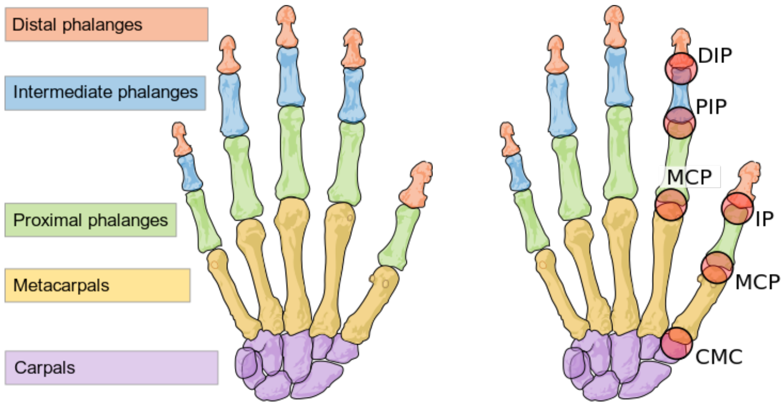
Hand finger modeling and simulating

1 Hand anatomy

The carpals and metacarpals for the fingers (index through pinky) can be assumed to be as fixed in the wrist frame. Squeezing the pinky and index fingers together will reveal a little amount of metacarpal abduction, which is ignored by the hand tracking algorithms since it is a relatively insignificant part of the overall hand motion. The proximal phalange, intermediate phalange, and distal phalange are the three bones of the finger. Only two phalangeal bones—the proximal and distal phalanges—make up the thumb's metacarpal, which is actually free to move.



(A) (B)

FIGURE 1 – (A) Bone structure of the hand and (B) Joints of the hand [5]

As they have different bone structures, the finger and thumb joints are also named differently. First, the finger joints (applies to the index, middle, ring, and pinky fingers).

- Metacarpal-phalangeal joint (MCP): First joint of the finger; connects the metacarpal bone to the proximal phalange.
- Proximal interphalangeal joint (PIP): Second joint of the finger; connects the proximal phalange to the intermediate phalange.
- Distal interphalangeal joint (DIP): Third and final joint of the finger; connects the intermediate phalange to the distal phalange.

Following, the thumb joints:

- Carpal-metacarpal joint (CMC): First joint of the thumb; connects the metacarpal to the carpal bones at the base of the wrist.
- Metacarpal-phalangeal joint (MCP): Second joint of the thumb; connects the metacarpal to the proximal phalange.
- Interphalangeal joint (IP) :Connects the proximal phalange to the distal phalange.

2 Index finger

A model for human index finger with 3-bar linkage with 3 revolute joints is presented in Figure 2.

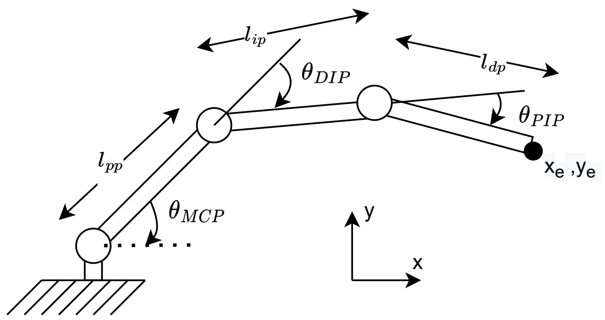


FIGURE 2 – Model of a human index finger.

This model can be employed to analyze kinematics. The angles θ_{MCP}, θ_{DIP} and θ_{PIP} represents joint angles of MCP, DIP and PIP joints. l_{pp},l_{ip} and l_{dp} stand for lengths of 3 phalanges (the distal, intermediate, and proximal). Note that, a human finger (except the thumb), without abduction/adduction movements, can be shown as in Figure 2. Without considering the anatomical constraints of human tendons:

- Derive the forward kinematics equations ($f(\theta) \rightarrow (x, y)$)
- How many solutions are there to the kinematic equations of the human finger model?
- Derive the velocity equations of finger tips ($f(\theta, \dot{\theta}) \rightarrow \dot{x}, \dot{y}$)
- If the orientation of the finger tip θ_e is also specified, how many solutions are there?

