EE475 Homework #4

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I. MOIRE NOISE REMOVAL.

A. Plotting the Magnitude Spectrum

The magnitude and log-magnitude (for better visualization) spectrums of the newspaper image with moiré pattern ar given below:

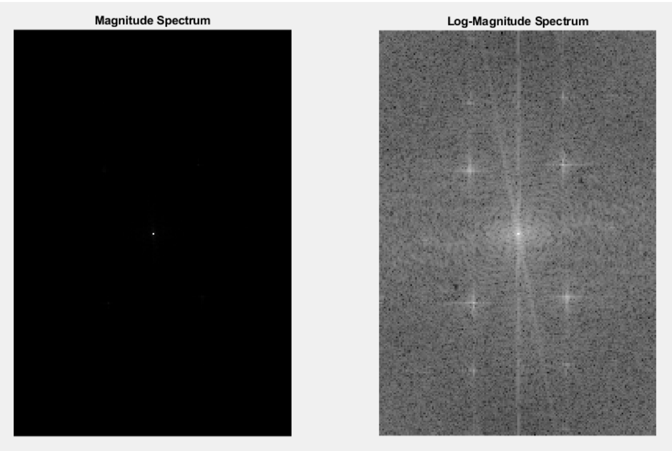

Fig 1. Magnitude and Log-Magnitude Spectrum of the Image

B. Finding the Center Points and Diameters of Prominent Spectral Peaks

From Fig. 1, it can be observed that to identify the prominent spectral peaks we should work with the log-magnitude spectrum of the image.

First, I have found the minimum and maximum values of the log-magnitude spectrum to determine a threshold value. According to the findings, I have set the threshold value as 10 and created a binary image, that shows prominent spectral peaks, from the log-magnitude spectrum with respect to the threshold.

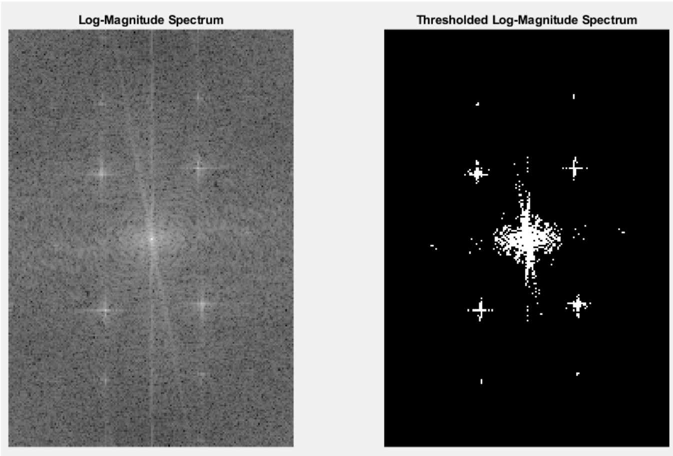

Fig 2. Log-Magnitude Spectrum and Its Thresholded Version

Since the white region in the middle are nothing to do with Moire Noise pattern, I went further and 0'ed this region as well to have better visualization. The result is:

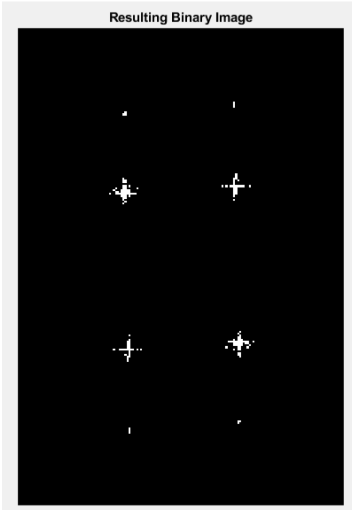

Fig 3. The Resulting Binary Image

After all this, I have used MATLAB's imagesc function to both visualize the final binary image and learn the coordinates of the spectral peaks with their approximate diameters.

