To get smooth(non jerky) movement a gradual increase of the velocity is necessary. The stepper motor accelerates and decelerates following a trapezoidal speed profile. Since the stepper motor turns in discrete steps a algorithm is utilized which can generate the pulses neccesary to approximate the linear increase of the velocity.

1 theory

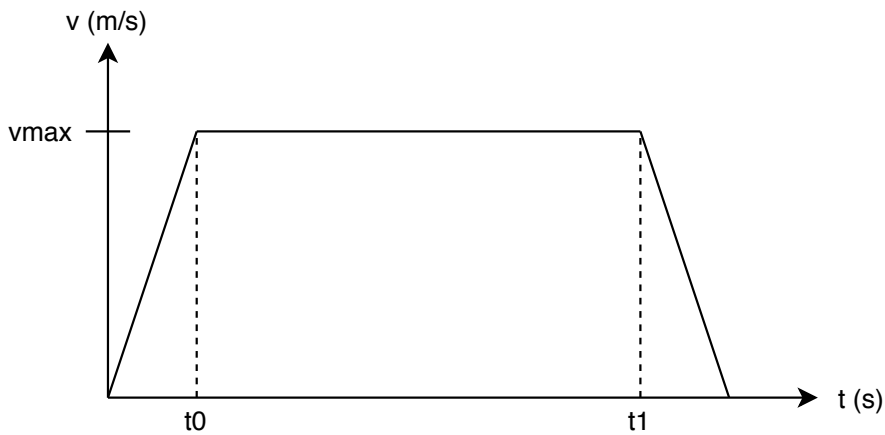

Figure 1: Trapezoidal speed profile for the stepper motor.

As can be seen in figure 1. The idea is that the stepper motor rotate n amount of turns in t seconds with a angular acceleration α. For simplicity the acceleration duration and deceleration duration are equally as long. In order to fufill these requirements the maximum velocity, ω_{max}, needs to be determined at which the stepper motor rotates.

The paramterization of the trapezoid is given by:

$$\omega(t) = \begin{cases} \alpha t & 0 < t \leq t_0 \\ \alpha t_0 & t_0 < t \leq t_1 \\ -\alpha(t - t_1) & t_1 < t \leq \text{stop} \end{cases}$$

By integrating the paremeterization the area is $\theta = \alpha t_0 t_1 = \omega_{max} t_1$ with some algebra formulae for the t_0 and t_1 can be found:

$$t_0 = \frac{w_{max}}{\alpha}, \;\; t_1 = \frac{\theta}{w_{max}} \tag{1}$$

With these equations the total time is given

$$t_d = t_0 + t_1 \tag{2}$$

Consequently the maximum velocity can be obtained as function of the total duration, angle and acceleration. Combining equation 1 and 2 the following quadratic solution is obtained.

$$\omega_{max} = \frac{\alpha t_d \pm \sqrt{(\alpha t_d)^2 - 4\alpha\theta}}{2} \tag{3}$$

but only one of the solutions is valid.

$$N = \frac{1}{\text{gear train} \cdot spr}[\text{deg}^{-1}] \tag{4}$$

$$\#\text{steps} = \theta N \tag{5}$$

$$t_0 = \frac{\omega_{max}^2 \#steps}{2\theta\alpha} \tag{6}$$

$$t_1 = \#\text{steps} - t_0 \tag{7}$$

the anglular velocity is given by the frequency and the step angle.

$$\frac{c}{f} = \Delta t \tag{8}$$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\Delta\theta f}{c} \tag{9}$$

$$\frac{1}{2}\alpha t^2 = \frac{n}{N} \tag{10}$$

$$t = \sqrt{\frac{2n}{\alpha N}} \tag{11}$$

the difference between to consecutive steps is given by:

$$c_n = c_0(\sqrt{n+1} - \sqrt{n}) \tag{12}$$

$$c_0 = f\sqrt{\frac{2}{\alpha N}} \tag{13}$$

using taylor series the expression is turned into

$$c_n = c_{n-1} - \frac{2c_{n-1}}{4n+1} \tag{14}$$