Macroeconomics II

Chapter 9: The New Keynesian Model

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1. Motivation, and organization

- Saw implications of nominal rigidities. Real effects of money. Muted effects of technological shocks.
- No dynamics. So time to introduce them. Two types.
 - -C/S, N/L, M/B choices. Can build on earlier models.
 - Richer structure of price setting: Introduce staggering. Not all price or wage decisions taken simultaneously.
- Will get a modern version of IS-LM-AD model. "New Keynesian model"
- Can use it to look at the behavior of inflation and output, dynamic effects of shocks, and role of monetary policy.

Organization

- Price staggering. Taylor, Calvo, and others. Time versus statedependent rules.
- Introducing monopolistic competition and Calvo price staggering in a GE model with consumption/saving, work/leisure, and money/bond choices.
- The New Keynesian model.
- Limitations and implications.

2. Price staggering

Go back to a model of monopolistic competition. Ignore nominal rigidities, and assume price setters choose:

$$p_i = p + b + ay - z$$

All variables in logs, y is aggregate output, and z is a common technological shock. (y shifts position of demand curve, z position of marginal cost curve). Aggregate demand is given in turn by:

$$y = (m - p)$$

The effects of staggering. An intuitive argument.

- The smaller a, the less each price setter wants to increase its price given others in response to
- If staggering, each adjustment is small. Thus, the price level adjusts slowly.
- Role of "real rigidities" (small a) and nominal rigidities (staggering of nominal price setting).

Equilibrium under flexible prices

Under flexible prices, the equilibrium level of output (the second best level of output, as the economy still suffers from the distortion coming from the markup), is given by:

$$\hat{y} = \frac{1}{a}(-b+z)$$

so we can rewrite the price setting equation as:

$$p_i = p + a(y - \hat{y}) = p + ax$$

where x is defined as the output gap the difference between actual output and equilibrium (second best) output.

(Note that, given the output gap, technological shocks or energy price changes do not appear anymore directly in the equation.)

Taylor staggering

Assume:

- Each period, 1/n of the price setters set their price for the current and the next n-1 periods. Their price is fixed between readjustments.
- Let q_t be (log) price chosen in t for t to t+n-1. Set equal to the average of the expected desired prices for periods t to t+n-1: (fast and loose on objective function, treatment of expectations.)

$$q_t = \frac{1}{n} \sum_{i=0}^{n-1} Ep_{t+i}^*$$
 where $p_t^* = p_t + ax_t$

The (log) price level is in turn given by:

$$p_t = \frac{1}{n} \sum_{i=0}^{n-1} q_{t-i}$$

Can solve the general case (Taylor 1981). The case where n=2 is convenient and useful:

$$q_t = 0.5 (p_t + Ep_{t+1}) + 0.5 a (x_t + Ex_{t+1})$$

and

$$p_t = 0.5(q_t + q_{t-1})$$

Replace p_t and Ep_{t+1} from the second equation in the first, and reorganize to get:

$$q_t = 0.5 \ q_{t-1} + 0.5 \ Eq_{t+1} + a(x_t + Ex_{t+1})$$

or

$$(q_t - q_{t-1}) = (Eq_{t+1} - q_t) + 2a(x_t + Ex_{t+1})$$

The first equation:

$$q_t = 0.5q_{t-1} + 0.5Eq_{t+1} + a(x_t + Ex_{t+1})$$

implies price level inertia (for q, and by implication for p.)

• Define $\pi_{q,t} = q_t - q_{t-1}$. Then rewrite the second equation as:

$$\pi_{q,t} = E\pi_{q,t+1} + 2a(x_t + Ex_{t+1})$$

Solving forward gives:

$$\pi_{q,t} = 2a \sum_{0}^{\infty} (Ex_{t+i} + Ex_{t+i+1})$$

(New price) inflation is fully forward looking. It does not depend on the past.

Strong conclusions: Price inertia. (Nearly no) inflation inertia.

Calvo staggering

• Each price is readjusted with probability δ each period (a "Poisson" assumption). The price is set according to:

$$q_t = \delta \sum_{0}^{\infty} (1 - \delta)^i E p_{t+i}^*$$
 where $p_t^* = p_t + ax_t$

The price q_t chosen in period t depends on all future expected desired prices, with weights corresponding to the probability that the price is still in place at each future date.

