IE 313 Assignment 3

Travelling Salesperson Problem for Turkey

Halit Taşdemir 2018402198
Batuhan Özkan 2018402033

Problem Definiton

The Travelling Salesperson problem (TSP) is a significant problem in various fields, particularly in operations research. It revolves around finding the shortest route to visit a given set of cities without revisiting any city. Despite its seemingly simple nature, solving this problem involves tackling a computationally challenging NP-hard problem.

Since its formulation, the TSP has garnered considerable interest, leading to numerous attempts at finding solutions. As a result, a variety of heuristics and exact-solution methods have been developed. These methods now enable us to solve the TSP for thousands of cities while achieving an optimality bound of 1%.

Solution Method:

In this study, a linear programming approach is utilized to solve the Travelling Salesperson Problem (TSP) for the cities of Turkey. Linear programming methods for the TSP can be viewed as assignment models with an additional constraint to eliminate subtours, which are suboptimal routes that do not cover all cities.

There are various linear models available for solving the TSP, each differing in how they incorporate subtour elimination constraints. Two well-known linear models for the TSP are the Dantzig-Fulkerson-Johnson formulation and the Miller-Tucker-Zemlin (MTZ) formulation.

In this particular study, the formulation proposed by Desrochers and Laporte in 1991, which is a slightly improved version of the MTZ formulation, is employed. This formulation aims to address the subtour elimination constraint more effectively, enhancing the performance and efficiency of the solution approach. By utilizing the Desrochers and Laporte formulation, the study aims to determine the optimal solution for the TSP considering the cities in Turkey.

Formulation

The resulting formulation is:

$$
\begin{array}{ll}
\min & \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \\
\text{s.t.} & \sum_{j:j\neq i} x_{ij} = 1 \qquad j = 1, 2, \cdots, n \\
 & \sum_{i:i\neq j} x_{ij} = 1 \qquad i = 1, 2, \cdots, n
\end{array}
$$

$$
u_i - u_j + (n-1)x_{ij} + (n-3)x_{ji} \leq n-2, \qquad i \neq j, i, j = 2, 3, \cdots n
$$

$$
x_{ij} \in \{0, 1\} \qquad \forall i, j
$$

Solution:

For the implementation of this solution, the Python programming language is utilized, and the "pulp" package is used as a tool for mathematical optimization. To solve the linear programming problem, the CPLEX solver is employed, which is integrated into the pulp package.

The Python code, which includes the necessary algorithms and functions for solving the TSP using linear programming, is provided in the appendix of the study. This code outlines the step-by-step process to formulate and solve the TSP as a linear program using the chosen linear model.

To obtain the intercity distance data required for the TSP, the researchers have collected it from KGM (Karayolları Genel Müdürlüğü). The dataset, containing the distances between the cities in Turkey, is also provided in the appendix of the study for reference.

Results:

To randomly select 15 cities out of a total of 81 cities, the Python random() function is utilized. This function generates a random number between 0 and 1, and based on this random number, a subset of 15 cities is created. Randomly chosen cities were:

Eskişehir - Gaziantep - İzmir - Nevşehir - Yozgat - Muğla - Uşak - Erzurum - Elazığ - Sakarya - Siirt - Antalya - Aksaray - Ağrı - Bilecik

Once the subset of 15 cities is obtained, the TSP problem is solved using the previously mentioned linear programming approach. The solution provides the optimal route for visiting each city without revisiting any city.

The sequence of visited cities for the best route, obtained after solving the TSP for the randomly selected 15 cities, is provided below:

Eskişehir - Nevşehir - Erzurum - Yozgat- Siirt - Ağrı - Sakarya- Gaziantep - Elazığ - Aksaray - Antalya - İzmir - Uşak - Muğla - Bilecik - Eskişehir

Total distance taken by traveling salesman is found **4542**.

Solving the problem for a smaller sample size required less time compared to solving it for 81 cities. This can be attributed to both the increased complexity of the problem when dealing with larger instances and the inherent time complexity of the problem.

The optimal solution for solving the TSP for all 81 cities resulted in an objective function value of **9938**. This means that the traveling salesman starts the journey in Adana, travels a total distance of 9938 km, and completes the route in Osmaniye, visiting all the cities while minimizing the distance traveled. The specific route, in sequential order, is provided below:

Adana - Mersin - Karaman - Konya - Aksaray - Nevşehir - Niğde - Kayseri - Sivas - Tokat - Amasya - Çorum - Yozgat - Kırşehir - Kırıkkale - Çankırı - Ankara - Eskişehir - Kütahya - Afyonkarahisar - Uşak - Isparta - Burdur - Antalya - Denizli - Muğla - Aydın - İzmir - Manisa - Balıkesir - Çanakkale - Edirne - Kırklareli - Tekirdağ - İstanbul - Kocaeli - Yalova - Bursa - Bilecik - Sakarya - Bolu - Düzce - Zonguldak - Bartın - Karabük - Kastamonu - Sinop - Samsun - Ordu - Giresun - Trabzon - Rize - Artvin - Ardahan - Kars - Iğdır - Ağrı - Erzurum - Bayburt - Gümüşhane - Erzincan - Tunceli - Elazığ - Malatya - Bingöl - Muş - Bitlis - Van - Hakkari - Şırnak - Siirt - Batman - Diyarbakır - Mardin - Şanlıurfa - Adıyaman - Kahramanmaraş - Gaziantep - Kilis - Hatay - Osmaniye