Macroeconomics II

Chapter 7: Real Business Cycles

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Aims of this lecture

- To extend the Ramsey model by endogenizing the labour supply decision of households

- To turn the model into an RBC model by assuming stochastic technology shocks

  – theory of fluctuations at business cycle frequencies

  – impulse response functions

  – matching real world data [calibration]

  – evaluation of the RBC approach

The Lucas Research Program

- Key idea: macroeconomists should build so-called structural models,i.e. Models that Are based on microeconomic foundations [maximizing households and firms,flexible prices/wages,market clearing, etcetera]
- The Lucas Research Program(LRP) is the logical outcome of the Rational Expectations Revolution of the 1970s.
- Kydland&Prescott accepted the challenge posed by Lucas: they built the first Real Business Cycle(RBC) model. •Outline of the RBC methodology:

–construct a discrete-time stochastic model of the economy populated by maximizing households and firms

–typically the source of the stochastic fluctuations is the level of general Productivity [our Z in the production function].Since Z_t is unknown agents must Form expectations about it.They adopt the REH to do so.

–calibrate the model in a realistic fashion

–find the stochastic equilibrium process for the macroeconomic variables[output, employment,consumption,investment,the capital stock,and factor prices]

–compute basic statistics [correlations,and standard deviations] for the different Variables both for the artificial economy and for the actual economy. Compare How well the model economy matches the actual economy’s characteristics

Building an RBC model

- We have most of the ingredients allready. Only things to do:
  - reformulate model in discrete time [rather than continuous time]
  - introduce stochastic productivity shock
  - rederive firm and household behaviour

- **Firms**

  – Technology:

$$Y_\tau = F(Z_\tau, K_\tau, L_\tau) \equiv Z_\tau \; L_\tau^{L} K_\tau^{1-L}, \quad 0 < L < 1$$

  Where Z_τ is the index of general technology.

  – Firms rent factors of production from the household sector. The marginal Productivity conditions are:

$$F_L(Z_\tau, K_\tau, L_\tau) = W_\tau$$

$$F_K(Z_\tau, K_\tau, L_\tau) = R_\tau^K$$

  Where R_K is the rental charge on capital.

- **Households**

  – Preferences [expected life time utility]:

$$E_t \Lambda_t \equiv E_t \sum_{\tau=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{\tau - t} \left[\epsilon_C \log C_\tau + (1 - \epsilon_C) \log [1 - L_\tau] \right]$$

Where E_t is the expectations operator [i.e.information dated up to and including Period t is used]

– Budget identity : $C_\tau + I_\tau = W_\tau L_\tau + R_\tau^K K_\tau - T_\tau$

– Capital accumulation: $K_{\tau+1} = I_\tau + (1-\delta) K_\tau$

–The first-order conditions [for the planning period t] are:

$$W_t = \left(\frac{1-\epsilon_C}{1-L_t}\right) / \left(\frac{\epsilon_C}{C_t}\right) \quad \text{(a)}$$

$$\left(\frac{\epsilon_C}{C_t}\right) = E_t\left(\frac{1+r_{t+1}}{1+\rho}\right)\left(\frac{\epsilon_C}{C_{t+1}}\right) \quad \text{(b)}$$

$$r_{t+1} \equiv R^K_{t+1} - \delta \quad \text{(c)}$$

(a) [static] The MRS between consumption and leisure should be equated to the Wage rate

(b) [dynamic] The stochastic consumption Euler equation: the MU of consumption

In the planning period (C_t) should be equated to the expected weighted MU of

Consumption one period later (C_{t+1}).

(c) [definition] The real interest rate is the rental rate minus the depreciation rate

- The full model is given in log-linearized form in **Table1**.

Table1.The log-linearized stochastic model

$$\tilde{K}_{t+1} - \tilde{K}_t = \delta \left[\tilde{I}_t - \tilde{K}_t \right] \tag{T4.1}$$

$$E_t \tilde{C}_{t+1} - \tilde{C}_t = \left(\frac{\rho}{1+\rho} \right) E_t \tilde{r}_{t+1} \tag{T4.2}$$

$$\tilde{G}_t = \tilde{T}_t \tag{T4.3}$$

$$\tilde{W}_t = \tilde{Y}_t - \tilde{L}_t \tag{T4.4}$$

$$\rho \tilde{r}_t = (\rho + \delta) \left[\tilde{Y}_t - \tilde{K}_t \right] \tag{T4.5}$$

$$\tilde{Y}_t = \omega_C \tilde{C}_t + \omega_I \tilde{I}_t + \omega_G \tilde{G}_t \tag{T4.6}$$

$$\tilde{L}_t = \omega_{LL} \left[\tilde{W}_t - \tilde{C}_t \right] \tag{T4.7}$$

$$\tilde{Y}_t = \tilde{Z}_t + \epsilon_L \tilde{L}_t + (1 - \epsilon_L) \tilde{K}_t \tag{T4.8}$$

- a part from the fact that the model is now in discrete time, it looks virtually identical to the deterministic model.

- because general technology is stochastic, so is the future interest rate. For that Reason, $E_t \tilde{r}_{t+1}$ Appears in the log-linearized Euler equation. Recall:

$$r_{t+1} = F_K\left[\underbrace{Z_{t+1}}_{(a)}, \underbrace{K_{t+1}}_{(b)}, \underbrace{L_{t+1}}_{(c)}\right] - \delta$$

(a) future general technology; unknown in period t [but maybe partially forecastable If the shock is persistent (seebelow)]

(b) future capital stock; known in period t As it depends only on present accumulation decisions

(c) future labour supply; unknown in period t as it depends on W_{t+1} and C_{t+1} and thus on Z_{t+1}

- The specification of the model is completed once the stochastic process for general productivity is specified. The commonly used specifications is first-order autoregressive:

$$\begin{aligned} \log Z_t &= \alpha_Z + \rho_Z \log Z_{t-1} + \epsilon_t^Z, \qquad 0 < \rho_Z < 1, \qquad \Longrightarrow \\ \tilde{Z}_t &= \rho_Z \tilde{Z}_{t-1} + \epsilon_t^Z \end{aligned}$$

where $\tilde{Z}_t \equiv \log[Z_t/Z]$ and:

  - ρ_Z is the *degree of persistence* of the shock [special cases: $\rho_Z = 0$ purely transitory shock; $\rho_Z = 1$ permanent shock]
  - ϵ_t^Z is the stochastic *innovation term* [identically and independently distributed with mean zero and variance σ_Z^2]
  - if ρ_Z is nonzero, general productivity in the next period is partially forecastable. Under REH the agents best forecast is:

$$E_t \tilde{Z}_{t+1} = \rho_Z \tilde{Z}_t$$

(since $E_t \epsilon_{t+1}^Z = 0$)

- The loglinearized model in Table1 can be solved under the REH. We can use two methods.The easiest of these looks directly at so-called impulse-response Functions for the different variables. Key idea:

  - assume that the system is initially in steady state and trace the effect of a single innovation at time $t = 0$: $\epsilon_0^Z > 0$ and $\epsilon_t^Z = 0$ for $t = 1, 2, \ldots$. We call ϵ_0^Z the *impulse* hitting the economic system.
  - compute the implied *response* of the different variables to the impulse.
  - in the text we derive the general case for which $0 < \rho_Z < 1$. To understand the general result it pays to look at the special cases.

- **A purely temporary shock**: ρ_Z =0 .The impulse-response functions for this type of shock Are given in **Figure 2**. Salient features:

  –no long-run effect on general productivity and thus no long-run effect on any variable

  –productivity only higher than normal in period t=0

  –agents are a little richer and thus $C_0 \uparrow$,and $I_0 \uparrow$ [agents spread gain over present and Future consumption]

  –strong incentive to work when productivity is high: $W_0 \uparrow$, $(1-L_0)\downarrow$, $L_0 \uparrow$, $Y_0 \uparrow$ (See **Figure 1**)

  – for t=1,2,3... general productivity back to normal. Agent gradually runs down extra Savings by consuming more than normal: $C_t \downarrow$, $K_t \downarrow$, Y_t, L_t,and I_t Almost back to Normal

  –NOTE: output response looks virtually identical to impulse [lack of internal propagation]