The price level is given in turn by:

$$p_t = \delta \sum_{0}^{\infty} (1 - \delta)^i q_{t-i}$$

The price level is in turn a weighted average of current and past individual prices, still in place today.

To solve the model, rewrite the two equations in recursive form as:

$$q_t = \delta(p_t + ax_t) + (1 - \delta)Eq_{t+1}$$
$$p_t = \delta q_t + (1 - \delta)p_{t-1}$$

Use the second equation to express q_t as a function of p_t and p_{t-1} and Eq_{t+1} as a function of Ep_{t+1} and p_t , and replace in the first equation to get:

$$p_t - (1 - \delta)p_{t-1} = \delta^2(p_t + ax_t) + (1 - \delta)(Ep_{t+1} - (1 - \delta)p_t)$$

Reorganizing:

$$p_t = \frac{1}{2}p_{t-1} + \frac{1}{2}Ep_{t+1} + \frac{1}{2}\frac{\delta^2}{1-\delta} \ a \ x_t$$

or, equivalently,

$$(p_t - p_{t-1}) = (Ep_{t+1} - p_t) + \frac{\delta^2}{1 - \delta} a x_t$$

Coefficient on the output gap depends not only on a, but also on δ , the frequency of adjustment.

Two differences with the Taylor formalization (both more convenient):

- ullet Equations in terms of the price level p_t rather than in terms of the new price q_t
- Only the current output gap appears, instead of the current and expected output gaps.

Implications. Price versus inflation stickiness

Staggering leads to **price stickiness**, and dynamic effects of money on output. Clear in both Taylor and Calvo specifications.

Take Calvo equation for p_t and close the model by assuming $b=z_t=0$ so $\hat{y_t}=0$, and $y_t=m_t-p_t$, so

$$x_t = y_t - \hat{y_t} = m_t - p_t$$

In this case, the price level is given by:

$$p_t = bp_{t-1} + bEp_{t+1} + (1-2b)m_t, \qquad b \equiv \frac{1-\delta}{2(1-\delta) + a(1-\delta)^2} < \frac{1}{2}$$

Solving this equation gives:

$$p_t = \lambda p_{t-1} + (1 - \lambda)^2 \sum_{i=1}^{\infty} \lambda^i E m_{t+i}, \qquad \lambda \equiv \frac{1 - \sqrt{1 - 4b^2}}{2b} \le 1$$

$$p_t = \lambda p_{t-1} + (1-\lambda)^2 \sum \lambda^i Em_{t+i}$$
 where $\lambda \equiv (1-\sqrt{1-4b^2})/2b \le 1$

- The closer a is to 0, the closer b is to 1/2, and the closer λ is to 1: The flatter the marginal cost, the more price stickiness.
- The closer δ is to 0, the closer b is to 1/2, and the closer λ is to 1: The less often prices are adjusted, the higher the price stickiness.

Consider the case where $m_t = m_{t-1} + \epsilon_t$. In this case, the price level follows:

$$p_t = \lambda p_{t-1} + (1 - \lambda)m_t$$

So output is given by:

$$y_t = \lambda y_{t-1} + \lambda \epsilon_t$$

The closer λ is to one, the longer lasting the effects of an unanticipated shock in money on output.

Staggering does not however lead to **stickiness in inflation**. Take Calvo specification:

$$\pi_t = E\pi_{t+1} + \left[\frac{\delta^2}{1-\delta}a\right]x_t$$

Inflation depends on current and expected output gaps, not (directly) on the past.

Strong warning: Price stickiness does not imply inflation stickiness (stickiness in a level does not necessarily imply stickiness in a derivative). Example:

- Assume $x_t = m_t p_t$ and money grows at rate g_m .
- Verify that, for given g_m , $p_t = m_t$, so $\pi_t = g_m$ and $x_t = 0$.
- Suppose that, at time t, the rate of money growth from t on decreases from g_m to $g'_m < g_m$. Verify inflation from t on decreases from g_m to g'_m , and x = 0.
- In words: Disinflation is achieved without any output loss.