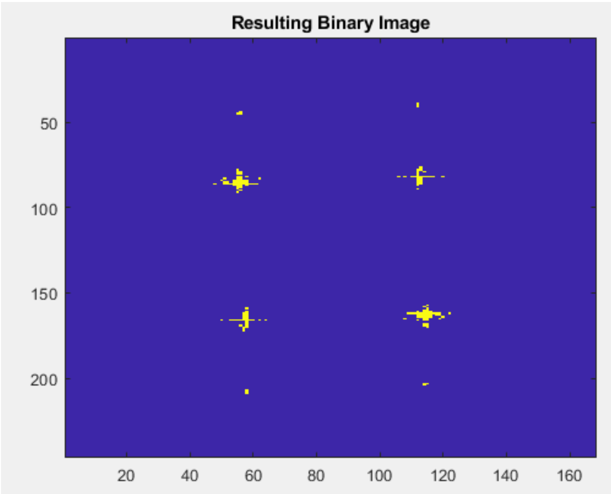

Fig 4. Visualization of the Final Binary Image via imagesc

I also used MATLAB's mesh function to visualize and find the coordinates of the peak points. After noting down the coordinates and approximate diameters of the center points of the spectral peaks, I have continued with designing a Notch Filter.

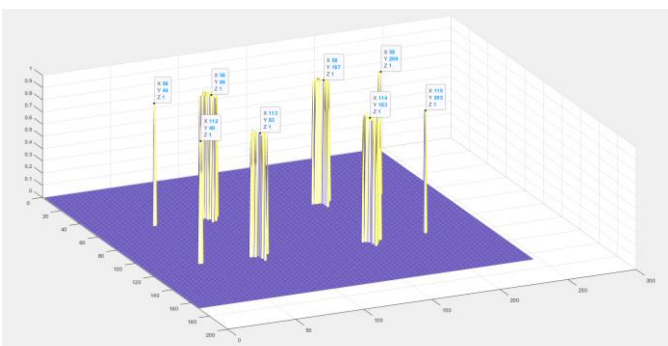

Fig 5. Visualization of the Final Binary Image via mesh function

C. *Designing a Notch Filter and Plotting the Moiré Pattern*

To design a Notch Filter, I have created my own function, called myNotchFilter. This function takes parameters as radius (for setting the D0 value), distance_u and distance_v (for shifting in frequency domain) and returns a Notch filter with n=4. For each spectral point (and its symmetric version, wrt the origin,), we generate a special Notch filter and apply it on the magnitude spectrum of the image. In total, 8 Notch filters are applied on the frequency spectrum of the image. The log-magnitude spectrum of the image after notch filtering process and the Moire pattern are given in the figure below.

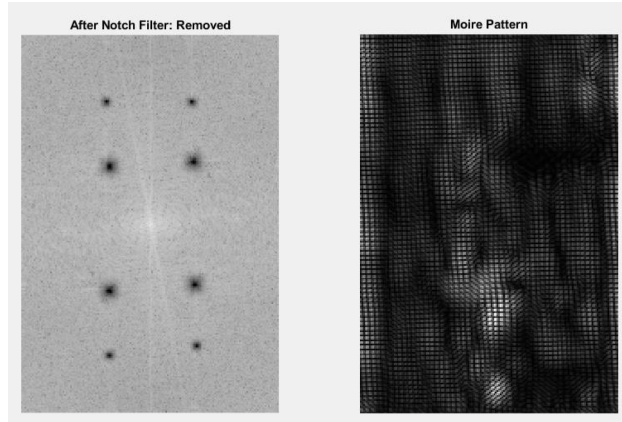

Fig 6. Log-magnitude spectrum after Notch filtering and the Extracted Moire Pattern in the Image

D. *Plotting Images with Moiré Pattern Removed and Only Moiré Pattern*

In this part, the Moiré Pattern removed version of the image and the extracted Moiré Pattern are given below.

