Proceedings of 2015 IEEE
International Conference on Mechatronics and Automation
August 2 - 5, Beijing, China

Passivity Analysis and Design of a Haptic Interface

Yanwen Liu, Xiaoxue Liu, Zhigang Qi, Yuanhui Wang

College of Automation

Harbin Engineering University

Harbin 150001, China

zhwlyw@163.com

Abstract **- The haptic interface provides the virtual environment for the remote servo manipulator, and it can make the operator feel and manipulate the remote or virtual environment. For arbitrary passive operator (human), the stability not only relates to the parameters of the haptic device and the virtual environment, but also relates to the type of the sampling signal of the system. The passivity conditions for haptic interface systems with position sampling are given in existing works, but the parameters scope for passivity design is very small. In view of this, velocity of the haptic device is used for the sampling signal in this paper, and a wider passive region is derived based on the frequency response of a sampled-data control system. At last, through overall consideration of the bandwidth, damping ratio and passivity requirements, a reasonable design result is given in the paper.**

Index Terms - Haptic Interface, Virtual Environment, Passivity Condition, Sampled-data Control System.

I. INTRODUCTION

The haptic interface can make the operator feel and manipulate the remote or virtual environment. For the case of interacting with a virtual environment, the interface is directly connected with the virtual environment, and the whole system constitutes a feedback connection structure. The mechanical part (haptic device) of the haptic system is continuous, yet the virtual environment (realized by computer) is discrete, so the whole system is a typical sampled-data control system.

Because the operator manipulates the virtual environment through the interface, and the interface simultaneously feed back to the operator a virtual environment signal (force), so the operator and the interface form a relationship of closed-loop system, the main performance of the closed-loop system including the operator is stability. Because the operator has objective initiative, improper operation may cause oscillation, so the passivity design method should be used for the stability analysis of this kind of system.

For the haptic interface system, the output signal of the haptic device can be position signal, and it can also be velocity signal. This continuous output signal is connected to the virtual environment (computer) through a sampler, and different sampling signal type has different effect on the stability of the system. Literature [1~3] consider the haptic interface system with position sampling. The passivity condition is given in these literatures, but its result is conservative [4], and there are some problems in the derivation process of this passivity condition. literature [4] gives a non-conservative passivity condition based on the frequency response of the sampled-data control system, but the parameters scope for passivity design is very small. Passivity conditions are derived for the haptic interface systems in literature [5] by using the position and velocity signal respectively as the sampling signal, but because the passivity conditions in this literature are given from a purely discrete system angle, the corresponding results are conservative and are not advisable. In order to obtain a wider passive region, the velocity is used as the sampling signal in this paper, and a not conservative passivity condition is derived in the paper based on the frequency response of a sampled-data control system. Finally, by combining the passivity condition with the bandwidth and damping requirements, a reasonable design result is given in the paper.

II. PASSIVITY CONDITION OF THE HAPTIC INTERFACE: POSITION SAMPLING

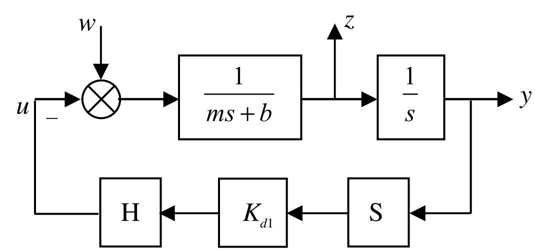

Fig.1: Model of a haptic system: position sampling

Fig.1 shows the model of a single degree of freedom haptic interface system with position sampling, where m denotes the mass of the actuator and handle, b is the viscous friction coefficient; K_{d1} is the discrete controller, which is used to simulate the virtual environment; S denotes the sampler, and H is the zero order hold (ZOH), T is defined as the sampling period of the system.

For a haptic interface system, the input power is the product of the applied force $f(t)$ (w in Fig.1) and the velocity $v(t)$(z in Fig.1). If this haptic interface system is passive, then the integral of the power will be greater than or equal to zero, that is

$$\int_0^t f(\tau)v(\tau)d\tau \geq 0 \qquad \forall f(t), t \geq 0 \qquad (1)$$

From the system point of view, if the input is $f(t)$ and output is $v(t)$, then the system satisfying inequality (1) is called passive system. So $v(t)$ is chosen for the output of the haptic system.

