

```

clc
clear
syms P D kd dd Jm bm kf w T bf t
assume(Jm, 'real')
assume(bm, 'real')
assume(kd, 'real')
assume(dd, 'real')
assume(P, 'real')
assume(D, 'real')
assume(bf, 'real')
assume(kf, 'real')
assume(w, 'real')
assume(t, 'real')
Jm=str2sym('Jm');
imp=poly2sym(str2sym('[dd kd]'));
cf=poly2sym(str2sym('[P+D*t P*t]'));
Tact=poly2sym(str2sym('[bf kf]'));
robot=(poly2sym(str2sym('[Jm bm]')));
tau1=poly2sym(str2sym('[1 2*t t^2]'));
tau2=poly2sym(str2sym('[1 t]'));
Tsens=poly2sym(str2sym('[kf]'));

x=j*w

```

$$x = wi$$

```
envnum=Tact*(imp*(cf+Tsens*tau2*(cf+tau2)))
```

$$\text{envnum} = (kf + bf x) (kd + dd x) (Pt + x (P + Dt) + kf (t + x) (t + x + Pt + x (P + Dt)))$$

```
envden=tau2*robot*poly2sym(str2sym('[1 0 0]'))
```

$$\text{envden} = x^2 (t + x) (bm + Jm x)$$

```

envnum1=expand(eval(envden));
envden1=expand(eval(envnum));
envnum2=expand(envnum1*conj(envden1));
envden2=expand(conj(envden1)*envden1);

pretty(real(envnum2))

```

$$\begin{aligned}
& \text{bm dd kf}^2 w^6 - \text{Jm kd kf}^2 w^6 - \text{Jm P kd kf}^2 w^6 + \text{P bm dd kf}^2 w^6 - \text{Jm dd kf}^2 t w^6 - \text{bm kd kf}^2 t w^4 - \text{Jm dd kf}^2 t w^3 \\
& - \text{Jm kd kf}^2 t w^4 + \text{bm dd kf}^2 t w^4 - \text{Jm P bf kd w}^6 - \text{Jm P dd kf w}^6 - \text{bm kd kf}^2 t w^3 + \text{P bf bm dd w}^6 \\
& + \text{Jm bf dd kf w}^8 - \text{P bm kd kf w}^4 + \text{bf bm kd kf w}^6 + \text{Jm P bf dd kf w}^8 - \text{D Jm bf kd t w}^6 - \text{D Jm dd kf t w}^6 \\
& + \text{D bf bm dd t w}^6 + \text{P bf bm kd kf w}^6 - \text{D bm kd kf t w}^4 - \text{Jm bf kd kf t w}^6 + \text{bf bm dd kf t w}^6 + \text{D Jm bf dd t w}^2
\end{aligned}$$

$$\begin{aligned}
& - D Jm \, kd \, kf \, t \, w \, - D Jm \, kd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, t \, w \, - Jm \, P \, dd \, kf \, t \, w \, - Jm \, P \, dd \, kf \, t \, w \, + D \, bf \, bm \, kd \, t \, w \\
& + D \, bm \, dd \, kf \, t \, w \, + D \, bm \, dd \, kf \, t \, w \, + P \, bf \, bm \, dd \, t \, w \, + Jm \, bf \, dd \, kf \, t \, w \, - Jm \, bf \, kd \, kf \, t \, w \, - P \, bm \, kd \, kf \, t \, w \\
& - P \, bm \, kd \, kf \, t \, w \, + bf \, bm \, dd \, kf \, t \, w \, + bf \, bm \, kd \, kf \, t \, w \, - D Jm \, kd \, kf \, t \, w \, - Jm \, P \, dd \, kf \, t \, w \\
& - Jm \, P \, kd \, kf \, t \, w \, + D \, bm \, dd \, kf \, t \, w \, + P \, bm \, dd \, kf \, t \, w \, - P \, bm \, kd \, kf \, t \, w \, + D Jm \, bf \, dd \, kf \, t \, w \\
& - Jm \, P \, bf \, kd \, kf \, t \, w \, + D \, bf \, bm \, kd \, kf \, t \, w \, + P \, bf \, bm \, dd \, kf \, t \, w \, + D Jm \, bf \, dd \, kf \, t \, w \, + Jm \, P \, bf \, dd \, kf \, t \, w \\
& - Jm \, P \, bf \, kd \, kf \, t \, w \, + D \, bf \, bm \, kd \, kf \, t \, w \, + P \, bf \, bm \, dd \, kf \, t \, w \, + P \, bf \, bm \, kd \, kf \, t \, w
\end{aligned}$$

```

a=coeffs(real(eval(envnum2)),w);
for i=1:length(a)
    b(i,1:length(coeffs(a(i),t)))=coeffs(a(i),t);
end

