EE552 – Power System Dynamics

Individual Project – Fall 2018

Deadline:

Submit through Canvas by midnight on Sunday December 9, 2018

Tasks:

In the attached extract from their book "Power System Analysis", Bergen and Vittal discuss a simplified version of the time domain simulation of multi-machine stability.

1. Write a Python or Matlab program that replicates the results obtained by Bergen and Vittal.
2. Enhance this program by replacing the second order classical model by the third order model described on page 456 of the book by Machowski et al.
3. Study the influence of the following factors on the stability of the system:
    a. Fault clearing time
    b. Damping
    c. Loading
4. Optional extra credit: further enhance your program by replacing the third order model by a fourth order model or by considering the dependence of one or more loads on the voltage.

Deliverables:

- A report describing
    - The design of your program
    - The tests that you performed
    - The results that you obtained
- A copy of your program. You must include enough comments in your program to make it easily readable.

The sketch of P_A^0 and P_{AB} is shown in Figure E14.5(b). Clearly, $A_a < A_{d\max}$ and the equal-area criterion predicts stability. In the sketch we show the balancing deceleration area $A_d = A_a$.

What makes this case so different from part (a) is the presence of a relatively strong line ($b_3 = 4$) which is initially lightly loaded. Thus, the line is capable of making a strong contribution to the deceleration area.

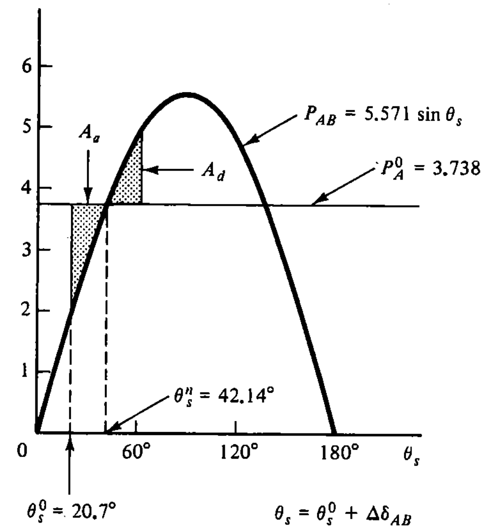

Figure E14.5(b)

Exercise 6. Suppose for the case under consideration in this section (i.e., two coherent groups of generators) that we lose all the lines connecting a hypothetical network A to network B at $t = 0$, and then all the breakers reclose after T seconds. All other factors being equal (or disregarded), what is the most favorable grouping of the M_i to minimize $\Delta\delta_{AB}$ after a fixed time T? More specifically, if M_T is the sum of all the M_i and $M_A = \lambda M_T$, $M_B = (1 - \lambda)M_T$, how would you pick λ? λ is a number between 0 and 1.

In concluding this section we make a few general comments.

Although the assumption of coherency is not expected to be fully realized in practice, we can obtain useful qualitative results, nevertheless, which help in thinking about stability problems in the system; these results can then be checked by more precise models and/or by simulation. Thus we can consider the effects of increasing (or decreasing) moments of inertia. We can consider the effects of readjusting generation versus load distributions (this will change P_A^0 and the θ_{ij}^0 across the lines joining the two networks A and B), the effect of losing particular lines between the two networks A and B (thereby changing b_s and θ_s^0), and the effect of low voltages during fault-on periods.

Using even our very simple system models and stability analysis techniques, we can appreciate certain transient stability enhancement techniques that are used in practice. The basic objective, whether we have two generators or two coherent groups of generators, is to reduce the acceleration area and increase the maximum available deceleration area. The importance of rapid removal of faulty lines to allow the remaining parallel line(s) to deliver more power is seen in Figure 14.16. This figure also illustrates the importance of maintaining voltage during fault-on conditions. During a fault, voltages tend to drop throughout the system; this is one of the reasons for the low $P_G(\delta)$ during the fault. Fast-acting voltage control systems at generator buses counter this drop by increasing generator excitation. One of the effects is to increase the transferred power during the fault. This then tends to decrease the acceleration area. In other cases, when the fault is temporary, the importance of rapid reclosing of circuit breakers may be inferred from the very simple case considered in Figure 14.8.

Fast-closing turbine steam valves actuated during faults can help reduce the mechanical input P_M and thereby both decrease the accelerating area and increase the decelerating area. It may also be possible to use up some of the excess accelerating power by momentarily activating resistance loads (braking resistances) when fault conditions are detected. Switchable series capacitors to cancel some of the series line inductance and thereby increase the transferred power is another possibility.