Literatures [1,2] consider the haptic interface system of Fig.1, the passivity condition is given in these literatures, as shown in inequality (2):

$$b \geq \frac{KT}{2} + B \tag{2}$$

By defining the variables $\beta = b/B$, $\alpha = B/(KT)$, inequality (2) can be also described as

$$\beta \geq \frac{1}{2\alpha} + 1 \tag{3}$$

The passivity condition in inequality (2) and (3) has been almost universally accepted since its given time[6,7], but in fact this result is conservative, so it is not advisable[4]. The reason is there are some problems in the derivation process of literatures [1,2]. In the discussion of the passive condition in literatures [1,2], the sampled-data control system is broken in the discrete signal point, and then the relationship between discrete signals is used for the discussion of the passive condition. In addition, its result is derived based on a not-existent inverse theorem of passivity theorem. In view of this, the lifting technique which can take into account the continuity between sampling signals is used in literature [3], but because there are some limitations and conditions when using the lifting technique, the lifting technique is essentially a conservative analysis and design approach[8,9], so the passivity condition obtained by using the lifting technique is also conservative.

In order to derive an accurate passivity condition, literature [4] also considers the haptic system of Fig.1. Based on the real frequency response of the sampled-data control system, a not conservative passivity condition for the haptic system is derived in literature [4], as shown in inequality (4):

$$b \geq \frac{KT + B}{2} \tag{4}$$

Inequality (4) can be further described as

$$\beta \geq \frac{1}{2\alpha} + \frac{1}{2} \tag{5}$$

Although the passivity condition given by inequality (4) and (5) is not conservative, the parameters scope for passivity design is small (see Fig.4 in the next section). In fact, for the haptic interface system, the velocity of the haptic device can be also used for the sampling signal. Literature [5] considers the case of velocity sampling for a haptic system, yet the passivity condition given by this literature is questionable and undesirable. In view of this, a wider range and accurate passivity condition for the haptic system with velocity sampling will be derived in the next section based on the frequency response of a sampled-data control system.

III. Passivity Condition of The Haptic Interface: Velocity Sampling

Fig.2 shows the model of a single degree of freedom haptic interface system with velocity sampling, where K_d denotes the virtual environment (discrete controller). Because the input of the controller is velocity, the z-transfer function of K_d is different with K_{d1} in Fig.1. Other variables in Fig.2 are the same as in Fig.1. In the following calculations, we assume m=0.5 Kg, b=0.1 N·s/m.

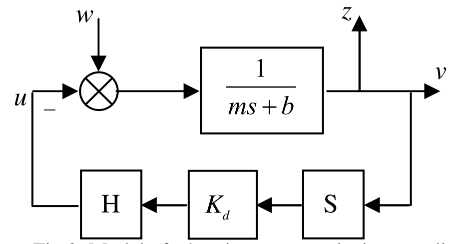

Fig.2: Model of a haptic system: velocity sampling

In this paper, the virtual wall is used as the virtual environment. Because the input of the virtual environment is velocity, when using the typical spring-damping structure to describe the wall, its continuous transfer function can be described as the following PI structure, i.e.,

$$K(s) = \frac{K}{s} + B \tag{6}$$

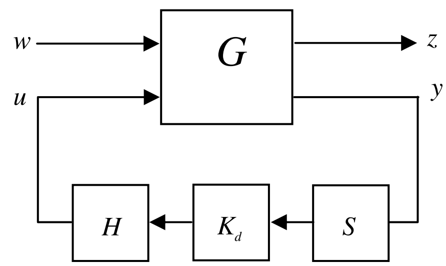

Fig.3 The closed-loop sampled-data control system

Fig.2 can be converted into the structure of a standard sampled-data control system as shown in Fig.3, where G is the continuous generalized plant, it satisfies

$$\begin{bmatrix} z \\ y \end{bmatrix} = G \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

where

$$\begin{aligned} G_{11}(s) &= -G_{12}(s) = \frac{1}{ms+b} \\ G_{21}(s) &= -G_{22}(s) = \frac{1}{ms+b} \end{aligned} \tag{7}$$

