Passivity and unconditional stability analysis of BIC of SCAs via two port representation

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Abstract

This paper is a report for a passivity and unconditional stability analysis of series compliant actuators. In this project, Basic Impedance Controller is employed to achieve design constraints to obtain passivity and/or absolute stability. These two conditions are main requirements for safe physical human and machine interaction. Passivity is a more conservative stability condition than unconditional stability. 180 systems are defined in the MATLAB in order to find optimum design for series compliant actuation. Virtual couplers are defined to the systems to relax conditions for unconditional stability and passivity. Virtual coupler's improvement on the conditions are shown.

Contents

1 Introduction

Force controlled compliant actuators are popular components of physical human-robot interaction(pHRI) applications. The most widely used one is series elastic actuator(SEA) contains an elastic element in series with actuation. This compliance provides several unique properties such as low mechanical output impedance, passive mechanical energy storage and tolerance to impact loads.[1][2] A haptic simulation provides a kinesthetic communication by stimulating the motion in a virtual environment or a remote operation. Many of haptic devices consist of a mechanism which is operated physically by manipulating the position of the end point.

In the research field of haptics there are two major assumptions. Both of the operator and the remote or virtual environment are passive[3]. Human operator will not destabilize a system that is designed with guaranteed passivity. According to Hollerbach[4], human operators tend to adapt to a haptic device's design characteristics. Dynamic impedance and admittance that can be simulated by the device, should be analyzed based upon the mechanical performance characteristics. This analysis must be completed while designing both the physical components and virtual coupler.

1.1 Two-port characterization

Two port models, are preferred way of describing force and velocity relations for the analyzing the stability and performance in bilateral teleoperation.[5][6]. The relationship between the efforts and flows is commonly described as an immitance matrix.[7]. The immitance matrix will be defined as following

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \tag{1}$$

1.2 Series Elastic Actuation

Due to requirements of pHRI, series elastic actuation mechanism with an electric motor is used as actuation module to satisfy both safety and weight requirements. Such mechanism provides measuring the transmitted force by the spring deflection between the actuator and the human side. Defined mechanical stiffness is combined with proper sensors to measure force exerted by the external environment to the mechatronic system. SEA stores an energy load during the interaction, and provides the safety in human machine interaction[8]. Series elastic actuator ensures a good torque or force control with low mass for exoskeletons.

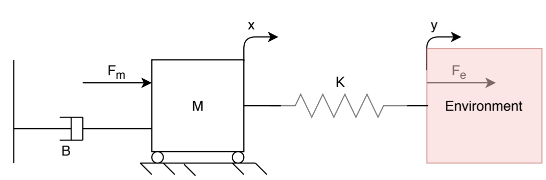


Figure 1: Kinematic model of a series elastic actuator

1.3 Series Compliant Actuation

A series compliant actuator(SCA) contains a motor to produce mechanical power and a compliant element to sense the force. These components can be selected for different design with various trade-offs. This trade-offs are defined according to constraints obtained from different analysis. The immetance matrix of a series

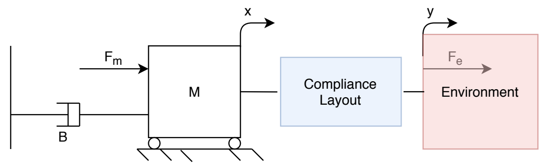


Figure 2: Kinematic model of a series compliant actuator

compliant actuator will be

$$\begin{bmatrix} \dot{x} \\ F_m \end{bmatrix} = \begin{bmatrix} 1 & Y(s) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{y} \\ -F_e \end{bmatrix} \tag{2}$$

where the Y(s) represents admittance of the compliance unit.

Remark 1. *Series elastic actuator is a series compliant actuator with compliance lay-out of a spring.*

1.4 Passivity

A two port system is passive if and only if the immitance matrix satisfies the following conditions[5]

$$Re(p_{11}) \geq 0 \tag{3}$$

$$Re(p_{22}) \geq 0 \tag{4}$$

$$Re(p_{11})Re(p_{22}) - \left| \frac{p_{21}^* + p_{12}}{2} \right|^2 \geq 0 \tag{5}$$

with condition of $\forall \omega \geq 0$

1.5 Unconditional stability

A two port system is unconditionally stable if it's immitance matrix satisfies the following necessary and sufficient conditions obtained by Llewelly's stability criteria.[9]

$$Re(p_{11}) \geq 0, \tag{6}$$

$$Re(p_{22}) \geq 0 \tag{7}$$

$$2Re(p_{11})Re(p_{22}) \geq |p_{12}p_{21}| + Re(p_{12}p_{21}) \tag{8}$$

with condition of $\forall \omega \geq 0$

A passive two port network is always unconditionally stable. However an unconditionally stable network does not have to be passive. Such systems are called potentially stable.[7] In this project, unconditional stability and passivity analysis are performed on BIC of series compliant actuators. Controller and coupler design constraints are derived for the virtually coupled impedance controlled haptic interfaces.

2 Notation

2.1 Plant

An admittance type plant is employed in the analysis.

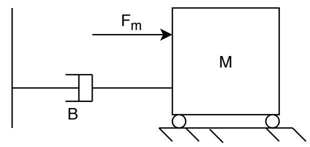


Figure 3: Mechanical model of plant

with a transfer function of

$$P_1(s) = \frac{1}{Ms + B} \tag{9}$$

2.1.1 Motion controlled plant

If we apply a PI motion control on the proposed plant

$$P_2(s) = \frac{P_1}{P_1 C_v + 1} = \frac{1}{C_v + \frac{1}{P_1}} = \frac{1}{P_c + \frac{I_c}{s} + Ms + B} = \frac{s}{Ms^2 + (B + P_c)s + I_c} \tag{10}$$

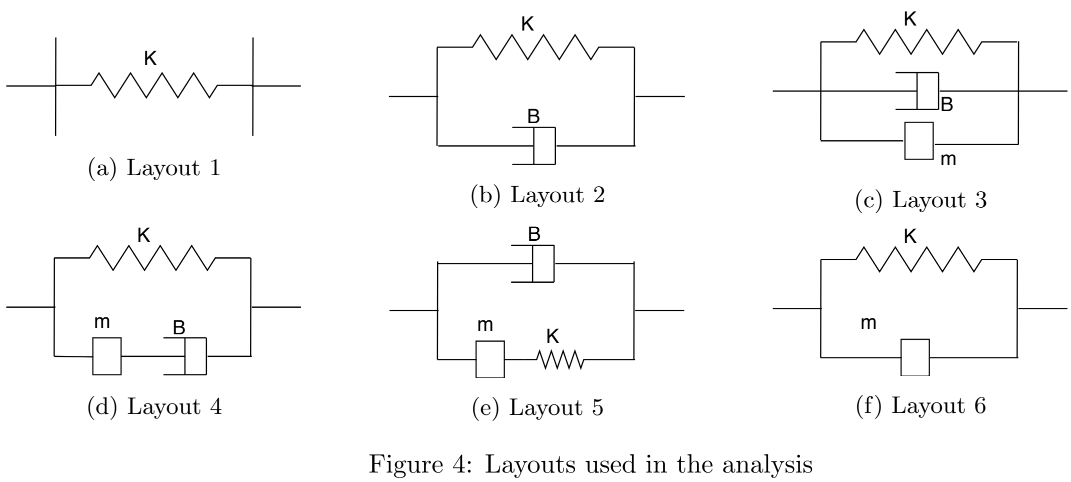
finally, a motion controlled plant will be employed in the analysis with a transfer function of

$$P_2(s) = \frac{s}{Ms^2 + B_m s + K_m} \tag{11}$$

Remark 2. *Applying a PI motion control on the plant provides an additional damping to the plant with the amount of proportional gain in the PI control.*

2.2 Compliance Layouts

Several compliance layouts are used. This compliance layouts are modeled as different combinations of spring, damper and inerter.