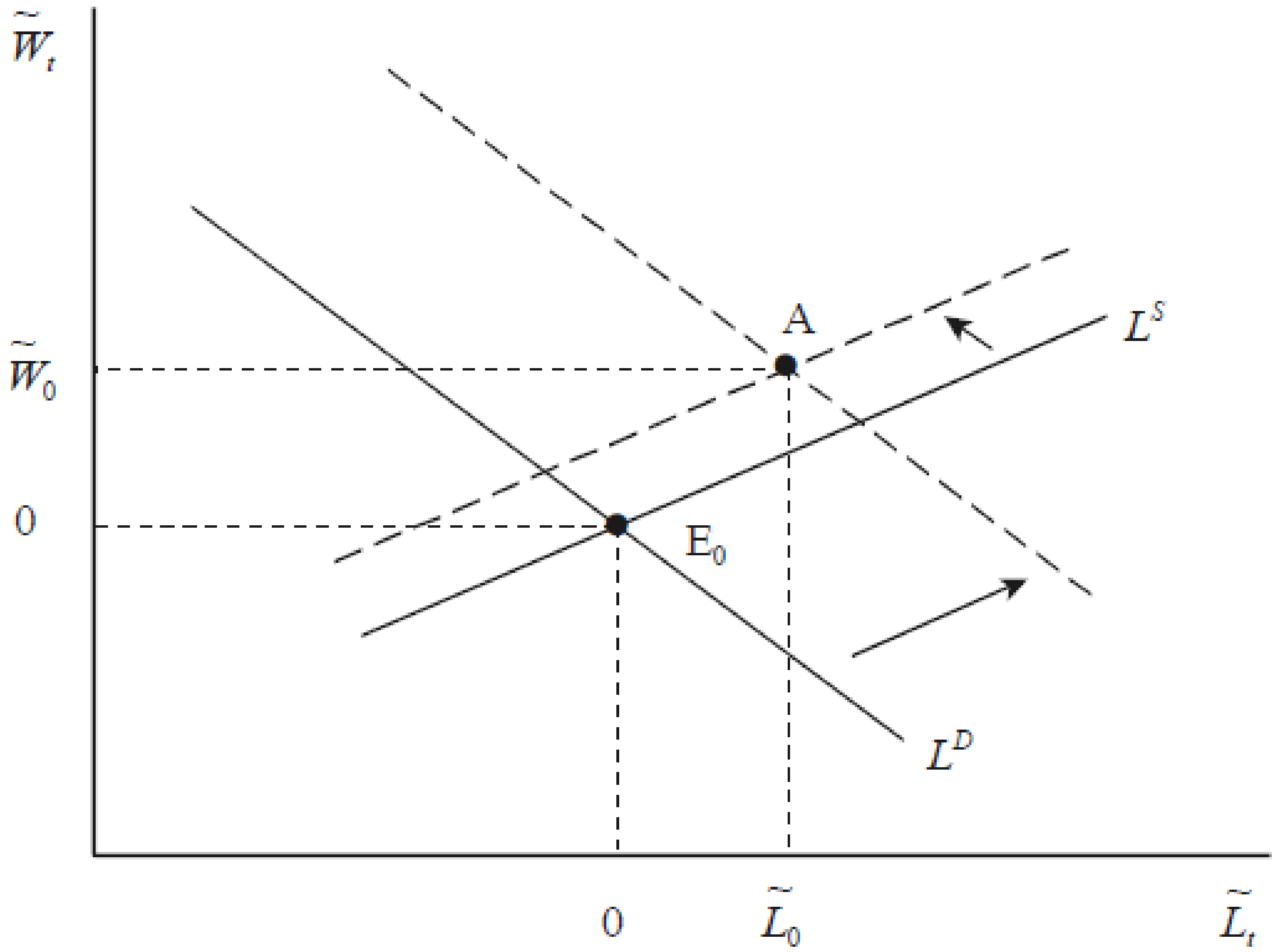


Figure1:A Shock to Technology and the Labour Market

Figure 2 Purely Transitory Productivity Shock

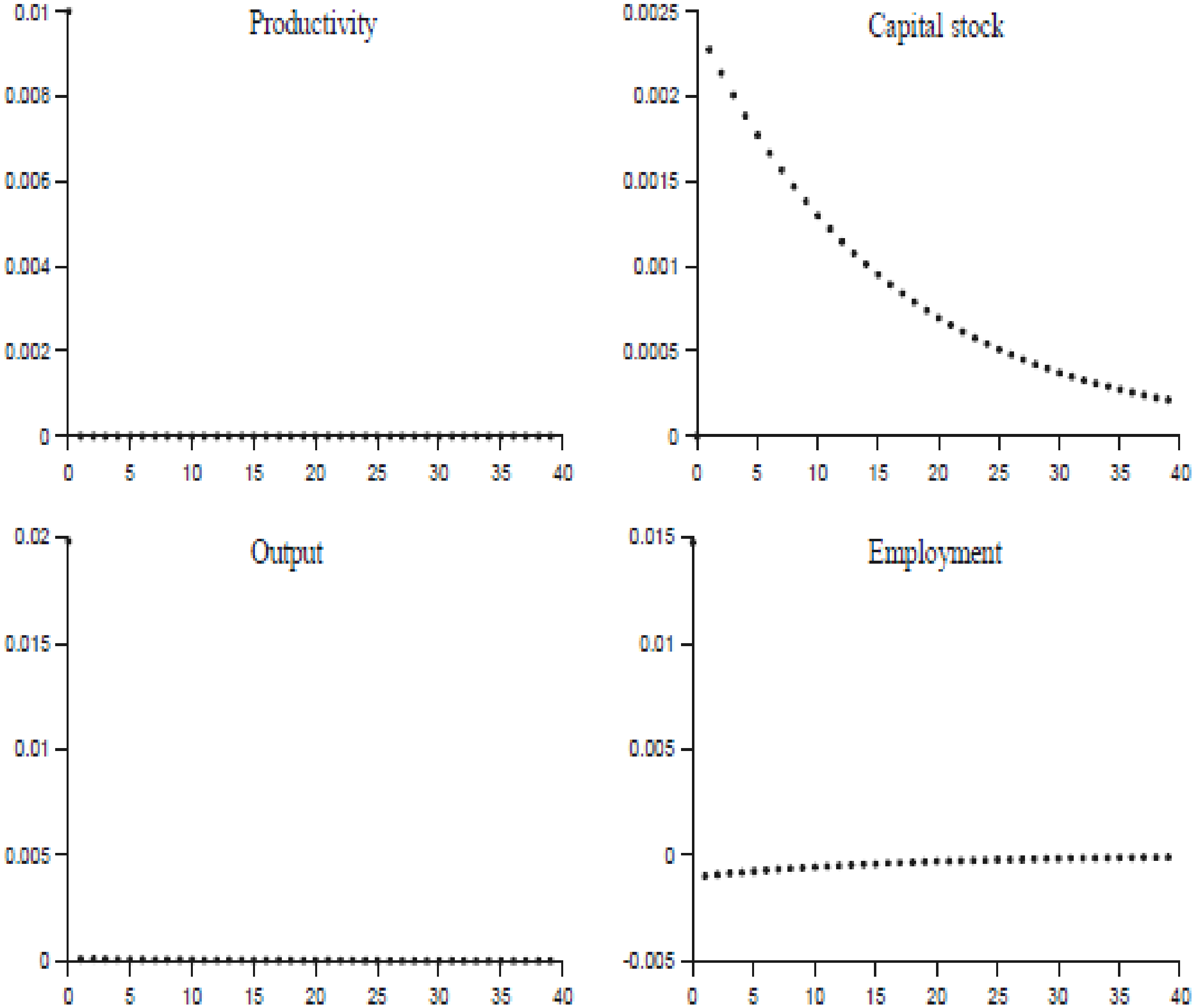
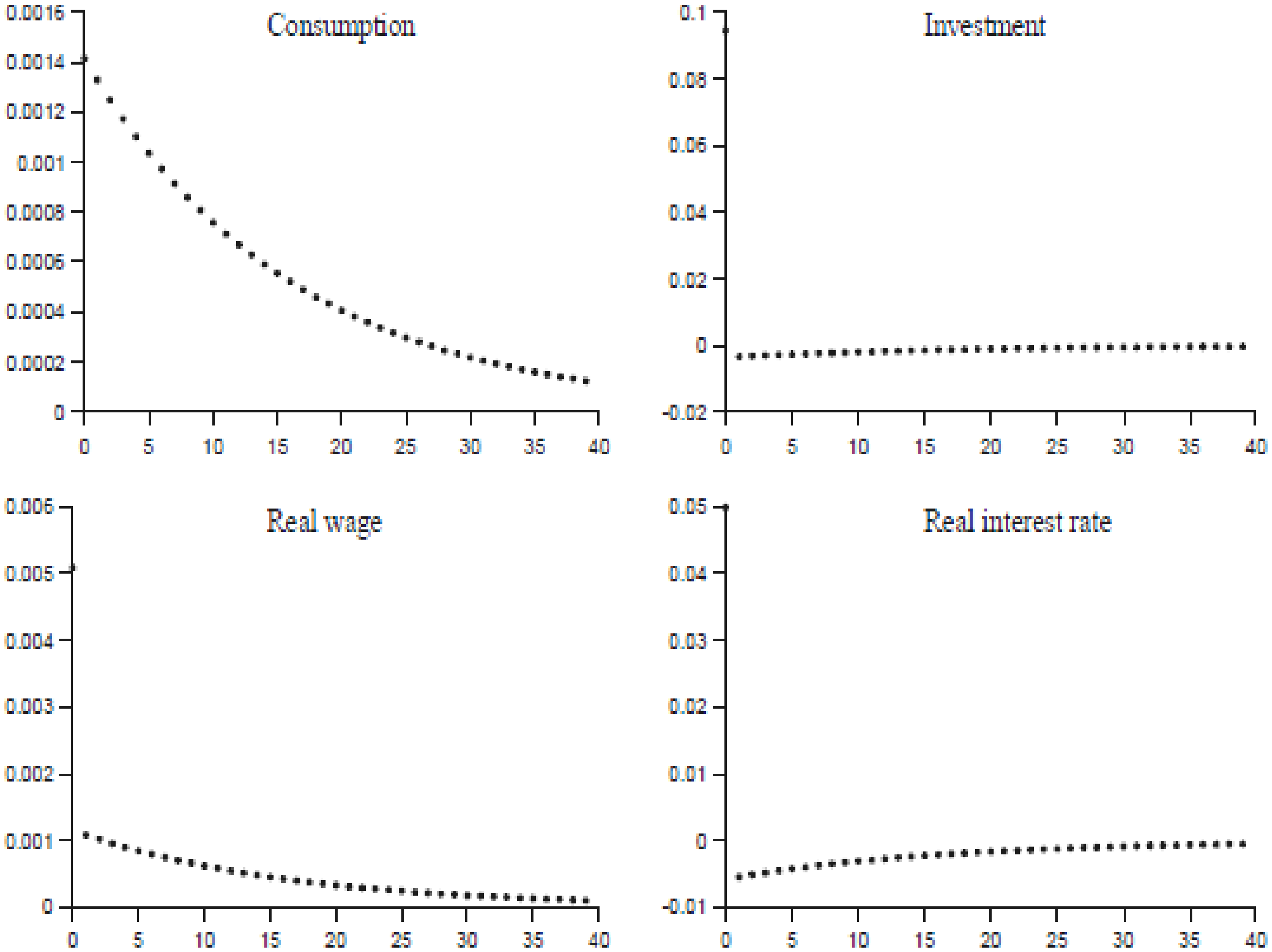


