Terminology

Random Experiment

- An experiment is a procedure that is
  - carried out under controlled conditions, and
  - executed to discover an unknown result.
- An experiment that results in different outcomes even when repeated in the same manner every time is a random experiment.

Sample Spaces

- The set of all possible outcomes of a random experiment is called the sample space, *S*.
- *S* is discrete if it consists of a finite or countable infinite set of outcomes.
- *S* is continuous if it contains an interval of real numbers.

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Example: Defining Sample Spaces

- Randomly select a camera and record the recycle time of a flash. $S = R^+ = \{x \mid x > 0\}$, the positive real numbers.
- Suppose it is known that all recycle times are between 1.5 and 5 seconds. Then

  $S = \{x \mid 1.5 < x < 5\}$ is continuous.
- It is known that the recycle time has only three values(low, medium or high). Then $S = \{low, medium, high\}$ is discrete.
- Does the camera conform to minimum recycle time specifications?

  $S = \{yes, no\}$ is discrete.

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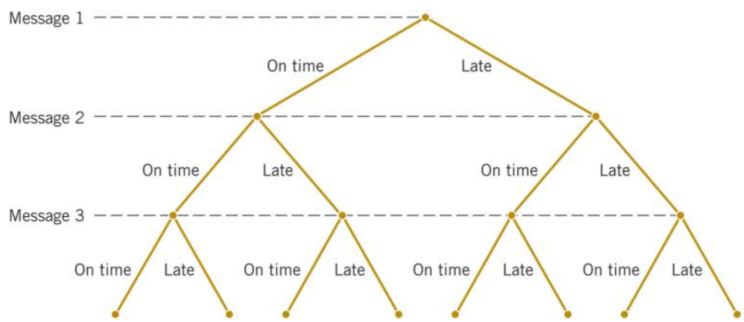
Sample Space Defined By A Tree Diagram

Example: Messages are classified as on-time(o) or late(l). Classify the next 3 messages.

S = {ooo, ool, olo, oll, loo, lol, llo, lll}



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Events are Sets of Outcomes

- An event (E) is a subset of the sample space of a random experiment.
- Event combinations
  - The Union of two events consists of all outcomes that are contained in one event or the other, denoted as $E_1 \cup E_2$.
  - The Intersection of two events consists of all outcomes that are contained in one event and the other, denoted as $E_1 \cap E_2$.
  - The Complement of an event is the set of outcomes in the sample space that are not contained in the event, denoted as E'.

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Example: Discrete Events

Suppose that the recycle times of two cameras are recorded.
Consider only whether or not the cameras conform to the manufacturing specifications
Is sample space discrete or continuous?

We abbreviate *yes* and *no* as *y* and *n*.
The sample space is $S = \{yy, yn, ny, nn\}$.

Let E_1 is the event that at least one camera conforms to specifications, then E_1 =?
$E_1 = \{yy, yn, ny\}$

Let E_2 denotes an event that no camera conforms to specifications, then E_2 ?
$E_2 = \{nn\}$

Let E_3 denotes an event that at least one camera <u>does not</u> conform, then E_3 =
$E_3 = \{yn, ny, nn\}$

$E_1 \cup E_3 = S$
$E_1 \cap E_3 = \{yn, ny\}$
$E_1' = \{nn\}$

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Example: Continuous Events

Measurements of the specifications of a manufactured part are modeled with the sample space: $S = R^+$.

Let $E_1 = \{x \mid 10 \leq x < 12\}$,
Let $E_2 = \{x \mid 11 < x < 15\}$

– Then $E_1 \cup E_2 = \{x \mid 10 \leq x < 15\}$
– Then $E_1 \cap E_2 = \{x \mid 11 < x < 12\}$
– Then $E_1' = \{x \mid 0 < x < 10 \text{ or } x \geq 12\}$
– Then $E_1' \cap E_2 = \{x \mid 12 \leq x < 15\}$

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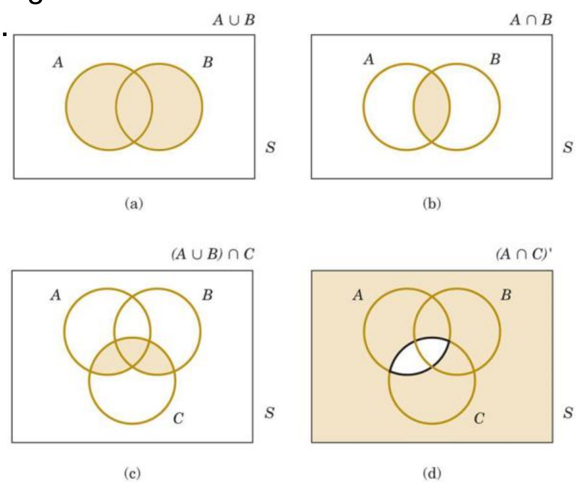
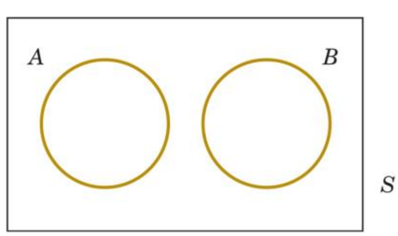
Venn Diagrams

Events *A* & *B* contain their respective outcomes. The shaded regions indicate the event relation of each diagram.



Mutually Exclusive Events

- Events *A* and *B* are mutually exclusive because they share no common outcomes.
- The occurrence of one event precludes the occurrence of the other.
- Symbolically, $A \cap B = \varnothing$



Probability

- Probability is the likelihood or chance that a particular outcome or event from a random experiment will occur.
- Probability is a number in the [0,1] interval.
- A probability of:
  - 1 means certainty
  - 0 means impossibility

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Types of Probability

- Subjective probability is a "degree of belief."