Robustness?

Alternative structures (much current work, theoretical and empirical).

- Chains of production. Amplification of nominal rigidities.
- Time versus state dependent rules: Caplin-Spulber. Caplin-Leahy. Caballero. Golosov-Lucas.

The one-sided Ss rule and the neutrality result.

- Alternative assumptions about expectations. Learning. In case of disinflations. Learning versus adaptive expectations.
- Alternative assumptions about the use of expectations. Rational inattention. Mankiw/Reiss.

Some empirical evidence. The difficulty of controlling for the (unobservable) output gap.

Annual data. 1960 to 2004. IV using 4 lags of inflation and the activity measure. Price is GDP deflator.

$$\pi_t = 0.65 \; \pi_{t-1} + 0.49 \; E \pi_{t+1} + 0.15 \; {
m ygap}$$
 $\pi_t = 0.65 \; \pi_{t-1} + 0.49 \; E \pi_{t+1} + 0.15 \; {
m ngap}$ $\pi_t = 0.63 \; \pi_{t-1} + 0.51 \; E \pi_{t+1} - 0.29 \; {
m u}$ u $\pi_t = 0.72 \; \pi_{t-1} + 0.41 \; E \pi_{t+1} - 0.54 \; {
m ugap}$

"Gap" variables are detrended variables, using a cubic trend. All estimated coefficients on inflation, lagged or expected, are significant at the 5% level.

Focused on price setting. But also, and perhaps primarily, wage setting (one-year contracts, even three-year contracts with pre-set nominal wage increases).

• Some equivalence. Back to Taylor model in initial incarnation: q_t as w_t .

$$w_t = 0.5 (p_t + Ep_{t+1}) + 0.5 a (x_t + Ex_{t+1})$$

and

$$p_t = 0.5(w_t + w_{t-1})$$

Or in terms of relative wages (different motivation):

$$w_t = 0.5 (w_t + w_{t+1}) + a (x_t + Ex_{t+1})$$

Relative importance of price versus wage stickiness:

Prices. Frequent individual adjustment. Amplification? Chains of production and horizontal/vertical interactions between price decisions,

Wages. Typically annual adjustments for existing contracts. New versus old matches.

3. The "New Keynesian" model

Required minimal ingredients:

- Consumption/saving choice. role of the interest rate.
- Leisure/work choice. employment fluctuations.
- Money/Bond choice. determination of the interest rate.
- Nominal rigidities. Choose Calvo specification.

Households

A continuum of households, indexed by i, who maximize:

$$\max E[\sum_{0}^{\infty} \beta^{k} (U(C_{it+k}) + V(\frac{M_{it+k+1}}{\bar{P}_{t+k}}) - Q(N_{it+k})) | \Omega_{t}]$$

subject to:

$$C_{it} \equiv \left[\int_{0}^{1} C_{ijt}^{(\sigma-1)/\sigma} dj \right]^{\sigma/(\sigma-1)} \qquad \bar{P}_{t} = \left[\int_{0}^{1} P_{jt}^{1-\sigma} dj \right]^{1/(1-\sigma)}$$

$$\int_0^1 P_{jt}C_{ijt} + M_{it+1} + B_{it+1} = W_tN_{it} + (1+i_t)B_{it} + M_{it} + \Pi_{it} + X_{it}$$

- Consumers derive utility from a consumption basket, real money balances, and leisure.
- The consumption basket is a CES function of different consumption goods. The price index associated with the consumption basket, the price level, is now denoted by a bar.
- Consumers supply labor in a competitive labor market and so receive labor income $W_t N_{it}$. (Convenient: All households the same, despite different price setting by firms.)
- Consumers equally own all the firms producing the individual consumption goods, and receive profits Π_{it} , and, possibly, transfers from the state.

The first order conditions are straightforward and familiar:

$$C_{ijt} = \left(\frac{P_{jt}}{\bar{P}_t}\right)^{-\sigma} C_{it}$$

$$U'(C_{it}) = E[\beta(1 + r_{t+1})U'(C_{it+1})|\Omega_t]$$

$$V'(\frac{M_{it+1}}{\bar{P}_t})/U'(C_{it}) = \frac{i_{t+1}}{1+i_{t+1}}$$

$$\frac{W_t}{\bar{P}_t} U'(C_{it}) = Q'(N_{it})$$

Firms. Production, and pricing.

