Finger modeling and simulating

1 Hand anatomy

The carpals and metacarpals for the fingers (index through pinky) can be assumed to be as fixed in the wrist frame. Squeezing the pinky and index fingers together will reveal a little amount of metacarpal abduction, which is ignored by the hand tracking algorithms since it is a relatively insignificant part of the overall hand motion. The proximal phalange, intermediate phalange, and distal phalange are the three bones of the finger. Only two phalangeal bones—the proximal and distal phalanges—make up the thumb's metacarpal, which is actually free to move.

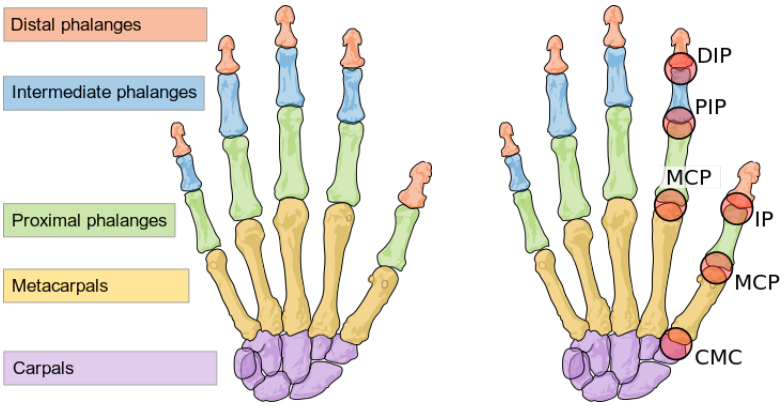


FIGURE 1 – (A) Bone structure of the hand and (B) Joints of the hand [4]

As they have different bone structures, the finger and thumb joints are also named differently. First, the finger joints (applies to the index, middle, ring, and pinky fingers).

- Metacarpal-phalangeal joint (MCP): First joint of the finger; connects the metacarpal bone to the proximal phalange.
- Proximal interphalangeal joint (PIP): Second joint of the finger; connects the proximal phalange to the intermediate phalange.
- Distal interphalangeal joint (DIP): Third and final joint of the finger; connects the intermediate phalange to the distal phalange.

Following, the thumb joints:

- Carpal-metacarpal joint (CMC): First joint of the thumb; connects the metacarpal to the carpal bones at the base of the wrist.
- Metacarpal-phalangeal joint (MCP): Second joint of the thumb; connects the metacarpal to the proximal phalange.
- Interphalangeal joint (IP) :Connects the proximal phalange to the distal phalange.

2 Thumb

In the literature, 6 DoF [2] and 7 DoF [1] models can be used for kinematic analysis of the thumb. To reduce the complexity, a model with three DoFs can be employed, as shown in Figure 2. Note that, in this case CMC and MCP joints are represented as a revolute joint.

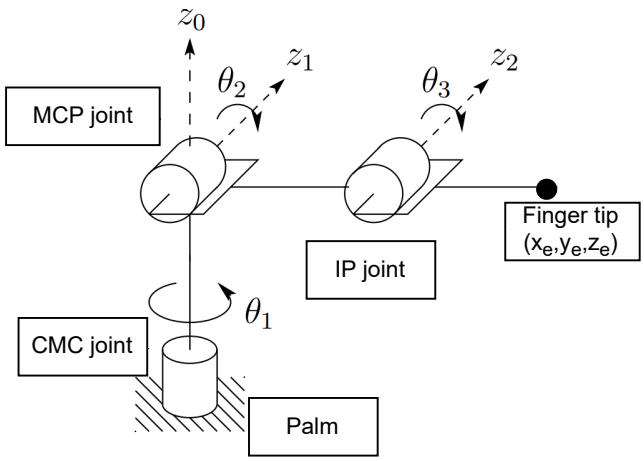


FIGURE 2 – Simplified model of a thumb finger (figure adapted from Spong et al [3])

In the figure above, z_0, z_1 and z_2 are the axes of rotations of the joints. New axes x' and y' can be introduced to the analysis.