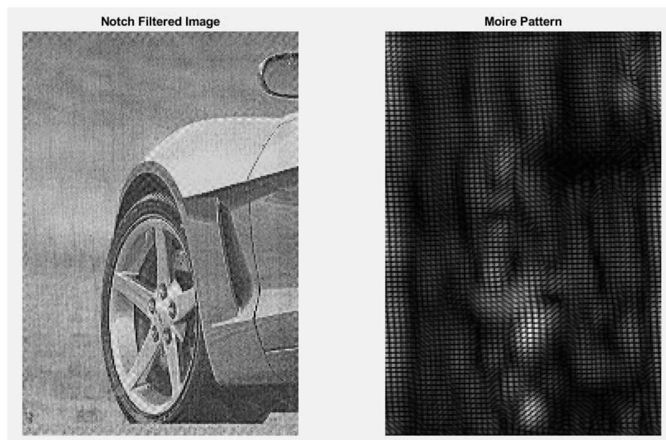

Fig 7. Moiré Pattern removed version and Extracted Moiré Pattern

II. IMPORTANCE OF THE PHASE

In this section, we will try to observe and understand the importance of the phase spectrum for reconstruction of an image from the frequency domain. The images used in section are given below:

Fig 8. Berkeley_girl and Berkeley_horses images

We know that the phase is a measure of displacement of the various sinusoids with respect to their origin. Therefore, the magnitude spectrum holds information about the intensities in the image, whereas the phase spectrum carries much of the information about where discernible objects are located in the image (shape features or structural information). Keeping this in mind, we will do our experiments.

A. *Reconstruct Berkeley_girl from its phase-only spectrum and its magnitude-only spectrum*

In part A, Berkeley_girl image is reconstructed from both its phase-only spectrum and magnitude-only spectrum separately. The results are given below:

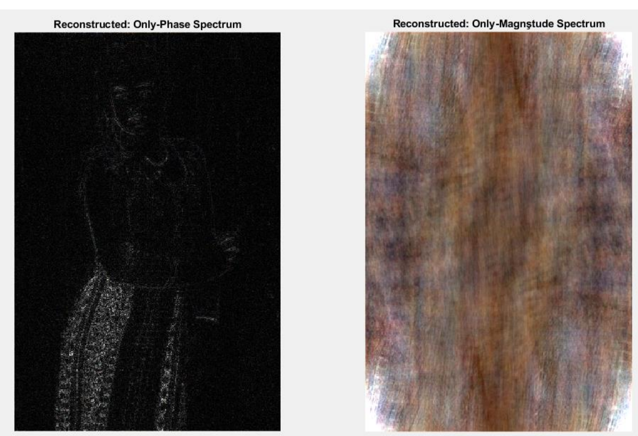

Fig 9. Berkeley_girl reconstructed from its phase-only spectrum and its magnitude-only spectrum

From the figure, it can be observed that the reconstructed image from only-phase spectrum has lost much of the intensity information but still has the shape features of the original image. On the other hand, the reconstructed image from only-magnitude spectrum only contains the intensity information and has lost all of the shape information. Therefore, reconstructing an image from only its magnitude spectrum is meaningless.

B. *Reconstruct Berkeley_horses from its phase-only spectrum and its magnitude-only spectrum*

In part B, Berkeley_horses image is reconstructed from both its phase-only spectrum and magnitude-only spectrum separately. The results are given below:

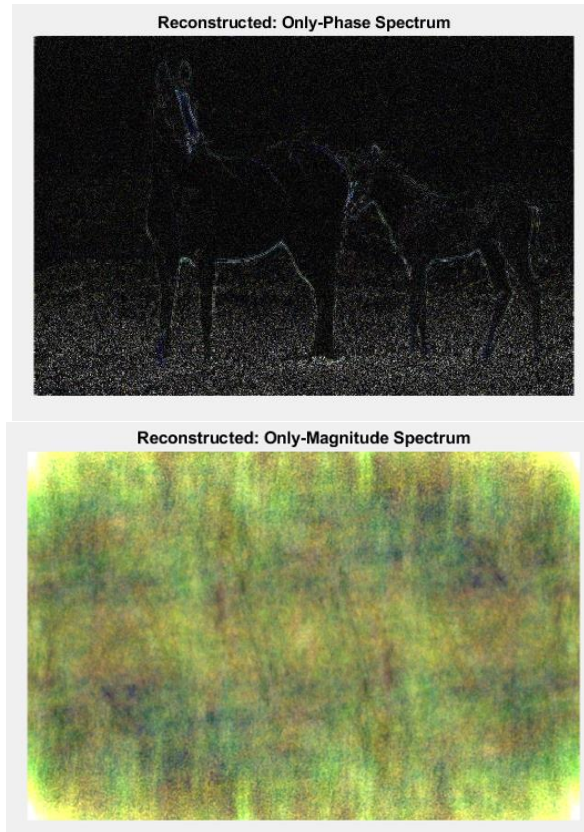

Fig 10. Berkeley_horses reconstructed from its phase-only spectrum and its magnitude-only spectrum

From the figure, it can be observed that the reconstructed image from only-phase spectrum has lost much of the intensity information but still has the shape features of the original image. On the other hand, the reconstructed image from only-magnitude spectrum only contains the intensity information and has lost all of the shape information. Therefore, reconstructing an image from only its magnitude spectrum is meaningless.

C. *Reconstruct Berkeley_girl from Berkeley_horse's phase spectrum and Berkeley_girl's magnitude spectrum*

In part C, Berkeley_girl image is reconstructed from Berkeley_horse's phase spectrum and Berkeley_girl's magnitude spectrum. The result is given below:

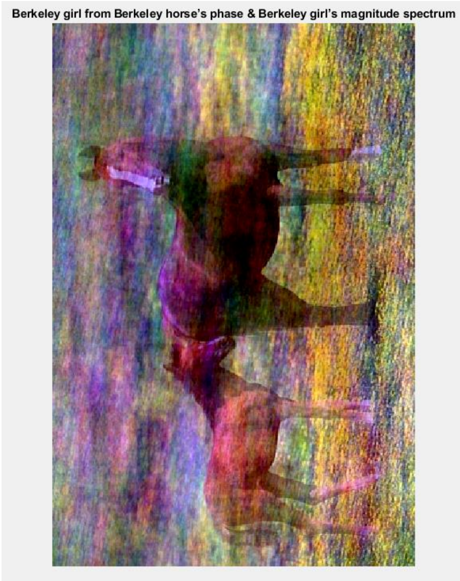

Fig 11. Berkeley_girl image is reconstructed from Berkeley_horse's phase spectrum and Berkeley_girl's magnitude spectrum

As I have mentioned earlier, the shape features of an image are contained in its phase spectrum. Therefore, when we try to reconstruct Berkeley_girl from its magnitude spectrum and Berkeley_horse image's phase spectrum, we expect to get an image with intensity information of Berkeley_girl and shape features of Berkeley_horse. As a proof for this, in the resulting image we observe horses instead of a girl.

D. ***Reconstruct Berkeley_horse from Berkeley_girl's phase spectrum and Berkeley_horse's magnitude spectrum***

In part D, Berkeley_horse image is reconstructed from Berkeley_girl's phase spectrum and Berkeley_horse's magnitude spectrum. The result is given below:

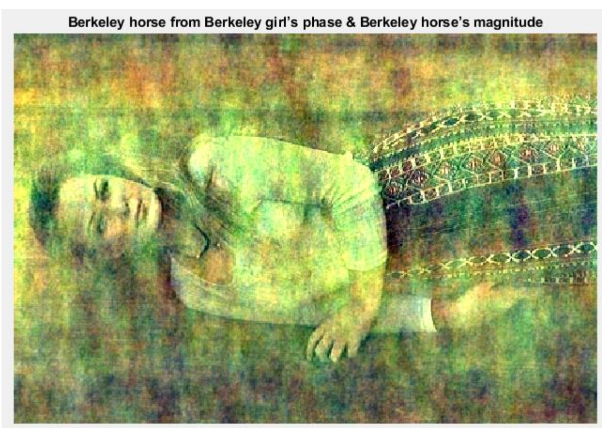

Fig 12. Berkeley_horse is reconstructed from Berkeley_girl's phase spectrum and Berkeley_horse's magnitude spectrum

Like the previous case, due to the same reason, we observe a girl in the reconstructed image instead of horses.