Individual consumption goods are produced by firms, according to a linear production technology:

$$Y_{jt} = Z_t N_{jt}$$

They take the wage as given, and set prices a la Calvo, with probability δ of changing the price every period. The price chosen by firm j at time t, P_{jt} is the solution to:

$$\max E\left[\sum_{k} \beta^{k} \frac{U'(C_{t+k})}{U'(C_{t})} (1-\delta)^{k} \left(\frac{P_{jt}}{\bar{P}_{t+k}} Y_{jt+k} - \frac{W_{t+k}}{\bar{P}_{t+k}} \frac{Y_{jt+k}}{Z_{t+k}}\right) |\Omega_{t}\right]$$

subject to

$$Y_{jt+k} = \left(\frac{P_{jt}}{\bar{P}_{t+k}}\right)^{-\sigma} Y_{t+k}$$

- Firms maximize the expected present value of profits they will get at the chosen price P_{jt} .
- Profit is equal to real revenues minus real costs.
- Profit at time t + k is discounted at the MRS of consumers (can drop the index i as all consumers face the same maximization problem and thus have the same consumption), times the probability that the price chosen at time t is still in effect at time t + k, $(1 \delta)^k$.

Maximization yields the following expression for P_{jt} :

$$P_{jt} = P_t = \frac{\sigma}{\sigma - 1} \frac{E[\sum_k A(k) (W_{t+k}/Z_{t+k}) | \Omega_t]}{E[\sum_k A(k) | \Omega_t]}$$

where

$$A(k) \equiv \beta^{k} \frac{U'(C_{t+k})}{U'(C_{t})} (1 - \delta)^{k} (\bar{P}_{t+k})^{\sigma - 1} Y_{t+k}$$

So the price chosen by price setters is a weighted average of current and expected future marginal costs W_{t+k}/Z_{t+k} .

Like Calvo, except more complex discounting: Expectations of products of random variables. Also, future periods are also discounted by the discount factor β^k .

The price level is given in turn by:

$$\bar{P}_t = \left[(1 - \delta) \bar{P}_{t-1}^{1-\sigma} + \delta P_t^{1-\sigma} \right]^{(1/(1-\sigma))}$$

General equilibrium

In general equilibrium, $C_{it}=C_t=Y_t,\ N_t=Y_t/Z_t,\ M_{it}=M_t$, so the first two FOC become:

IS:
$$U'(Y_t) = E[\beta(1 + r_{t+1})U'(Y_{t+1})|\Omega_t]$$

LM:
$$V'(\frac{M_{t+1}}{\bar{P}_t})/U'(Y_t) = \frac{i_{t+1}}{1+i_{t+1}}$$

Interpretation.

- IS: Demand for goods (consumption) depends on expected future income and the real interest rate. So (too large) role of expectations of future output/income
- LM: Equilibrium in financial markets determines the nominal interest rate as a function of real money and income.

The other FOC are given by:

$$LS: \frac{W_t}{\overline{P}_t} U'(Y_t) = Q'(Y_t/Z_t)$$

$$PS: P_t = \frac{\sigma}{\sigma - 1} \frac{E[\sum_k A(k) (W_{t+k}/Z_{t+k}) | \Omega_t]}{E[\sum_k A(k) | \Omega_t]}$$

where

$$A(k) \equiv \beta^{k} \frac{U'(Y_{t+k})}{U'(Y_{t})} (1 - \delta)^{k} (\bar{P}_{t+k})^{\sigma - 1} Y_{t+k}$$

and

$$\bar{P}_t = [(1 - \delta)\bar{P}_{t-1}^{1-\sigma} + \delta P_t^{1-\sigma}]^{(1/(1-\sigma))}$$

$$\mathsf{PF}:\ N_t = Y_t/Z_t$$

- LS (labor supply). The real wage depends on employment and income.
- PS (price setting). The price is set as a markup over current and future expected marginal cost.

