CS 189: HW6 Report

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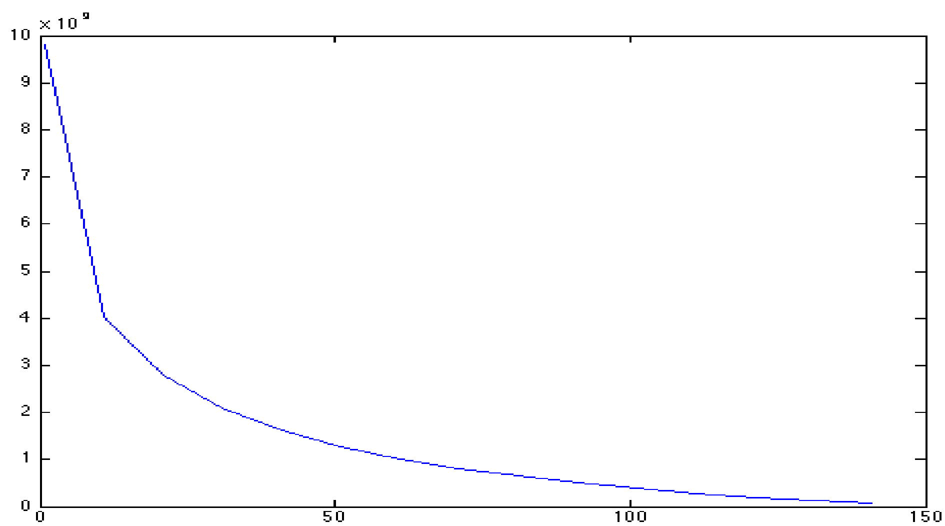
1 Eigenfaces

1. The variations correspond to facial hair, hair, gender, skin color, eye color, hair color, lighting, smile type and eye positioning/wrinkles. You can visualize these by running visualize_eigenfaces()

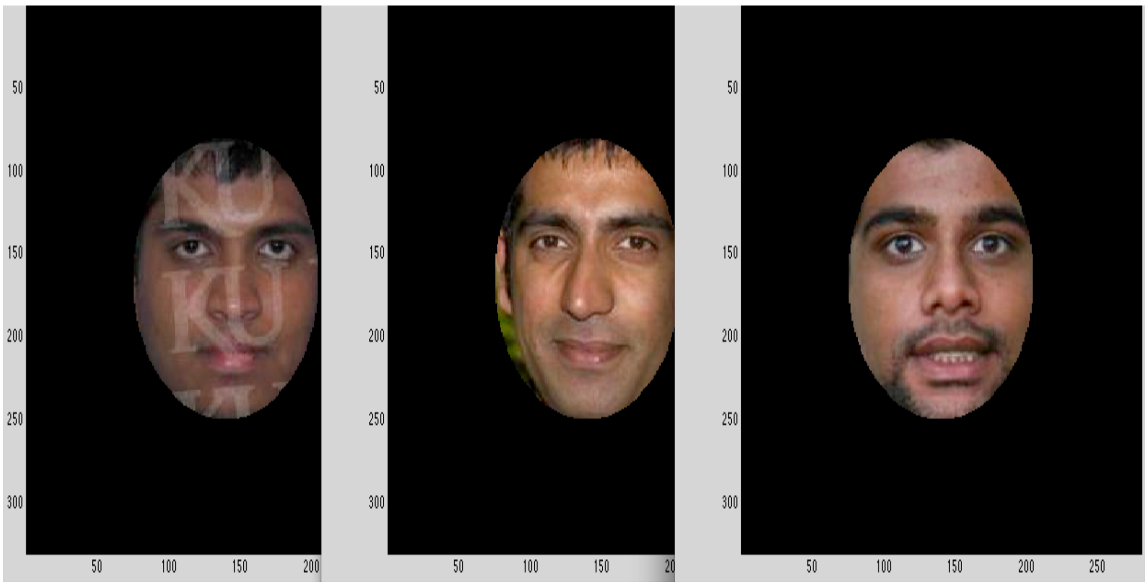
2. To visualize the faces side by side using 10 eigenfaces, run compare_faces(index), where index is the ith celebrity face to reconstruct

   Note, I didn't have very much success with 10 eigenfaces. I found that using more around 50 produced better results in reconstruction.
   You can change this number within the compare_faces function, just pass another parameter,
   compare_faces(index, num_eigenfaces)



3. To find the closest celebrity faces using euclidean distance, simply run
   euclidean_dist(i, true)
   aka euclidean_dist(5, true)

   where i is the ith student_image
   true sets the function to display the images, otherwise it just returns the row vectors of the celebrities



4. We can use the matrix of eigenvalues as our Σ because, obvious there exists a decomposition of $X = U\Sigma V^T$. Of course, this isn't in our eigenface basis. So we can preform a change of basis into eigenface coordinates. This change of basis will affect the eigenvectors but not the eigenvalues, aka Σ will stay the same, so we can use Σ as our Σ when computing the Mahalanobis distance.

   The display the student and nearest celebrity faces next to each other, simply run

   mahalanobis_distance(i, true)
   aka: mahalanobis_distance(5, true)
   where i is the ith student_image
   true sets the function to display the image, rather than return the celebrities.

2 SVD

1. $AA^T = \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 2 \end{pmatrix}$
   eigenvalues$= 8, 2$
   eigenvectors $= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
   $AA^T = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 8 & 4 \\ 4 & 2 \end{pmatrix}$
   $(8-\lambda)(2-\lambda) - 16 = 0$
   $\lambda_1 = 10, \lambda_2 = 0$
   $\begin{pmatrix} 8 & 4 \\ 4 & 2 \end{pmatrix} v = 10v$
   $8v_1 + 4v_2 = 10v_1$
   $4v_1 + 2v_2 = 10v_2$
   $8v_1 + 4v_2 = 0$
   $4v_1 + 2v_2 = 0$
   eigenvectors $= \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

2. First one: eigenvalues are $8, 2$ so $\Sigma = \begin{pmatrix} \sqrt{8} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$

   Second one: eigenvalues are $10, 0$ so $\Sigma = \begin{pmatrix} \sqrt{10} & 0 \\ 0 & 0 \end{pmatrix}$

3. $A^TA = \begin{pmatrix} 2 & 1 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$
$(5-\lambda)(5-\lambda) - 9 = 0$
eigenvalues: $8, 2$
$5v_1 + 3v_2 = 8v_1$
$3v_1 + 5v_2 = 8v_2$
$5v_1 + 3v_2 = 2v_1$
$3v_1 + 5v_2 = 2v_2$
eigenvectors: $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$
$A^TA = \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 5 \\ 5 & 5 \end{pmatrix}$
$(5-\lambda)^2 - 25 = 0$
eigenvalues: $10, 0$
eigenvectors: $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix}$

4. does: $A = U\Sigma V^T$
$\begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{8} & 0 \\ 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$
Yep!
$\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2/\sqrt{5} & -1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{pmatrix} \begin{pmatrix} \sqrt{10} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$

Yep!!