The frequency response of the sampled-data control system in Fig.3 can be computed by using the following linear fractional transformation[10]:

$$T_{zw} = G_{11} + G_{12}K_d(I - G_{22d}K_d)^{-1}G_{21d} \tag{8}$$

The subscript d in equation (8) denotes the corresponding discretized transfer function. Because the system we now dealing with is a SISO (single input single output) system, the inverse in equation (8) can be expressed as the division of the numerator and denominator, i.e.

$$T_{zw} = G_{11}\left(1 - \frac{K_d G_{21d}}{1 - G_{22d}K_d}\right) \tag{9}$$

According to equation (7), $G_{22d} = -G_{21d}$, so the above equation (9) can be further expressed as

$$T_{zw} = \frac{G_{11}}{1 - G_{22d} K_d} \tag{10}$$

Where G_{22d} in equation (10) is the ZOH discretization of G_{22}.

For the linear haptic system in Fig.2 and Fig.3, passivity is equivalent to positive real, so the passivity condition is in fact the positive real condition of T_{zw}, i.e., the passivity condition of the haptic system is:

$$T_{zw} + T_{zw}^{*} \geq 0, \quad \forall \omega \in [0, \pi / T]$$

This is equivalent to

$$\mathrm{Re}\left(T_{zw}\right) \geq 0, \forall \theta \in [0, \pi], \theta = \omega T \tag{11}$$

Where $\mathrm{Re}\left(T_{zw}\right)$ denotes the real part of T_{zw}.

We now derive the passivity condition of the haptic system according to $\mathrm{Re}\left(T_{zw}\right) \geq 0$.

The ZOH discretization G_{22d} of G_{22} can be described as

$$G_{22d} = \frac{k_1}{z - k_2} \tag{12}$$

Where $k_1 = \frac{1}{b}\left(1 - e^{-(b/m)T}\right), k_2 = e^{-(b/m)T}$.

Substitute $z = e^{j\omega T} = \cos\theta + j\sin\theta$ into formula (12), after simplification, we have

$$G_{22d}(\theta) = -C_{1\theta}(a_1 + b_1 j) \tag{13}$$

Where

$$\begin{cases} C_{1\theta} = \dfrac{k_1}{(\cos\theta - k_2)^2 + (\sin\theta)^2} \\ a_1 = \cos\theta - k_2 \\ b_1 = -\sin\theta \end{cases} \tag{14}$$

Similarly, substitute $s = j\omega = j\theta / T$ in to G_{11}, we can get

$$G_{11}(\theta) = C_{2\theta}(a_2 + b_2 j) \tag{15}$$

Where

$$\begin{cases} C_{2\theta} = \dfrac{1}{m^2\theta^2 / T^2 + b^2} \\ a_2 = b \\ b_2 = -m\theta / T \end{cases} \tag{16}$$

Because the virtual environment is implemented by computer, the continuous transfer function of the virtual wall in equation (6) need to be discretized. we now use the backward difference method to obtain the discrete K_d.

Substitute $s = \frac{1 - z^{-1}}{T}$ into equation (6), we have

$$\begin{aligned} K_d(z) &= \frac{K}{s} + B \bigg| s = \frac{1 - z^{-1}}{T} \\ &= \frac{KTz}{z - 1} + B \end{aligned} \tag{17}$$

Similarly, substitute $z = \cos\theta + j\sin\theta$ into $K_d(z)$, we can get

$$K_d(\theta) = a_3 + b_3 j \tag{18}$$

Where

$$\begin{cases} a_3 = \left(\dfrac{KT}{2} + B\right) \\ b_3 = \dfrac{KT\sin\theta}{2(1 - \cos\theta)} \end{cases} \tag{19}$$

Finally, by substituting equation (13)、(15) and (18) into T_{zw} of (10), we have

$$T_{zw}(\theta) = \frac{C_{2\theta}(a_2 + b_2 j)}{1 + C_{1\theta}(a_1 + b_1 j)(a_3 + b_3 j)}$$

After further simplification, we can get

$$\begin{aligned} \mathrm{Re}(T_{zw}) = C_\theta[&a_2(1 + C_{1\theta}a_1a_3 - C_{1\theta}b_1b_3) + \\ &b_2C_{1\theta}(a_1b_3 + b_1a_3)] \end{aligned}$$

