

# Algebraic Causal Block Diagrams

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October 29, 2017

# 1 Discrete Time CBD simulator

## 1.1 getDependencies

The getDependencies function of the basicBlock simply returns a list of blocks connected to the input ports of the basicBlock.

## 1.2 Compute functions

The compute functions will generally take the input signals and execute a computation on these signals. It will then add the result to its own output signal. The ConstantBlock does not take an input, and will repeatedly add its constant value to its signal. The genericBlock uses the python function *eval* to evaluate whatever function the user assigned to the block as long as the function is known by python and it is written correctly. Correctly means without braces or parameters. For example, the sinus-function would be written as *sin*, not as *sin()* or *sin(x)*.

## 1.3 getDependencies DelayBlock

The dependencies of the delayblock change during the execution. During the first iteration, the block is dependent on its IC. After this first iteration, it is dependent on nothing.

## 1.4 \_\_createDepGraph

The \_\_createDepGraph function will first add all blocks to the dependencygraph. It will then go over all blocks and use the previously defined getdependencies function to add this blok's dependencies to the graph. We have to take care of sub-models. To do this, we keep a list containing all blocks in the current "level". Whenever we encounter a sub-model, we add all blocks inside of this sub-model to the list. Whenever we add the dependencies for a certain block to the graph, we remove this block from the list. All blocks and their dependencies will be added when the list is empty.

## 1.5 \_\_isLinear

By definition, constant blocks are not in strong components. Thus, if both input 1 and input 2 is in strong components, then it means that they violate the linear loop constraint. Because it means that two inputs are variable such as  $a*b$  in a loop. Therefore, for that cases the function return false and for all other cases it returns true.

## 2 CBD Simulator

For the entire CBD model, refer to Appendix ??

As a CBD simulator, we calculated kinetic energy for each velocity given in a list. We assume that mass is constant and should be given as input. Refer to equation (1) for the kinetic energy formula.

$$E = mv^2 - E \quad (1)$$

Before calculating the formula, we created velocity values by using counter block. The counter block has a 0 IC value at first and by using delay and sum block it is incremented by 1. After the value is incremented, the output of counter block is used at the Energy Calculator block as velocity.

Energy Calculator block takes mass as a constant block and takes output of counter block as velocity. Then, these inputs are used by product block. The output of product block is also used at another product block as IN1 and the velocity is used as IN2. After these operations we obtained equation (2)

$$mv^2 \quad (2)$$

To satisfy the equation (1), we used linear algebraic loop and obtained the kinetic energy value for the given mass and velocity values.

For the example output of this CBD model refer to Appendix ??. Note that in this example, there is 10 steps starting from 0. Therefore, the velocity changes from 0 to 9 discretely and we considered mass as 1.

## 3 Discrete Time CBD Denotational Semantics

The latex writer will follow a few steps. In what follows we use the following conventions:

$x^n$  = the signal at port  $x$  during iteration  $n$

$Input1, Input2$  = Input ports with a unique id

$Output$  = Output port with unique id

It will first describe all connections between blocks. These will be written as follows:

$$Input1^{i+1} = Output^{i+1}$$

Then it will add equations for each elementary block. The equations are:

$$Constant : Output^{i+1} = c$$

$$Negation : Output^{i+1} = -Input1^{i+1}$$

$$\begin{aligned}
\textit{Inverse} : \textit{Output}^{i+1} &= 1/\textit{Input1}^{i+1} \\
\textit{Sum} : \textit{Output}^{i+1} &= \textit{Input1}^{i+1} + \textit{Input2}^{i+1} \\
\textit{Product} : \textit{Output}^{i+1} &= \textit{Input1}^{i+1} * \textit{Input2}^{i+1} \\
\textit{Generic} : \textit{Output}^{i+1} &= \textit{Function}(\textit{Input1}^{i+1}) \\
\textit{Root} : \textit{Output}^{i+1} &= \sqrt[\textit{Input2}^{i+1}]{\textit{Input1}^{i+1}} \\
\textit{Modulo} : \textit{Output}^{i+1} &= \textit{Input1}^{i+1} \% \textit{Input2}^{i+1} \\
\textit{Delay} &= \begin{cases} \textit{Output}^0 = \textit{IC}^0 \\ \textit{Output}^{i+1} = \textit{Input1}^i \end{cases}
\end{aligned}$$

The output of our example CBD from exercise 2 can be found in the appendix ???. To facilitate the translation of this CBD (and other CBD's with outputs that are not connected to anything), we had to update the flatten function a bit. There was a bug where if your top-level CBD has outputs, it would throw an error. This can be seen by calling the original flatten function on the given *evenNumbersCBD.py*.

