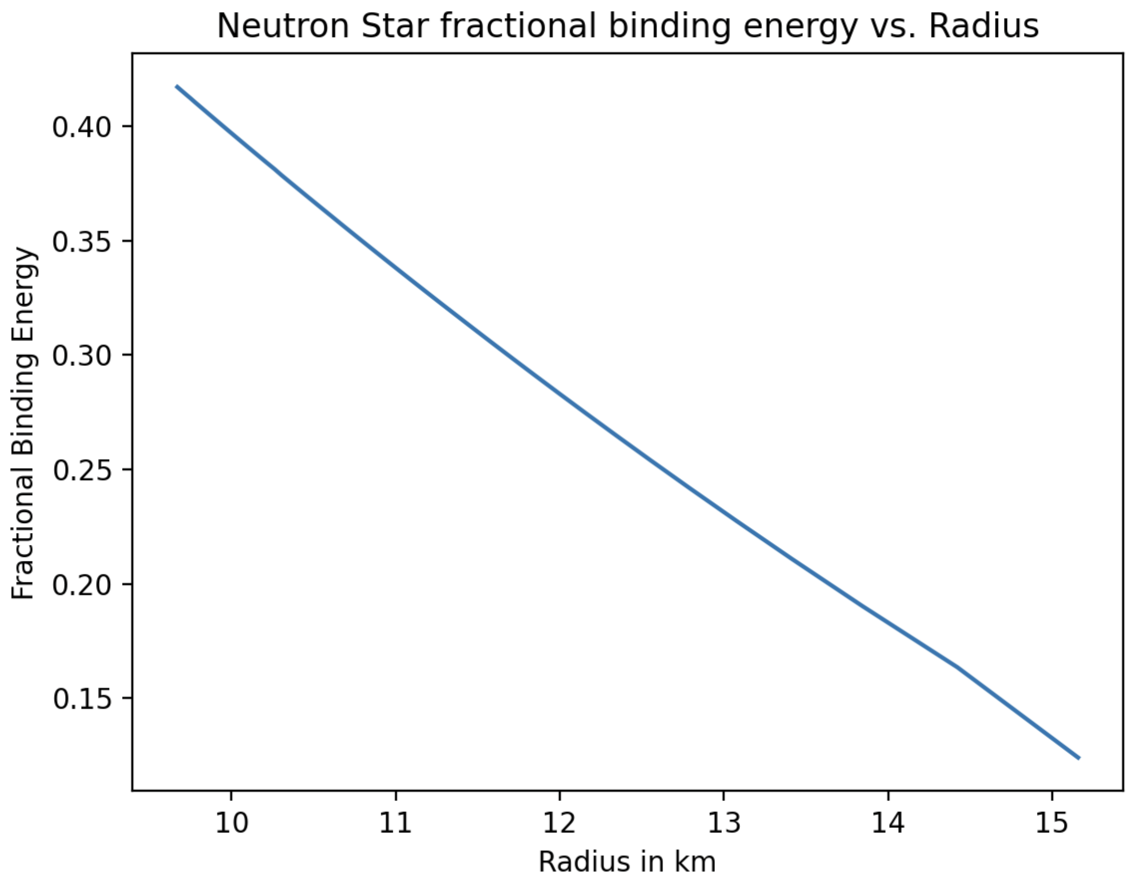
## Part A

In this analysis, I solve the Tolman-Oppenheimer-Volkoff (TOV) equations using the shooting method. The initial central pressure p(0) is set to 1e-4, with an increment of 0.0001 for each of the 50 iterations. This approach generates a Mass-Radius curve for neutron stars, as shown in Figure 6, providing insight into their structural characteristics.



