fuzzyclara: Efficient Medoid-based Clustering Algorithms for Large and Fuzzy Data

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Summary

Cluster analysis identifies optimal groupings of observations that share similar characteristics. One popular approach is to use medoid-based methods where each cluster center is represented by one *typical* observation (Kaufman & Rousseeuw, 2009). The R package fuzzyclara makes a wide range of clustering algorithms conveniently available. It not only covers classical *hard clustering* methods (where one observation belongs to exactly one cluster) and alternative *fuzzy clustering* methods (where the characteristics of each observation are shaped by its partial membership to different clusters), but specifically also implements the option to combine such algorithms with subsampling-based estimation techniques to make the estimation on large data feasible. The package additionally provides convenience functionalities and visualization techniques to cover the whole workflow for real-world clustering applications.

Statement of Need

Partitioning clustering algorithms aim to find reasonable groupings (*clusters*) of a set of observations based on a predefined number of clusters. Medoid-based versions of this strategy build clusters based on *medoids*, i.e. one observation per cluster best representing its typical characteristics. The most prominent representative of medoid-based clustering is the *partitioning around medoids* (PAM) algorithm, which is robust and applicable to many data situations (Kaufman & Rousseeuw, 2009).

The PAM algorithm, however, has two drawbacks. First, its estimation is often only hardly or not at all feasible in large data situations with thousands of observations. The algorithm requires the computation of a (dis)similarity matrix between all observations, scaling quadratically ($O(n^2)$) in terms of runtime and memory usage. Sampling-based algorithms such as CLARA (Kaufman & Rousseeuw, 1986) or CLARANS (Ng & Han, 2002) make the estimation feasible in such situations. Second, PAM is a hard clustering algorithm where each observation is rigidly assigned to a single cluster. This assumption is not best resembling reality in many data situations where observations may share characteristics of several *typical* clusters. Such structures are taken into account by *fuzzy clustering* methods which compute membership scores for each observation to every cluster.

The statistical software R already provides a wide range of packages implementing clustering algorithms for large or fuzzy data. The package cluster (Maechler et al., 2022) contains diverse clustering routines developed by Kaufman & Rousseeuw (2009) including the CLARA algorithm for large data and the FANNY algorithm for fuzzy data. The CLARANS algorithm as an extension of CLARA is implemented in the package qtcat (Klasen, 2022). The package fastkmedoids (Li, 2021) provides computationally more efficient versions of the CLARA and CLARANS algorithms. A variety of medoid-based fuzzy clustering methods are further available in packages vegclust (De Caceres et al., 2010) and fclust (Ferraro et al., 2019).

All of the above implementations have in common that they either allow for the application of fuzzy clustering or of subsampling approaches, but not both simultaneously. While the `fuzzyclara` package includes the option to perform hard clustering using the classical PAM algorithm and to use all implemented algorithms with any kind of (dis)similarity metric, (which are used by `proxy::dist`) or use a self-defined metric.−> the package also allows for simultaneously analyzing large and fuzzy data, by combining the CLARA / CLARANS algorithms with the fuzzy-k-medoids algorithm by Krishnapuram et al. (1999). Beyond this, the package provides routines to cover the whole workflow for real-world clustering applications, including the use of user-defined distance functions, the choice of the optimal number of clusters and diverse visualization techniques.

Combination of fuzzy and CLARA clustering

To combine the CLARA strategy with the principle of fuzzy clustering, we utilize the original CLARA clustering algorithm by Kaufman & Rousseeuw (1986). The algorithm consists of the following steps, given a predefined number of J clusters:

1. Determination of K random subsamples of the data

2. For each subsample $k = 1, ..., K$:

    (a) Application of PAM clustering on the subsample.

    (b) Assignment of each observation in the complete dataset to the cluster with the closest medoid.

    (c) Computation of the average distance of each observation to its closest clustering medoid as clustering criterion C_p:

$$C_p = \frac{1}{n} \sum_{i=1}^{n} d_{ij_{min}p},$$ (1)

where d_{ijp} denotes the distance of observation i to its closest medoid (i.e. observation j_{min}) based on the clustering solution on subsample k.

