

```

clc
clear
syms P D kd dd Jm bm kf w T bf t
assume(Jm, 'real')
assume(bm, 'real')
assume(kd, 'real')
assume(dd, 'real')
assume(P, 'real')
assume(D, 'real')
assume(bf, 'real')
assume(kf, 'real')
assume(w, 'real')
assume(t, 'real')
Jm=str2sym('Jm');
imp=poly2sym(str2sym(' [kd+dd*t kd*t]'));
cf=poly2sym(str2sym(' [P+D*t P*t]'));
Tact=poly2sym(str2sym(' [bf kf]'));
robot=(poly2sym(str2sym(' [Jm bm]')));
tau1=poly2sym(str2sym(' [1 2*t t^2]'));
tau2=poly2sym(str2sym(' [1 t]'));
Tsens=poly2sym(str2sym(' [kf]'));

x=j*w

```

$x = w i$

```
envnum=Tact*(imp*(cf+Tsens*tau2*(cf+tau2)))
```

$$\text{envnum} = (kf + bf x) (kd t + x (kd + dd t)) (P t + x (P + D t) + kf (t + x) (t + x + P t + x (P + D t)))$$

```
envden=tau1*robot*poly2sym(str2sym(' [1 0 0]'))
```

$$\text{envden} = x^2 (bm + Jm x) (t^2 + 2 t x + x^2)$$

```

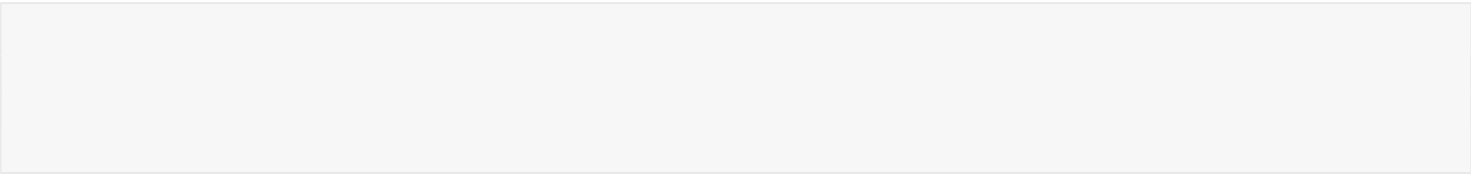
envnum1=expand(eval(envden));
envden1=expand(eval(envnum));
envnum2=expand(envnum1*conj(envden1));
envden2=expand(conj(envden1)*envden1);

pretty(real(envnum2))

```

$$\begin{aligned}
& bf^8 bm^2 kd^8 kf^2 w^8 - Jm^2 P^2 kd^8 kf^2 w^8 - Jm^2 dd^2 kf^2 t^8 w^8 - bm^2 kd^2 kf^2 t^6 w^8 - 2 Jm^2 dd^2 kf^2 t^3 w^6 - Jm^2 dd^2 kf^2 t^5 w^4 \\
& - 2 Jm^2 kd^2 kf^2 t^2 w^6 - Jm^2 kd^2 kf^2 t^4 w^4 - Jm^2 P^2 bf^8 kd^8 w^8 - 2 bm^2 kd^2 kf^2 t^3 w^4 - bm^2 kd^2 kf^2 t^5 w^2 - P^2 bm^2 kd^2 kf^2 w^6 \\
& - Jm^2 kd^2 kf^2 t^8 w^8 - D^2 Jm^2 bf^8 kd^8 t^8 w^8 - Jm^2 P^2 bf^8 dd^8 t^8 w^8 + P^2 bf^8 bm^2 kd^8 kf^8 w^8 - D^2 bm^2 kd^2 kf^2 t^6 w^6 - P^2 bm^2 dd^2 kf^2 t^8 w^6 \\
& - Jm^2 bf^8 kd^8 kf^2 t^8 w^8 + bf^8 bm^2 dd^8 kf^2 t^8 w^8 - D^2 Jm^2 bf^8 dd^8 t^8 w^8 + D^2 Jm^2 bf^8 dd^8 t^4 w^6 - D^2 Jm^2 bf^8 kd^8 t^3 w^6 - 2 D^2 Jm^2 dd^8 kf^2 t^3 w^6
\end{aligned}$$