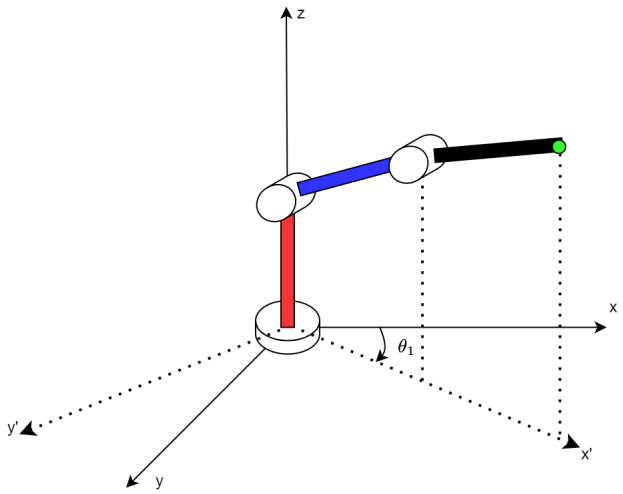
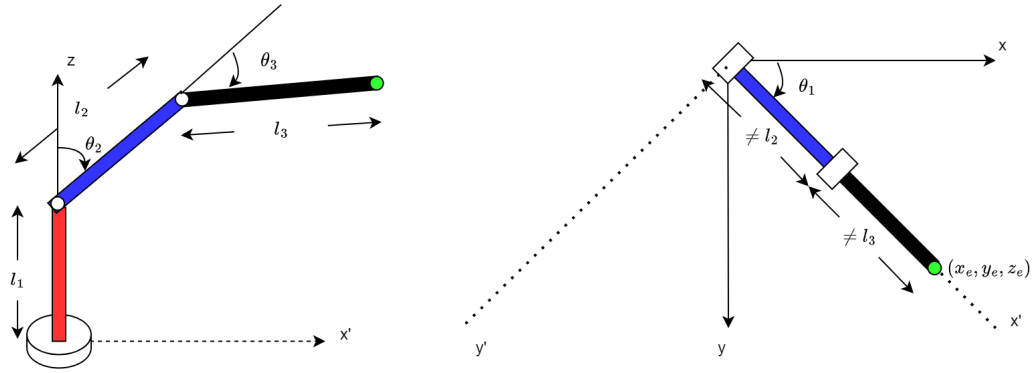


FIGURE 3 – Model of a thumb finger in 3D reference frame

Note that, x' axis will be helpful during the definitions of positions at x and y axes.



(A) (B)

FIGURE 4 – (A) Model in zx' plane (B) Model in xy plane (top view)

Using the above model of the thumb,

- Derive the forward kinematics equations ($f(\theta) \rightarrow (x, y, z)$)
- How many solutions are there to the kinematic equations of the human finger model?
- Derive the velocity equations of finger tip ($\dot{x_e}, \dot{y_e}, \dot{z_e}$)

All finger joints have some anatomical constraints. Assume that joints are limited with following ranges of motion:

$$40^\circ \leq \theta_1 \leq 60^\circ \tag{1}$$
$$0^\circ \leq \theta_2 \leq 60^\circ \tag{2}$$
$$-10^\circ \leq \theta_3 \leq 80^\circ \tag{3}$$

- Plot the workspace of the finger tip, locating the start of the metacarpal bone at $x = 0, y = 0, z = 0$
- Animate the finger movement for the joint angle motions of $\theta_1 : 40^\circ \rightarrow 60^\circ$, $\theta_2 : 0^\circ \rightarrow 60^\circ$, $\theta_3 : -10^\circ \rightarrow 80^\circ$.

References

[1] Visakha Nanayakkara et al. "Kinematic Analysis of the Human Thumb with Foldable Palm". In: vol. 9716. June 2016, pp. 226–238. ISBN: 978-3-319-40378-6. DOI: 10.1007/978-3-319-40379-3_23.

[2] Visakha Nanayakkara et al. "Kinematic Analysis of the Human Thumb with Foldable Palm to Understand its Role in Grasp Affordances". In: Sept. 2015.

[3] M.W. Spong, S. Hutchinson, and M. Vidyasagar. *Robot Modeling and Control*. Wiley select coursepack. Wiley, 2005. ISBN: 9780471765790. URL: https://books.google.de/books?id=muCMAAAACAAJ.

[4] Wikipedia. *Hand — Wikipedia, The Free Encyclopedia*. http://en.wikipedia.org/w/index.php?title=Hand&oldid=1100202007. [Online; accessed 17-October-2022]. 2022.