Where

$$C_\theta = \frac{C_{2\theta}}{(1 + C_{1\theta}a_1a_3 - C_{1\theta}b_1b_3)^2 + [C_{1\theta}(a_1b_3 + b_1a_3)]^2}$$

Obviously, the coefficient $C_\theta > 0$ is hold for $\forall \theta \in [0, \pi]$, so the condition of $\mathrm{Re}\left(T_{zw}\right) \geq 0$ becomes to

$$a_2(1 + C_{1\theta}a_1a_3 - C_{1\theta}b_1b_3) + b_2C_{1\theta}(a_1b_3 + b_1a_3) \geq 0 \tag{20}$$

By substituting the specific parameters into equation (20), we can get the following inequality:

$$b + k_1(\theta)\frac{KT}{2} + k_2(\theta)B \geq 0, \forall \theta \in [0, \pi] \tag{21}$$

Where $k_1(\theta)$ and $k_2(\theta)$ are both expressions about variable θ.

The coefficient $k_1(\theta)$ of $KT/2$ in equation (21) is:

$$\begin{aligned} k_1(\theta) &= C_{1\theta}\left[-b\left(1 + e^{-(b/m)T}\right) + \frac{m\theta\left(1 - e^{-(b/m)T}\right)\sin\theta}{T(1 - \cos\theta)}\right] \\ &= \frac{\frac{1}{b}\left(1 - e^{-(b/m)T}\right)}{(\cos\theta - e^{-(b/m)T})^2 + (\sin\theta)^2} \times \\ &\quad \left[-b\left(1 + e^{-(b/m)T}\right) + \frac{m\theta\left(1 - e^{-(b/m)T}\right)\sin\theta}{T(1 - \cos\theta)}\right] \end{aligned}$$

The coefficient $k_2(\theta)$ of *B* in equation (21) is:

$$k_2(\theta) = C_{1\theta}\left[b\left(\cos\theta - e^{-(b/m)T}\right) + \frac{m\theta}{T}\sin\theta\right]$$

It can be verified that $k_1(\theta)$ is monotone decreasing, and $k_1(\theta) < 0$ can be satisfied for $\theta \in [0,\pi]$. Similarly, $k_2(\theta)$ is also monotone decreasing for $\theta \in [0,\pi]$. Thus, inequality (21) can be rewritten as

$$b \ge -k_1(\theta)\frac{KT}{2} - k_2(\theta)B, \forall\theta \in [0,\pi] \tag{22}$$

In view of the monotonicity of the functions, when θ is changing in $[0,\pi]$, the right side of the inequality (22) achieves the maximum value at $\theta = \pi$. In order to ensure the inequality (22) be satisfied for $\forall\theta \in [0,\pi]$, the following condition should be satisfied:

$$b \ge -k_1(\theta)\frac{KT}{2} - k_2(\theta)B\bigg|_{\theta=\pi}$$

After substituting $\theta = \pi$ into $k_1(\theta)$ and $k_2(\theta)$, we can get

$$b \ge \frac{1-e^{-(b/m)T}}{1+e^{-(b/m)T}}\left(\frac{KT}{2} + B\right) \tag{23}$$

Inequality (22) can be also described as

$$\beta \ge \frac{1-e^{-(b/m)T}}{1+e^{-(b/m)T}}\left(\frac{1}{2\alpha} + 1\right) \tag{24}$$

Fig.4 shows the $\alpha - \beta$ passive boundary curve for different cases. The passivity condition of inequality (24) is related to the sampling period *T*, the smaller the sampling period is, the wider is the passive region, and correspondingly, the bigger is the parameters selection range of the controller. The three solid curves in Fig.4 are the passive boundary with different sampling period for velocity sampling haptic system [see inequality (24)], the dashed curve in Fig.4 is the passive boundary [see inequality (3)] given by literatures [1~3], the dotted curve in Fig.4 is the passive boundary [see inequality (5)] given by literature [4], and the region above each curve is its own passive region. By comparison, it can be seen that the passive region for the haptic system with position sampling is small, parameters of the controller is limited to a small range, moreover, the passive condition of inequality (3) is too conservative. The passive region for the haptic system with velocity sampling is wide, this passive region is in fact the reasonable parameters design region, and the parameters in this region can meet the robust stability, bandwidth and damping requirements of the system at the same time.