Figure 2 Purely Transitory Productivity Shock (continued)



- **A purely permanent shock**: ρ_Z =1 .The impulse-response functions for this type of shock Are given in **Figure 3**. Salient features:
  - there is a long-run effect on productivity and thus on most macro variables: the great ratios explain that $Y_\infty \uparrow, C_\infty \uparrow, K_\infty \uparrow, I_\infty \uparrow$,and $L_\infty \downarrow$ (if $\omega_G > 0$ So that IE effect Dominates SE in labour supply)
  - agents are a lot richer and thus $C_0 \uparrow$,and $I_0 \uparrow$ [agents spread gain over present and Future consumption]
  - even though $W0 \uparrow$and SE says $L_0 \uparrow$, There is a smaller upward jump in employment (than for temporary shock) because IE says $L_0 \downarrow$
  - for t=1,2,3... general productivity stays high. Agent gradually keep accumulating Capital and consumption continues to rise: $C_t \uparrow$, $K_t \uparrow$, $L_t \downarrow$,and $I_t \downarrow$
  - NOTE: output response again looks virtually identical to impulse [lack of internal propagation]

Figure 3. Permanent Productivity Shock

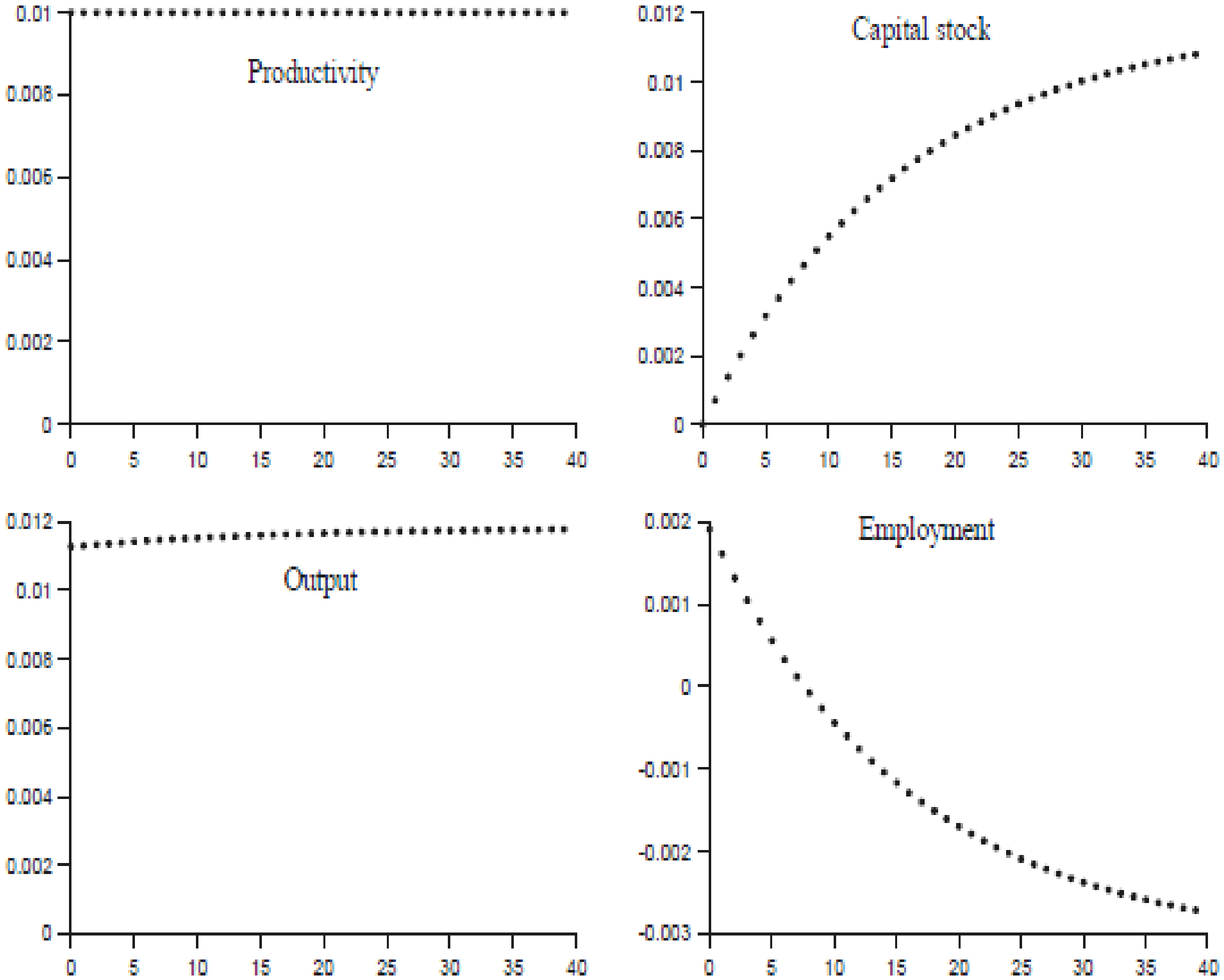
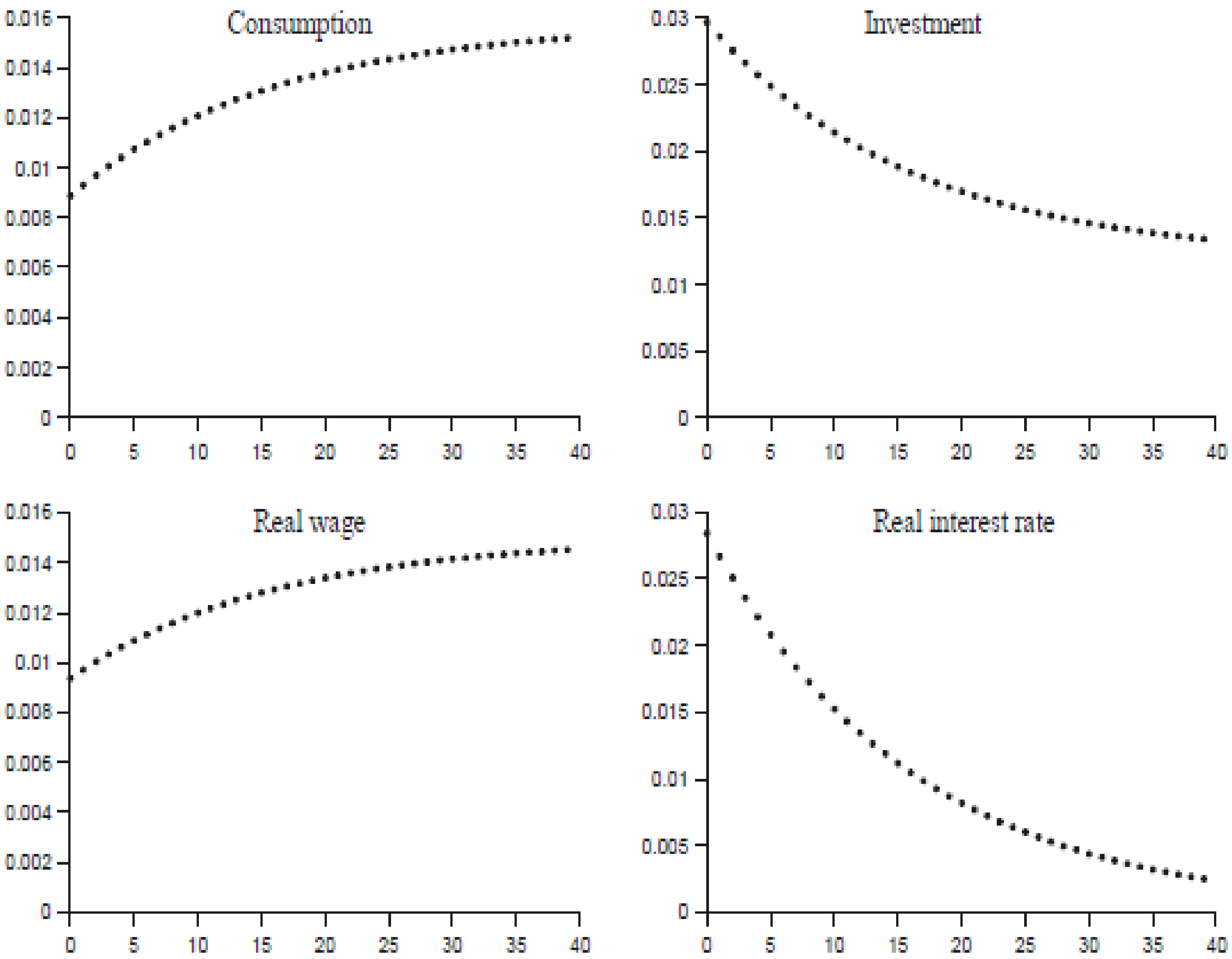


