Due Date: Mar 7, 2018, Beginning of the class
How to submit: Hard copy in the Class

2.1 Show that $f(x) = e^{\alpha x^T A x}$ is convex, where A is a positive semidefinite symmetric $n \times n$ matrix and α is a positive scalar.

2.2 Show that $f(x,t) = -\log(t^2 - x^T x)$ with $\mathbf{dom}\, f = \{(x,t) \in \mathbf{R}^n \times \mathbf{R} | t > \|x\|_2\}$ is convex. Hint: you can use composition rules here and use convexity of the quadratic over linear function.

2.3 Show that $f(x) = \frac{x^T x}{(\prod_{i=1}^n x_i)^{\frac{1}{n}}}$ is convex $\mathbf{dom}\, f = \mathbf{R}^n_{++}$.

Hint: Perspective Composition Rule. Suppose that $f: \mathbf{R}^n \to \mathbf{R}$ is a closed proper convex function satisfying $f(0) \leq 0$ and $g: \mathbf{R}^m \to \mathbf{R}$ be a closed proper concave function which is nonnegative on its effective domain, the function $h(x) = g(x) f(x/g(x))$ is convex with $\mathbf{dom}\, h = \{x \in \mathbf{dom}\, g | \, x/g(x) \in \mathbf{dom}\, f\}$.

2.4 Show the following:

(a) If f and g are convex, both nondecreasing (or nonincreasing), and positive functions on an interval, then fg is convex.

(b) Suppose that $f: \mathbf{R}^n \to \mathbf{R}$ is nonnegative and convex, and $g: \mathbf{R}^n \to \mathbf{R}$ is positive and concave. Show that the function $\frac{f^2}{g}$, with domain $\mathbf{dom}\, f \cap \mathbf{dom}\, g$ is convex.