E. ***Reconstruct Berkeley_girl again***

In part E, Berkeley_girl image is reconstructed from $|M_{Tr}(u,v)|\exp\{j\theta_{Tr}(u,-v)\}$. The result is given below:

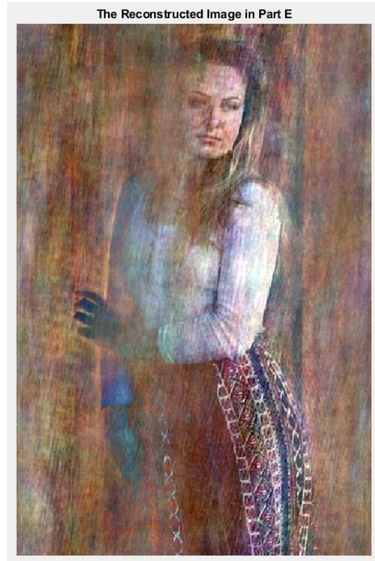

Fig 13. Reconstructed Berkeley_girl Image in Part E

The operation done in this case is to take the symmetry of the phase spectrum of the image wrt to u-axis. In the spatial domain, this corresponds to taking the symmetry of the shape features in image wrt x-axis (vertical axis). Therefore, the reconstructed image looks like the vertically mirrored version of the original image.

III. LAPLACIAN OPERATOR IN FREQUENCY DOMAIN

The two masks that approximate the Laplacian operator are given below.

1	1	1
1	-8	1
1	1	1

Q(x,y)

0	1	0
1	-4	1
0	1	0

K(x,y)

Fig 14. Two Different Approximations of Laplacian Operator

A. The Fourier Transform of the Laplacian of a function

Q3.a

$$\mathcal{F}\{f(x,y)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+\omega y)}\, dx\, dy$$

$$= F(u,\omega)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \iint_{-\infty}^{\infty} F(u,\omega) \cdot \frac{d^2}{dx^2}\left(e^{j2\pi(ux+\omega y)}\right) du\, d\omega$$

$$= \iint_{-\infty}^{\infty} F(u,\omega)\left(-4\pi^2 u^2\right) e^{j2\pi(ux+\omega y)}\, du\, d\omega$$

$$= \left(-4\pi^2 u^2\right) \mathcal{F}^{-1}\{F(u,\omega)\}$$

$$\frac{\partial^2 f}{\partial y^2} = \left(-4\pi^2 \omega^2\right) \mathcal{F}^{-1}\{F(u,\omega)\}$$

$$\nabla^2 f = \left(-4\pi^2\right)\left(u^2+\omega^2\right) \mathcal{F}^{-1}\{F(u,\omega)\}$$

$$\mathcal{F}\{\nabla^2 f\} = \underbrace{\left(-4\pi^2\right)\left(u^2+\omega^2\right)}\, F(u,\omega)$$

$$L(u,\omega) = \left(-4\pi^2\right)\left(u^2+\omega^2\right)$$

Fig 15. Solution for Q3 Part A

B. Frequency Domain Expression of the Two Masks

Q3.b

$$Q(x,y) = \begin{bmatrix} 0 & +1 & 0 \\ +1 & -4 & +1 \\ 0 & +1 & 0 \end{bmatrix}$$

$$f(x,y) * Q(x,y) = f(x-1,y) + f(x+1,y) + f(x,y-1) + f(x,y+1) - 4f(x,y)$$

$$\mathcal{F}\{f(x,y) * Q(x,y)\} = \underbrace{\left(e^{-j2\pi u} + e^{j2\pi u} + e^{-j2\pi\omega} + e^{j2\pi\omega} - 4\right)} \cdot F(u,\omega)$$

