```
clc
clear
syms P D kd dd Jm bm kf w T bf t
assume(Jm, 'real')
assume(bm, 'real')
assume(kd, 'real')
assume(dd, 'real')
assume(P, 'real')
assume(D, 'real')
assume(bf, 'real')
assume(kf, 'real')
assume(w, 'real')
assume(t, 'real')
Jm=str2sym('Jm');
imp=poly2sym(str2sym('[dd kd]'));
cf=poly2sym(str2sym('[P+D*t P*t]'));
Tact=poly2sym(str2sym('[bf kf]'));
robot=(poly2sym(str2sym('[Jm bm]')));
tau1=poly2sym(str2sym('[1 2*t t^2]'));
tau2=poly2sym(str2sym('[1 t]'));
Tsens=poly2sym(str2sym('[kf]'));

x=j*w
```

$$x = w\,\mathrm{i}$$

```
envnum=Tact*(imp*(cf+Tsens*tau2*(cf+tau2)))
```

$$\mathrm{envnum} = (\mathrm{kf}+\mathrm{bf}\,x)\,(\mathrm{kd}+\mathrm{dd}\,x)\,(P\,t+x\,(P+\mathrm{D}\,t)+\mathrm{kf}\,(t+x)\,(t+x+P\,t+x\,(P+\mathrm{D}\,t)))$$

```
envden=tau2*robot*poly2sym(str2sym('[1 0 0]'))
```

$$\mathrm{envden} = x^2\,(t+x)\,(\mathrm{bm}+\mathrm{Jm}\,x)$$

```
envnum1=expand(eval(envden));
envden1=expand(eval(envnum));
envnum2=expand(envnum1*conj(envden1));
envden2=expand(conj(envden1)*envden1);

pretty(real(envnum2))
```

```
        2  6           2  6            2  6            2  6           2    6           2    4           2  3  4
bm dd kf  w  - Jm kd kf  w  - Jm P kd kf  w  + P bm dd kf  w  - Jm dd kf  t w  - bm kd kf  t w  - Jm dd kf  t  w

             2  2  4           2  2  4              6              6           2  3  2              6
   - Jm kd kf  t  w  + bm dd kf  t  w  - Jm P bf kd w  - Jm P dd kf w  - bm kd kf  t  w  + P bf bm dd w

                8              4               6                  8                6                6
   + Jm bf dd kf w  - P bm kd kf w  + bf bm kd kf w  + Jm P bf dd kf w  - D Jm bf kd t w  - D Jm dd kf t w

                 6                 6                4                  6                  6              2  6
   + D bf bm dd t w  + P bf bm kd kf w  - D bm kd kf t w  - Jm bf kd kf t w  + bf bm dd kf t w  + D Jm bf dd t  w
```

```
              2  4            2    6              2  4              2  4            2    6              2
- D Jm kd kf t  w  - D Jm kd kf  t w  - Jm P bf kd t  w  - Jm P dd kf t  w  - Jm P dd kf  t w  + D bf bm kd t  w

              2  4            2    6              2  4              2  6              3  4              2
+ D bm dd kf t  w  + D bm dd kf  t w  + P bf bm dd t  w  + Jm bf dd kf t  w  - Jm bf kd kf t  w  - P bm kd kf t

            2    4               3  4              2  4            2  3  4            2  3  4
- P bm kd kf  t w  + bf bm dd kf t  w  + bf bm kd kf t  w  - D Jm kd kf  t  w  - Jm P dd kf  t  w

            2  2  4            2  3  4            2  2  4            2  3  2                  8
- Jm P kd kf  t  w  + D bm dd kf  t  w  + P bm dd kf  t  w  - P bm kd kf  t  w  + D Jm bf dd kf t w

                  6                  6                  6                3  6                2  6
- Jm P bf kd kf t w  + D bf bm kd kf t w  + P bf bm dd kf t w  + D Jm bf dd kf t  w  + Jm P bf dd kf t  w

                3  4                3  4                3  4                2  4
- Jm P bf kd kf t  w  + D bf bm kd kf t  w  + P bf bm dd kf t  w  + P bf bm kd kf t  w
```

```
a=coeffs(real(eval(envnum2)),w);
for i=1:length(a)
    b(i,1:length(coeffs(a(i),t)))=coeffs(a(i),t);
end
```