```

Case 0

bm=0

bm = 0

pretty(real(envnum2))

$$\begin{aligned}
& bm \, dd \, kf \, w \, - Jm \, kd \, kf \, w \, - Jm \, P \, kd \, kf \, w \, + P \, bm \, dd \, kf \, w \, - Jm \, dd \, kf \, t \, w \, - bm \, kd \, kf \, t \, w \, - Jm \, dd \, kf \, t \, w \\
& - Jm \, kd \, kf \, t \, w \, + bm \, dd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, w \, - Jm \, P \, dd \, kf \, w \, - bm \, kd \, kf \, t \, w \, + P \, bf \, bm \, dd \, w \\
& + Jm \, bf \, dd \, kf \, w \, - P \, bm \, kd \, kf \, w \, + bf \, bm \, kd \, kf \, w \, + Jm \, P \, bf \, dd \, kf \, w \, - D Jm \, bf \, kd \, t \, w \, - D Jm \, dd \, kf \, t \, w \\
& + D \, bf \, bm \, dd \, t \, w \, + P \, bf \, bm \, kd \, kf \, w \, - D \, bm \, kd \, kf \, t \, w \, - Jm \, bf \, kd \, kf \, t \, w \, + bf \, bm \, dd \, kf \, t \, w \, + D Jm \, bf \, dd \, t \, w \\
& - D Jm \, kd \, kf \, t \, w \, - D Jm \, kd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, t \, w \, - Jm \, P \, dd \, kf \, t \, w \, - Jm \, P \, dd \, kf \, t \, w \, + D \, bf \, bm \, kd \, t \, w \\
& + D \, bm \, dd \, kf \, t \, w \, + D \, bm \, dd \, kf \, t \, w \, + P \, bf \, bm \, dd \, t \, w \, + Jm \, bf \, dd \, kf \, t \, w \, - Jm \, bf \, kd \, kf \, t \, w \, - P \, bm \, kd \, kf \, t \, w \\
& - P \, bm \, kd \, kf \, t \, w \, + bf \, bm \, dd \, kf \, t \, w \, + bf \, bm \, kd \, kf \, t \, w \, - D Jm \, kd \, kf \, t \, w \, - Jm \, P \, dd \, kf \, t \, w \\
& - Jm \, P \, kd \, kf \, t \, w \, + D \, bm \, dd \, kf \, t \, w \, + P \, bm \, dd \, kf \, t \, w \, - P \, bm \, kd \, kf \, t \, w \, + D Jm \, bf \, dd \, kf \, t \, w \\
&
\end{aligned}$$

$$\begin{aligned}
& - Jm P bf kd kf t w + D bf bm kd kf t w + P bf bm dd kf t w + D Jm bf dd kf t w + Jm P bf dd kf t w \\
& - Jm P bf kd kf t w + D bf bm kd kf t w + P bf bm dd kf t w + P bf bm kd kf t w
\end{aligned}$$

```

a0=coeffs(real(eval(envnum2)),w);
for i=1:length(a0)
    b0(i,1:length(coeffs(a0(i),t)))=coeffs(a0(i),t);
end
clear bm
syms bm
assume(bm, 'real')

```

Case 1 dd=0

```

dd=0;

pretty(real(eval(envnum2)))