  Example: "There is a 50% chance that I'll study tonight."

- Relative frequency probability is based on how often an event occurs over a very large sample space.

  Example:

$$\lim_{n \to \infty} \frac{n(A)}{n}$$

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Probability Based on Equally-Likely Outcomes

- Whenever a sample space consists of N possible outcomes that are equally likely, the probability of each outcome is $1/N$.
- Example: Throwing a dice. The probability of getting, say 2, is 1/6, because each outcome in the sample space is equally likely.

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Probability of an Event

- For a discrete sample space, the *probability of an event E*, denoted by $P(E)$, equals the sum of the probabilities of the outcomes in E.

- Remember: Event is defined as "a subset of the sample space of a random experiment"

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Example: Probabilities of Events

- A random experiment has a sample space {*a,b,c,d*}. These outcomes are not equally-likely; their probabilities are: 0.1, 0.3, 0.5, 0.1.
- Let Event A = {*a,b*}, B = {*b,c,d*}, and C = {*d*}
  - $P(A) = 0.1 + 0.3 = 0.4$
  - $P(B) = 0.3 + 0.5 + 0.1 = 0.9$
  - $P(C) = 0.1$
  - $P(A') = 0.6$ and $P(B') = 0.1$ and $P(C') = 0.9$
  - Since event $A \cap B = \{b\}$, then $P(A \cap B) = 0.3$
  - Since event $A \cup B$ = {a,*b,c,d*}, then $P(A \cup B) = 1.0$
  - Since event $A \cap C$ = {null}, then $P(A \cap C) = 0$

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Axioms of Probability

- Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

  If S is the sample space and E is any event in the random experiment,

  1. $P(S) = 1$
  2. $0 \leq P(E) \leq 1$
  3. For any two events E_1 and E_2 with $E_1 \cap E_2 = \varnothing$,

  $$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

- The axioms imply that:
  - $P(\varnothing) = 0$ and $P(E') = 1 - P(E)$
  - If E_1 is contained in E_2, then $P(E_1) \leq P(E_2)$.

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Addition Rules

- Joint events are generated by applying basic set operations to individual events, specifically:
  - Unions of events, $A \cup B$ *(i.e., OR)*
  - Intersections of events, $A \cap B$ *(i.e., AND)*
  - Complements of events, A'
- Probabilities of <u>joint events</u> can often be determined from the probabilities of the individual events that comprise them.

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Probability of a Union

- For any two events *A* and *B*, the probability of union is given by:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If events *A* and *B* are mutually exclusive, then

$$P(A \cap B) = \varphi,$$

and therefore:

$$P(A \cup B) = P(A) + P(B)$$

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Addition Rule: 3 or More Events

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Note the alternating signs.

If a collection of events E_i are pairwise mutually exclusive; that is $E_i \cap E_j = \phi$, for all i, j

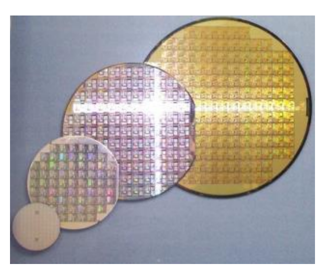
$$\text{Then}: P(E_1 \cup E_2 \cup ... \cup E_k) = \sum_{i=1}^{k} P(E_i)$$

Example: Semiconductor Wafers

A wafer is randomly selected from a batch that is classified by contamination and location.

- Let *H* be the event of high concentrations of contaminants. Then *P*(*H*) = 358/940.
- Let *C* be the event of the wafer being located at the center of a sputtering tool. Then *P*(*C*) = 626/940.
- *P*(*H*∩*C*) = 112/940

Contamination	Location of Tool		Total
	Center	Edge	
Low	514	68	582
High	112	246	358
Total	626	314	940



- *P*(*H*∪*C*) = *P*(*H*) + *P*(*C*) – *P*(*H*∩*C*)
  = (358 + 626 – 112)/940

  This is the addition rule.

Conditional Probability

- $P(B \mid A)$ is the probability of event B occurring, given that event A has already occurred.
- A communications channel has an error rate of 1 per 1000 bits transmitted. Errors are rare, but do tend to occur in bursts. If a bit is in error, the probability that the next bit is also in error is greater than 1/1000.

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Conditional Probability Rule

- The conditional probability of an event B given an event A, denoted as $P(B \mid A)$, is:

  $P(B \mid A) = P(A \cap B) / P(A)$ for $P(A) > 0$.
- From a relative frequency perspective of n equally likely outcomes:
  - $P(A)$ = (number of outcomes in A) / n
  - $P(A \cap B)$ = (number of outcomes in $A \cap B$) / n
  - $P(B \mid A)$ = number of outcomes in $A \cap B$ / number of outcomes in A

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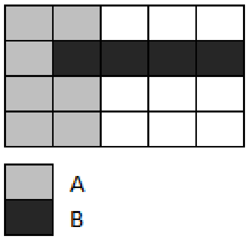
Conditional Probability Rule



$$P(A) = \frac{8}{20} = 0.40$$

$$P(B) = \frac{4}{20} = 0.20$$

$$Probability\ of\ event\ B\ given\ that\ event\ A\ has\ occured = P(B|A)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1}{8} = 0.125$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \rightarrow P(A \cap B) = P(B|A)P(A)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow P(A \cap B) = P(A|B)P(B)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

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