However, there are some anatomical constraints for the finger joints. Assume that, index finger has the following ranges of motion [3]:

$$-45^\circ \leq \theta_{MCP} \leq 90^\circ \tag{1}$$
$$0^\circ \leq \theta_{DIP} \leq 100^\circ \tag{2}$$
$$0^\circ \leq \theta_{PIP} \leq 80^\circ \tag{3}$$

Using the constraints, and the average joint lengths of $l_{pp} = 45$ mm, $l_{ip} = 30$ mm, and $l_{dp} = 25$ mm [3]:

- Plot the workspace of the index finger tip, locating the start of proximal phalanges at $x = 0, y = 0$
- Animate the finger movement for the joint angle motions of $\theta_{MCP} : -20^\circ \rightarrow 20^\circ$, $\theta_{DIP} : 0^\circ \rightarrow 30^\circ$ and $\theta_{PIP} : 0^\circ \rightarrow 10^\circ$

3 Thumb

In the literature, 6 DoF [2] and 7 DoF [1] models were used for kinematic analysis of thumb. To reduce the complexity, a model with three DoFs can be employed as shown in Figure 3. Note that, in human anatomy, CMC joint does not actuate the thumb as a revolute joint.

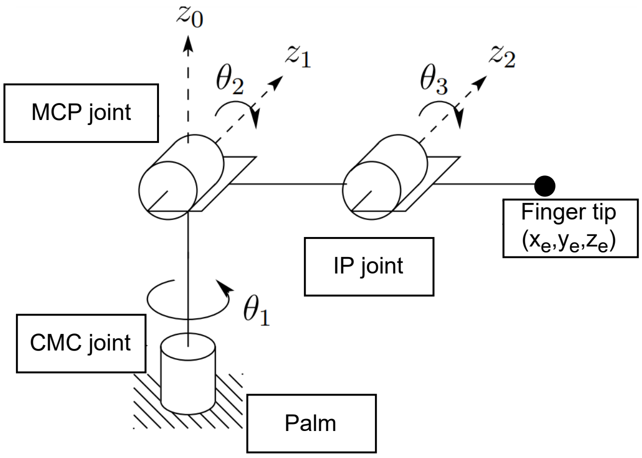


FIGURE 3 – Model of a thumb finger, (figure adopted from Spong et al [4], joint names are changed)

Using the above model of thumb,

- Derive the forward kinematics equations ($f(\theta) \to (x, y, z)$)
- How many solutions are there to the kinematic equations of the human finger model?
- Derive the velocity equations of finger tips ($\dot{x_e}, \dot{y_e}, \dot{z_e}$)

All finger joints has some anatomical constraints. Assume that joints are limited with following ranges of motion:

$$0^\circ \leq \theta_1 \leq 45^\circ \tag{4}$$
$$-45^\circ \leq \theta_2 \leq 90^\circ \tag{5}$$
$$0^\circ \leq \theta_3 \leq 80^\circ \tag{6}$$

- Plot the workspace of the finger tip, locating the start of the metacarpal bone at $x = 0, y = 0, z = 0$
- Animate the finger movement for the joint angle motions of $\theta_1 : 0^\circ \to 15^\circ$, $\theta_2 : -30^\circ \to 70^\circ$, $\theta_3 : 30^\circ \to 0^\circ$.

A kinematic singularity is a configuration in which there may exists many or no inverse kinematic solutions. There exists no solution at a full extension of a joint, and the end-effector is asked to move beyond where it can be positioned. More than one solution may exists when the mechanism can reach the same end-effector position with different joint configurations. By analyzing the Jacobian matrix of a manipulator we can find the singular positions of the model. Jacobian matrix is the matrix which relates changes in joint velocities to Cartesian velocities.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \tag{7}$$

- Find the jacobian matrix,
- By solving det (J)=0, find singularities in the robot workspace
- Find the singularities in the workspace plots of index and thumb fingers.

References

[1] Visakha Nanayakkara et al. "Kinematic Analysis of the Human Thumb with Foldable Palm". In: vol. 9716. June 2016, pp. 226–238. ISBN: 978-3-319-40378-6. DOI: 10.1007/978-3-319-40379-3_23.

[2] Visakha Nanayakkara et al. "Kinematic Analysis of the Human Thumb with Foldable Palm to Understand its Role in Grasp Affordances". In: Sept. 2015.

[3] Jimson Ngeo, Tomoya Tamei, and Tomohiro Shibata. "Continuous and simultaneous estimation of finger kinematics using inputs from an EMG-to-muscle activation model". In: *Journal of neuroengineering and rehabilitation* 11 (Aug. 2014), p. 122. DOI: 10.1186/1743-0003-11-122.

[4] M.W. Spong, S. Hutchinson, and M. Vidyasagar. *Robot Modeling and Control*. Wiley select coursepack. Wiley, 2005. ISBN: 9780471765790. URL: https://books.google.de/books?id=muCMAAAACAAJ.

[5] Wikipedia. *Hand — Wikipedia, The Free Encyclopedia*. http://en.wikipedia.org/w/index.php?title=Hand&oldid=1100202007. [Online; accessed 17-October-2022]. 2022.