Case 0

```
bm=0
```

```
bm = 0
```

```
pretty(real(envnum2))
```

```
       2  6          2  6            2  6            2  6          2    6          2    4          2  3  4
bm dd kf  w  - Jm kd kf  w  - Jm P kd kf  w  + P bm dd kf  w  - Jm dd kf  t w  - bm kd kf  t w  - Jm dd kf  t  w

           2  2  4          2  2  4              6              6          2  3  2              6
- Jm kd kf  t  w  + bm dd kf  t  w  - Jm P bf kd w  - Jm P dd kf w  - bm kd kf  t  w  + P bf bm dd w

               8              4               6                 8               6               6
+ Jm bf dd kf w  - P bm kd kf w  + bf bm kd kf w  + Jm P bf dd kf w  - D Jm bf kd t w  - D Jm dd kf t w

                6                 6                4                 6                 6              2  6
+ D bf bm dd t w  + P bf bm kd kf w  - D bm kd kf t w  - Jm bf kd kf t w  + bf bm dd kf t w  + D Jm bf dd t  w

              2  4            2    6              2  4              2  4            2    6              2
- D Jm kd kf t  w  - D Jm kd kf  t w  - Jm P bf kd t  w  - Jm P dd kf t  w  - Jm P dd kf  t w  + D bf bm kd t  w

              2  4            2    6              2  4              2  6              3  4              2
+ D bm dd kf t  w  + D bm dd kf  t w  + P bf bm dd t  w  + Jm bf dd kf t  w  - Jm bf kd kf t  w  - P bm kd kf t

            2    4               3  4              2  4            2  3  4            2  3  4
- P bm kd kf  t w  + bf bm dd kf t  w  + bf bm kd kf t  w  - D Jm kd kf  t  w  - Jm P dd kf  t  w

            2  2  4            2  3  4            2  2  4            2  3  2                  8
- Jm P kd kf  t  w  + D bm dd kf  t  w  + P bm dd kf  t  w  - P bm kd kf  t  w  + D Jm bf dd kf t w

                  6                  6                  6                3  6                2  6
```

```
   - Jm P bf kd kf t w  + D bf bm kd kf t w  + P bf bm dd kf t w  + D Jm bf dd kf t  w  + Jm P bf dd kf t  w

                  3  4                 3  4                 3  4                 2  4
   - Jm P bf kd kf t  w  + D bf bm kd kf t  w  + P bf bm dd kf t  w  + P bf bm kd kf t  w
```

```
a0=coeffs(real(eval(envnum2)),w);
for i=1:length(a0)
    b0(i,1:length(coeffs(a0(i),t)))=coeffs(a0(i),t);
end
clear bm
syms bm
assume(bm, 'real')
```

Case 1 dd=0

```
dd=0;

pretty(real(eval(envnum2)))
```

```
            6             2  6          2    4          2  2  4              6          2  3  2
bf bm kd kf w  - Jm P kd kf  w  - bm kd kf  t w  - Jm kd kf  t  w  - Jm P bf kd w  - bm kd kf  t  w  - P bm kd kf w

            2  6                6                  6                 4                  6              2  4
   - Jm kd kf  w  - D Jm bf kd t w  + P bf bm kd kf w  - D bm kd kf t w  - Jm bf kd kf t w  - D Jm kd kf t  w

              2    6              2  4              2  4              3  4              2  2            2
   - D Jm kd kf  t w  - Jm P bf kd t  w  + D bf bm kd t  w  - Jm bf kd kf t  w  - P bm kd kf t  w  - P bm kd kf  t

               2  4            2  3  4            2  2  4            2  3  2                  6
   + bf bm kd kf t  w  - D Jm kd kf  t  w  - Jm P kd kf  t  w  - P bm kd kf  t  w  - Jm P bf kd kf t w

                  6                3  4                3  4                2  4
   + D bf bm kd kf t w  - Jm P bf kd kf t  w  + D bf bm kd kf t  w  + P bf bm kd kf t  w
```

```
a1=coeffs(real(eval(envnum2)),w);
for i=1:length(a1)
    b1(i,1:length(coeffs(a1(i),t)))=coeffs(a1(i),t);
end
```

Case 1.1 bm=0 dd=0

```
dd=0;


bm=0;
pretty(real(eval(envnum2)))
```

```
          2  6            2  6          2  2  4              6                6                6
 - Jm kd kf  w  - Jm P kd kf  w  - Jm kd kf  t  w  - Jm P bf kd w  - D Jm bf kd t w  - Jm bf kd kf t w

               2  4            2    6              2  4              3  4            2  3  4            2
   - D Jm kd kf t  w  - D Jm kd kf  t w  - Jm P bf kd t  w  - Jm bf kd kf t  w  - D Jm kd kf  t  w  - Jm P kd kf  t

                  6                3  4
```

```
   - Jm P bf kd kf t w  - Jm P bf kd kf t  w
```

```
a11=coeffs(real(eval(envnum2)),w);
for i=1:length(a11)
    b11(i,1:length(coeffs(a11(i),t)))=coeffs(a11(i),t);
end
clear dd bm
syms dd bm
assume(bm, 'real')
assume(dd, 'real')
```

