Appendix 4.A Continuous-Time Fourier Transform : Properties

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega \quad \longleftrightarrow \quad X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Property	Signal	Fourier transform
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha X(\omega) + \beta Y(\omega)$
Shift in time	$x(t - t_0)$	$e^{-jt_0\omega}X(\omega)$
Shift in frequency	$e^{j\omega_0 t}x(t)$	$X(\omega - \omega_0)$
Scaling in time and frequency	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Time reversal	$x(-t)$	$X(-\omega)$
Differentiation in time	$\frac{d^n}{dt^n}x(t)$	$(j\omega)^n X(\omega)$
Differentiation in frequency	$(-jt)^n x(t)$	$\frac{d^n}{d\omega^n}X(\omega)$
Integration in time	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{j\omega}X(\omega)$, assuming $X(0) = 0$.
Convolution in time	$(x * y)(t)$	$X(\omega)Y(\omega)$
Convolution in frequency	$x(t)y(t)$	$\frac{1}{2\pi}(X * Y)(\omega)$
Conjugate	$x^*(t)$	$X^*(-\omega)$
Conjugate, time-reversed	$x^*(-t)$	$X^*(\omega)$
Conjugate symmetry	$x(t)$ real-valued	$X(\omega) = X^*(-\omega)$ which implies $\lvert X(\omega)\rvert = \lvert X(-\omega)\rvert$
	$x(t)$ real and even *i.e.*, $x(t) = x(-t)$	$X(\omega)$ real and even *i.e.*, $X(\omega) = X(-\omega)$
Parseval's Equality	$\int_{-\infty}^{+\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \lvert X(\omega)\rvert^2 d\omega$	

Appendix 4.B Continuous-Time Fourier Transform : Pairs

	Signal	**Fourier transform**
Dirac delta function	$x(t) = \delta(t)$ $x(t) = \delta(t - t_0)$	$X(\omega) = 1$ $X(\omega) = e^{-jt_0\omega}$
Dirac comb	$x(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$X(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$
Constant function Harmonics	$x(t) = 1$ $x(t) = e^{j\omega_0 t}$ $x(t) = \cos(\omega_0 t)$ $x(t) = \sin(\omega_0 t)$	$X(\omega) = 2\pi\delta(\omega)$ $X(\omega) = 2\pi\delta(\omega - \omega_0)$ $X(\omega) = \pi\left(\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right)$ $X(\omega) = \frac{\pi}{j}\left(\delta(\omega - \omega_0) - \delta(\omega + \omega_0)\right)$
Step function	$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0. \end{cases}$	$U(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$
One-sided exponential with $Re(a) > 0$ for integers $n \geq 2$	$x(t) = e^{-at}u(t)$ $x(t) = \frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$X(\omega) = \frac{1}{a + j\omega}$ $X(\omega) = \frac{1}{(a + j\omega)^n}$
Two-sided exponential	$x(t) = e^{-a\lvert t\rvert}$ with $Re(a) > 0$	$X(\omega) = \frac{2a}{a^2 + \omega^2}$
Sinc function	$x(t) = \sqrt{\frac{\omega_0}{2\pi}}\,\mathrm{sinc}\left(\frac{\omega_0}{2\pi}t\right)$ where $\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$	$X(\omega) = \begin{cases} \sqrt{\frac{2\pi}{\omega_0}}, & \lvert\omega\rvert \leq \frac{1}{2}\omega_0 \\ 0, & \text{otherwise.} \end{cases}$
Box function	$b(t) = \begin{cases} \frac{1}{\sqrt{t_0}}, & \lvert t\rvert \leq \frac{1}{2}t_0, \\ 0, & \text{otherwise.} \end{cases}$	$B(\omega) = \sqrt{t_0}\,\mathrm{sinc}\left(\frac{t_0}{2\pi}\omega\right)$

Appendix 4.C Discrete-Time Fourier Transform : Properties

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega \quad \longleftrightarrow \quad X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

Property	**Signal**	**Fourier transform**
Linearity	$\alpha x[n] + \beta y[n]$	$\alpha X(e^{j\omega}) + \beta Y(e^{j\omega})$
Shift in time	$x[n-n_0]$	$e^{-j\omega n_0}X(e^{j\omega})$
Shift in frequency	$e^{j\omega_0 n}x[n]$	$X(e^{j(\omega-\omega_0)})$
Time Reversal	$x[-n]$	$X(e^{-j\omega})$
Differentiation in Frequency	$nx[n]$	$j\dfrac{dX(e^{j\omega})}{d\omega}$
Convolution in time	$(x*y)[n]$	$X(e^{j\omega})Y(e^{j\omega})$
Circular convolution in frequency	$x[n]y[n]$	$\dfrac{1}{2\pi}\displaystyle\int_{2\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$
Conjugate	$x^*[n]$	$X^*(e^{-j\omega})$
Conjugate, time-reversed	$x^*[-n]$	$X^*(e^{j\omega})$
Conjugate symmetry	$x[n]$ real-valued	$X(e^{j\omega}) = X^*(e^{-j\omega})$ which implies $\lvert X(e^{j\omega})\rvert = \lvert X(e^{-j\omega})\rvert$
	$x[n]$ real and even *i.e.*, $x[n] = x[-n]$	$X(e^{j\omega})$ real and even *i.e.*, $X(e^{j\omega}) = X(e^{-j\omega})$