In the design phase the importance of paralleling lines to strengthen weak transmission links is clear. It would also be desirable to increase the inertia constants, H, and decrease the internal impedances of the generating units. Unfortunately, current manufacturing practice runs contrary to this need.

We note that there are many natural extensions of the technique we have been describing to more general systems with more general assumptions. One powerful technique, the so-called *second method of Liapunov,* is a generalization of the energy technique we used in Section 14.4. In fact, (14.28) is an example of a so-called *Liapunov function,* and its use is demonstrated in the material through (14.33). With a more general Liapunov function we can undertake the investigation of the stability of multimachine systems without the coherency restriction. There are other restrictions however, and in any case, we never achieve the simplicity and intuitive understanding that we get with the equal area criterion.

For a description of the application of the second method of Liapunov to multimachine power system stability using more advanced models, the interested reader is referred to Pai (1981), and Fouad and Vittal. We turn next to the investigation of stability by solving the power system differential and algebraic equations directly. In so doing, we eliminate the many restrictions of the preceding methods.

14.8 MULTIMACHINE STABILITY STUDIES

In North America, the electric power network is operated and designed based on reliability criteria set by the North American Electric Reliability Council (NERC). For more information about NERC see http://www.nerc.com/. Current NERC planning and operating standards call for strict compliance with a wide range of guidelines to ensure the reliability of the interconnected system. An important component of the standards deals with transient stability assessment of the interconnected system and the associated modeling requirements. This analysis ensures that following a large disturbance the synchronous machines remain in synchronism in the transition following the disturbance, and the system is able to achieve a satisfactory steady-state operating condition after the protection system has operated to isolate the disturbance.

As compared to the previous sections, we turn to more realistic and complete generator models and also include in the power system model, the associated exciters, governors, stabilizers, high-voltage dc links, etc. The loads are also modeled more realistically.

The synchronous generators are represented in varying levels of detail depending on the phenomena being studied. These models range from representation of the rotor cicuits, damper circuits, and stator circuits using the Park's transformation and associated differential equations describing the behavior of flux linkages, or induced electromotive forces, or currents, to the simplest model that represents the electromechanical characteristics of the synchronous generators. In addition, a wide range of models exist for various control components like exciters, governors, high-voltage dc links, etc.

The synchronous generators are interconnected by the transmission network. For transient stability studies dealing with the electromechanical behavior of the power system, the electrical transients in the network involving the inductances and capacitances are neglected, and the network is assumed to be in a quasi steady state. As a result, the network is represented by a system of algebraic equations that govern the relationship between the injected complex power at the generator buses and the complex voltages at all other buses based on the fundamental principles of the power flow analysis presented in Chapter 10. The important difference in stability studies is that at each instant in time during the transient the complex power injected into the network by the synchronous generators varies. As a result, a new solution of the voltages at all other buses has to be determined.

Load modeling is another important feature of transient stability studies. A variety of load models is used. These range from static nonlinear loads that represent the overall behavior of a load as a combination of constant impedance, constant current, and constant power, to detailed dynamic models for induction motors.

We are led to consider a coupled set of differential and algebraic equations. These are solved numerically to obtain the condition of the system at each instant in time. Various important quantities like relative rotor angles, voltages at key buses, and power flows on critical transmission lines are plotted and observed. The behavior of these variables is then used to judge stability or instability. The starting point for any transient stability simulation is the predisturbance power flow solution of the kind obtained in Chapter 10. The system is always assumed to be in a steady state when the initiating disturbance occurs. As a result, using the predisturbance power flow solution, all the initial values of the state variables that govern the differential equations are calculated. After these are obtained, the disturbance is simulated. Different disturbances can occur on the system. These can be short circuits, outages of critical components like transmission lines or generators, or sudden application or loss of large loads. The simulation of the disturbance causes a mismatch between the mechanical power input to the generators and the electrical power output of the generators. Hence, the equilibrium is upset, and the state variables governed by the various differential equations coupled with algebraic equations change their values. The change in behavior is tracked by numerically integrating the coupled set of differential and algebraic equations. Two broad categories of numerical techniques are commonly used to integrate the coupled set of equations: implicit integration techniques and explicit integration techniques. The time evolution of the state variables and other system variables is observed to determine the behavior of the system. For a detailed description of the formulation of these equations and their solution, the interested reader is referred to Anderson and Fouad, Kundur, and Sauer and Pai.

