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CS791-hw3

Question 1.

3.1 Consider the optimization problem

$$\begin{array}{ll} \text{minimize} & f_0(x_1, x_2) \\ & 2x_1 + x_2 \geq 1 \\ \text{subject to} & x_1 + 3x_2 \geq 1 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

Make a sketch of the feasible set. For each of the following objective functions, give the optimal set and the optimal value. Then use CVX to verify the optimal values you obtained.

(a) $f_0(x_1, x_2) = x_1 + x_2$
(b) $f_0(x_1, x_2) = -x_1 - x_2$
(c) $f_0(x_1, x_2) = x_1$
(d) $f_0(x_1, x_2) = \max\{x_1, x_2\}$
(e) $f_0(x_1, x_2) = {x_1}^2 + 9{x_2}^2$

Answer:

Feasible set is set of point which are in domain of f_0 and satisfies to our constraint functions. So for this problem, convex hull of (0, ∞), (∞,0), (0,1), (1,0), (2/5,1/5) is our feasible set.

a) p^* = (2/5, 1/5)
b) p^* = -∞ (unbounded below)
c) Since here our object function's output is only x_1 even if it takes x_2 as input, we have set of optimal values where our x_2 can be any number in domain.
   So $X_{opt} = \{(0,x_2) \mid x_2 \geq 1\}$

d) p^* = (1/3,1/3)
e) p^* = (1/2,1/6)

Question 2.

3.2 Solve the optimal activity level problem described in exercise 4.17 in Convex Optimization, for the instance with problem data

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 3 & 1 & 1 \\ 2 & 1 & 2 & 5 \\ 1 & 0 & 3 & 2 \end{bmatrix}, c^{\max} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}, p = \begin{bmatrix} 3 \\ 2 \\ 7 \\ 6 \end{bmatrix}, p^{\text{disc}} = \begin{bmatrix} 2 \\ 1 \\ 4 \\ 2 \end{bmatrix}, q = \begin{bmatrix} 4 \\ 10 \\ 5 \\ 10 \end{bmatrix}$$

You can do this by forming the LP you found in your solution of exercise 4.17, or more directly, using CVX. Give the optimal activity levels, the revenue generated by each one, and the total revenue generated by the optimal solution. Also, give the average price per unit for each activity level, i.e., the ratio of the revenue associated with an activity, to the activity level. (These numbers should be between the basic and discounted prices for each activity.) Give a very brief story explaining, or at least commenting on, the solution you find. **You also need to submit your CVX matlab code.**

Answer:

I was not sure how to submit CVX code, so here is the screenshot. To run it on Matlab you should install CVX.

```matlab
A=[1 2 0 1; 0 0 3 1; 0 3 1 1; 2 1 2 5; 1 0 3 2];
cmax=[100; 100; 100; 100; 100];
pbase=[3;2;7;6];
pdiscount=[2;1;4;2];
q=[4;10;5;10];
cvx_begin
    variable x(4)
    revenue=min(pbase.*x,pbase.*q+pdiscount.*(x-q))
    totalrevenue=sum(revenue)
    maximize(totalrevenue)
    subject to
        A*x<=cmax
        x>=0
cvx_end
x
averagePrice=revenue./x
revenue
totalrevenue
```

We can see that highest revenue is 3rd level and lowest revenue is 4th level. Third level has highest basic price, discount price and highest activity level. So its contribution to total revenue is highest. 4th level also has high basic price however, there is a considerable amount of difference in basic price and discount price so it has less contribution to total revenue.

```
Command Window
Optimal value (cvx_optval): +192.5


x =

    4.0000
   22.5000
   31.0000
    1.5000


averagePrice =

    3.0000
    1.4444
    4.4839
    6.0000


revenue =

   12.0000
   32.5000
  139.0000
    9.0000


totalrevenue =

  192.5000

fx >>
```

Question 3

3.3 The illumination problem. In lecture 1 we encountered the function

$$f(p) = \max_{i=1,\dots,n} \left|\log a_i^T p - \log I_{des}\right|$$

where $a_i \in \mathbf{R}^m$, and $I_{des} > 0$ are given, and $p \in \mathbf{R}_+^m$

(a) Show that $\exp f$ is convex on $\{p | a_i^T p > 0, i = 1, \dots, n\}$.

(b) Show that the constraint 'no more than half of the total power is in any 10 lamps' is convex (i.e., the set of vectors p that satisfy the constraint is convex).

(c) Show that the constraint 'no more than half of the lamps are on' is (in general) not convex.

Answer:

a) let's say X = $a_i^T p$

we know X is convex (affine function), since I_{des} is positive $\log(X/I_{des})$ is also convex, since it is convex, maximum of convex function is also convex so $\max(\log(X/I_{des}))$ is convex. Since exp is increasing function exp of convex function is convex, so exp(f) is convex.

b) From lecture notes we know that feasible set of convex problem is convex. Since p is set of vectors that satisfy the constraint we know that it is in feasible set and since our problem is convex we can conclude that “no more than half of the total power is in any 10 lamps” is convex.

c) “No more than half of the lamps are on” means that “at least half of the lamps have no contribution to total power”. This constraint contradicts with $p \in \mathbf{R}_+^m$ so it is not convex.