(a) Layout 1

(b) Layout 2

(c) Layout 3

(d) Layout 4

(e) Layout 5

(f) Layout 6

Figure 4: Layouts used in the analysis

2.3 Physical coupler

Force sensing unit in the actuator is defined in this section. Z_c defines the corresponding impedance.

$$Z_c = \frac{G_{pc}}{s} \tag{12}$$

Physical Coupler	Corresponded layout	G_{pc}
1	1	K
2	2	$K + b_f\, s$
3	3	$m_f\, s^2 + b_f\, s + K$
4	4	$\frac{K m_f s + K b_f + m_f b_f s^2}{m_f s^2 + b_f s}$
5	5	$\frac{b_f m_f s^3 + m_f K s^2 + K b_f s}{m_f s^2 + K}$
6	6	$K + m_f\, s^2$

2.4 Controllers

Several controllers are employed through the analysis.

Controller	Name	Model
1	Proportional	P_c
2	Proportional-Derivative	$P_c + D_c\, s$
3	Proportional-Integral	$P_c + \frac{I_c}{s}$
4	Proportional-Filtered Derivative	$P_c + D_c \frac{s\tau}{s+\tau}$
5	Proportional-Integral-Second integral	$P_c + \frac{I_c}{s} + \frac{A_c}{s^2}$

2.4.1 Proportional-Integral-Second integral

In velocity sourced impedance control, a motion controlled plant is controlled by a proportional-integral-second integral control. Proof will be given in the one of following section.

2.5 Virtual coupler

Virtual coupler networks ensure stability while they can be designed to give maximum performance[5].

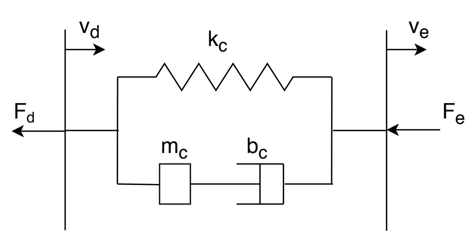


Figure 5: Mechanical analogue of 4^{th} virtual coupler

$$F_d = F_e \tag{13}$$

$$-v_e = -v_d + Y_c F_e \tag{14}$$

where admittance of the virtual coupler is defined as

$$Y_c = \frac{s}{G_{vc}} \tag{15}$$

For fourth lay-out

$$G_{vc4} = k_c + \frac{1}{\frac{1}{m_c s^2} + \frac{1}{b_c s}} \tag{16}$$

and the fifth one can be derived by

$$G_{vc5} = b_c s + \frac{1}{\frac{1}{m_c s^2} + \frac{1}{k_c}} \tag{17}$$

Virtual Coupler	Corresponded layout	G_{vc}
1	1	k_c
2	2	$k_c + b_c\, s$
3	3	$m_c\, s^2 + b_c\, s + k_c$
4	4	$\frac{k_c m_c s + k_c b_c + m_c b_c s^2}{m_c s + b_c}$
5	5	$\frac{b_c m_c s^3 + m_c k_c s^2 + k_c b_c s}{m_c s^2 + k_c}$
6	6	$k_c + m_c\, s^2$

2.6 Systems and Conditions

Conditions for passivity and unconditional stability were analyzed for all combinations of defined virtual couplers, physical couplers, controllers, and plants. Passivity and unconditional stability requires equation 3 and 4 to be satisfied for any of immitance matrix. The equation 5 must be satisfied to ensure the passivity. Less conservative stability criterion for given immitance matrix, unconditional stability, is achievable if equation 8 is satisfied.

For a hybrid matrix of H:

$$H_{i,j,k} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \tag{18}$$

where i is corresponded virtual coupler, j is corresponded controller, k is the physical coupler, n is the plant employed in the analysis. Conditions for that hybrid matrix will be represented as

$$_{n}\text{Condition}^{m}_{i,j,k} \tag{19}$$

where

Condition	Satisfied equation
$_{n}\text{Condition}^{1}_{i,j,k}$	$Re(h_{11}) \geq 0$
$_{n}\text{Condition}^{2}_{i,j,k}$	$Re(h_{22}) \geq 0$
$_{n}\text{Condition}^{3}_{i,j,k}$	$Re(h_{11})Re(h_{22}) - \left\lvert \frac{h_{21}+h_{12}}{2} \right\rvert^2 \geq 0$
$_{n}\text{Condition}^{4}_{i,j,k}$	$2Re(h_{11})Re(h_{22}) - \lvert h_{12}h_{21} \rvert - Re(h_{12}h_{21}) \geq 0$

3 Basic Impedance Control

Basic Impedance Controller is a cascaded controller architecture for series compliant actuator. The cascaded control architecture is given in the Figure 6. One port analysis is applied on this controller[10] in order to derive the conditions to passively render pure stiffness and voigt model environments.

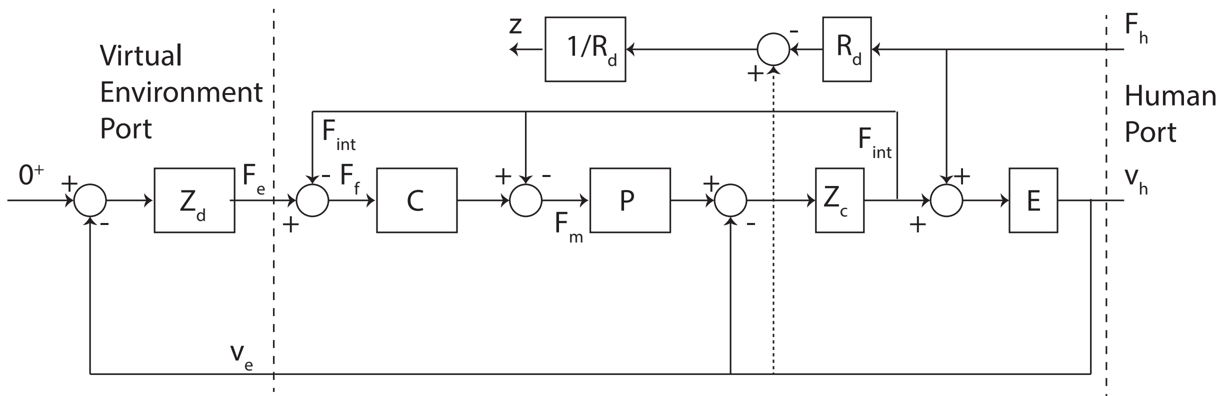


Figure 6: Control diagram of Basic Impedance Control

Driven equations for this control diagram are represented below

$$\begin{pmatrix} F_e = Z_d\, v_e \\ F_f = F_e - F_{\text{int}} \\ F_c = C\, F_f \\ v_m = F_m\, P \\ F_m = F_c - F_{\text{int}} \\ F_{\text{int}} = -Z_c\, (v_h - v_m) \end{pmatrix} \tag{20}$$

3.1 Comments on Velocity Sourced Impedance Control

Velocity sourced impedance controller is a cascaded controller architecture for series compliant actuator. The cascaded control architecture is given in the Figure 7

This controller is made of an inner velocity controlled plant and outer impedance control. The inner velocity control provides a robust motion to plant to compensate the unmodeled dynamics of friction, stiction and slip. The intermediate loop provides force feedback to ensure good force tracking. Effective impedance of the system is controlled by the outer loop.