Figure 3. Permanent Productivity Shock (continued)



- What would a **realistic shock** look like? The seminal work by Solow(1957) has been used to estimate the nature of technological change. Solow residual : compute how much of output growth can be explained by growth in inputs.The remainder is Now called the Solow residual.
  - In our model the Solow residual is equal to the general productivity index Z_t:

$$\log SR_t \equiv \log Y_t - \epsilon_L \log L_t - (1 - \epsilon_L) \log K_t = \log Z_t.$$

We can obtain estimates for α_Z, ρ_Z, and σ_Z^2 [the variance of ϵ_t^Z] by regressing:

$$\log SR_t = \alpha_Z + \rho_Z \log SR_{t-1} + \epsilon_t^Z$$

  - For the US one finds:

$$\hat{\rho}_Z = 0.979$$

which means that the technology shocks are not permanent but nevertheless display a very high degree of persistence.

– In **Figures 4- 10** we show the different impulse-response functions for a whole range of ρ_Z Values (including the realistic one).The key thing to note is the highly nonlinear behaviour of the IR functions for values of ρ_Z Near unity.

- Although the impulse-response functions display a lot of information about the Model, most RBC modellers judge the performance of their model by looking at the Match between model-generated and actual statistics. In **Table 2** we show an Example of this approach. The standard model yields the results in panel (b) whilst Actual statistics for the US economy are found in panel (a). Salient features:

  – model captures that $\sigma(I_t)$ higher $\sigma(Y_t)$, $\sigma(C_t) < \sigma(Y_t)$

  – model more or less matches $\rho(C_t, Y_t)$, $\rho(I_t, Y_t)$, $\rho(K_t, Y_t)$, and $\rho(L_t, Y_t)$, but Overstates $\rho(Y_t/L_t, Y_t)$.

  – given the simple structure of the model, the fit is quite impressive

  – ....but recall the lack of propagation [explanation is almost entirely exogenous]