We will now deal with a much simpler representation of the power system, which is commonly referred to as the *classical model.* The classical model is used to study the transient stability of a power system for a period of time during which the dynamic behavior of the system is dependent largely on the stored energy in the rotating inertias. This is the simplest model used in stability studies and requires a minimum amount of data. The assumptions made in developing the classical model are as follows (Anderson and Fouad, Chapter 2):

1. The mechanical power input to each synchronous machine is constant.
2. Damping or asynchronous power is negligible.
3. The synchronous machines are represented electrically, by constant-voltage-behind-transient-reactance models.
4. The motion of each synchronous machine rotor (relative to a synchronously rotating reference frame) is at a fixed angle relative to the angle of the voltage behind the transient reactance.
5. Loads are represented by constant impedances.

While assumptions 1-4 look similar to those used in Section 14.7, there are some important differences; the number of generators in arbitrary (with no coherency restriction), and the generators are connected to an arbitrary transmission network.

The model is useful for stability analysis but is limited to the study of transient for periods on the order of one second. This type of analysis is usually called first swing analysis. Assumption 2 can be relaxed by assuming a linear damping characteristic. A damping torque $D\dot{\delta}$ is included in the swing equation. Note that this was also considered in the one-machine–infinite bus case in (14.19). In assumption 3, the steady-state model of the synchronous machine given in Figure 6.5 is modified as shown in Figure 14.18 for the purpose of transient stability analysis. The reactance x'_d is the direct axis transient reactance. The constant voltage source $|E|\angle\delta$ is determined from the initial conditions (i.e., predisturbance power flow conditions).

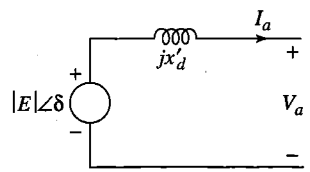

Figure 14.18 Representation of a synchronous machine by constant voltage behind transient reactance.

During the transient the magnitude $|E|$ is held constant, while the variation of angle δ is governed by (14.19).

Assumption 5, dealing with the representation of loads as constant impedances, is usually made for simplicity. This assumption allows us to eliminate the algebraic network equations and reduce the system of equations for the multimachine system to a system consisting of only differential equations. It is important to note, however, that loads have their own dynamic behavior. In many studies loads are modeled as a combination of constant impedance, constant current, and constant MVA, together with several critical loads modeled in detail using induction motor models. Load representation can have a marked effect on stability results.

Considering these assumptions, we will now derive the equations governing the motion of the multimachine power system. The assumptions provide a representation of the power system shown in Figure 14.19 for an n-generator system. Nodes $1, 2, \ldots, n$ are referred to as the *internal* machine nodes. These are the nodes or buses to which the voltages behind transient reactances are applied. The transmission network, together with transformers modeled as impedances, connects the various nodes. The loads, modeled as impedances, also connect the load buses to the reference node.

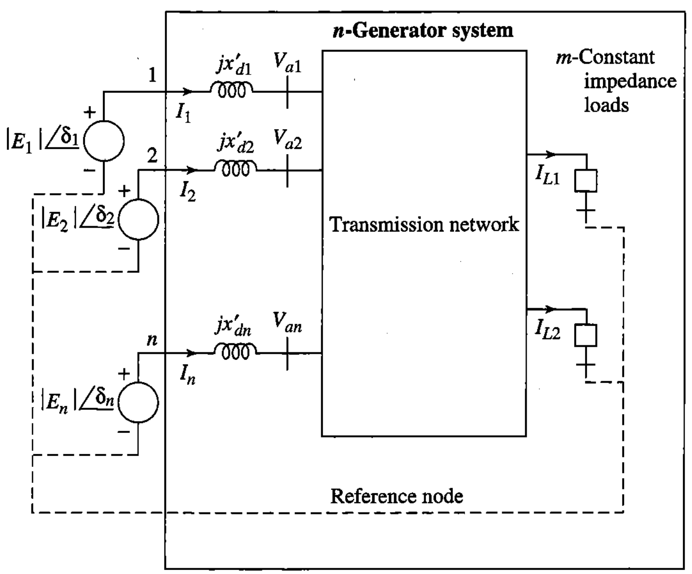

Figure 14.19 Multimachine system representation (classical model).