Remark 3. *If a block diagram reduction applied on the controller architecture as given in the Figure 7, one can see that velocity sourced impedance controller is nothing but basic impedance controller with motion controlled plant.*

Remark 4. *VSIC can be analyzed with the same algorithm by defining the plant as motion controlled plant*

$$P_2(s) = \frac{P_1}{P_1 C_v + 1} = \frac{1}{C_v + \frac{1}{P_1}} = \frac{1}{P_c + \frac{I_c}{s} + Ms + B} = \frac{s}{Ms^2 + (B + P_c)s + I_c} \tag{21}$$

and the controller as proportional - integral - second integral controller as given C_5

$$C_f C_v = (P_f + \frac{I_f}{s})(P_v + \frac{I_v}{s}) = P_f P_v + \frac{P_f I_v + P_v I_f}{s} + \frac{I_f I_v}{s^2} \tag{22}$$

$$C_5 = C_f C_v = P_c + \frac{I_c}{s} + \frac{A_c}{s^2} \tag{23}$$

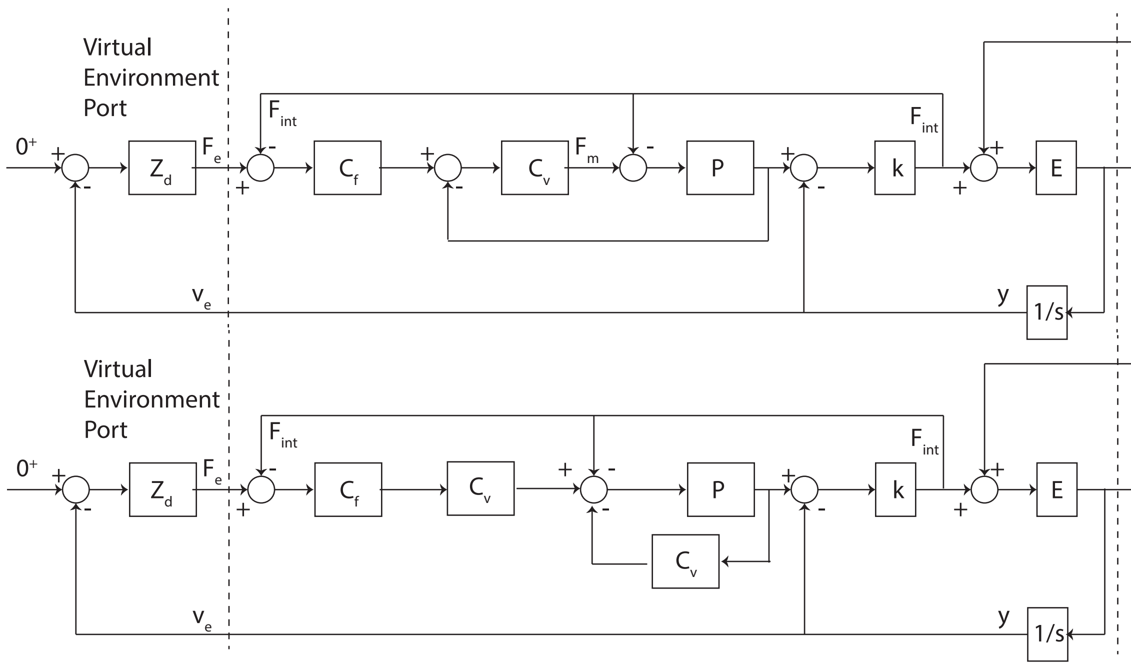


Figure 7: Control diagram of Velocity Sourced Impedance Control(above), equivalent control diagram after block diagram reduction(below)

4 Two port analysis of BIC

4.1 Haptic Simulation

Haptic simulation is devided into 5 terminals as following

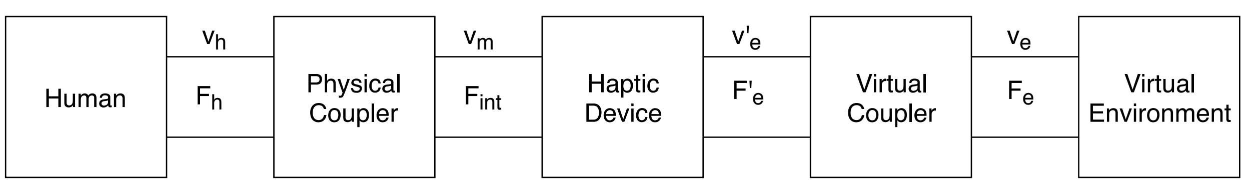


Figure 8: Proposed haptic simulation is made up of a physical and virtual coupler which connects a haptic device with human operator and virtual environment

A physical coupler is a passive terminal which is used to sense the force with any compliance layout in series compliant actuation. This passive terminal is defined individually in order to simplify the two port network analysis.

Hypothesis 1. *If the force sensing unit is required to simulated as a different terminal, a three port network model should be defined.*

Remark 5. *An impedance controlled cascaded connection of physical coupler and haptic device(plant) is defined as the haptic interface. This haptic interface is not passive(obviously dissipates energy) and might be unconditionally stable for bounded controller gains and physical characteristics.*

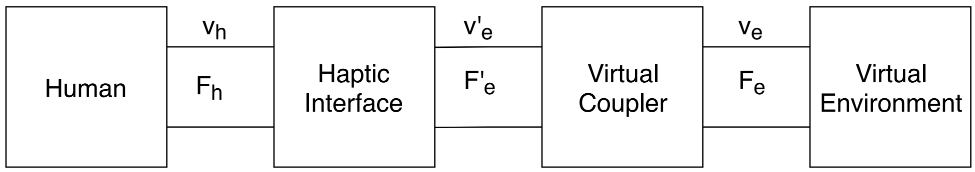


Figure 9: Haptic simulation is made up of a impedance controlled haptic interface and virtual coupler which connects a haptic interface with virtual environment

4.2 Implementation of virtual couplers

4.2.1 Impedance controlled haptic interface

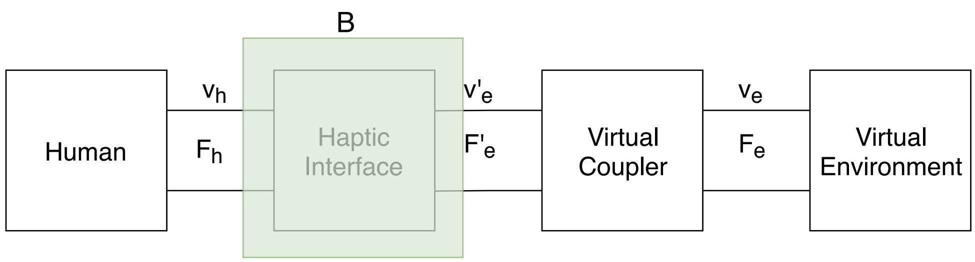


Figure 10: Impedance controlled haptic interface is defined with matrix B

$$\begin{bmatrix} F_{int} \\ v'_e \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} v_h \\ F'_e \end{bmatrix} \tag{24}$$

$$\begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} -\frac{Z_c}{P\,Z_c + C_f\,P\,Z_c + 1} & \frac{C_f\,P\,Z_c}{P\,Z_c + C_f\,P\,Z_c + 1} \\ -1 & 0 \end{bmatrix} \tag{25}$$

4.2.2 Virtual Coupler

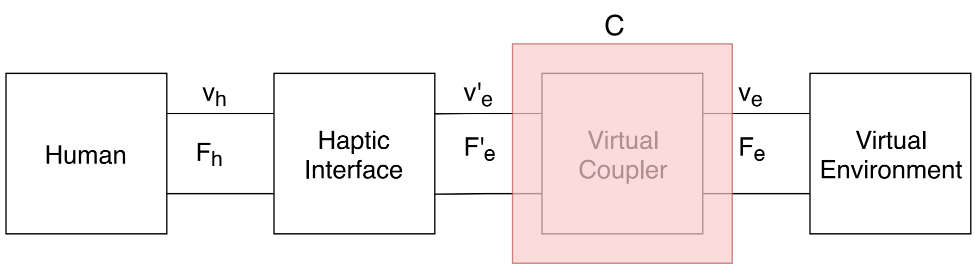


Figure 11: Virtual coupler is defined with matrix C

$$\begin{bmatrix} F_e \\ v'_e \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} v_e \\ F'_e \end{bmatrix} \tag{26}$$

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -Y_c \end{bmatrix} \tag{27}$$

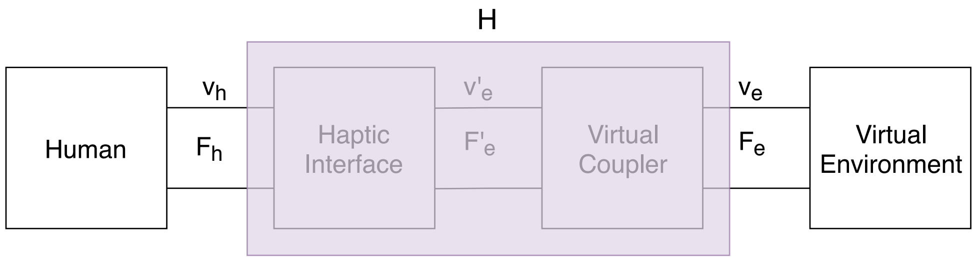


Figure 12: Impedance controlled haptic interface and virtual coupler is defined with matrix H

$$\begin{bmatrix} F_{int} \\ v_e \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} v_h \\ F_e \end{bmatrix} \tag{28}$$