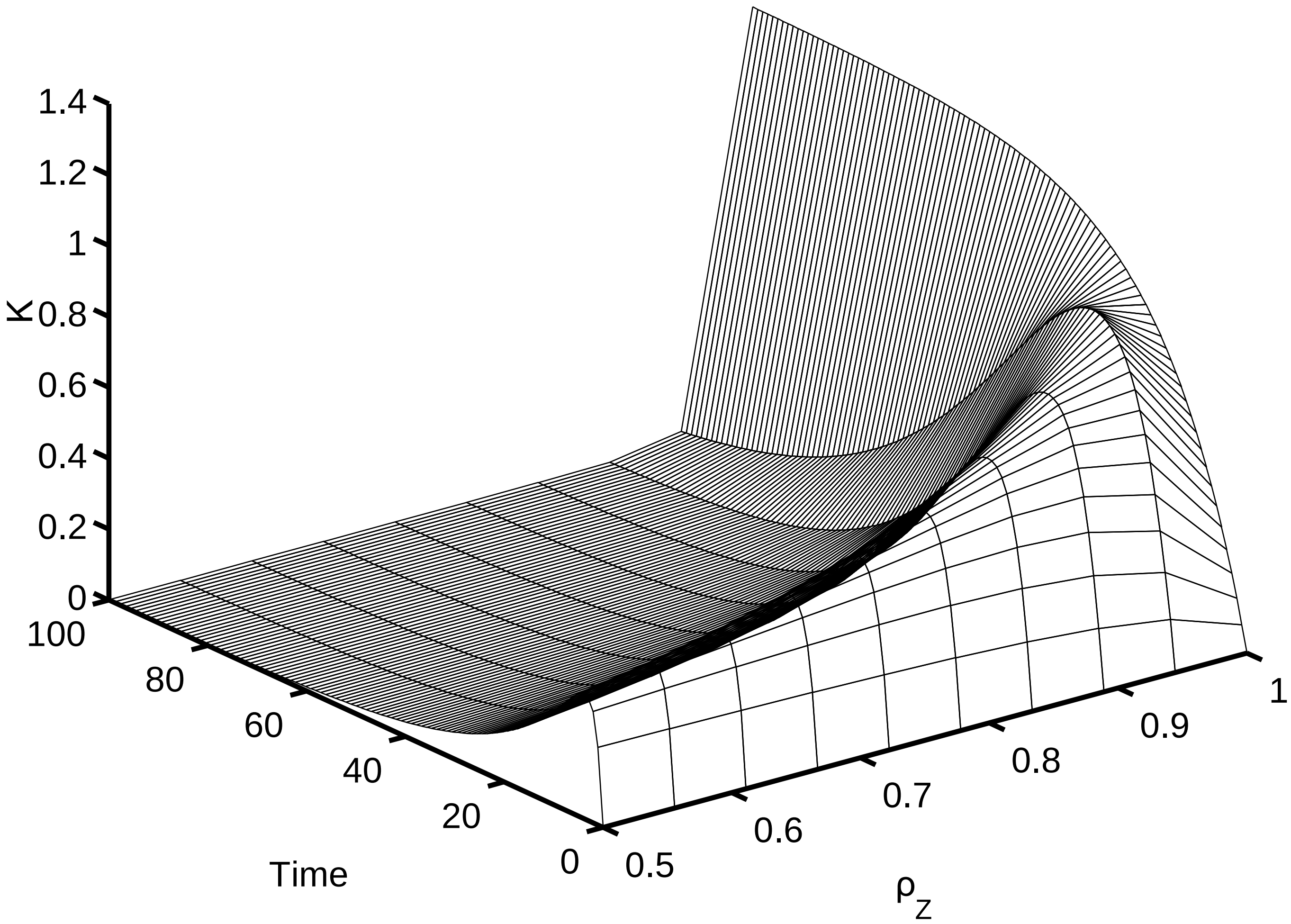


Figure 4:Capital stock

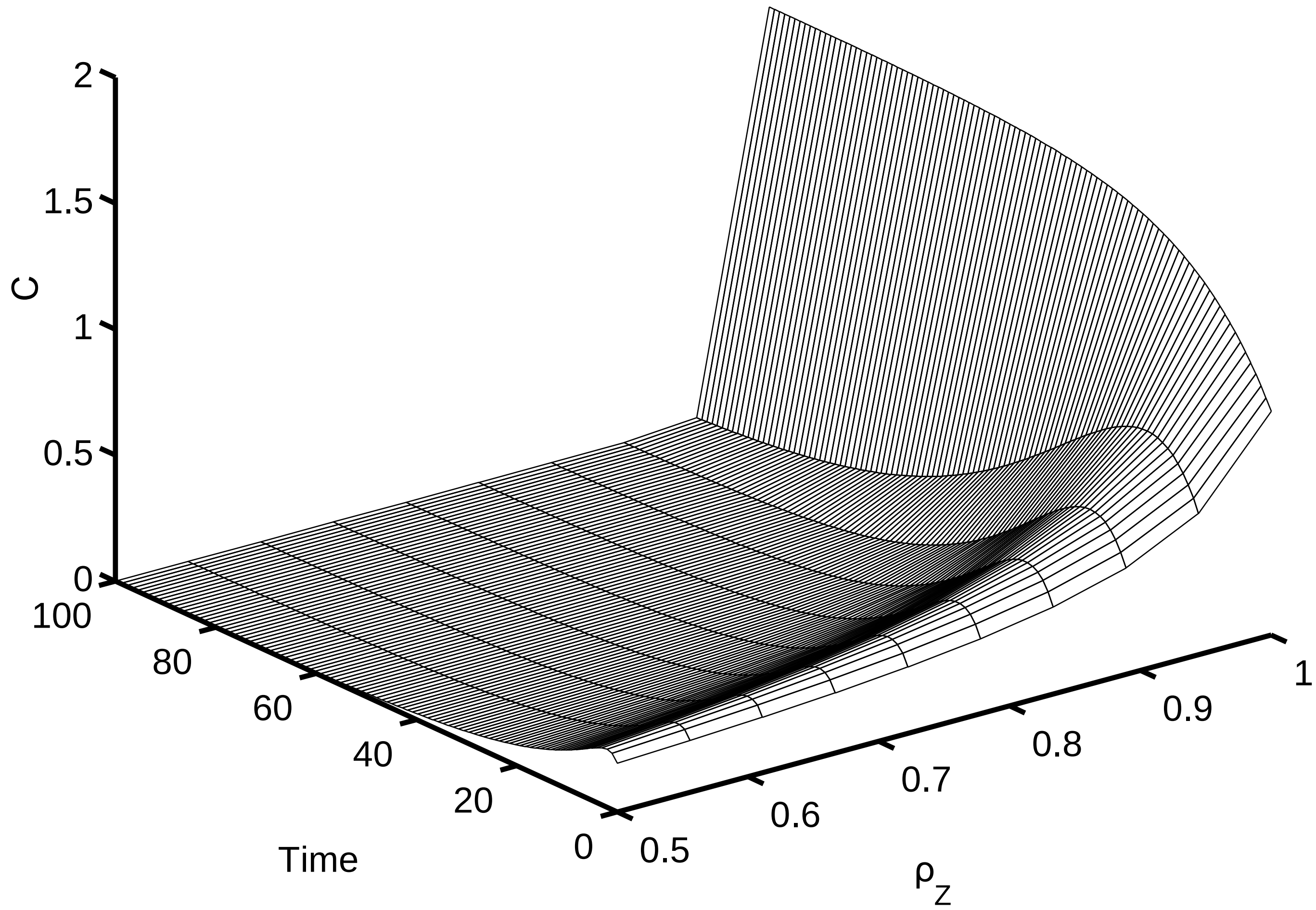


Figure 5: Consumption

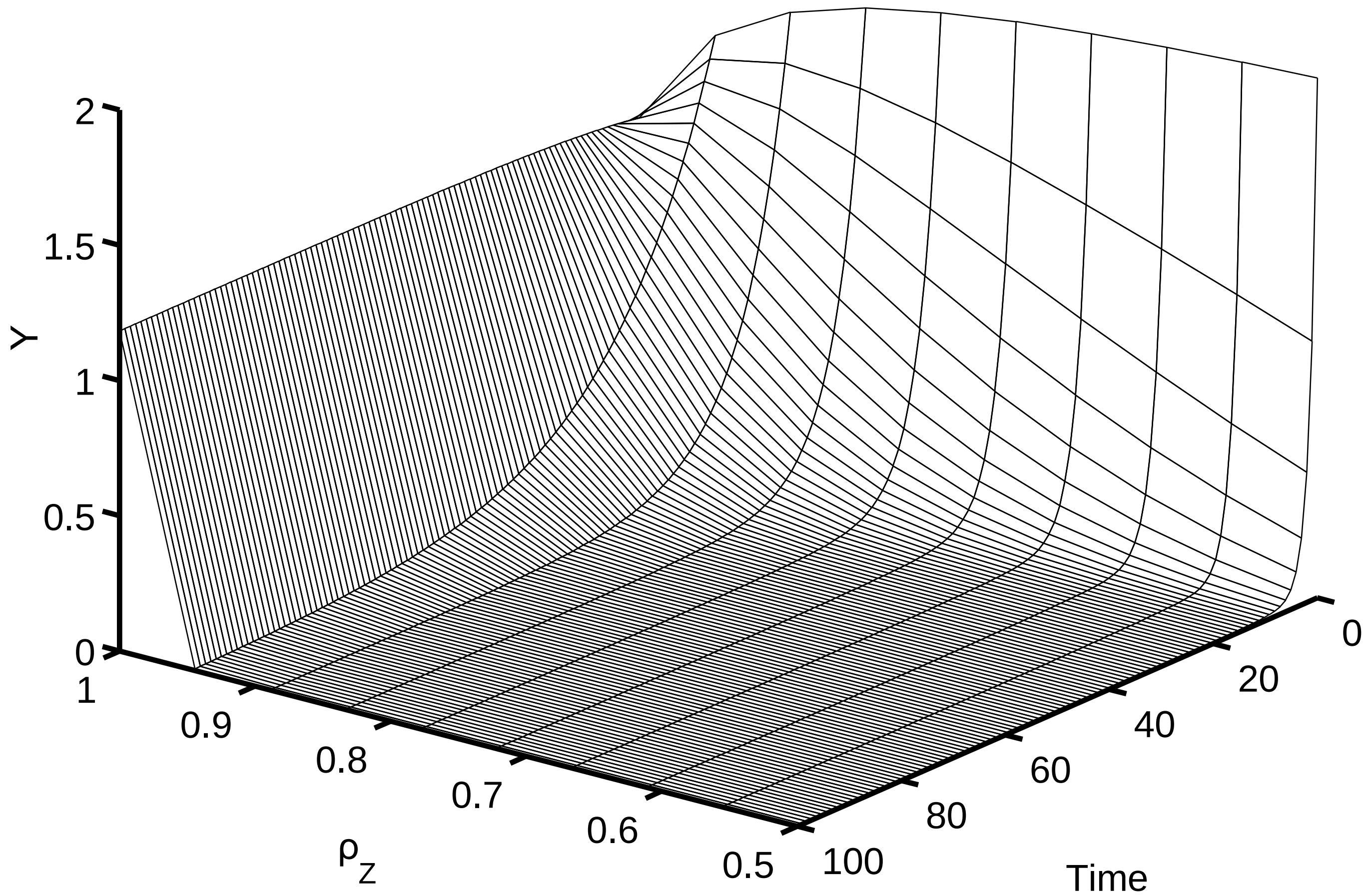


Figure 6: Output

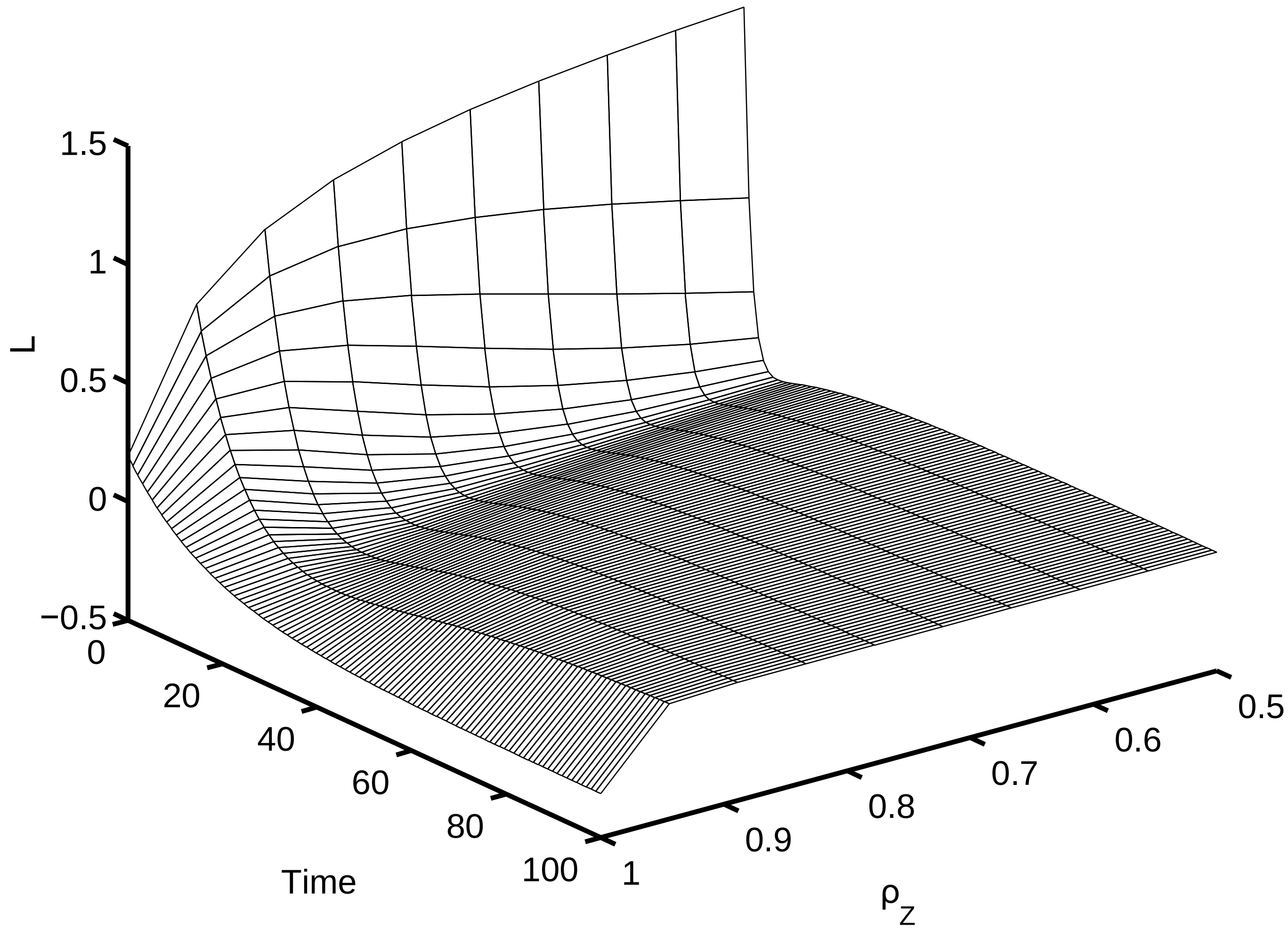


Figure 7: Employment

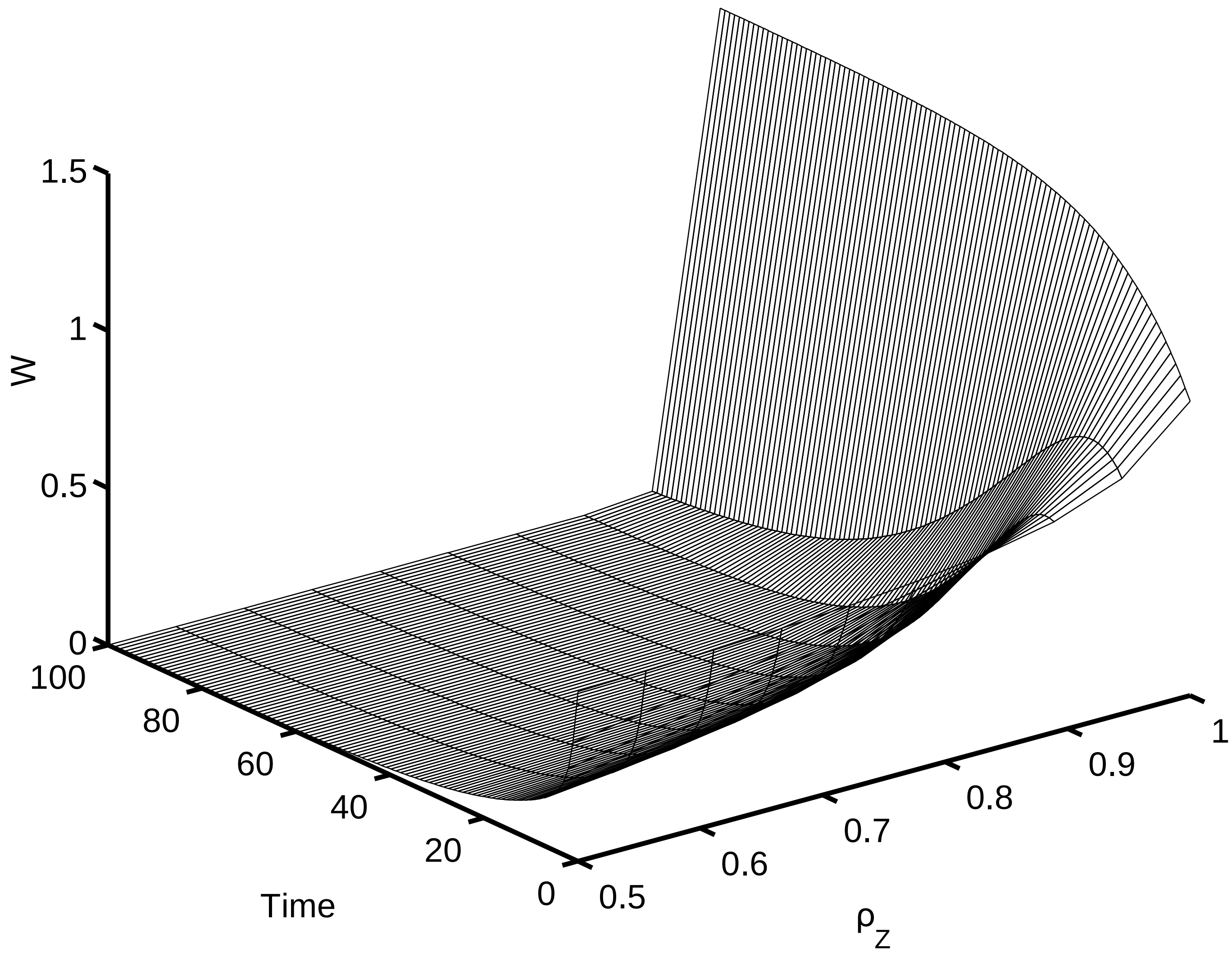


Figure 8: Wage

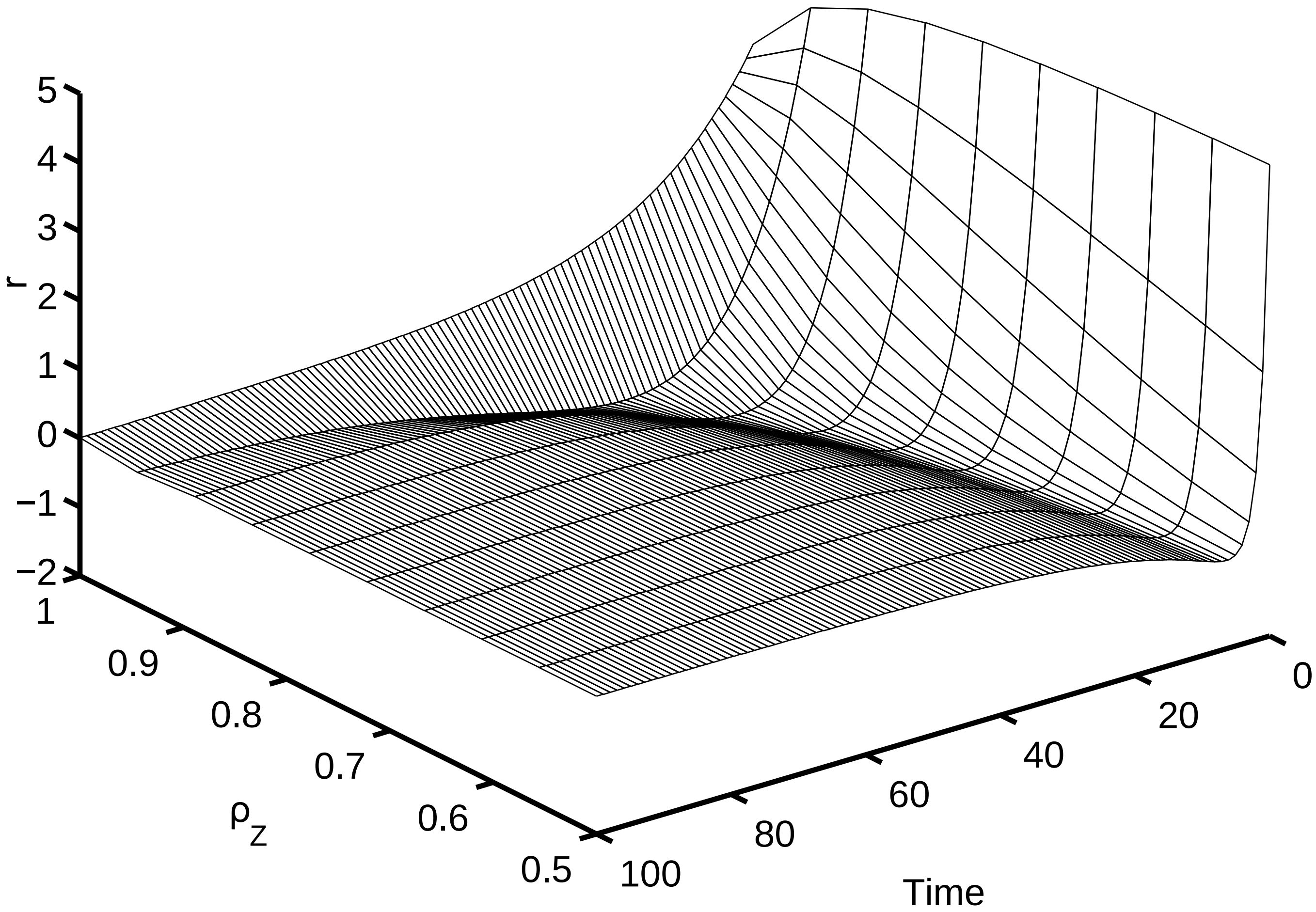


Figure 9: Interest Rate

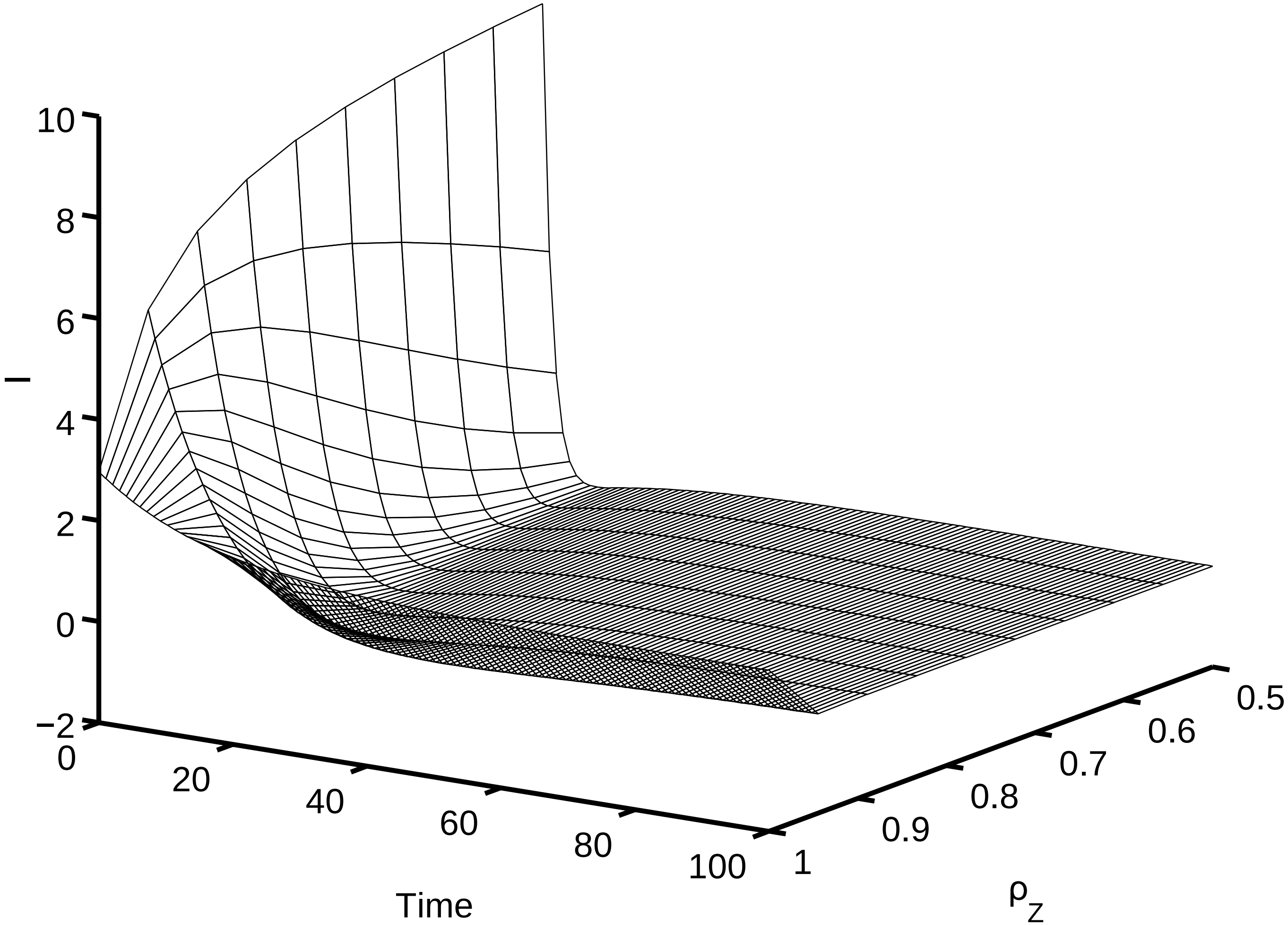


Figure 10: Investment

Table 2.The unit-elastic RBC model

	(a)US economy		(b)Model economy I		(c)Model economy II	
X_t:	$\sigma(x_t)$	$\rho(x_t, Y_t)$	$\sigma(x_t)$	$\rho(x_t, Y_t)$	$\sigma(x_t)$	$\rho(x_t, Y_t)$
Y_t	1.76		1.35		1.76	
C_t	1.29	0.85	0.42	0.89	0.51	0.87
I_t	8.60	0.92	4.24	0.99	5.71	0.99
K_t	0.63	0.04	0.36	0.06	0.47	0.05
L_t	1.66	0.76	0.70	0.98	1.35	0.98
Y_t/L_t	1.18	0.42	0.68	0.98	0.50	0.87