3. Selection of the optimal set of clusters according to the minimal clustering criterion.

We further allow for fuzziness in the algorithm by adapting it as follows. Instead of a hard clustering method, we apply the fuzzy-k-medoids algorithm (Krishnapuram et al., 1999) on each subsample of the data in step 2a. Each observation of the complete dataset is assigned a membership score to all clusters j according to the fuzzy-k-medoids algorithm:

$$u_{ijp} = \frac{\left(\frac{1}{d_{ijp}}\right)^{\frac{1}{m-1}}}{\sum_{j=1}^{J} \left(\frac{1}{d_{ijp}}\right)^{\frac{1}{m-1}}},$$ (2)

where m denotes the fuzziness exponent controlling the degree of fuzziness. The clustering criterion C_p is accordingly defined as the weighted sum of all distances to the medoids of all clusters, with weights according to the membership scores:

$$C_p = \frac{1}{n} \sum_{i=1}^{n} \sum_{k=1}^{K} u_{ijp}^{m} d_{ijp}.$$ (3)

The optimal cluster solution is the subsample solution which minimizes this average weighted distance. Note that this adapted CLARA algorithm simplifies to the original, hard clustering CLARA algorithm, when only allowing membership scores of 0 and 1.

As an alternative, and in contrast to the CLARA algorithm, the CLARANS algorithm does not evaluate a set of random samples, but random pairs of medoids and non-medoids, to iteratively check for a potential improvement of the current clustering solution (Ng & Han, 2002). The implementation of its fuzzy version follows the same scheme as the fuzzy CLARA algorithm with the computation of membership scores according to (2) and the selection of the best clustering solution over all local clusterings based on the minimal weighted average distance.

General Routine of Cluster Analysis

Beyond the efficient estimation of (fuzzy) clustering through subsampling approaches, `fuzzyclara` comprises functionalities that cover the whole clustering workflow. The package implements a range of routines for (visually) evaluating the clustering solutions, including principal components plots, the analysis of silhouette scores or the determination of the number of clusters. Using the implemented visualizations, the fuzziness of a clustering solution can either be visualized based on the estimated membership scores or by restricting on *core cluster observations* with some minimal membership score threshold.

Application

We demonstrate the functionality of the `fuzzyclara` package by clustering German tourists based on the included `travel` data. The data originates from an annual cross-sectional study on pleasure travel and contains information on 11,871 travelers between 2009 to 2018. Apart from the travel year, included variables are the number of trips made within a year, the overall amount of travel expenses and the maximum travel distance.

Due to the rather large data size of over 10,000 observations, we apply the CLARA algorithm with 20 randomly drawn samples of size 1,000. As tourists are likely to share characteristics of several tourist types, we use the fuzzy version with a membership exponent of $m = 1.5$. Based on the minimum average weighted dissimilarity, the elbow criterion (see Figure 1) suggests the use of the solution with five different clusters.
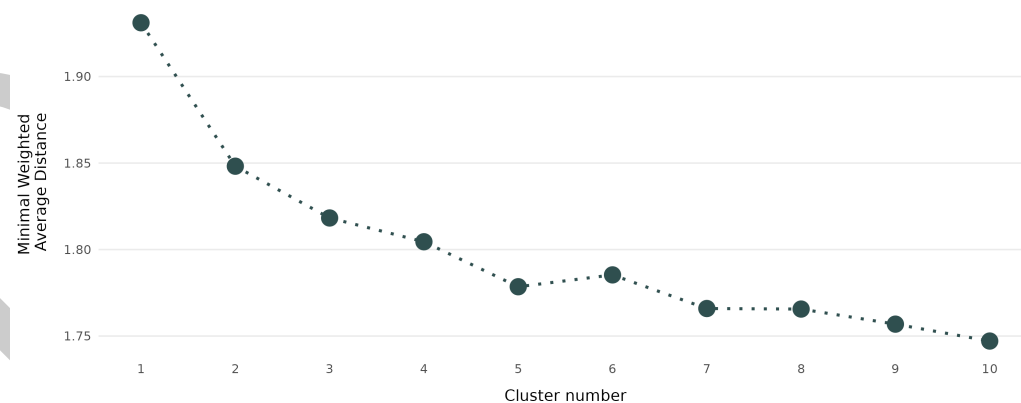
Figure 1: Elbow plot of clustering solutions with 1 to 10 clusters according to the minial average weighted distance.

Figure 2 highlights the characteristics of the different clusters by visualizing the individual paths of 500 randomly sampled observations plus the medoid. While cluster 1 shows a tendency to trips of shorter length, lower distance and lower costs, the tourists of cluster 2 tend to travel most frequently and spend the most money for travelling. Between the clusters 3 and 4, there are only minor differences regarding travel distances and expenses.

Figure 2: Parallel coordinate plot showing characteristics of 500 randomly sampled observations over the standardized variables. The characteristics of medoids are highlighted with bold black lines. The transparency of the line represents the membership score of the observation to the assigned cluster where less transparency encodes clearer membership, i.e. a lower degree of fuzziness.

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