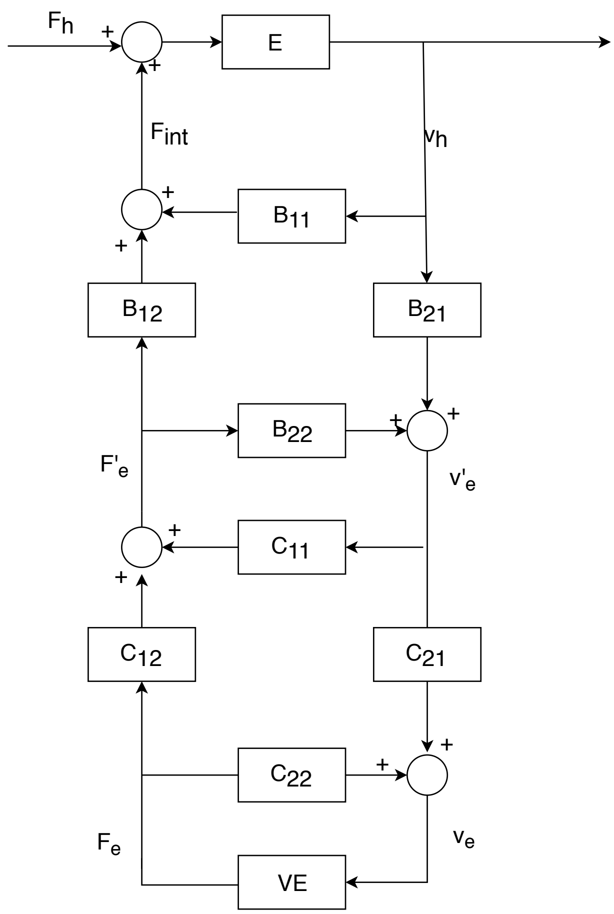


Figure 13: Control diagram of virtually coupled haptic interface

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} = \begin{bmatrix} -\frac{B_{11}-B_{11}\,B_{22}\,C_{11}+B_{12}\,B_{21}\,C_{11}}{B_{22}\,C_{11}-1} & -\frac{B_{12}\,C_{12}}{B_{22}\,C_{11}-1} \\ -\frac{B_{21}\,C_{21}}{B_{22}\,C_{11}-1} & -\frac{C_{22}-B_{22}\,C_{11}\,C_{22}+B_{22}\,C_{12}\,C_{21}}{B_{22}\,C_{11}-1} \end{bmatrix} \tag{29}$$

Hybrid matrix of this interface will be

$$\begin{bmatrix} F_{int} \\ -v_e \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ -H_{21} & -H_{22} \end{bmatrix} \begin{bmatrix} v_h \\ F_e \end{bmatrix} \tag{30}$$

4.3 Hybrid matrix

The resulting hybrid matrix of virtually coupled, impedance controlled haptic interface will be

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} -\frac{Z_c}{P\,Z_c + C_f\,P\,Z_c + 1} & \frac{C_f\,P\,Z_c}{P\,Z_c + C_f\,P\,Z_c + 1} \\ 1 & Y_c \end{bmatrix} \tag{31}$$

5 Results

5.1 First condition on defined systems

Passivity and unconditional stability requires

$$Re(h_{11}) \geq 0 \tag{32}$$

where

$$h_{11}(j\omega) = \frac{num(j\omega)}{den(j\omega)} = \frac{num(j\omega)}{den(j\omega)} \frac{den(-j\omega)}{den(-j\omega)} \tag{33}$$

First condition will be satisfied if

$$Re(\frac{num_{h_{11}}(j\omega)}{den_{h_{11}}(j\omega)} \frac{den_{h_{11}}(-j\omega)}{den_{h_{11}}(-j\omega)}) \geq 0 \tag{34}$$

Denominator of the equation above is positive definite, however numerator of that equation can be represented as

$$num_{h_{11}}(jw)den_{h_{11}}(-jw) = A\omega^n + B\omega^{n-1}j + C\omega^{n-1} \cdots + D\omega^0 \tag{35}$$

Proposition 1. *For a given hybrid matrix element h_{11}, condition of positive realness of $Re(h_{11})$, can be expressed as*

$$Re(num_{h_{11}}(jw)den_{h_{11}}(-jw)) \geq 0 \equiv Re(h_{11}) \geq 0 \tag{36}$$

Proposition 2. *Necessary condition of positive realness of $Re(h_{11})$, for a given hybrid matrix element h_{11},*

$$A \geq 0 \tag{37}$$

$$D \geq 0 \tag{38}$$

Proposition 3. *Sufficient condition of positive realness of ${}_{n}Condition^{1}_{i,j,k}$ for a given hybrid matrix element h_{11}*

$${}_{n}Condition^{1}_{i,j,k} = a_m\omega^m + a_{m-2}\omega^{m-2} + \cdots + a_2\omega^2 + a_0\omega^0 \tag{39}$$

where $\forall\ a_i \geq 0$ for $i = 1, 2, \ldots m$, m is the highest order power.

Remark 6. *First condition is not correlated to the dynamics of virtual coupling.*

Remark 7. *Passivity and unconditional stability cannot be achievable by employing proportional, proportional-derivative, and proportional-filtered derivative controllers. However compliance layouts of 2, 4 and 5 should not be employed as physical coupler to achieve passivity or unconditional stability.*

Physical Coupler	Controller	$Re(h_{11}) \geq 0$
1	1,2,4	not satisfied
	3	if ${}_n\text{Condition}^1_{i,3,1} \geq 0$
	5	if ${}_n\text{Condition}^1_{i,5,1} \geq 0$
2	all controllers	not satisfied
3	1,2,4	not satisfied
	3	if ${}_n\text{Condition}^1_{i,3,3} \geq 0$
	5	if ${}_n\text{Condition}^1_{i,5,3} \geq 0$
4	all controllers	not satisfied
5	all controllers	not satisfied
6	1,2,4	not satisfied
	3	if ${}_n\text{Condition}^1_{i,3,6} \geq 0$
	5	if ${}_n\text{Condition}^1_{i,5,6} \geq 0$

Table 1: Achievability of the first condition

5.1.1 Plant 1

First condition of passivity and unconditional stability of basic impedance controlled series elastic actuator, virtually coupled to virtual environment with controller $C_3 = P_c + \frac{I_c}{s}$, $Z_{c1} = \frac{K}{s}$ represented as below

$$
{}_1\text{Condition}^1_{i,3,1} = I_c\,K^2\,M - B\,K^2\,P_c - B\,K^2 \tag{40}
$$

where i is the number of virtual coupler.

This condition implies a limitation between controller gains and the actuator's mass and damping.

$$
I_c M - B(P_c + 1) \geq 0 \tag{41}
$$

and then

$$
\frac{I_c}{P_c + 1} \geq \frac{B}{M} \tag{42}
$$

First condition of passivity and unconditional stability of basic impedance controlled series compliant actuator with layout 3, virtually coupled to virtual environment with controller $C_3 = P_c + \frac{I_c}{s}$, $Z_{c3} = \frac{m_c\,s^2 + b_c\,s + k_c}{s}$ represented as below

$$
\begin{aligned}
{}_1\text{Condition}^1_{i,3,3} = {} & I_c\,K^2\,M - B\,K^2 - w^4\,\left(B\,{m_f}^2 + M^2\,b_f + B\,P_c\,{m_f}^2 - I_c\,M\,{m_f}^2\right) - B\,K^2\,P_c \\
& - w^2\,\left(B\,{b_f}^2 + B^2\,b_f - 2\,B\,K\,m_f + B\,P_c\,{b_f}^2 - I_c\,M\,{b_f}^2 - 2\,B\,K\,P_c\,m_f + 2\,I_c\,K\,M\,m_f\right)
\end{aligned} \tag{43}
$$