- A number of puzzles remain. Solving these puzzles is at the forefront of current Research in the area:

(A) employment variability puzzle

(B) pro-cyclical real wage

(C) productivity puzzle

(D) unemployment

(E) monetary aspects

(A) Employment variability puzzle

- Key idea: In reality $\sigma(Y_t)$ Close to $\sigma(L_t)$,employment strongly pro-cyclical [$\rho(L_t, Y_t)$ near unity], and wages a-cyclical or mildly pro-cyclical [$\rho(W_t, Y_t)$ near Zero]. In the model:
  - With productivity shocks: ϵ_t^Z shifts labour demand, given upward sloping labour supply both W_t and L_t should be pro-cyclical.
  - With low labour supply elasticity [micro-evidence] $\sigma(L_t)$ should be low and $\sigma(W_t)$ should be high
  - Hence, model under- predicts $\sigma(L_t)$ By quite a margin!
  - see **Figures A and B** to visualize correlations

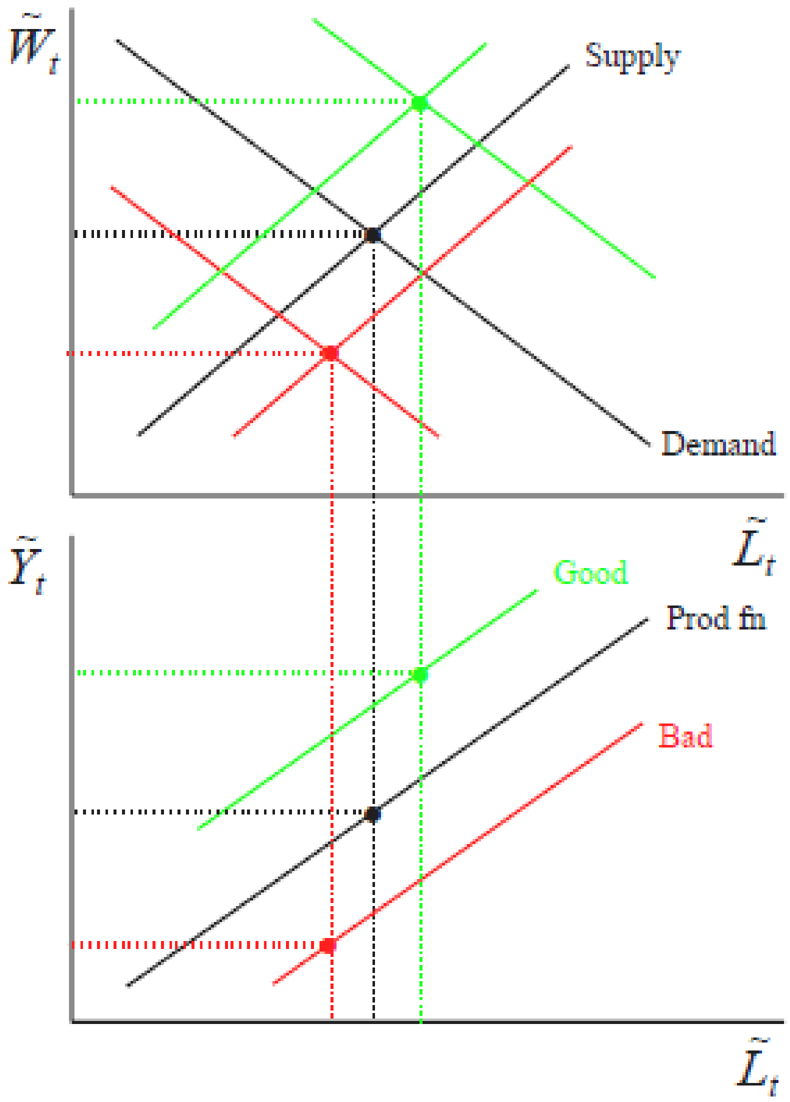


Figure A: TheGood, theBad, and the Average

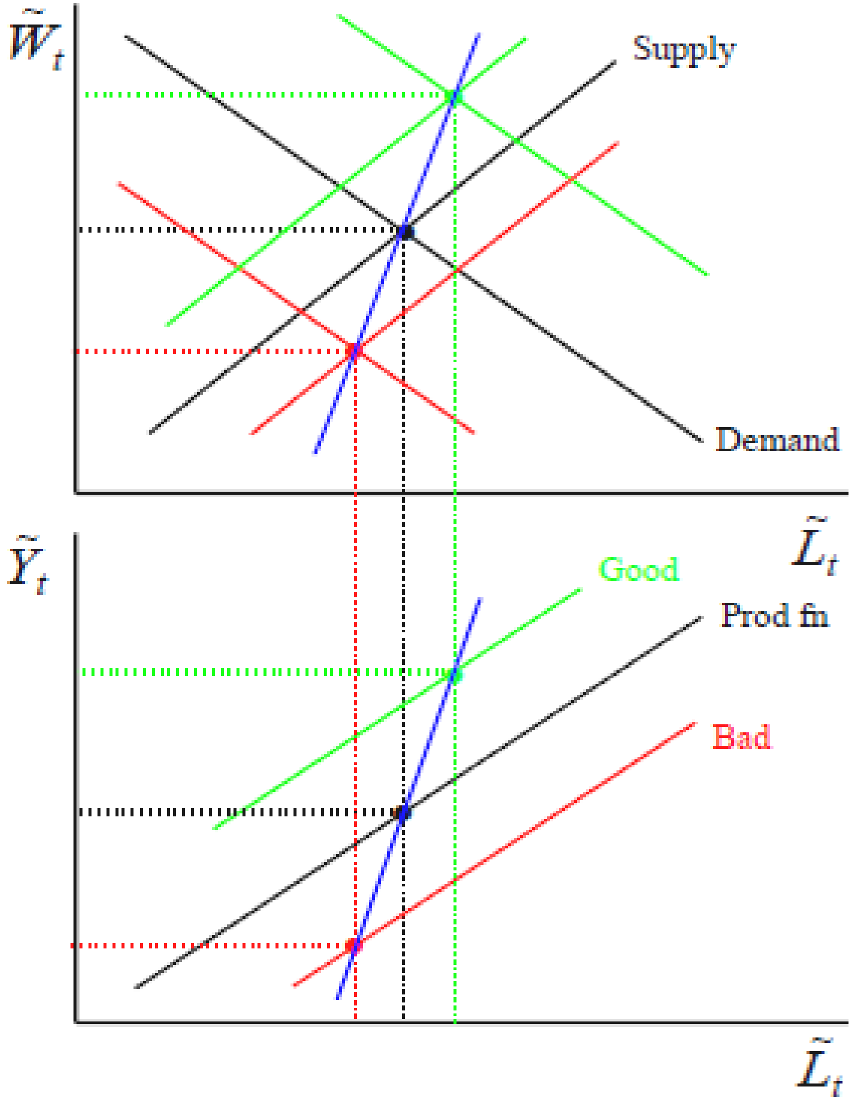


Figure B: Visualizing Contemporaneous Correlations

- Solution to the puzzle: we need a high substitution elasticity of labour supply [near Horizontal labour supply curve] despite micro-evidence to the contrary. Indivisible Labour model:
  - –you either work 8 hours per day or 0 hours per day.
  - – lottery determines which is which each period
  - – firm provides full insurance to the worker, and aggregate outcome is as if the Representative agent has an infinite intertemporal substitution elasticity of labour Supply
  - –see **Figureb C**
  - –In Table2 panel (c), we observe that the lottery [or indivisible labour] model does better than the unit elastic model at matching $\sigma(L_t)$