```

$$\begin{aligned}
& bf bm kd kf w^6 - Jm P kd kf w^2 6 - bm kd kf t w^2 4 - Jm kd kf t w^2 2 4 - Jm P bf kd w^6 - bm kd kf t w^2 3 2 - P bm kd kf w \\
& - Jm kd kf w^2 6 - D Jm bf kd t w^6 + P bf bm kd kf w^6 - D bm kd kf t w^4 - Jm bf kd kf t w^6 - D Jm kd kf t w^2 4 \\
& - D Jm kd kf t w^2 6 - Jm P bf kd t w^2 4 + D bf bm kd t w^2 4 - Jm bf kd kf t w^3 4 - P bm kd kf t w^2 2 - P bm kd kf t w^2 \\
& + bf bm kd kf t w^2 4 - D Jm kd kf t w^2 3 4 - Jm P kd kf t w^2 2 4 - P bm kd kf t w^2 3 2 - Jm P bf kd kf t w^6 \\
& + D bf bm kd kf t w^6 - Jm P bf kd kf t w^3 4 + D bf bm kd kf t w^3 4 + P bf bm kd kf t w^2 4
\end{aligned}$$

```

a1=coeffs(real(eval(envnum2)),w);
for i=1:length(a1)
    b1(i,1:length(coeffs(a1(i),t)))=coeffs(a1(i),t);
end

```

Case 1.1 bm=0 dd=0

```

dd=0;

bm=0;
pretty(real(eval(envnum2)))

```

$$\begin{aligned}
& - Jm kd kf w^2 6 - Jm P kd kf w^2 6 - Jm kd kf t w^2 2 4 - Jm P bf kd w^6 - D Jm bf kd t w^6 - Jm bf kd kf t w^6 \\
& - D Jm kd kf t w^2 4 - D Jm kd kf t w^2 6 - Jm P bf kd t w^2 4 - Jm bf kd kf t w^3 4 - D Jm kd kf t w^2 3 4 - Jm P kd kf t w^2 \\
& 6 \qquad 3 4
\end{aligned}$$

- Jm P bf kd kf t w - Jm P bf kd kf t w

```
a11=coeffs(real(eval(envnum2)),w);
for i=1:length(a11)
    b11(i,1:length(coeffs(a11(i),t)))=coeffs(a11(i),t);
end
clear dd bm
syms dd bm
assume(bm, 'real')
assume(dd, 'real')
```