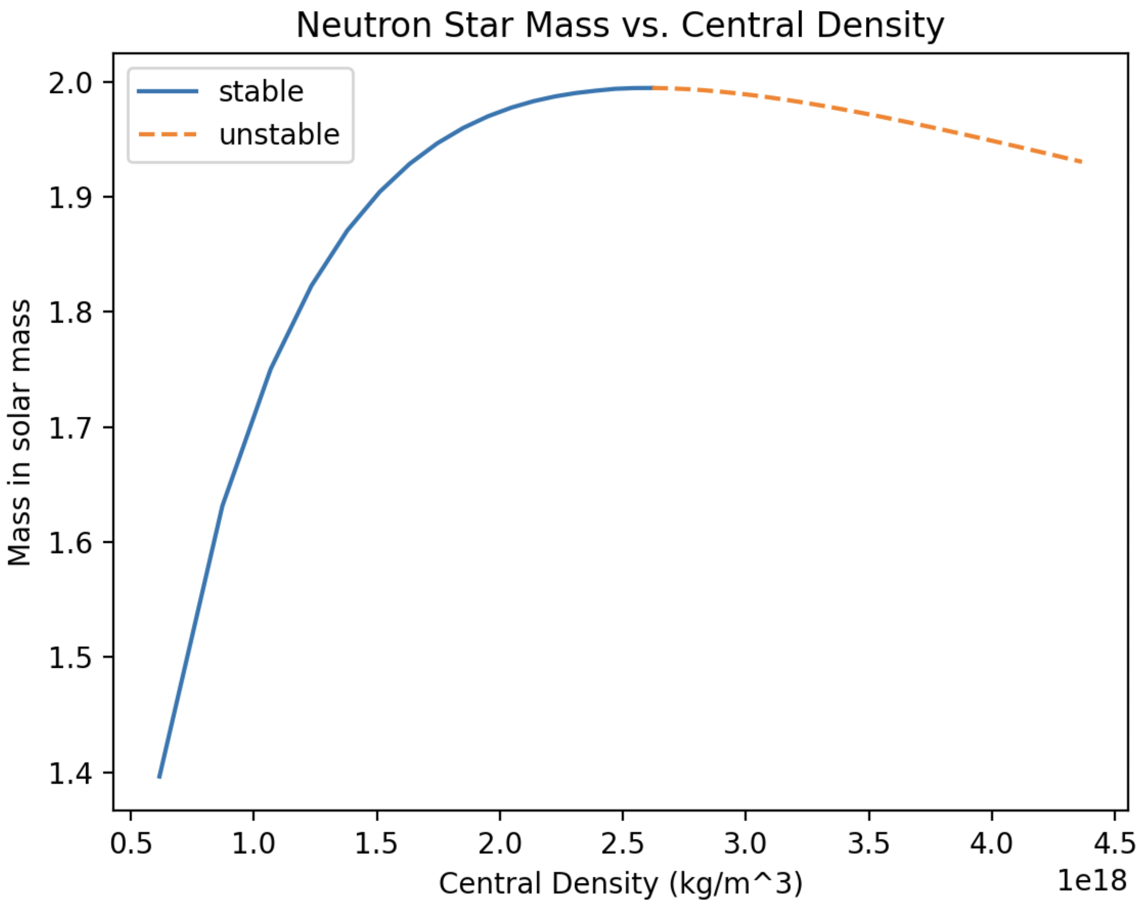

## Part B

In the second part of the analysis, I include baryonic mass as an additional variable in the shooting method. This approach involves iterating over initial central pressures and integrating the extended equations to establish a relationship between fractional binding energy and neutron star radius. The resulting graph from this analysis provides data on the stability and composition of neutron stars.




## Part C

In the analysis, the Mass (M) versus Central Density (ρc) graph presents the relationship at neutron star stability. Stability determination is the mass corresponding to the zero derivative of this curve, essentially the inflection point where the graph starts to descend. This specific mass, identified at approximately 1.9948 solar units, represents the maximum mass sustainable by a neutron star under the given model. Prior to this peak, as the central density rises, so does the mass, indicating stable configurations. Beyond this peak, any further increase in density results in a decrease in mass, suggesting instability due to the inability of the internal pressure to counteract gravitational forces, leading to potential collapse. So, the peak at 1.9947 solar masses marks the boundary between stable and unstable neutron star configurations.




## Part E

When we integrate v’ by using python we can see that

$$\nu(r > R) = \ln\left(1 - \frac{2M}{r}\right) - \ln\left(1 - \frac{2M}{R}\right) + \nu(R)\ .$$

is satisfied.

**Output:**

```
v(R) + log(R) - log(r) - log(-2*M + R) + log(-2*M + r)
```

You can look at the code in github, function is `part_e()`