$$\begin{aligned}
& - \text{Jm P bf dd t w} - \text{D Jm kd kf t w} - \text{D Jm kd kf t w} - \text{D Jm kd kf t w} - 2 \text{Jm P bf kd t w} - \text{Jm P bf kd t w} \\
& - \text{Jm P dd kf t w} - \text{Jm P dd kf t w} - \text{Jm P dd kf t w} + 2 \text{D bf bm dd t w} + \text{D bf bm kd t w} + \text{D bf bm kd t w} \\
& - \text{D bm dd kf t w} + \text{D bm dd kf t w} + \text{P bf bm dd t w} + \text{P bf bm dd t w} - \text{D bm kd kf t w} - \text{P bm dd kf t w} \\
& - 2 \text{Jm bf kd kf t w} - \text{Jm bf kd kf t w} - 2 \text{P bm kd kf t w} - \text{P bm kd kf t w} - \text{P bm kd kf t w} \\
& + 2 \text{bf bm dd kf t w} + \text{bf bm dd kf t w} + 2 \text{bf bm kd kf t w} + \text{bf bm kd kf t w} - \text{D Jm dd kf t w} \\
& - \text{D Jm dd kf t w} - 2 \text{D Jm kd kf t w} - \text{D Jm kd kf t w} - 2 \text{Jm P dd kf t w} - \text{Jm P dd kf t w} \\
& - 2 \text{Jm P kd kf t w} - \text{Jm P kd kf t w} + \text{D bm dd kf t w} + \text{D bm dd kf t w} - 2 \text{P bm kd kf t w} \\
& - \text{P bm kd kf t w} - \text{Jm P bf kd kf t w} + \text{D bf bm kd kf t w} + \text{P bf bm dd kf t w} + \text{D Jm bf dd kf t w} \\
& + \text{D Jm bf dd kf t w} - 2 \text{Jm P bf kd kf t w} - \text{Jm P bf kd kf t w} + \text{D bf bm dd kf t w} + \text{D bf bm dd kf t w} \\
& + 2 \text{D bf bm kd kf t w} + \text{D bf bm kd kf t w} + 2 \text{P bf bm dd kf t w} + \text{P bf bm dd kf t w} + 2 \text{P bf bm kd kf t w} \\
& + \text{P bf bm kd kf t w}
\end{aligned}$$



Case 0

bm=0

bm = 0

pretty(real(envnum2))

$$\begin{aligned}
& \text{bf bm kd kf w} - \text{Jm P kd kf w} - \text{Jm dd kf t w} - \text{bm kd kf t w} - 2 \text{Jm dd kf t w} - \text{Jm dd kf t w} \\
& - 2 \text{Jm kd kf t w} - \text{Jm kd kf t w} - \text{Jm P bf kd w} - 2 \text{bm kd kf t w} - \text{bm kd kf t w} - \text{P bm kd kf w} \\
& - \text{Jm kd kf w} - \text{D Jm bf kd t w} - \text{Jm P bf dd t w} + \text{P bf bm kd kf w} - \text{D bm kd kf t w} - \text{P bm dd kf t w} \\
& - \text{Jm bf kd kf t w} + \text{bf bm dd kf t w} - \text{D Jm bf dd t w} + \text{D Jm bf dd t w} - \text{D Jm bf kd t w} - 2 \text{D Jm dd kf t w} \\
& - \text{Jm P bf dd t w} - \text{D Jm kd kf t w} - \text{D Jm kd kf t w} - \text{D Jm kd kf t w} - 2 \text{Jm P bf kd t w} - \text{Jm P bf kd t w}
\end{aligned}$$