Case 2 $D=0$

```
D=0;
pretty(real(eval(envnum2)))
```

$$\begin{aligned}
& \overset{2}{bm}\overset{6}{dd}\overset{2}{kf}\overset{6}{w}-\overset{2}{Jm}\overset{6}{kd}\overset{2}{kf}\overset{6}{w}-\overset{2}{Jm}\overset{6}{P}\overset{2}{kd}\overset{6}{kf}\overset{2}{w}+\overset{2}{P}\overset{6}{bm}\overset{2}{dd}\overset{6}{kf}\overset{2}{w}-\overset{2}{Jm}\overset{6}{dd}\overset{2}{kf}\overset{6}{t}\overset{2}{w}-\overset{2}{bm}\overset{6}{kd}\overset{2}{kf}\overset{6}{t}\overset{2}{w}-\overset{2}{Jm}\overset{6}{dd}\overset{2}{kf}\overset{6}{t}\overset{2}{w}\\
& -\overset{2}{Jm}\overset{2}{kd}\overset{4}{kf}\overset{2}{t}\overset{4}{w}+\overset{2}{bm}\overset{2}{dd}\overset{4}{kf}\overset{2}{t}\overset{4}{w}-\overset{6}{Jm}\overset{6}{P}\overset{6}{bf}\overset{6}{kd}\overset{6}{w}-\overset{6}{Jm}\overset{6}{P}\overset{6}{dd}\overset{6}{kf}\overset{6}{w}-\overset{2}{bm}\overset{3}{kd}\overset{2}{kf}\overset{2}{t}\overset{2}{w}+\overset{6}{P}\overset{6}{bf}\overset{6}{bm}\overset{6}{dd}\overset{6}{w}\\
& +\overset{8}{Jm}\overset{8}{bf}\overset{8}{dd}\overset{8}{kf}\overset{8}{w}-\overset{4}{P}\overset{4}{bm}\overset{4}{kd}\overset{4}{kf}\overset{4}{w}+\overset{6}{bf}\overset{6}{bm}\overset{6}{kd}\overset{6}{kf}\overset{6}{w}+\overset{8}{Jm}\overset{8}{P}\overset{8}{bf}\overset{8}{dd}\overset{8}{kf}\overset{8}{w}+\overset{6}{P}\overset{6}{bf}\overset{6}{bm}\overset{6}{kd}\overset{6}{kf}\overset{6}{w}-\overset{6}{Jm}\overset{6}{bf}\overset{6}{kd}\overset{6}{kf}\overset{6}{t}\overset{6}{w}\\
& +\overset{6}{bf}\overset{6}{bm}\overset{6}{dd}\overset{6}{kf}\overset{6}{t}\overset{6}{w}-\overset{2}{Jm}\overset{4}{P}\overset{4}{bf}\overset{4}{kd}\overset{4}{t}\overset{4}{w}-\overset{2}{Jm}\overset{4}{P}\overset{4}{dd}\overset{4}{kf}\overset{4}{t}\overset{4}{w}-\overset{2}{Jm}\overset{6}{P}\overset{6}{dd}\overset{6}{kf}\overset{6}{t}\overset{6}{w}+\overset{2}{P}\overset{4}{bf}\overset{4}{bm}\overset{4}{dd}\overset{4}{t}\overset{4}{w}+\overset{2}{Jm}\overset{2}{bf}\overset{2}{dd}\overset{2}{kf}\overset{2}{t}\overset{2}{w}\\
& -\overset{3}{Jm}\overset{4}{bf}\overset{3}{kd}\overset{4}{kf}\overset{4}{t}\overset{4}{w}-\overset{2}{P}\overset{2}{bm}\overset{2}{kd}\overset{2}{kf}\overset{2}{t}\overset{2}{w}-\overset{2}{P}\overset{4}{bm}\overset{4}{kd}\overset{4}{kf}\overset{4}{t}\overset{4}{w}+\overset{3}{bf}\overset{4}{bm}\overset{3}{dd}\overset{4}{kf}\overset{4}{t}\overset{4}{w}+\overset{2}{bf}\overset{4}{bm}\overset{2}{kd}\overset{4}{kf}\overset{4}{t}\overset{4}{w}-\overset{2}{Jm}\overset{2}{P}\overset{2}{dd}\overset{2}{kf}\overset{2}{t}\overset{2}{w}\\
& -\overset{2}{Jm}\overset{2}{P}\overset{4}{kd}\overset{4}{kf}\overset{4}{t}\overset{4}{w}+\overset{2}{P}\overset{2}{bm}\overset{4}{dd}\overset{4}{kf}\overset{4}{t}\overset{4}{w}-\overset{2}{P}\overset{3}{bm}\overset{2}{kd}\overset{3}{kf}\overset{2}{t}\overset{2}{w}-\overset{6}{Jm}\overset{6}{P}\overset{6}{bf}\overset{6}{kd}\overset{6}{kf}\overset{6}{t}\overset{6}{w}+\overset{6}{P}\overset{6}{bf}\overset{6}{bm}\overset{6}{dd}\overset{6}{kf}\overset{6}{t}\overset{6}{w}\\
& +\overset{2}{Jm}\overset{6}{P}\overset{6}{bf}\overset{6}{dd}\overset{6}{kf}\overset{6}{t}\overset{6}{w}-\overset{3}{Jm}\overset{4}{P}\overset{4}{bf}\overset{4}{kd}\overset{4}{kf}\overset{4}{t}\overset{4}{w}+\overset{3}{P}\overset{4}{bf}\overset{4}{bm}\overset{3}{dd}\overset{4}{kf}\overset{4}{t}\overset{4}{w}+\overset{2}{P}\overset{4}{bf}\overset{4}{bm}\overset{2}{kd}\overset{4}{kf}\overset{4}{t}\overset{4}{w}
\end{aligned}$$

```
a2=coeffs(real(eval(envnum2)),w);
for i=1:length(a2)
    b2(i,1:length(coeffs(a2(i),t)))=coeffs(a2(i),t);
end
```