This condition implies a limitation between controller gains and the actuator's mass and damping. The upper bound on controller gains must satisfy the following conditions

$$
I_c M - B(P_c + 1) \geq \frac{M^2 b_f}{m_f^2} \tag{44}
$$

and

$$
I_c M - B(P_c + 1) \geq \frac{B^2 b_f}{b_f^2 - 2K m_f} \tag{45}
$$

First condition of passivity and unconditional stability of basic impedance controlled series compliant actuator with layout 6, with the same PI controller

$$
\begin{aligned}
{}_1\text{Condition}^1_{i,3,6} = {} & w^2\,\left(2\,B\,K\,m_f + 2\,B\,K\,P_c\,m_f - 2\,I_c\,K\,M\,m_f\right) - w^4\,\left(B\,{m_f}^2 + B\,P_c\,{m_f}^2 - I_c\,M\,{m_f}^2\right) \\
& - B\,K^2 - B\,K^2\,P_c + I_c\,K^2\,M
\end{aligned} \tag{46}
$$

will promise the same constraint with series elastic actuator.

$$I_cM - B(P_c + 1) \geq 0 \tag{47}$$

First condition of passivity and unconditional stability of basic impedance controlled series elastic actuator, virtually coupled to virtual environment with controller $C_5 = P_c + \frac{I_c}{s} + \frac{A_c}{s^2}$, $Z_{c1} = \frac{K}{s}$ represented as below

$${}_1\text{Condition}^1_{i,5,1} = A_c\, B\, K^2 - w^2\, \left(B\, K^2 + B\, K^2\, P_c - I_c\, K^2\, M\right) \tag{48}$$

will promise the same constraints with PI controlled series elastic actuator.

$$I_cM - B(P_c + 1) \geq 0 \tag{49}$$

The next system is defined with C_5 and Z_{c3}

$$\begin{aligned} {}_1\text{Condition}^1_{i,5,3} = A_c\, B\, K^2 - w^6\, \left(B\, {m_f}^2 + M^2\, b_f + B\, P_c\, {m_f}^2 - I_c\, M\, {m_f}^2\right) \\ - w^2\, \left(B\, K^2 + B\, K^2\, P_c - I_c\, K^2\, M - A_c\, B\, {b_f}^2 + 2\, A_c\, B\, K\, m_f\right) \\ - w^4\, \left(B\, {b_f}^2 + B^2\, b_f - 2\, B\, K\, m_f - A_c\, B\, {m_f}^2 + B\, P_c\, {b_f}^2 - I_c\, M\, {b_f}^2 - 2\, B\, K\, P_c\, m_f + 2\, I_c\, K\, M\, m_f\right) \end{aligned} \tag{50}$$

will have the constraints of

$$I_cM - B(P_c + 1) \geq \frac{M^2 b_f}{m_f^2} \tag{51}$$

To simplify further analysis, a new variable is defined as:

$$\xi = I_cM - B(P_c + 1) \tag{52}$$

other conditions implied by the first condition

$$\xi \geq \frac{BA_c(2Km_f - b_f^2)}{K^2} \tag{53}$$

$$\xi \geq \frac{b_f^2 - 2Km_f}{B^2 b_f - A_c B m_f} \tag{54}$$

First condition of a system defined with C_5 and Z_{c6} has to satisfy the following condition to be passive

$$\begin{aligned} {}_1\text{Condition}^1_{i,5,6} = w^4\, \left(2\, B\, K\, m_f + A_c\, B\, {m_f}^2 + 2\, B\, K\, P_c\, m_f - 2\, I_c\, K\, M\, m_f\right) - w^6\, \left(B\, {m_f}^2 + B\, P_c\, {m_f}^2 - I_c\, M\, {m_f}^2\right) \\ - w^2\, \left(B\, K^2 + B\, K^2\, P_c - I_c\, K^2\, M + 2\, A_c\, B\, K\, m_f\right) + A_c\, B\, K^2 \end{aligned} \tag{55}$$

This condition implies

$$\xi \geq 0 \tag{56}$$

$$\xi \geq \frac{A_c B m_f}{2K} \tag{57}$$

$$\xi \geq \frac{2A_c B m_f}{K} \tag{58}$$

5.1.2 Plant 2

Plant 2 is defined as motion controlled plant, which has a transfer function of

$$P_2(s) = \frac{s}{M s^2 + B_m s + K_m} \tag{59}$$

By using that plant model and a PI force controller with physical coupler layouts of 1-3-6, first condition of passivity and unconditional stability implies conditions below

$${}_2\mathrm{Condition}^1_{i,3,1} = -w^2\,\left(B_m\,K^2 + B_m\,K^2\,P_c - I_c\,K^2\,M\right) - I_c\,K^2\,K_m \tag{60}$$

$$\begin{aligned}{}_2\mathrm{Condition}^1_{i,3,3} &= -w^2\,\left(B_m\,K^2 + {K_m}^2\,b_f + B_m\,K^2\,P_c - I_c\,K^2\,M + I_c\,K_m\,{b_f}^2 - 2\,I_c\,K\,K_m\,m_f\right) \\ &- w^4\,\left(B_m\,{b_f}^2 + {B_m}^2\,b_f - 2\,B_m\,K\,m_f - 2\,K_m\,M\,b_f + B_m\,P_c\,{b_f}^2 - I_c\,M\,{b_f}^2 + I_c\,K_m\,{m_f}^2 - 2\,B_m\,K\,P_c\,m_f\right) \\ &- w^4\,\left(+2\,I_c\,K\,M\,m_f\right) - w^6\,\left(M^2\,b_f + B_m\,{m_f}^2 + B_m\,P_c\,{m_f}^2 - I_c\,M\,{m_f}^2\right) - I_c\,K^2\,K_m\end{aligned} \tag{61}$$

$$\begin{aligned}{}_2\mathrm{Condition}^1_{i,3,6} &= w^4\,\left(2\,B_m\,K\,m_f - I_c\,K_m\,{m_f}^2 + 2\,B_m\,K\,P_c\,m_f - 2\,I_c\,K\,M\,m_f\right) \\ &- w^2\,\left(B_m\,K^2 + B_m\,K^2\,P_c - I_c\,K^2\,M - 2\,I_c\,K\,K_m\,m_f\right) - w^6\,\left(B_m\,{m_f}^2 + B_m\,P_c\,{m_f}^2 - I_c\,M\,{m_f}^2\right) - I_c\,K^2\,K_m\end{aligned} \tag{62}$$

These conditions cannot be satisfied. At low frequencies negative coefficient of ω^0 does not allow the condition to be positive real.

Remark 8. *Passivity and unconditional stability are not achievable by applying integral control on motion controlled plant model.*

A motion controlled plant with controller C_5 with Z_{c1} is basically a velocity sourced impedance control of a series elastic actuator.

$${}_2\mathrm{Condition}^1_{i,5,1} = A_c\,B_m\,K^2 - w^2\,\left(B_m\,K^2 + B_m\,K^2\,P_c - I_c\,K^2\,M\right) - I_c\,K^2\,K_m \tag{63}$$

This condition set implies the following conditions

$$\xi_2 = I_c M - B_m(P_c + 1) \geq 0 \tag{64}$$

$$\beta = A_c B_m - I_c K_m \geq 0 \tag{65}$$

First condition of passivity and unconditional stability of basic impedance controlled series compliant actuator with layout 3, virtually coupled to virtual environment with controller C_5, $Z_{c3} = \frac{m_c\,s^2 + b_c\,s + k_c}{s}$ represented as below

$$\begin{aligned}{}_2\mathrm{Condition}^1_{i,5,3} &= A_c\,B_m\,K^2 - w^6\,\left(M^2\,b_f + B_m\,{m_f}^2 + B_m\,P_c\,{m_f}^2 - I_c\,M\,{m_f}^2\right) \\ &- w^2\,\left(B_m\,K^2 + {K_m}^2\,b_f + B_m\,K^2\,P_c - I_c\,K^2\,M - A_c\,B_m\,{b_f}^2 + I_c\,K_m\,{b_f}^2 + 2\,A_c\,B_m\,K\,m_f - 2\,I_c\,m_f\,K_m\,K\right) \\ &- w^4\,\left(B_m\,{b_f}^2 + {B_m}^2\,b_f - 2\,B_m\,K\,m_f - 2\,K_m\,M\,b_f - A_c\,B_m\,{m_f}^2 + B_m\,P_c\,{b_f}^2 - I_c\,M\,{b_f}^2 + I_c\,K_m\,{m_f}^2\right) \\ &- w^4\,\left(-2\,B_m\,K\,P_c\,m_f + 2\,I_c\,K\,M\,m_f\right) - I_c\,K^2\,K_m\end{aligned} \tag{66}$$