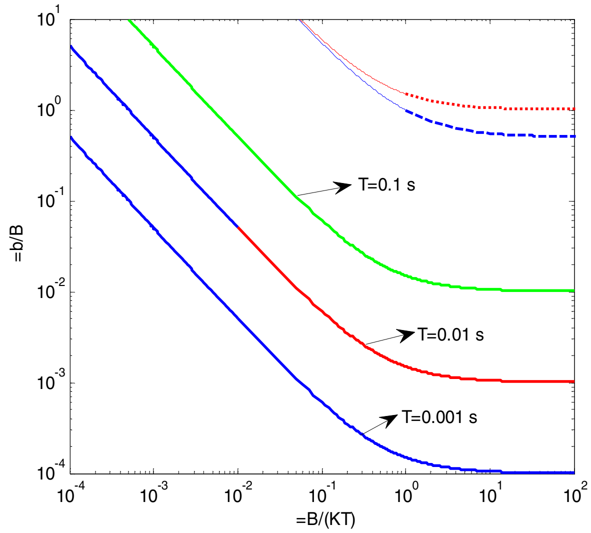

Fig.4: The passive regions for different cases

IV. System Design

The above passivity requirement is just one of the properties of the haptic interface system. The haptic system is also a negative feedback system, as a feedback system, some basic characteristics such as bandwidth and damping should be considered. For a system with a pair of complex dominant poles, even if it is passive, the overshoot may appear in its time domain response, and thus vibrate can be felt in actual operations. And if the bandwidth is too wide, the oscillation mode of the unmodeled dynamics may be excited in the actual system, thus the true feeling of operating the virtual environment may be affected. In view of this, some limitations should be added to the passive regions in Fig.4 from the angle of negative feedback.

In order to make the results more simple and clear, the discussion here will use some concepts of continuous system. For example, for the continuous system, the *B*/*K* in controller (6) is just the time constant $T_d = B/K$ of the differential correction element, so the variable α can be rewritten as:

$$\alpha > \frac{B}{KT} = \frac{T_d}{T} = \frac{1}{\pi}\frac{\omega_N}{\omega_d}$$

Where $\omega_d = 1/T_d$, $\omega_N = \pi/T$. Generally, the cross-over frequency of the sampled-data control system should satisfy the condition of $\omega_c < 0.1\omega_N$, and the corner frequency of the differential correction element is $\omega_d \approx 0.5\omega_c$, thus we can get

$$\alpha > 6 \tag{25}$$

So from the point of view of servo design, the region of $\alpha > 6$ is the passive region for controller parameters design. In this design, the sampling period is chosen as *T*=0.001 s. Combining the above design requirement with the passive condition, the final controller parameters is designed as *B*=100 N.s/m, *K*=200 N/m.

Fig.5 shows the closed-loop frequency characteristic T_{zw} of the haptic system, as can be seen that the frequency characteristics are located in the right half plane of *S* plane, i.e., the passivity condition $\mathrm{Re}(T_{zw}) \geq 0$ is satisfied, so the system is passive.

It can be computed that the closed-loop poles of the system are z=1, z=0.998, z=0.8014, which are all real poles. Fig.6 shows the step response of the feedback force, as can be seen from Fig.6, the system can achieve steady state in a very short time without overshoot. As can be seen from the open-loop Bode plot of the system (figure omitted), the bandwidth of the system also meet the design requirements.