Parseval's Relation for Aperiodic Signals

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{2\pi} |X(e^{j\omega})|^2 d\omega$$

Appendix 4.D Discrete-Time Fourier Transform : Pairs

	Signal	Fourier transform
Kronecker delta	$\delta[n]$ $\delta[n-n_0]$	1 $e^{-jn_0\omega}$
Constant	$x[n]=1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega-2\pi l)$
Harmonics	$x[n]=e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega-\omega_0-2\pi l)$
Step function	$u[n]=\begin{cases} 1, & n\geq 0 \\ 0, & \text{otherwise.}\end{cases}$	$\frac{1}{1-e^{-j\omega}}+\sum_{k=-\infty}^{+\infty} \pi\delta(\omega-2\pi k)$
One-sided exponential	$x[n]=\begin{cases} \alpha^n, & n\geq 0 \\ 0, & \text{otherwise.}\end{cases}$ with $\lvert\alpha\rvert<1$	$\frac{1}{1-\alpha e^{-j\omega}}$
"Arithmetic- -geometric"	$x[n]=\begin{cases} n\alpha^n, & n\geq 0 \\ 0, & \text{otherwise.}\end{cases}$ with $\lvert\alpha\rvert<1$	$\frac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2}$
Sinc sequence	$\sqrt{\frac{\omega_0}{2\pi}}\,\mathrm{sinc}\left(\frac{\omega_0}{2\pi}n\right)$ where $\mathrm{sinc}(x)=\frac{\sin\pi x}{\pi x}$	$X(e^{j\omega})=\begin{cases} \sqrt{\frac{2\pi}{\omega_0}}, & \lvert\omega\rvert\leq\frac{1}{2}\omega_0 \\ 0, & \text{otherwise.}\end{cases}$
Box sequence	$x[n]=\begin{cases} \frac{1}{\sqrt{n_0}}, & \lvert n\rvert\leq\frac{1}{2}(n_0-1), \\ 0, & \text{otherwise.}\end{cases}$ where n_0 is odd	$\sqrt{n_0}\frac{\mathrm{sinc}\left(\frac{n_0}{2\pi}\omega\right)}{\mathrm{sinc}\left(\frac{1}{2\pi}\omega\right)}$

Appendix 4.F Continuous-Time Fourier Series : Properties

$$x(t) = \sum_{k=-\infty}^{+\infty} X_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} X_k e^{jk(2\pi/T)t}$$

$$X_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt$$

Let $x(t)$ and $y(t)$ both be periodic signals of fundamental period T.

Property	**Periodic Signal**	**Fourier Series Coefficients**
Linearity	$Ax(t) + By(t)$	$AX_k + BY_k$
Time-Shifting	$x(t - t_0)$	$X_k e^{-jk\omega_0 t_0} = X_k e^{-jk(2\pi/T)t_0}$
Frequency-Shifting	$e^{jM(2\pi/T)t}x(t)$	X_{k-M}
Conjugation	$x^*(t)$	X^*_{-k}
Time Reversal	$x(-t)$	X_{-k}
Time Scaling	$x(\alpha t), \alpha > 0$	X_k
Periodic Convolution	$\int_T x(\tau)y(t-\tau)d\tau$	TX_kY_k
Multiplication	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} X_l Y_{k-l}$
Differentiation	$\frac{dx(t)}{dt}$	$jk\omega_0 X_k = jk\frac{2\pi}{T}X_k$
Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\left(\frac{1}{jk\omega_0}\right)X_k = \left(\frac{1}{jk(2\pi/T)}\right)X_k$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X_k = X^*_{-k} \\ Re(X_k) = Re(X_{-k}) \\ Im(X_k) = -Im(X_{-k}) \\ \|X_k\| = \|X_{-k}\| \\ \arg X_k = -\arg X_{-k} \end{cases}$
Real and Even Signals	$x(t)$ real and even	X_k real and even
Real and Odd Signals	$x(t)$ real and odd	X_k purely imaginary and odd