$$\xi_2 \geq \frac{M^2 b_f}{m_f^2} \tag{67}$$

$$\beta \geq 0 \tag{68}$$

$$K^2 \xi_2 + K_m^2 b_f \leq \beta(b_f^2 - 2K m_f) \tag{69}$$

Condition	Plant 1	Condition	Plant 2
$\text{Condition}^1_{i,3,1}$	$\xi = I_c M - B(P_c+1) \geq 0$	$\text{Condition}^1_{i,3,1}$	≤ 0
$\text{Condition}^1_{i,3,3}$	$\xi \geq \frac{M^2 b_f}{m_f^2}$ $\xi \geq \frac{B^2 b_f}{b_f^2 - 2Km_f}$	$\text{Condition}^1_{i,3,3}$	≤ 0
$\text{Condition}^1_{i,3,6}$	$\xi \geq 0$	$\text{Condition}^1_{i,3,6}$	≤ 0
$\text{Condition}^1_{i,5,1}$	$\xi \geq 0$	$\text{Condition}^1_{i,5,1}$	$\xi_2 = I_c M - B_m(P_c+1) \geq 0$ $\beta = A_c B_m - I_c K_m \geq 0$
$\text{Condition}^1_{i,5,3}$	$\xi \geq \frac{M^2 b_f}{m_f^2}$ $\xi \geq \frac{BA_c(2Km_f - b_f^2)}{K^2}$ $\xi \geq \frac{b_f^2 - 2Km_f}{B^2 b_f - BA_c m_f}$	$\text{Condition}^1_{i,5,3}$	$\xi_2 \geq \frac{M^2 b_f}{m_f^2}$ $\beta \geq 0$ $\beta \geq \frac{K^2 \xi_2 + K_m^2 b_f}{b_f^2 - 2Km_f}$ $b_f \geq \frac{\xi_2(2Km_f - b_f^2) - m_f^2 \beta}{(2K_m M - B_m^2)}$
$\text{Condition}^1_{i,5,6}$	$\xi \geq 0$ $\xi \geq \frac{(BA_c m_f)}{2K}$ $\xi \geq \frac{2A_c B m_f}{K}$	$\text{Condition}^1_{i,5,6}$	$\xi_2 \geq 0$ $\beta \geq 0$ $m_f \geq \frac{\xi_2}{\beta} \geq \frac{2m_f}{K}$

Table 2: Boundaries implied by first condition

and finally

$$\xi_2(2Km_f - b_f^2) - m_f^2\beta \leq b_f(2K_m M - B_m^2) \tag{70}$$

For C_5 and Z_{c6}

$$\begin{aligned}
{}_2\text{Condition}^1_{i,5,6} = w^4\ &\left(2\,B_m\,K\,m_f + A_c\,B_m\,{m_f}^2 - I_c\,K_m\,{m_f}^2 + 2\,B_m\,K\,P_c\,m_f - 2\,I_c\,K\,M\,m_f\right)\\
&- w^6\ \left(B_m\,{m_f}^2 + B_m\,P_c\,{m_f}^2 - I_c\,M\,{m_f}^2\right)\\
- w^2\ &\left(B_m\,K^2 + B_m\,K^2\,P_c - I_c\,K^2\,M + 2\,A_c\,B_m\,K\,m_f - 2\,I_c\,K\,K_m\,m_f\right)\\
&+ A_c\,B_m\,K^2 - I_c\,K^2\,K_m
\end{aligned} \tag{71}$$

This condition will be positive real for any parameters that will satisfy following conditions

$$\xi_2, \beta \geq 0 \tag{72}$$

$$m_f \geq \frac{\xi_2}{\beta} \geq \frac{2m_f}{K} \tag{73}$$

5.2 Second condition on defined systems

Passivity and unconditional stability requires

$$Re(h_{22}) \geq 0 \tag{74}$$

All of the systems have the hybrid matrix of

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} -\frac{Z_c}{P\,Z_c + C_f\,P\,Z_c + 1} & \frac{C_f\,P\,Z_c}{P\,Z_c + C_f\,P\,Z_c + 1} \\ 1 & Y_c \end{bmatrix} \tag{75}$$

Remark 9. *For any configuration basic impadence controlled haptic interface, with no virtual coupling* $Re(h_{22}) = 0$.

Remark 10. *For virtual coupled haptic interfaces, $Re(h_{22}) = Re(Y_c) \geq 0$, for any virtual coupler layout.*

Virtual Coupler	$Re(h_{22})$	G_{vc}
1	0	k_c
2	$b_c\,w^2$	$k_c + b_c\,s$
3	$b_c\,w^2$	$m_c\,s^2 + b_c\,s + k_c$
4	$b_c\,{m_c}^2\,w^4$	$\frac{k_c m_c s + k_c b_c + m_c b_c s^2}{m_c s + b_c}$
5	$b_c\,{k_c}^2 - 2\,b_c\,k_c\,m_c\,w^2 + b_c\,{m_c}^2\,w^4$	$\frac{b_c m_c s^3 + m_c k_c s^2 + k_c b_c s}{m_c s^2 + k_c}$
6	0	$k_c + m_c\,s^2$

5.2.1 Comments on inerter employment

According to results given in the previous section, inerters do not provide any improvement on passivity/unconditional stability if they are used in the system with lay-out of 6. Real part of admittance function of the fourth lay-out is $b_c m_c^2 w^4$. Real part of admittance function of the fifth lay-out is $b_c(m_c w^2 - k_c)^2$.

Remark 11. *Inerter in the fourth lay-out increases the virtual coupler's damping with square of it's coefficient. However the fifth-layout provides more relaxed condition. Series and/or parallel connections of the spring/damper/inerter should be selected as lay-out 4 instead of lay-out 5, when it will be employed as a virtual coupler.*

5.3 Third condition on defined systems

Passivity of a hybrid matrix can be achieved if

$$Re(h_{11})Re(h_{22}) - \left|\frac{h_{21}^* + h_{12}}{2}\right|^2 \geq 0 \tag{76}$$

Proposition 4. *Haptic interfaces with no virtual coupling cannot satisfy conditions of passivity and unconditional stability*

Proposition 5. *Using a spring as a virtual coupler lay-out cannot satisfy conditions of passivity for any set of variables. For virtual coupled haptic interfaces, $Re(h_{22}) = 0$, for a virtual coupler of lay-out 1, spring.*

$${}_1Condition^3_{i,j,1} = -\left|\frac{h_{21}^* + h_{12}}{2}\right|^2 \leq 0 \tag{77}$$

Proof. Square of the modulus of any complex number is positive real. □

Third condition of passivity of basic impedance controlled series elastic actuator, virtually coupled to virtual environment with a spring and controller $C_3 = P_c + \frac{I_c}{s}$, $Z_{c1} = \frac{K}{s}$ represented as below

$${}_1\mathrm{Condition}^3_{1,3,1} = -\lambda = -\left|\frac{K\,(I_c + P_c\,w\,1\mathrm{i})}{2\,w^2\,(B + M\,w\,1\mathrm{i})\left(\frac{K\,(I_c + P_c\,w\,1\mathrm{i})}{w^2\,(B + M\,w\,1\mathrm{i})} - 1 + \frac{K\,1\mathrm{i}}{w\,(B + M\,w\,1\mathrm{i})}\right)} + \frac{1}{2}\right| \leq 0 \tag{78}$$

where

$$\lambda = \frac{1}{2}\sqrt{\frac{B^2\,w^4 - 4\,B\,I_c\,K\,w^2 + 4\,{I_c}^2\,K^2 + 4\,K^2\,{P_c}^2\,w^2 + 4\,K^2\,P_c\,w^2 + K^2\,w^2 - 4\,K\,M\,P_c\,w^4 - 2\,K\,M\,w^4 + M^2\,w^6}{B^2\,w^4 - 2\,B\,I_c\,K\,w^2 + {I_c}^2\,K^2 + K^2\,{P_c}^2\,w^2 + 2\,K^2\,P_c\,w^2 + K^2\,w^2 - 2\,K\,M\,P_c\,w^4 - 2\,K\,M\,w^4 + M^2\,w^6}} \tag{79}$$