therefore $\boxed{Q(u,\omega) = -4 + 2\cos(2\pi u) + 2\cos(2\pi\omega)}$

$$K(x,y) = \begin{bmatrix} +1 & +1 & +1 \\ +1 & -8 & +1 \\ +1 & +1 & +1 \end{bmatrix}$$

$$f(x,y) * K(x,y) = f(x-1,y) + f(x+1,y) + f(x,y-1) + f(x,y+1) + f(x-1,y-1) + f(x+1,y-1) + f(x-1,y+1) + f(x+1,y+1) - 8f(x,y)$$

$$\mathcal{F}\{f(x,y) * K(x,y)\} = \left(e^{-j2\pi u} + e^{j2\pi u} + e^{-j2\pi\omega} + e^{j2\pi\omega} + e^{-j2\pi(u+\omega)} + e^{-j2\pi(u-\omega)} + e^{-j2\pi(-u+\omega)} + e^{-j2\pi(-u-\omega)} - 8\right) \cdot F(u,\omega)$$

$$\boxed{K(u,\omega) = -8 + 2\cos(2\pi u) + 2\cos(2\pi\omega) + 2\cos(2\pi(u+\omega)) + 2\cos(2\pi(u-\omega))}$$

Fig 16. Solution for Q3 Part B

C. Plotting (in 3D) the Magnitude Spectra of the Three Operators

The magnitude spectra of the three operators, $\mathcal{L}(u,v), \mathcal{Q}(u,v), \mathcal{K}(u,v)$ are generated wrt formulas found in Part A and B using M = 256 points, that is with angular separation $\frac{2\pi}{M}$, and then plotted in 3D using MATLAB's surf function. The resulting plots are given below.

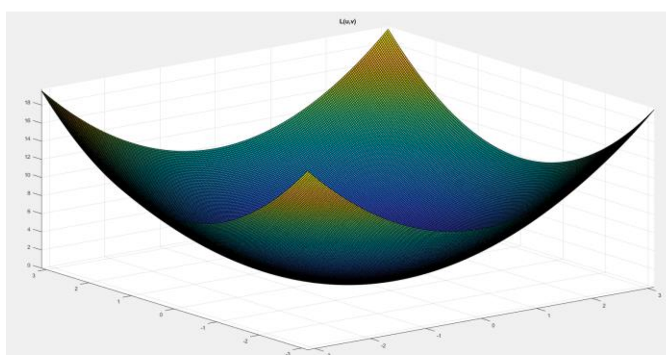

Fig 16. L(u,v)

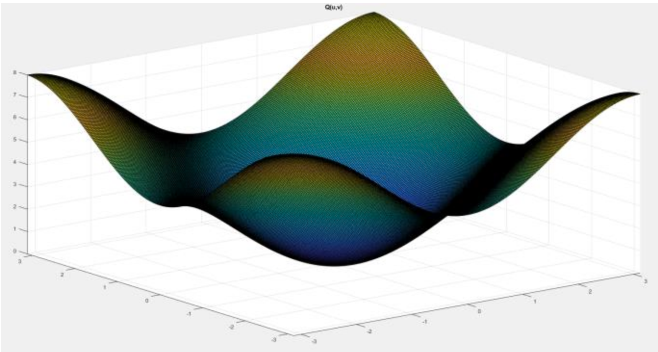

Fig 17. Q(u,v)

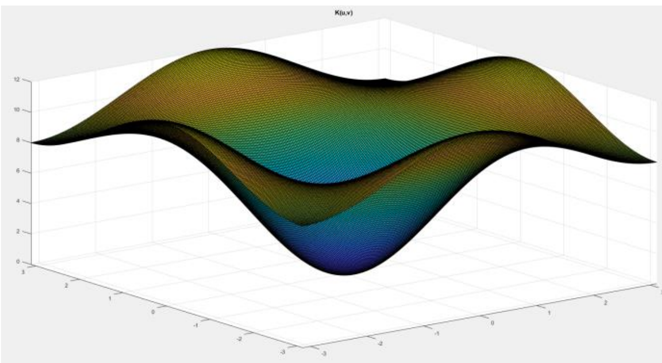

Fig 18. K(u,v)

Then, the absolute value of the difference $|\mathcal{L}(u,v) - \mathcal{K}(u,v)|$ *and* $|\mathcal{L}(u,v) - Q(u,v)|$ are also plotted in 3D using MATLAB's mesh function. The resulting plots are given below.