Case 2 D=0

```
D=0;
pretty(real(eval(envnum2)))
```

```
        2  6           2  6            2  6            2  6          2    6          2    4          2  3  4
bm dd kf  w  - Jm kd kf  w  - Jm P kd kf  w  + P bm dd kf  w  - Jm dd kf  t w  - bm kd kf  t w  - Jm dd kf  t  w

             2  2  4          2  2  4              6              6          2  3  2              6
   - Jm kd kf  t  w  + bm dd kf  t  w  - Jm P bf kd w  - Jm P dd kf w  - bm kd kf  t  w  + P bf bm dd w

                8              4              6                  8                 6                  6
   + Jm bf dd kf w  - P bm kd kf w  + bf bm kd kf w  + Jm P bf dd kf w  + P bf bm kd kf w  - Jm bf kd kf t w

                  6              2  4              2  4            2    6              2  4               2
   + bf bm dd kf t w  - Jm P bf kd t  w  - Jm P dd kf t  w  - Jm P dd kf  t w  + P bf bm dd t  w  + Jm bf dd kf t

                3  4              2  2            2    4               3  4               2  4            2
   - Jm bf kd kf t  w  - P bm kd kf t  w  - P bm kd kf  t w  + bf bm dd kf t  w  + bf bm kd kf t  w  - Jm P dd kf

             2  2  4            2  2  4            2  3  2                    6                  6
   - Jm P kd kf  t  w  + P bm dd kf  t  w  - P bm kd kf  t  w  - Jm P bf kd kf t w  + P bf bm dd kf t w

                  2  6                3  4                3  4                2  4
   + Jm P bf dd kf t  w  - Jm P bf kd kf t  w  + P bf bm dd kf t  w  + P bf bm kd kf t  w
```

```
a2=coeffs(real(eval(envnum2)),w);
for i=1:length(a2)
    b2(i,1:length(coeffs(a2(i),t)))=coeffs(a2(i),t);
end
```

Case 2.1 D=0 bm=0

```
D=0;
bm=0;
pretty(real(eval(envnum2)))
```

```
             8            2  6          2    6          2  3  4          2  2  4              6
Jm bf dd kf w  - Jm P kd kf  w  - Jm dd kf  t w  - Jm dd kf  t  w  - Jm kd kf  t  w  - Jm P bf kd w  - Jm P dd kf w

             2  6                8                6              2  4              2  4            2    6
```

```
   - Jm kd kf  w  + Jm P bf dd kf w  - Jm bf kd kf t w  - Jm P bf kd t  w  - Jm P dd kf t  w  - Jm P dd kf  t w

                2  6                3  4             2  3  4             2  2  4                    6
   + Jm bf dd kf t  w  - Jm bf kd kf t  w  - Jm P dd kf  t  w  - Jm P kd kf  t  w  - Jm P bf kd kf t w

                 2  6                  3  4
   + Jm P bf dd kf t  w  - Jm P bf kd kf t  w
```

```
a21=coeffs(real(eval(envnum2)),w);
for i=1:length(a21)
    b21(i,1:length(coeffs(a21(i),t)))=coeffs(a21(i),t);
end
clear bm
syms bm
assume(bm, 'real')
```

Case 3 D=0 dd=0

```
D=0;
dd=0;

pretty(real(eval(envnum2)))
```

```
            6             2  6           2    4           2  2  4               6           2  3  2
bf bm kd kf w  - Jm P kd kf  w  - bm kd kf  t w  - Jm kd kf  t  w  - Jm P bf kd w  - bm kd kf  t  w  - P bm kd kf w

            2  6                  6                  6               2  4               3  4              2  2
   - Jm kd kf  w  + P bf bm kd kf w  - Jm bf kd kf t w  - Jm P bf kd t  w  - Jm bf kd kf t  w  - P bm kd kf t  w

              2    4                2  4             2  2  4             2  3  2                    6
   - P bm kd kf  t w  + bf bm kd kf t  w  - Jm P kd kf  t  w  - P bm kd kf  t  w  - Jm P bf kd kf t w

                  3  4                  2  4
   - Jm P bf kd kf t  w  + P bf bm kd kf t  w
```

```
a3=coeffs(real(eval(envnum2)),w);
for i=1:length(a3)
    b3(i,1:length(coeffs(a3(i),t)))=coeffs(a3(i),t);
end
```

Case 3.1 D=0 dd=0 bm=0

```
D=0;
dd=0;
bm=0;
pretty(real(eval(envnum2)))
```

```
          2  6             2  6           2  2  4               6                   6               2  4
 - Jm kd kf  w  - Jm P kd kf  w  - Jm kd kf  t  w  - Jm P bf kd w  - Jm bf kd kf t w  - Jm P bf kd t  w

                3  4             2  2  4                    6                   3  4
   - Jm bf kd kf t  w  - Jm P kd kf  t  w  - Jm P bf kd kf t w  - Jm P bf kd kf t  w
```

```
a31=coeffs(real(eval(envnum2)),w);
for i=1:length(a31)
    b31(i,1:length(coeffs(a31(i),t)))=coeffs(a31(i),t);
end
clear dd bm
syms dd bm
assume(bm, 'real')
assume(dd, 'real')
```