$$\begin{aligned}
& - Jm P \ddot{d} k f t w - Jm P \ddot{d} k f t w - Jm P \ddot{d} k f t w + 2 D b f b m \ddot{d} t w + D b f b m k d t w + D b f b m k d t w \\
& - D b m \ddot{d} k f t w + D b m \ddot{d} k f t w + P b f b m \ddot{d} t w + P b f b m \ddot{d} t w - D b m k d k f t w - P b m \ddot{d} k f t w \\
& - 2 Jm b f k d k f t w - Jm b f k d k f t w - 2 P b m k d k f t w - P b m k d k f t w - P b m k d k f t w \\
& + 2 b f b m \ddot{d} k f t w + b f b m \ddot{d} k f t w + 2 b f b m k d k f t w + b f b m k d k f t w - D Jm \ddot{d} k f t w \\
& - D Jm \ddot{d} k f t w - 2 D Jm k d k f t w - D Jm k d k f t w - 2 Jm P \ddot{d} k f t w - Jm P \ddot{d} k f t w \\
& - 2 Jm P k d k f t w - Jm P k d k f t w + D b m \ddot{d} k f t w + D b m \ddot{d} k f t w - 2 P b m k d k f t w \\
& - P b m k d k f t w - Jm P b f k d k f t w + D b f b m k d k f t w + P b f b m \ddot{d} k f t w + D Jm b f \ddot{d} k f t w \\
& + D Jm b f \ddot{d} k f t w - 2 Jm P b f k d k f t w - Jm P b f k d k f t w + D b f b m \ddot{d} k f t w + D b f b m \ddot{d} k f t w \\
& + 2 D b f b m k d k f t w + D b f b m k d k f t w + 2 P b f b m \ddot{d} k f t w + P b f b m \ddot{d} k f t w + 2 P b f b m k d k f t w \\
& + P b f b m k d k f t w
\end{aligned}$$

```
clear bm
syms bm
assume(bm, 'real')
```

Case 1 dd=0

```
dd=0;
pretty(real(eval(envnum2)))
```

$$\begin{aligned}
& b f b m k d k f w - J m P k d k f w - b m k d k f t w - 2 J m k d k f t w - J m k d k f t w - J m P b f k d w \\
& - 2 b m k d k f t w - b m k d k f t w - P b m k d k f w - J m k d k f w - D J m b f k d t w + P b f b m k d k f w \\
& - D b m k d k f t w - J m b f k d k f t w - D J m b f k d t w - D J m k d k f t w - D J m k d k f t w - D J m k d k f t w \\
& - 2 J m P b f k d t w - J m P b f k d t w + D b f b m k d t w + D b f b m k d t w - D b m k d k f t w \\
& - 2 J m b f k d k f t w - J m b f k d k f t w - 2 P b m k d k f t w - P b m k d k f t w - P b m k d k f t w \\
& + 2 b f b m k d k f t w + b f b m k d k f t w - 2 D J m k d k f t w - D J m k d k f t w - 2 J m P k d k f t w \\
& - J m P k d k f t w - 2 P b m k d k f t w - P b m k d k f t w - J m P b f k d k f t w + D b f b m k d k f t w
\end{aligned}$$

$$\begin{aligned}
& - 2 \text{ Jm P bf kd kf t } w^3 w^6 - \text{ Jm P bf kd kf t } w^5 w^4 + 2 \text{ D bf bm kd kf t } w^3 w^6 + \text{ D bf bm kd kf t } w^5 w^4 + 2 \text{ P bf bm kd kf t } w^2 \\
& + \text{ P bf bm kd kf t } w^4 w^4
\end{aligned}$$

Case 1.1 bm=0 dd=0

dd=0;

bm=0;
pretty(real(eval(envnum2)))

$$\begin{aligned}
& - \text{ Jm kd kf } w^2 w^8 - \text{ Jm P kd kf } w^2 w^8 - 2 \text{ Jm kd kf t } w^2 w^2 w^6 - \text{ Jm kd kf t } w^2 w^4 w^4 - \text{ Jm P bf kd w }^8 - \text{ D Jm bf kd t } w^8 \\
& - \text{ Jm bf kd kf t } w^8 - \text{ D Jm bf kd t } w^3 w^6 - \text{ D Jm kd kf t } w^2 w^6 - \text{ D Jm kd kf t } w^4 w^4 - \text{ D Jm kd kf t } w^2 w^8 - 2 \text{ Jm P bf kd t } w^2 \\
& - \text{ Jm P bf kd t } w^4 w^4 - 2 \text{ Jm bf kd kf t } w^3 w^6 - \text{ Jm bf kd kf t } w^5 w^4 - 2 \text{ D Jm kd kf t } w^2 w^3 w^6 - \text{ D Jm kd kf t } w^2 w^5 w^4 \\
& - 2 \text{ Jm P kd kf t } w^2 w^2 w^6 - \text{ Jm P kd kf t } w^2 w^4 w^4 - \text{ Jm P bf kd kf t } w^8 - 2 \text{ Jm P bf kd kf t } w^3 w^6 - \text{ Jm P bf kd kf t } w^5 w^4
\end{aligned}$$

```
clear dd bm
syms dd bm
assume(bm, 'real')
assume(dd, 'real')
```