Second best level of output

Can derive the level of output that would prevail, absent nominal rigidities. From LS:

$$\frac{W_t}{\bar{P}_t} = \frac{Q'(Y_t/Z_t)}{U'(Y_t)}$$

From PS, without nominal rigidities:

$$\bar{P}_t = \frac{\sigma}{\sigma - 1} \frac{W_t}{Z_t}$$

or rewriting:

$$\frac{W_t}{\bar{P}_t} = \frac{\sigma - 1}{\sigma} Z_t$$

Price setting determines the real wage paid by firms.

Putting the two together gives:

$$\frac{Q'(Y_t/Z_t)}{U'(Y_t)} = \frac{\sigma - 1}{\sigma} Z_t$$

$$\frac{Q'(Y_t/Z_t)}{U'(Y_t)} = \frac{\sigma - 1}{\sigma} Z_t$$

- Determines implicitly output as a function of the technological shock.
- If U(.) is log (balanced growth condition), then the equation becomes:

$$Q'(\frac{Y_t}{Z_t}) \frac{Y_t}{Z_t} = \frac{\sigma - 1}{\sigma}$$

- This determines a unique value of Y_t/Z_t , or equivalently a unique value of N_t . So second best employment is constant, and second best output varies one for one with Z_t .
- Why different from RBC? No capital accumulation, so tight link between consumption and wage.

The loglinearized system

To make progress, useful to log linearize around zero inflation steady state (zero inflation assumption important for log linearization of the price equation)

IS: $y_t = Ey_{t+1} - \sigma r_{t+1}$

LM: $m_{t+1} - \bar{p_t} = by_t - ci_{t+1}$

LS: $w_t - \bar{p_t} = \gamma n_t + z_t, \qquad \gamma = (1 + \phi)$

PS: $p_t = (1 - \delta)\beta E p_{t+1} + (1 - \beta(1 - \delta))(w_t - z_t)$

 $\bar{p_t} = (1 - \delta)\bar{p}_{t-1} + \delta p_t$

 $\mathsf{PF}: \qquad \qquad n_t = y_t - z_t$

Combine the last three equations to get a "Phillips curve relation":

 Use the labor supply relation to eliminate the wage in the price setting equation:

$$p_t = (1 - \delta)\beta E p_{t+1} + (1 - \beta(1 - \delta))(\bar{p_t} + \gamma n_t)$$

• Denote the log deviations of second best output and employment from steady state by $\hat{y_t}$ and $\hat{n_t}$.

Define the output gap, i.e. the difference between actual and second best log deviation of output $y_t - \hat{y_t}$ by x_t .

From above $\hat{y_t} = z_t$ and $\hat{n_t} = 0$ so

$$x_t = y_t - \hat{y_t} = n_t - \hat{n_t} = n_t$$

Replacing the output gap in the previous equation gives:

$$p_t = (1 - \delta)\beta E p_{t+1} + (1 - \beta(1 - \delta))(\bar{p_t} + \gamma x_t)$$

Combining with the equation for the price level and reorganizing gives:

PC:
$$\pi_t = \beta E \pi_{t+1} + \frac{\delta (1 - \beta (1 - \delta))}{1 - \delta} \gamma x_t$$

where $\pi_t \equiv p_t - p_{t-1}$

PC:
$$\pi_t = \beta E \pi_{t+1} + \frac{\delta (1 - \beta (1 - \delta))}{1 - \delta} \gamma x_t$$

- Inflation depends on itself expected, and on the output gap.
- If output is above its second best (or "natural") level, then, given expected inflation, inflation increases.
- As in the Calvo model, inflation is forward looking, and there is no inflation inertia.
- Unlike Calvo model, β rather than one. A (small) long run trade-off between π and x.

The New Keynesian model. A first assessment

Putting the equations together:

IS:
$$y_t = Ey_{t+1} - ar_{t+1}; r_{t+1} = i_{t+1} - E\pi_{t+1}$$

LM:
$$m_{t+1} - \bar{p_t} = by_t - ci_{t+1}$$

PC:
$$\pi_t = \beta E \pi_{t+1} + d x_t$$
; $x_t = y_t - \hat{y_t} = y_t - z_t$

These three equations constitute the New Keynesian model. (Sometimes, LM replaced by a Taylor rule, PC by a simpler price setting equation. More on this later).