Parseval's Relation for Periodic Signals

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |X_k|^2$$

Appendix 4.G Continuous-Time Fourier Series : Pairs

Signal	**Fourier series coefficients**
$\sum_{k=-\infty}^{+\infty} X_k e^{jk\omega_0 t}$	X_k
$e^{j\omega_0 t}$	$X_1 = 1$ $X_k = 0, \quad \text{otherwise}$
$\cos \omega_0 t$	$X_1 = X_{-1} = \frac{1}{2}$ $X_k = 0, \quad \text{otherwise}$
$\sin \omega_0 t$	$X_1 = -X_{-1} = \frac{1}{2j}$ $X_k = 0, \quad \text{otherwise}$
$x(t) = 1$	$X_0 = 1, \quad X_k = 0,\ k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0, & T_1 < \lvert t\rvert \leq \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$X_k = \frac{1}{T}$ for all k

Appendix 6.A Laplace Transform : Properties

Property	Signal	Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t)$	$X_1(s)$	R_1
	$x_2(t)$	$X_2(s)$	R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Shift in time	$x(t - t_0)$	$e^{-st_0}X(s)$	R
Shift in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $(s - s_0)$ is in R)
Scaling in time	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{s}{a}\right)$	"Scaled" ROC (i.e., s is in the ROC if (s/a) is in R)
Differentiation in time	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in time	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{Re(s) > 0\}$
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Conjugate symmetry	$x(t)$ real-valued	$X(s) = X^*(s^*)$	

Appendix 6.B Laplace Transform : Pairs

	Signal	Transform	ROC
Dirac delta function	$\delta(t)$ $\delta(t-T)$	1 e^{-sT}	All s All s
Step function	$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & \text{otherwise.} \end{cases}$ $-u(-t)$	$\frac{1}{s}$ $\frac{1}{s}$	$Re(s) > 0$ $Re(s) < 0$
	$\frac{t^{n-1}}{(n-1)!}u(t)$ $-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$ $\frac{1}{s^n}$	$Re(s) > 0$ $Re(s) < 0$
One-sided exponential	$e^{-\alpha t}u(t)$ $-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$ $\frac{1}{s+\alpha}$	$Re(s) > -Re(\alpha)$ $Re(s) < -Re(\alpha)$
	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$ $-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$ $\frac{1}{(s+\alpha)^n}$	$Re(s) > -Re(\alpha)$ $Re(s) < -Re(\alpha)$
One-sided Cosines and Sines	$[\cos\omega_0 t]u(t)$ $[\sin\omega_0 t]u(t)$ $[e^{-\alpha t}\cos\omega_0 t]u(t)$ $[e^{-\alpha t}\sin\omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$ $\frac{\omega_0}{s^2+\omega_0^2}$ $\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$ $\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	$Re(s) > 0$ $Re(s) > 0$ $Re(s) > -Re(\alpha)$ $Re(s) > -Re(\alpha)$

Appendix 7.A $\mathcal{Z}$-Transform : Properties

Property	Signal	$\mathcal{Z}$-Transform	ROC
	$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$	R R_1 R_2
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least the intersection of R_1 and R_2
Shift in time	$x[n - n_0]$	$z^{-n_0}X(z)$	R except for the possible addition or deletion of the origin
Scaling in the z-Domain	$e^{j\omega_0 n}x[n]$ $z_0^n x[n]$	$X(e^{-j\omega_0}z)$ $X\left(\frac{z}{z_0}\right)$	R $z_0 R$
Time reversal	$x[-n]$	$X(z^{-1})$	R^{-1} (i.e., the set of points z^{-1} where z is in R)
Convolution	$(x_1 * x_2)[n]$	$X_1(z)X_2(z)$	At least the intersection of R_1 and R_2
Differentiation in the z-Domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R
Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1-z^{-1}}X(z)$	At least the intersection of R and $\lvert z\rvert > 1$
Time expansion by some integer k	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$	$X(z^k)$	$R^{1/k}$ (i.e., the set of points $z^{1/k}$ where z is in R)
Conjugation	$x^*[n]$	$X^*(z^*)$	R
Conjugate symmetry	$x[n]$ real-valued	$X(z) = X^*(z^*)$	

Appendix 7.B Z-Transform : Pairs

	Signal	**Z-Transform**	**ROC**
Kronecker delta	$\delta[n]$	1	All z
	$\delta[n-m]$	z^{-m}	All z except 0 (if $m>0$) or ∞ (if $m<0$)
Step function	$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & \text{otherwise.} \end{cases}$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
	$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
One-sided exponential	$\alpha^n u[n]$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
	$-\alpha^n u[-n-1]$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert < \lvert\alpha\rvert$
	$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
	$-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert < \lvert\alpha\rvert$
One-sided cosines and sines	$[\cos\omega_0 n]u[n]$	$\frac{1-[\cos\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
	$[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1-[2\cos\omega_0]z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
	$[r^n\cos\omega_0 n]u[n]$	$\frac{1-[r\cos\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
	$[r^n\sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1-[2r\cos\omega_0]z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$