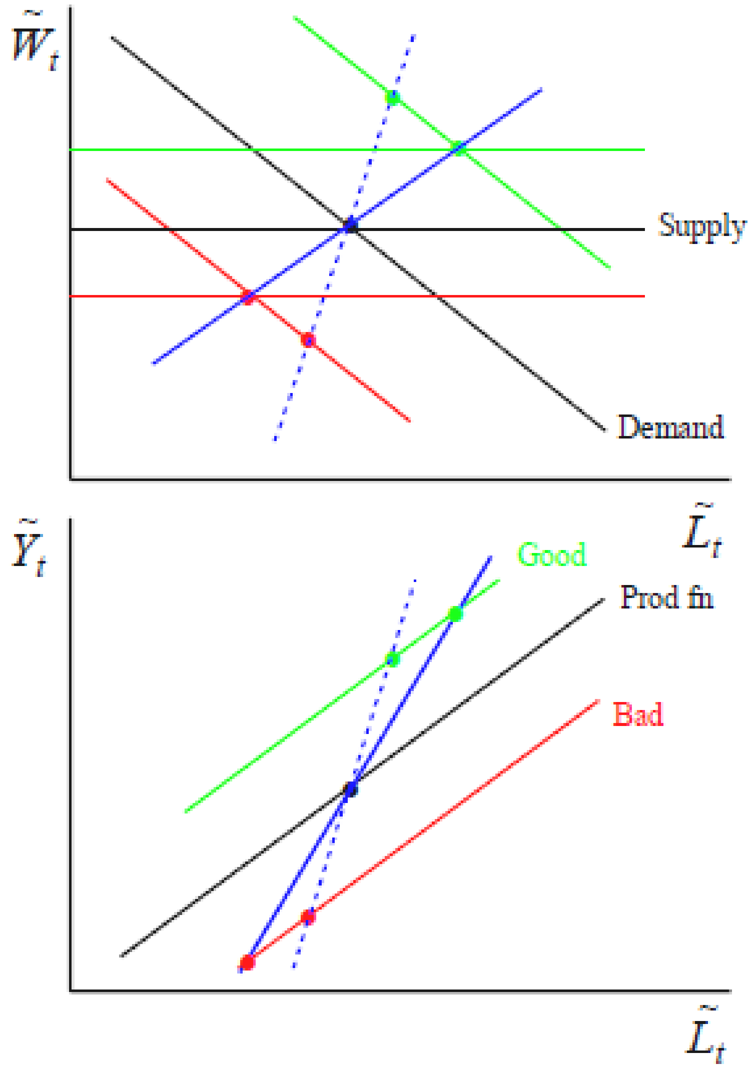


Figure C: Contemporaneous Correlations in the Lottery Model

(B) Pro-cyclical real wage

- Key idea: the unit-elastic model predicts too high correlation between labour Productivity [the wage] and output, $\rho(Y_t/L_t, Y_t)$=0.98 In Hansen model we have $\rho(Y_t/L_t, Y_t)$=0.78 In reality this correlation is much lower (0.42 For US).
- Solution(s) of the puzzle:
  - –introduce shift factors in the labour supply function [both L^D and L^S Shift out]
  - –see **FigureD**
  - –use any of the theories explaining real wage rigidity [efficiency wages, union model, etcetera]

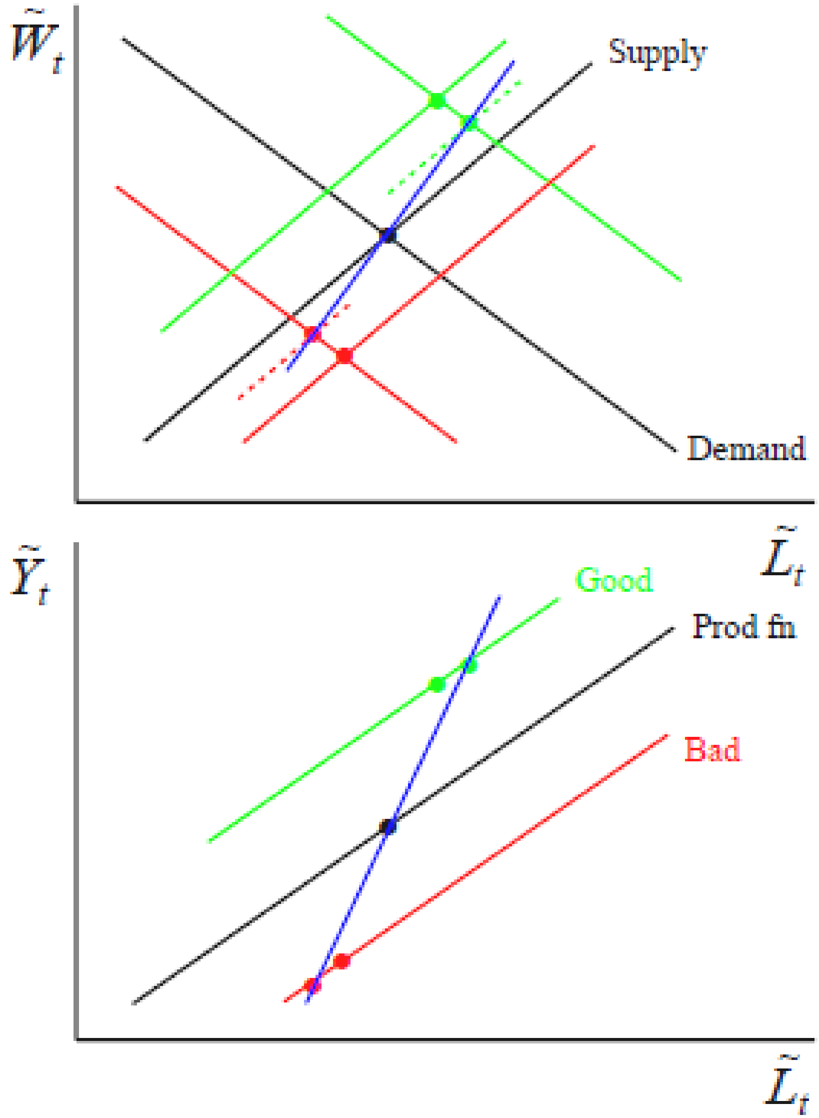


Figure D: Contemporaneous Correlations and Shift Factors

(C) Productivity puzzle

- Key idea: if productivity shocks are predominant then L^D Shifts explain high $\rho\,(Y_t/L_t, L_t)$ and $\rho(Y_t/L_t, Y_t)$.In reality $\rho(Y_t/L_t, L_t)\approx 0$ and $\rho(Y_t/L_t, Y_t)$ is Weaker than predicted.

- Solution(s) of the puzzle:
  - introduce shift factors in the labour supply function [both L^D and L^S Shift out]
  - nominal wage contracts and money supply shocks (nominal innovation shifts effective labour supply)
  - labour hoarding by firms
  - non-market sector also subject to technology shocks
  - preference shocks affecting labour supply
  - shocks to government spending

(D) Unemployment

- Key idea: the standard RBC models assume market clearing in the labour market. Hence all variation in employment is due to adjustment in hours worked. In reality 2/3 is explained by the extensive margin [in/out of employment] and 1/3 by the Intensive margin [over time etcetera].

- Solution(s) to the puzzle: introduce unemployment model in the RBC framework, such as:

  – search-theoretic approach

  – efficiency wage theory, union models

Evaluation of the RBC approach

- The standard RBC model has a hard time matching data for real economies
- It is difficult to believe that the productivity shocks explain all fluctuations in the economy: “if They are so important then why don’t we read about them in the Wall Street Journal”
- Link between micro-data and calibration values not strong
- Most important contribution of the approach is a methodological one:
  - approach is flexible
  - micro-foundations for macro are important and can be improved [alternative market structures,heterogeneous households, etcetera]
  - other shocks can be introduced [government spending shocks, tax shocks, changes in The real exchange rate, etcetera]
- Hence, the RBC approach is worth pursuing!!