Case 2 D=0

D=0;
pretty(real(eval(envnum2)))

$$\begin{aligned}
& \text{bf bm kd kf w }^8 - \text{ Jm P kd kf } w^2 w^8 - \text{ Jm dd kf t } w^2 w^8 - \text{ bm kd kf t } w^2 w^6 - 2 \text{ Jm dd kf t } w^2 w^3 w^6 - \text{ Jm dd kf t } w^2 w^5 w^4 \\
& - 2 \text{ Jm kd kf t } w^2 w^2 w^6 - \text{ Jm kd kf t } w^2 w^4 w^4 - \text{ Jm P bf kd w }^8 - 2 \text{ bm kd kf t } w^2 w^3 w^4 - \text{ bm kd kf t } w^2 w^5 w^2 - \text{ P bm kd kf w }^6 \\
& - \text{ Jm kd kf } w^2 w^8 - \text{ Jm P bf dd t } w^8 + \text{ P bf bm kd kf w }^8 - \text{ P bm dd kf t } w^6 - \text{ Jm bf kd kf t } w^8 + \text{ bf bm dd kf t } w^8 \\
& - \text{ Jm P bf dd t } w^3 w^6 - 2 \text{ Jm P bf kd t } w^2 w^6 - \text{ Jm P bf kd t } w^4 w^4 - \text{ Jm P dd kf t } w^2 w^6 - \text{ Jm P dd kf t } w^4 w^4 - \text{ Jm P dd kf t } w^2 \\
& + \text{ P bf bm dd t } w^2 w^6 + \text{ P bf bm dd t } w^4 w^4 - \text{ P bm dd kf t } w^3 w^4 - 2 \text{ Jm bf kd kf t } w^3 w^6 - \text{ Jm bf kd kf t } w^5 w^4 \\
& - 2 \text{ P bm kd kf t } w^2 w^4 - \text{ P bm kd kf t } w^4 w^2 - \text{ P bm kd kf t } w^2 w^6 + 2 \text{ bf bm dd kf t } w^3 w^6 + \text{ bf bm dd kf t } w^5 w^4
\end{aligned}$$

$$\begin{aligned}
& + 2 \text{ }^2 \text{ }^6 \text{ }^4 \text{ }^4 \text{ }^2 \text{ }^3 \text{ }^6 \text{ }^2 \text{ }^5 \text{ }^4 \text{ }^2 \text{ }^2 \text{ }^6 \\
& + 2 \text{ }^2 \text{ }^4 \text{ }^4 \text{ }^2 \text{ }^3 \text{ }^4 \text{ }^2 \text{ }^5 \text{ }^2 \text{ }^8 \text{ }^8 \\
& - \text{ }^2 \text{ }^4 \text{ }^4 \text{ }^2 \text{ }^3 \text{ }^4 \text{ }^2 \text{ }^5 \text{ }^2 \text{ }^8 \text{ }^8 \\
& - 2 \text{ }^3 \text{ }^6 \text{ }^5 \text{ }^4 \text{ }^3 \text{ }^6 \text{ }^5 \text{ }^4 \text{ }^2 \\
& + P \text{ }^4 \text{ }^4
\end{aligned}$$

Case 2.1 D=0 bm=0

```

D=0;
bm=0;
pretty(real(eval(envnum2)))