Embody the basic mechanisms we have seen in the course. Has become a benchmark model, to study the effects of shocks, of policy.

Can look not only at fluctuations, but also welfare.

4. Applications and implications

Back to technology shocks. Likely that there are such events as technology booms, but they are likely to be much more gradual than in the RBC model. What does the NK model imply? A simple example:

IS:
$$y_t = E_t y_{t+1} - a r_{t+1}$$
 (1)

PC:
$$p_t \mid E_{t-1}y_t = E_{t-1}\hat{y_t}$$
 (2)

Assumptions:

- r directly under the control of the central bank. So no need for an LM relation, except to back out the nominal interest rate and the nominal money stock.
- One-period nominal price rigidity (a la Fischer) rather than pricestaggering.

Initial steady state:

Assume z = 0, current and expected. Then: $\hat{y_t} = y_t = 0$. p is not well defined, as we have not specified the details of how monetary policy is conducted.

- Effect of a change in expected z from 0 to z>0, as of period t, for period t+1 on? (captures anticipated technology boom).
- Assume central bank keeps r constant. Then $E_t y_{t+i} = z$. Given fixed r, $y_t = E_t y_{t+1} = z$. A demand boom, in anticipation of the future productivity boom.
- What is the optimal central bank response? Best is to keep output at natural level, so $y_t = 0$, which requires $r_t = z/a$. Thus, optimal policy is to increase the real interest rate to replicate the flexible price outcome.

- Suppose central bank follows $r_t = b(y_t \hat{y_t})$. Then, replacing in the first equation, and solving gives $y_t = (1/(1+ab))z$. The higher b, the smaller the response of current output to the anticipated future technology boom.
- In the short run, the technology boom starts as a "demand boom", with output increasing above its natural level. Later on, it changes into a supply boom, with an increase in actual and natural output.
- Compare to the Blanchard-Quah Gali responses of output to a technological shock. Does it fit?

Slightly richer simulations

$$y_{t} = (1 - \alpha)y_{t-1} + \alpha E y_{t+1} - a r_{t+1}$$

$$r_{t+1} = i_{t+1} - E \pi_{t+1}$$

$$\pi_{t} = \beta E \pi_{t+1} + d x_{t}$$

$$x_{t} = y_{t} - \hat{y_{t}} = y_{t} - z_{t}$$

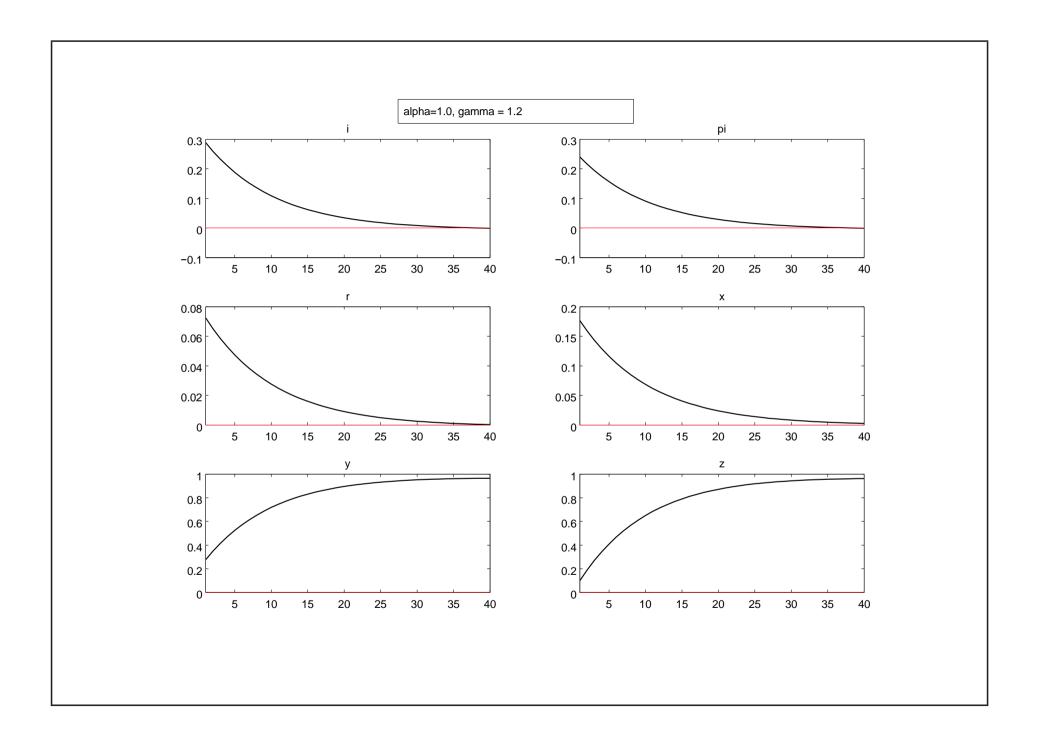
$$\Delta z_{t} = \rho \Delta z_{t-1} + \epsilon_{zt}$$