Case 2.1 $D=0$ $bm=0$

```
D=0;
bm=0;
pretty(real(eval(envnum2)))
```

8 2 6 2 6 2 3 4 2 2 4 6
Jm bf dd kf w - Jm P kd kf w - Jm dd kf t w - Jm dd kf t w - Jm kd kf t w - Jm P bf kd w - Jm P dd kf w

2 6 8 6 2 4 2 4 2 6

$$\begin{aligned}
& - Jm \, kd \, kf \, w \, + Jm \, P \, bf \, dd \, kf \, w \, - Jm \, bf \, kd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, t \, w \, - Jm \, P \, dd \, kf \, t \, w \, - Jm \, P \, dd \, kf \, t \, w \\
& + Jm \, bf \, dd \, kf \, t \, w \, - Jm \, bf \, kd \, kf \, t \, w \, - Jm \, P \, dd \, kf \, t \, w \, - Jm \, P \, kd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, kf \, t \, w \\
& + Jm \, P \, bf \, dd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, kf \, t \, w
\end{aligned}$$

```

a21=coeffs(real(eval(envnum2)),w);
for i=1:length(a21)
    b21(i,1:length(coeffs(a21(i),t)))=coeffs(a21(i),t);
end
clear bm
syms bm
assume(bm, 'real')

```

Case 3 D=0 dd=0

```

D=0;
dd=0;

pretty(real(eval(envnum2)))

```

$$\begin{aligned}
& bf \, bm \, kd \, kf \, w \, - Jm \, P \, kd \, kf \, w \, - bm \, kd \, kf \, t \, w \, - Jm \, kd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, w \, - bm \, kd \, kf \, t \, w \, - P \, bm \, kd \, kf \, w \\
& - Jm \, kd \, kf \, w \, + P \, bf \, bm \, kd \, kf \, w \, - Jm \, bf \, kd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, t \, w \, - Jm \, bf \, kd \, kf \, t \, w \, - P \, bm \, kd \, kf \, t \, w \\
& - P \, bm \, kd \, kf \, t \, w \, + bf \, bm \, kd \, kf \, t \, w \, - Jm \, P \, kd \, kf \, t \, w \, - P \, bm \, kd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, kf \, t \, w \\
& - Jm \, P \, bf \, kd \, kf \, t \, w \, + P \, bf \, bm \, kd \, kf \, t \, w
\end{aligned}$$

```

a3=coeffs(real(eval(envnum2)),w);
for i=1:length(a3)
    b3(i,1:length(coeffs(a3(i),t)))=coeffs(a3(i),t);
end

```

Case 3.1 D=0 dd=0 bm=0

```

D=0;
dd=0;
bm=0;
pretty(real(eval(envnum2)))

```

$$\begin{aligned}
& - Jm \, kd \, kf \, w \, - Jm \, P \, kd \, kf \, w \, - Jm \, kd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, w \, - Jm \, bf \, kd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, t \, w \\
& - Jm \, bf \, kd \, kf \, t \, w \, - Jm \, P \, kd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, kf \, t \, w \, - Jm \, P \, bf \, kd \, kf \, t \, w
\end{aligned}$$

```
a31=coeffs(real(eval(envnum2)),w);  
for i=1:length(a31)  
    b31(i,1:length(coeffs(a31(i),t)))=coeffs(a31(i),t);  
end  
clear dd bm  
syms dd bm  
assume(bm, 'real')  
assume(dd, 'real')
```