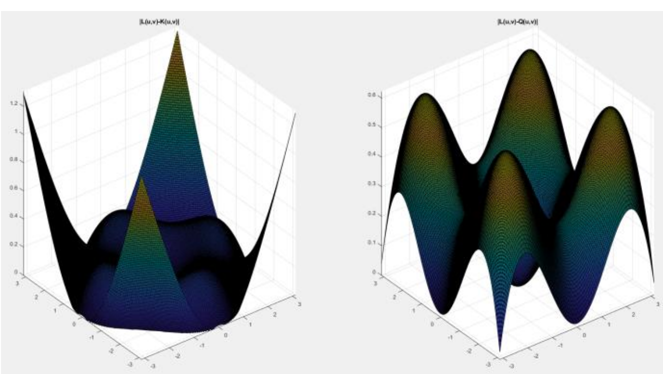

Fig 19. |L(u,v)-K(u,v)| and |L(u,v)-Q(u,v)|

First of all, the Laplacian kernel Q(x,y), in spatial domain,is isotropic for rotations in increments of 90° with respect to the x- and y-axes, and due to the 0's the differentiation will not be done in diagonal directions. However, the Laplacian kernel K(x,y) yields isotropic results in increments of 45° due to the fact that the -1's in the corners provide additional differentiation in the diagonal directions.

Now we can do our analysis in the frequency domain. Starting with Fig. 16, we can observe that L(u,v) has a spectrum that looks like a high-pass filter since its magnitude smoothly increases as we go to the higher frequency values and its peak points are located at the points farthest from the origin (corners). Considering the Fig. 17, Q(u,v) is very similar to L(u,v). One difference between the two is that, the magnitude of the Q(u,v) does not increase as smoothly as the magnitude of the L(u,v) as we go to the higher frequencies. From Fig. 18, we can observe that the magnitude of K(u,v) increases as we move to the higher frequencies but its peaks are not located at the corners but rather at the mid points of the edges.

In Fig. 19, we can observe the deviations of Q(u,v) and K(u,v) from the original Laplacian L(u,v) in the frequency domain. Considering the 3D plot for |L(u,v)-K(u,v)|, we can observe that the main deviation happens at the corners. Considering the 3D plot for |L(u,v)-Q(u,v)|, the main deviation happens at the midpoints of the diagonals. We can correlate those differences with the ones in the spatial domain.

IV. DEBLURRING

In this section, the book-cover image is degraded by the given atmospheric turbulence model and gaussian white noise. Then, we try to restore the image by using several methods.

A. Atmospheric Turbulence Model & Gaussian Noise

In this part, the book-cover image is first degraded with the given atmospheric turbulence model and then white Gaussian noise N(0, 625) is added to the already degraded image. To model the atmospheric turbulence, I have created my own function called myAtmosphericTurbulence.

B. Plotting the Original and the Blurred & Noisy Versions

The original and the blurred & noisy version of the book-cover image are given below:

Fig 20. The original and the blurred & noisy version of the book-cover image

C. Applying Inverse Filter and Butterworth Low-Pass Filter

To restore the image, first only the inverse filter is applied. Then, the inverse filter is applied with a cutoff frequency at radius 50 (70 was not giving a good result therefore changed to 50) using a Butterworth low-pass filter of order 10. The results are given below:

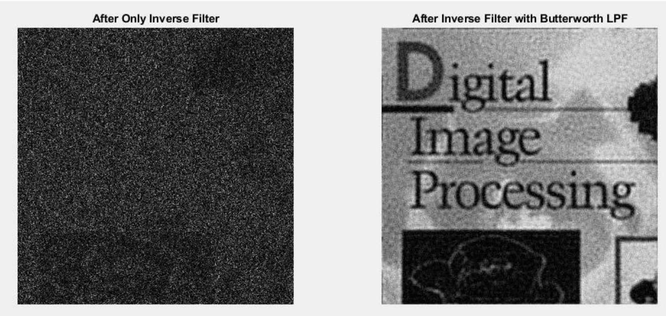

Fig 21. Restored Images with Only Inverse Filter and Inverse Filter with cutoff frequency at radius 50

The simplest approach to restore the image is direct inverse filtering, which is multiplying the Fourier transform of the degraded image G(u,v) with 1/(H(u,v)). The result we get after direct inverse filtering, not only consists of the Fourier transform of the original image F(u,v) but also the component N(u,v)/H(u,v) where N(u,v) is the Fourier transform of the Gaussian noise. Therefore, we cannot fully recover the undegraded image. Moreover, since the degradation function H(u,v) has very small values at high frequencies, the ratio N(u,v)/H(u,v) have very large values that dominate the F(u,v). Thus, the result after only inverse filtering is very bad. To reduce the likelihood of encountering zero values at the high frequencies, we apply inverse filtering with a cutoff frequency at radius 50 using a Butterworth low-pass filter of order 10. The effect of this can be observed in Fig. 21. Due to this operation, we got both deblurring effect (1/H) and we avoided the effect of the zero values in H.

*D. **Restoring the Image by Direct Inverse Filtering and by Wiener Filter***

In this part, to restore the image, first only the inverse filter is applied. Then, only the Wiener Filter is applied. The results are given below:

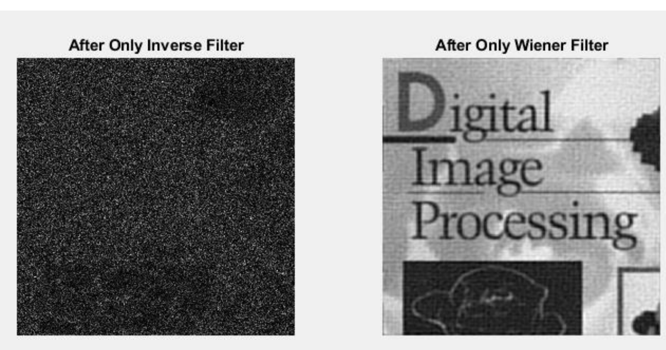

Fig 21. Restored Images with Only Inverse Filter and Only Wiener Filter

Wiener filtering method incorporates both the degradation function and statistical characteristics of noise into the restoration process therefore it gives better result than both only inverse filtering and inverse filtering with a cutoff frequency at radius 50 using a Butterworth low-pass filter of order 10, which can be observed at Fig 21.

V. IDENTIFY PATTERNS AND SPECTRA

Patches_cosines_digital image contains various sinusoidal patterns or line patterns. In this section, our purpose is to match sinusoidal patches with the corresponding formation in the spectrum. To observe the magnitude spectrum better, I took the logarithm of it, and then labeled the patches and corresponding spectral formations with the same numbers.

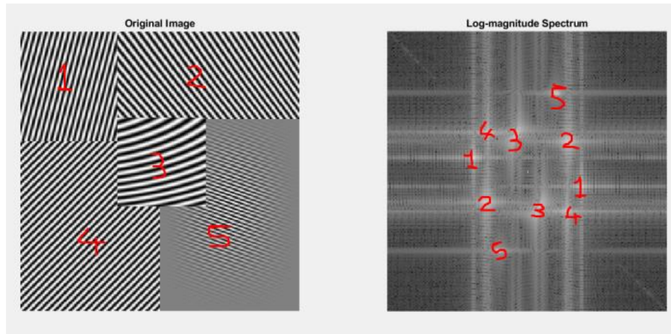

Fig 22. Numbered the patches and corresponding spectral formations