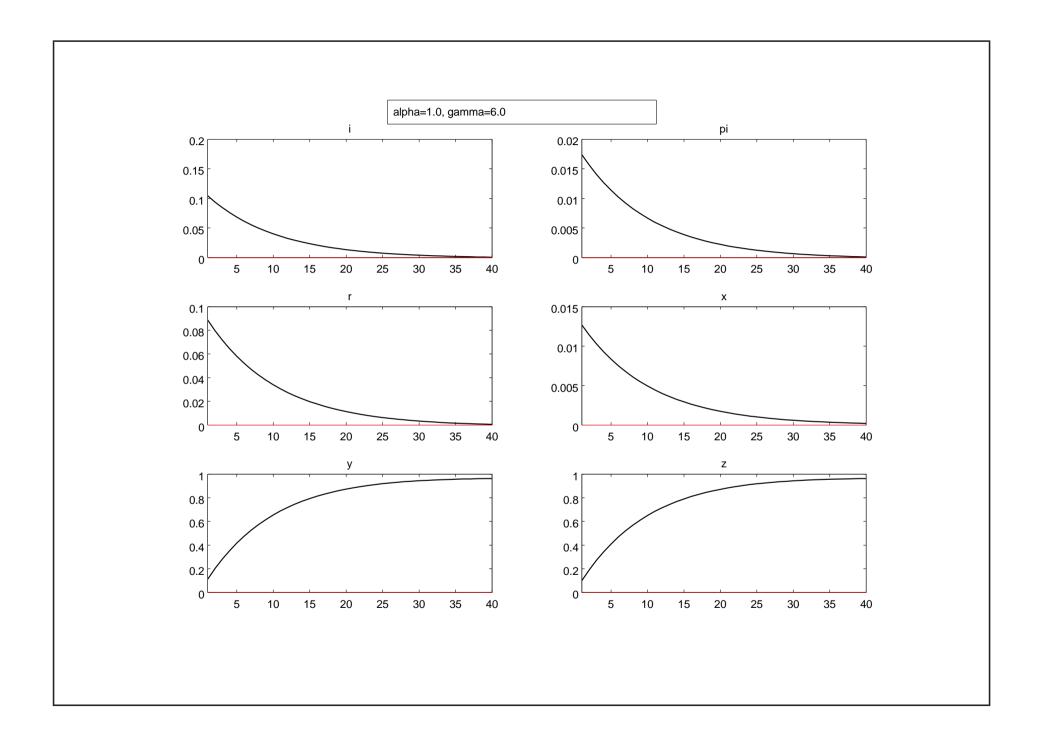
$$i_{t+1} = \gamma \pi_{t}$$

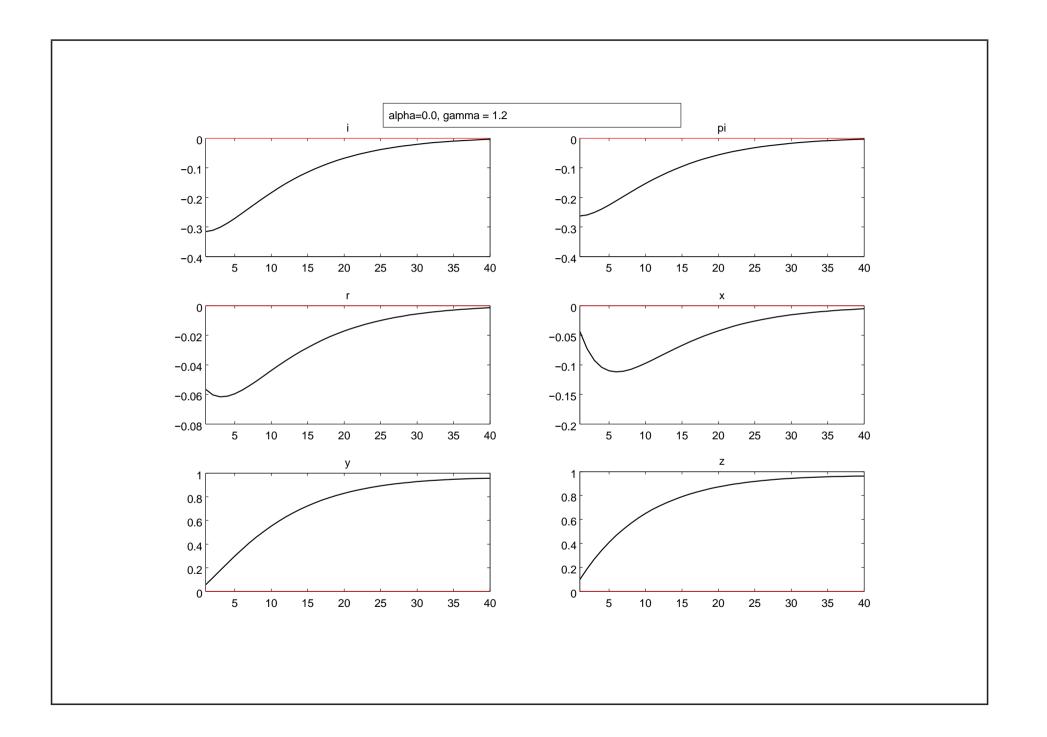
Note slight extension of model (α) . Three simulations:

(1)
$$\alpha = 1.0, \gamma = 1.2; \ \alpha = 1.0, \gamma = 6.0; \ \alpha = 0.0, \gamma = 1.2$$

Relation to BQ-Gali characterization of technological shocks?







Expectations, demand shocks, and technology shocks

- Role of expectations. Any change in Ey_{t+i} will affect y_t today—unless offset by monetary policy.
- Anticipations of a productivity boom—justified or not.
- Effects of such shocks depend very much on monetary policy.
- Decreases in money will lead to recessions. Decreases in money growth?
 Depends on specifics of price setting/credibility.
- Monetary transmission mechanism? Lower nominal interest rate, lower real interest rate, higher spending, larger output gap, inflation.

Limitations of the benchmark NK model?

Obviously. Many, and serious. Lacking:

- Nominal wage rigidities in addition to nominal price rigidities. (Erceg, Henderson, Levin, 2000)
- Investment and capital accumulation.
- Government spending and taxes.
 - Spending and taxes under Ricardian equivalence.
 - Non-Ricardian equivalence, and thus richer implications of fiscal policy, of tax versus deficit finance. (2-period OLG or finite horizons)
- Openness in goods and financial markets. See Obstfeld Rogoff

More imperfections.

Heterogeneity

Coming from incomplete markets. Incomplete insurance. Precautionary saving. Wealth distribution and consumption.

Coming from fixed costs. Lumpy investment decisions.

Imperfections in many markets.

Imperfect competition in goods markets (beyond Dixit-Stiglitz monopolistic competition) Schumpeterian growth.

Frictions in labor markets. Search/matching models. Involuntary unemployment rather than choice between leisure and work.

Imperfect information in financial/credit markets. Role of credit as opposed to the interest rate. Role of the banking system. Role of money, as opposed to the interest rate.

All these extensions have been and are studied. Either in isolation, or together in medium and large size DSGE models.

Issues of construction, estimation. Classical/Bayesian. Calibration versus estimation. Equation-by-equation versus system estimation.