```

$$\begin{aligned}
& - \text{ }^2 \text{ }^8 \text{ }^2 \text{ }^8 \text{ }^2 \text{ }^8 \text{ }^2 \text{ }^3 \text{ }^6 \text{ }^2 \text{ }^5 \text{ }^4 \text{ }^2 \text{ }^2 \text{ }^6 \\
& - \text{ }^2 \text{ }^4 \text{ }^4 \text{ }^8 \text{ }^8 \text{ }^8 \text{ }^3 \text{ }^6 \text{ }^2 \text{ }^6 \\
& - \text{ }^4 \text{ }^4 \text{ }^2 \text{ }^6 \text{ }^4 \text{ }^4 \text{ }^2 \text{ }^8 \text{ }^3 \text{ }^6 \\
& - 2 \text{ }^2 \text{ }^3 \text{ }^6 \text{ }^2 \text{ }^5 \text{ }^4 \text{ }^2 \text{ }^2 \text{ }^6 \text{ }^2 \text{ }^4 \text{ }^4 \text{ }^8 \\
& - 2 \text{ }^3 \text{ }^6 \text{ }^5 \text{ }^4
\end{aligned}$$

```

clear bm
syms bm
assume(bm, 'real')

```

Case 3 D=0 dd=0

```

D=0;
dd=0;
pretty(real(eval(envnum2)))

```

$$\begin{aligned}
& \text{ }^8 \text{ }^2 \text{ }^8 \text{ }^2 \text{ }^6 \text{ }^2 \text{ }^2 \text{ }^6 \text{ }^2 \text{ }^4 \text{ }^4 \text{ }^8 \\
& - 2 \text{ }^2 \text{ }^3 \text{ }^4 \text{ }^2 \text{ }^5 \text{ }^2 \text{ }^6 \text{ }^2 \text{ }^8 \text{ }^8 \text{ }^8 \\
& - 2 \text{ }^2 \text{ }^6 \text{ }^4 \text{ }^4 \text{ }^3 \text{ }^6 \text{ }^5 \text{ }^4 \text{ }^2 \text{ }^4
\end{aligned}$$

$$\begin{aligned}
& - P \text{ }^4 \text{ }^2 \text{ }^2 \text{ }^6 \text{ }^2 \text{ }^6 \text{ }^4 \text{ }^4 \text{ }^2 \text{ }^2 \text{ }^6 \\
& - P \text{ }^4 \text{ }^2 \text{ }^2 \text{ }^6 \text{ }^2 \text{ }^6 \text{ }^4 \text{ }^4 \text{ }^2 \text{ }^2 \text{ }^6 \\
& - Jm \text{ }^2 \text{ }^4 \text{ }^4 \text{ }^2 \text{ }^3 \text{ }^4 \text{ }^2 \text{ }^5 \text{ }^2 \text{ }^8 \text{ }^3 \text{ }^6 \\
& - Jm \text{ }^2 \text{ }^4 \text{ }^4 \text{ }^2 \text{ }^3 \text{ }^4 \text{ }^2 \text{ }^5 \text{ }^2 \text{ }^8 \text{ }^3 \text{ }^6 \\
& - Jm \text{ }^5 \text{ }^4 \text{ }^2 \text{ }^6 \text{ }^4 \text{ }^4 \\
& - Jm \text{ }^5 \text{ }^4 \text{ }^2 \text{ }^6 \text{ }^4 \text{ }^4
\end{aligned}$$

Case 3.1 D=0 dd=0 bm=0

```

D=0;
dd=0;
bm=0;
pretty(real(eval(envnum2)))

```

$$\begin{aligned}
& - Jm \text{ }^2 \text{ }^8 \text{ }^2 \text{ }^8 \text{ }^2 \text{ }^2 \text{ }^6 \text{ }^2 \text{ }^4 \text{ }^4 \text{ }^8 \text{ }^8 \\
& - Jm \text{ }^2 \text{ }^8 \text{ }^2 \text{ }^8 \text{ }^2 \text{ }^2 \text{ }^6 \text{ }^2 \text{ }^4 \text{ }^4 \text{ }^8 \text{ }^8 \\
& - 2 Jm \text{ }^2 \text{ }^6 \text{ }^4 \text{ }^4 \text{ }^3 \text{ }^6 \text{ }^5 \text{ }^4 \text{ }^2 \text{ }^2 \text{ }^6 \\
& - 2 Jm \text{ }^2 \text{ }^6 \text{ }^4 \text{ }^4 \text{ }^3 \text{ }^6 \text{ }^5 \text{ }^4 \text{ }^2 \text{ }^2 \text{ }^6 \\
& - Jm \text{ }^2 \text{ }^4 \text{ }^4 \text{ }^8 \text{ }^3 \text{ }^6 \text{ }^5 \text{ }^4
\end{aligned}$$