## 4 Kinetic Energy Calculator CBD

kinetic.jpg

## 5 Outputs of Kinetic Energy Calculator CBD

plot.jpg

## 6 Output denotational semantics CBD

$OutEnergy.IN1^{[s+1]} = multiplication.OutMult.OUT1^{[s+1]}$ ;  
 $counter.delay.IC^{[s+1]} = counter.zero.OUT1^{[s+1]}$ ;  
 $counter.delay.IN1^{[s+1]} = counter.sum.OUT1^{[s+1]}$ ;  
 $counter.delay.OUT1^{[s+1]} = counter.delay.IN1^{[s]}$ ;  
 $counter.delay.OUT1^{[0]} = counter.delay.IN2^{[0]}$ ;  
 $counter.sum.IN1^{[s+1]} = counter.delay.OUT1^{[s+1]}$ ;  
 $counter.sum.IN2^{[s+1]} = counter.one.OUT1^{[s+1]}$ ;  
 $counter.sum.OUT1^{[s+1]} = counter.sum.IN1^{[s+1]} + counter.sum.IN2^{[s+1]}$ ;  
 $counter.zero.OUT1^{[s+1]} = 0.0$ ;  
 $counter.one.OUT1^{[s+1]} = 1.0$ ;  
 $counter.OutCount.IN1^{[s+1]} = counter.delay.OUT1^{[s+1]}$ ;  
 $counter.OutCount.OUT1^{[s+1]} = counter.OutCount.IN1^{[s+1]}$ ;  
 $multiplication.mult1.IN1^{[s+1]} = multiplication.InNumber.OUT1^{[s+1]}$ ;  
 $multiplication.mult1.IN2^{[s+1]} = multiplication.mass.OUT1^{[s+1]}$ ;  
 $multiplication.mult1.OUT1^{[s+1]} = multiplication.mult1.IN1^{[s+1]} * multiplication.mult1.IN2^{[s+1]}$ ;  
 $multiplication.mult2.IN1^{[s+1]} = multiplication.mult1.OUT1^{[s+1]}$ ;  
 $multiplication.mult2.IN2^{[s+1]} = multiplication.InNumber.OUT1^{[s+1]}$ ;  
 $multiplication.mult2.OUT1^{[s+1]} = multiplication.mult2.IN1^{[s+1]} * multiplication.mult2.IN2^{[s+1]}$ ;  
 $multiplication.mass.OUT1^{[s+1]} = 1.0$ ;  
 $multiplication.adder.IN1^{[s+1]} = multiplication.mult2.OUT1^{[s+1]}$ ;  
 $multiplication.adder.IN2^{[s+1]} = multiplication.negator.OUT1^{[s+1]}$ ;  
 $multiplication.adder.OUT1^{[s+1]} = multiplication.adder.IN1^{[s+1]} + multiplication.adder.IN2^{[s+1]}$ ;  
 $multiplication.negator.IN1^{[s+1]} = multiplication.adder.OUT1^{[s+1]}$ ;  
 $multiplication.negator.OUT1^{[s+1]} = -multiplication.negator.IN1^{[s+1]}$ ;  
 $multiplication.InNumber.IN1^{[s+1]} = counter.OutCount.OUT1^{[s+1]}$ ;  
 $multiplication.InNumber.OUT1^{[s+1]} = multiplication.InNumber.IN1^{[s+1]}$ ;  
 $multiplication.OutMult.IN1^{[s+1]} = multiplication.adder.OUT1^{[s+1]}$ ;  
 $multiplication.OutMult.OUT1^{[s+1]} = multiplication.OutMult.IN1^{[s+1]}$ ;