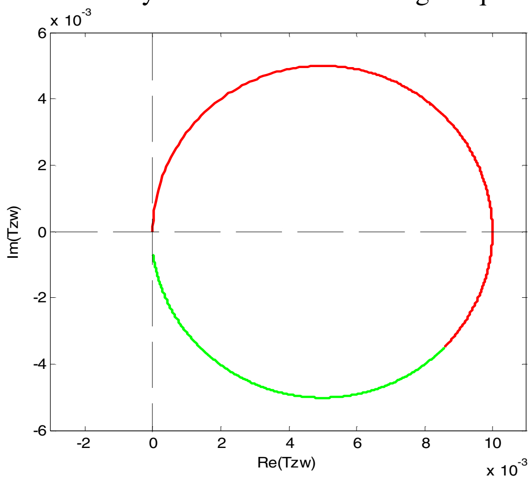

Fig.5: Closed-loop frequency characteristic T_{zw}

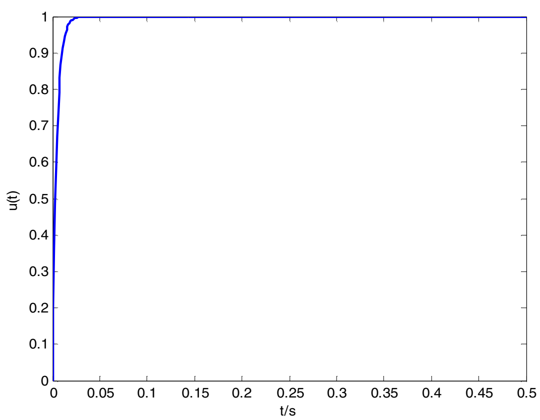

Fig.6: Step response of the feedback force

V. Conclusions

Passivity is the design basis of the haptic interface system, and as the input of the virtual environment, different sampling signal (position and velocity) will lead to different passivity condition. The passivity condition given in previous literatures are mostly based on the position sampling haptic system, and most of the results are conservative, even if the result is not conservative(see the result in literature [5]), the passive design region of the controller parameters is small. The velocity of the haptic device is used for the sampling signal in this paper, and a not conservative and wider range passivity condition is derived.

Passivity is just one of the properties of the interface. In the real design, enough bandwidth and damping are needed to ensure no vibrate appear in the process of operation, thus to achieve a virtual environment with the sense of reality.

Acknowledgment

This work is supported by National Natural Science Foundation under Grant 61304076 and Fundamental Research Funds for the Central Universities under Grant HEUCFX041501.

References

[1] J. E. Colgate, G. Schenkel, "Passivity of a class of sampled-data systems: Application to haptic interfaces," *Journal of Robotic Systems*, vol. 14, no. 1, pp. 37-47, 1997.

[2] J. E. Colgate, G. Schenkel, "Passivity of a class of sampled-data systems: Application to haptic interfaces," *Proceedings of The American Control Conference*, Baltimore, Maryland, 1994, pp.3236-3240.

[3] M. Fardad, B. Bamieh, "A Necessary and sufficient frequency domain criterion for the passivity of SISO sampled-data systems," *IEEE Transactions on Automatic Control*, vol. 54, no. 3, pp. 611-614, 2009.

[4] Y.W. Liu, J. Li, etc, "Passivity design of a haptic interface for the remote servo manipulator," *Control Theory & Applications*, vol. 28, no. 7, pp. 994-998, 2011.

[5] A. Haddadi, K. Hashtrudi-Zaad, "Stability Analysis of Haptic Interfaces for Different Types of Sampled Signals and Virtual Environment Implementations," *IEEE Haptics Symposium 2010*, Waltham, Massachusetts, USA, 2010.

[6] J.P. Kim, J. Ryu, "Energy bounding algorithm based-on passivity theorem for stable haptic interaction control," *Proceedings of the 12th International Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems*, Chicago, pp.351-357, 2004.

[7] J.J. Gil, A. Avello, etc, "Stability analysis of a 1 DOF haptic interface using the Routh–Hurwitz criterion," *IEEE Transactions on Control Systems Technology*, vol. 12, no. 4, pp. 583-588, 2004.

[8] G.X. Wang, Y.W. Liu, Z. He, "Ten years of the lifting technique for sampled-data control systems," *Control and Descision*, vol. 10, no. 1, pp. 44-48, 2006.

[9] G.X. Wang, Y.W. Liu, Z. He, "H∞ Design for sampled-data systems via lifting technique: conditions and limitation," *Acta Automatica Sinica*, vol. 10, no. 1, pp.1-5, 2006.

[10] Y.W. Liu, G.X. Wang, Z. He, "Frequency response and the L2-induced norm of sampled-data systems," *Control and Descision*, vol. 20, no. 10, pp. 1133 - 1136, 2005.