Additional damping to the virtual coupler dynamics will provide

$$ {}_1\text{Condition}^3_{2,3,1} = -\lambda_1 + \mu_1 \tag{80} $$

where

$$ \mu_1 = \frac{A}{B} \tag{81} $$

$$
\begin{aligned}
A =& K\, b_c\, w \left(\frac{-B\, K}{w\, \left(B^2 + M^2\, w^2\right)} - \frac{B\, K\, P_c}{w\, \left(B^2 + M^2\, w^2\right)} + \frac{I_c\, K\, M}{w\, \left(B^2 + M^2\, w^2\right)} \right) \\
B =& \left({b_c}^2\, w^2 + {k_c}^2\right) \left(\left(\frac{B\, K}{w\, \left(B^2 + M^2\, w^2\right)} + \frac{B\, K\, P_c}{w\, \left(B^2 + M^2\, w^2\right)} - \frac{I_c\, K\, M}{w\, \left(B^2 + M^2\, w^2\right)} \right)^2 \right. \\
&\left. + \left(\frac{K\, M}{B^2 + M^2\, w^2} + \frac{K\, M\, P_c}{B^2 + M^2\, w^2} + \frac{B\, I_c\, K}{w^2\, \left(B^2 + M^2\, w^2\right)} - 1 \right)^2 \right)
\end{aligned}
$$

Proposition 6. *For a given hybrid matrix H, the third passivity condition can be expressed as*

$$ {}_n\mathit{Condition}^3_{i,j,k} = ({}_n\mathit{Condition}^1_{i,j,k})({}_n\mathit{Condition}^2_{i,j,k}) +_n \mathit{Condition}^3_{1,j,1} \tag{82} $$

where i is corresponded virtual coupler, j is corresponded controller, k is the physical coupler, n is the plant employed in the analysis.

We design such systems that satisfy positive realness for first two condition. Application of virtual couplers has an improvement on third passivity condition

If we employ inerter in the virtual coupler, we will have

$$ {}_1\text{Condition}^3_{3,3,1} = -\lambda_1 + \mu_2 \tag{83} $$

$$ \mu_2 = \frac{A}{C} \tag{84} $$

$$
\begin{aligned}
C = \left(\left(k_c - m_c\, w^2\right)^2 + {b_c}^2\, w^2 \right) &\left(\left(\frac{B\, K}{w\, \left(B^2 + M^2\, w^2\right)} + \frac{B\, K\, P_c}{w\, \left(B^2 + M^2\, w^2\right)} - \frac{I_c\, K\, M}{w\, \left(B^2 + M^2\, w^2\right)} \right)^2 \right. \\
&\left. + \left(\frac{K\, M}{B^2 + M^2\, w^2} + \frac{K\, M\, P_c}{B^2 + M^2\, w^2} + \frac{B\, I_c\, K}{w^2\, \left(B^2 + M^2\, w^2\right)} - 1 \right)^2 \right)
\end{aligned}
$$

Remark 12. *If we compare second and third virtual coupler lay-outs, additional inertance to the virtual coupler does not improve the third passivity condition at high frequencies.. Additional inertance decreases additional term μ as ω increases. At low frequencies, additional inertance increases the system's passivity.*

5.4 Fourth condition on defined systems

For a hybrid matrix H,

$$ \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} -\frac{Z_c}{P\, Z_c + C_f\, P\, Z_c + 1} & \frac{C_f\, P\, Z_c}{P\, Z_c + C_f\, P\, Z_c + 1} \\ 1 & Y_c \end{bmatrix} \tag{85} $$

Unconditional stability of a hybrid matrix can be achieved if

$$ 2Re(h_{11})Re(h_{22}) - |h_{21}| - Re(h_{21}) \geq 0 \tag{86} $$

Proposition 7. *Last term $Re(h_{21})$ can be expressed as*

$$Re(h_{21}) = -Re(C_f(iw)P(iw)h_{11}(iw)) = -Re(C_f(iw)P(iw))Re(h_{11}(iw)) + Im(C_f(iw)P(iw))Im(h_{11}(iw)) \tag{87}$$

Proof. General rule for complex number multiplication implies

$$(x + yi)(u + vi) = (xu - yv) + (xv + yu)i \tag{88}$$

□

Proposition 8. *The term $|h_{21}|$ can be expressed as*

$$|h_{21}| = |C_f(iw)P(iw) - h_{11}(iw)| = |C_f(iw)P(iw)|\,|-h_{11}(iw)| \tag{89}$$

Proof. General rule for complex number multiplication implies

$$(x + yi)(u + vi) = (xu - yv) + (xv + yu)i \tag{90}$$

□

Third condition of unconditional stability of basic impedance controlled series elastic actuator, virtually coupled to virtual environment with a spring and controller $C_3 = P_c + \frac{I_c}{s}$, $Z_{c1} = \frac{K}{s}$ represented as below

$$ {}_1\mathrm{Condition}^4_{1,3,1} = -\lambda_2 = -\left|\frac{K\ (I_c + P_c\, w\, 1\mathrm{i})}{w^2\ (B + M\, w\, 1\mathrm{i})\ \left(\frac{K\,(I_c + P_c\, w\, 1\mathrm{i})}{w^2\,(B + M\, w\, 1\mathrm{i})} - 1 + \frac{K\, 1\mathrm{i}}{w\,(B + M\, w\, 1\mathrm{i})}\right)} + 1\right| \tag{91}$$

$$ - \mathrm{real}\left(\frac{K\ (I_c + P_c\, w\, 1\mathrm{i})}{w^2\ (B + M\, w\, 1\mathrm{i})\ \left(\frac{K\,(I_c + P_c\, w\, 1\mathrm{i})}{w^2\,(B + M\, w\, 1\mathrm{i})} - 1 + \frac{K\, 1\mathrm{i}}{w\,(B + M\, w\, 1\mathrm{i})}\right)}\right) \tag{92}$$

Additional damping to the virtual coupler dynamics will provide

$$ {}_1\mathrm{Condition}^4_{2,3,1} = -\lambda_2 + \mu_3 \tag{93}$$

where

$$\mu_3 = \frac{2A}{B} \tag{94}$$

$$\begin{aligned} A =& 2K\, b_c\, w\, \left(\frac{-B\,K}{w\ (B^2 + M^2\, w^2)} - \frac{B\,K\,P_c}{w\ (B^2 + M^2\, w^2)} + \frac{I_c\,K\,M}{w\ (B^2 + M^2\, w^2)}\right) \\ B =& \left({b_c}^2\, w^2 + {k_c}^2\right)\left(\left(\frac{B\,K}{w\ (B^2 + M^2\, w^2)} + \frac{B\,K\,P_c}{w\ (B^2 + M^2\, w^2)} - \frac{I_c\,K\,M}{w\ (B^2 + M^2\, w^2)}\right)^2\right. \\ &+ \left.\left(\frac{K\,M}{B^2 + M^2\, w^2} + \frac{K\,M\,P_c}{B^2 + M^2\, w^2} + \frac{B\,I_c\,K}{w^2\ (B^2 + M^2\, w^2)} - 1\right)^2\right) \end{aligned}$$

Remark 13. *Additional damping to the virtual coupler improves unconditional stability with twice of its improvement on unconditional stability.*

If we employ inerter, unconditional stability will be achieve if the condition below is satisfied.

$$ {}_1\mathrm{Condition}^4_{3,3,1} = -\lambda_2 + 2\mu_2 \tag{95}$$

Remark 14. *If we compare second and third virtual coupler lay-outs, additional inertance to the virtual coupler does not improve the third unconditional stability condition at high frequencies.. Additional inertance decreases additional term μ as ω increases. At low frequencies, additional inertance increases the system's unconditional stability.*

6 Case Study

Case study will be performed on the following physical device[11]:

	Symbol	Value	Units
Stage and motor inertia	M	176	[kg]
Stage damping	B	1051	[kg/s]
Physical spring stiffness	k	1300	[N/m]

For given physical characteristic, to satisfy first condition of passivity and unconditional stability, the controller gains should satisfy the conditions given in the Table 2. PI controller's gains will be selected as

$$\frac{I_c}{P_c+1} \geq \frac{B}{M} \tag{96}$$

if we substitute the plant characteristics

$$\frac{I_c}{P_c+1} \geq 5.9716 \tag{97}$$

Through the analysis, previously obtained conditions will be used in order to achieve passivity and unconditional stability

6.1 Effect of controller gains

The controller gains are limited by the physical characteristics of the series compliant actuator.

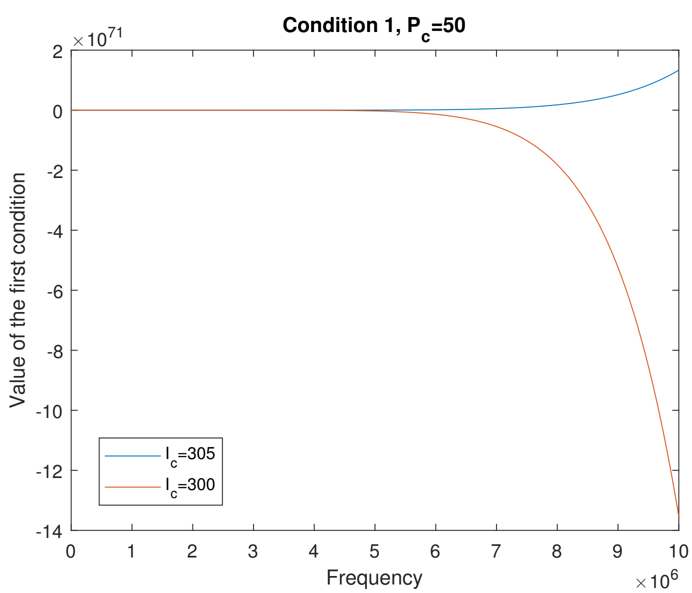


Figure 14: Comparison of value of the first condition for two different set of controller gains

In the Figure 14, it's shown that controller gains should be selected in properly in order to satisfy the first condition of passivity and unconditional stability. For $P_c = 50$, if we set $I_c = 300$

$$\frac{I_c}{P_c+1} = 5.8824 \leq 5.9716 \tag{98}$$

Stability of the system is violated and passivity and unconditional stability criterias are not satisfied. However, For $P_c = 50$, if we set $I_c = 305$

$$\frac{I_c}{P_c + 1} = 5.9804 \geq 5.9716 \tag{99}$$

and the first condition is satisfied $\forall \omega \geq 0$

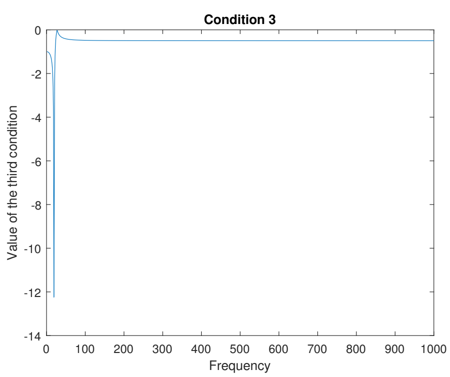
Remark 15. *The integral component sums the error of the system over time. This gain should be selected properly in order to drive the steady-state error to zero.*

6.2 Effect of virtual coupling

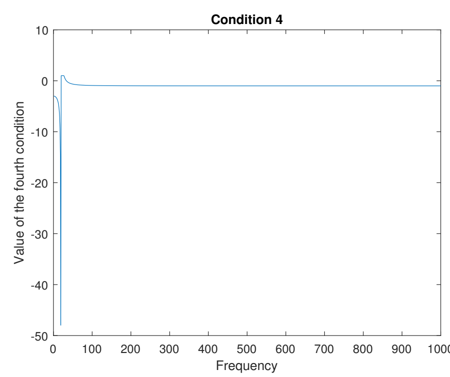
Virtual coupling is defined to the BIC in order to achieve passivity. Passivity and unconditional stability is achievable if and only if a virtual coupler is defined to the system. Virtual coupler dynamics is defined as following

	Symbol	Value	Units
Virtual coupler damping	b_c	1e-14	[kg/s]
Virtual coupler stiffness	k_c	1e-15	[N/m]

Remark 16. *At very high frequencies both passivity and unconditional stability is violated by virtual coupler dynamics as it is shown in the previous sections. Human limits are considered and the analysis is limited at certain frequency.*



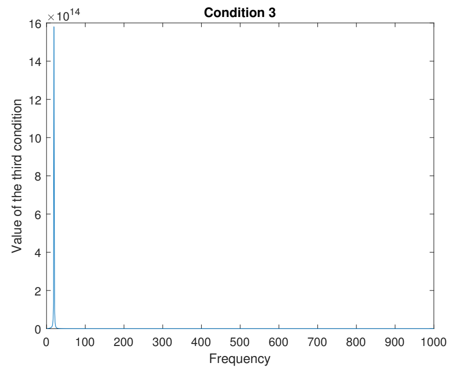
(a) Condition 3



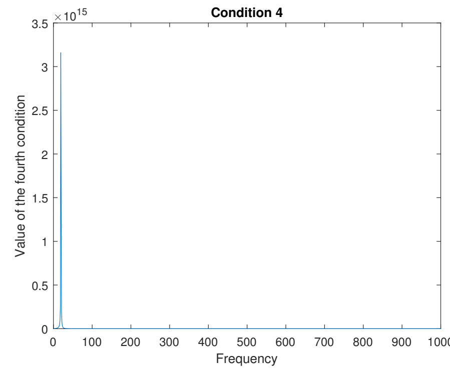
(b) Condition 4

Figure 15: Condition 3 and 4, when a spring is defined as virtual coupler

In the Figure 15, it is shown that, spring as virtual coupling lay-out does not provide an improvement on passivity and unconditional stability.



(a) Condition 3



(b) Condition 4

Figure 16: Condition 3 and 4, when spring-damper is defined as virtual coupler

In the Figure 16, it is shown that, spring-damper as virtual coupling lay-out can ensure passivity and unconditional stability $\forall \omega \leq 1000$Hz.

Remark 17. *Very low amount of virtual damping and stiffness is added to the system to achieve passivity and unconditional stability[5].*

7 Conclusion

Ensuring passivity of the control architecture is a necessary condition to ensure a stable interaction with any passive environment. In the pHRI systems, the human is assumed as a passive system[12]. In VSIC and BIC architecture, it is not possible to passively render any desired impedance(i.e a SEA with VSIC architecture cannot passively render a Voigt model[13]), which causes environment uncertainties[10]. This project was aiming to find trade-offs between physical coupler, virtual coupler and controllers.

All of the conditions for passivity and unconditional stability are derived for all of the systems defined. Without adding virtual coupler, BIC and VSIC cannot ensure passivity and unconditional stability. Benefits of addition of virtual coupler is proven numerically, with different lay-outs of virtual coupling. Recently introduced inerters are also employed in the compliance lay-outs to observe it's effect on stability criterias.

6 different model of series compliant actuator is analyzed with 5 different controllers. Some controllers cannot ensure passivity and unconditional stability. Series compliant actuators with PI and PI-second integral controllers are analyzed in terms of first two condition to derive constraints on controller gains and plant characteristics.Third and fourth conditions are quite complex to find constraints on virtual coupler dynamics. Fundamental effects of virtual coupling on those conditions are shown.

8 Future work

An experimental validation is necessary to prove derived conditions. HandsOn-SEA[14] will be employed in the experiments. New virtual coupler lay-outs will be analyzed and employed in order to obtain passivity and unconditional stability without violating system's performance. Physical characteristics of the HandsOn-SEA will be used in order to find virtual coupling dynamics that will satisfy the third and fourth conditions.

One-port modeling of Basic Impedance Controller with PI controller is not provided in the literature. That's why results could not be compared with previous one-port models.

9 Appendix

All of the codes used in this project are provided in the "MATLAB Codes" file.

References

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