To prepare the system data for a stability study, the following preliminary calculations are made:

1. The system data are converted to a common system base; a system base of 100 MVA is conventionally chosen. The procedure to perform this transformation has been described in Chapter 5.
2. The load data from the prefault power flow are converted to equivalent impedances or admittances. This procedure was described in Example 2.3. The necessary information for this step is obtained from the result of the power flow. If a certain load bus has a voltage solution V_{Li} and complex power demand $S_{Li} = P_{Li} + jQ_{Li}$, then using $S_{Li} = V_{Li}I_{Li}^*$ (which implies $I_{Li} = S_{Li}^*/V_{Li}^*$), we get

$$y_{Li} = \frac{I_{Li}}{V_{Li}} = \frac{S_{Li}^*}{|V_{Li}|^2} = \frac{P_{Li} - jQ_{Li}}{|V_{Li}|^2} \tag{14.69}$$

where $y_{Li} = g_{Li} + jb_{Li}$ is the equivalent shunt load admittance.
3. The internal voltages of the generators $|E_i|\angle\delta_i^0$ are calculated from the power flow data using the predisturbance terminal voltages $|V_{ai}|\angle\beta_i$ as follows. We will temporarily use the terminal voltage as reference as shown in Figure 14.20.

$$|E_i|\angle\delta_i' = |V_{ai}| + jx_{di}I_i \tag{14.70}$$

Expressing I_i in terms of S_{Gi} and V_{ai}, we have

$$\begin{aligned} |E_i|\angle\delta_i' &= |V_{ai}| + j\frac{x'_{di}S_{Gi}^*}{|V_{ai}|} = |V_{ai}| + j\frac{x'_{di}(P_{Gi} - jQ_{Gi})}{|V_{ai}|} \\ &= (|V_{ai}| + Q_{Gi}x'_{di}/|V_{ai}|) + j(P_{Gi}x'_{di}/|V_{ai}|) \end{aligned} \tag{14.71}$$

Thus, the angle difference between internal and terminal voltage in Figure 14.20 is δ_i'. Since the actual terminal voltage angle is β_i, we obtain the initial generator angle δ_i^0 by adding the predisturbance voltage angle β_i to δ_i', or

$$\delta_i^0 = \delta_i' + \beta_i \tag{14.72}$$

4. The $\mathbf{Y}_{\text{bus}}$ matrices for the prefault, faulted, and postfault network conditions are calculated. In obtaining these matrices, the following steps are involved:
   a. The equivalent load admittances calculated in step 2 are connected between the load buses and the reference node. Additional nodes are provided for the internal generator nodes (nodes $1, 2, \ldots, n$ in Figure 14.19) and the

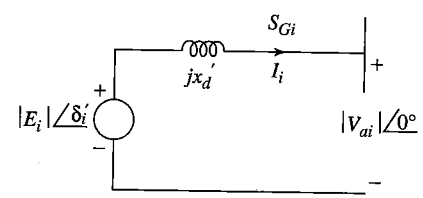

Figure 14.20 Generator representation for computing initial angle.

appropriate values of admittances corresponding to x'_d are connected between these nodes and the generator terminal nodes. The procedure to augment new nodes in the $\mathbf{Y}_{\text{bus}}$ has been described in Chapter 9.

b. In order to obtain the $\mathbf{Y}_{\text{bus}}$ corresponding to the faulted system, we usually only consider three-phase to ground faults. The faulted $\mathbf{Y}_{\text{bus}}$ is then obtained by setting the row and column corresponding to the faulted node to zero.

c. The postfault $\mathbf{Y}_{\text{bus}}$ is obtained by removing the line that would have been switched following the protective relay operation.

5. In the final step we eliminate all the nodes except the internal generator nodes using Kron reduction. The procedure for performing Kron reduction was described in Chapter 9. After Kron reduction we obtain the reduced $\mathbf{Y}_{\text{bus}}$ matrix, which we will denote by $\hat{\mathbf{Y}}$. The reduced matrix can also be derived as follows. The system $\mathbf{Y}_{\text{bus}}$ for each network condition provides the following relationship between the voltages and currents:

$$\mathbf{I} = \mathbf{Y}_{\text{bus}}\mathbf{V} \tag{14.73}$$

where the current vector $\mathbf{I}$ is given by the injected currents at each bus. In the classical model considered, injected currents exist only at the n-internal generator buses. All other currents are zero. As a result, the injected current vector has the form

$$\mathbf{I} = \begin{bmatrix} \mathbf{I}_n \\ \cdots \\ \mathbf{0} \end{bmatrix}$$

We now partition the matrices $\mathbf{Y}_{\text{bus}}$ and $\mathbf{V}$ appropriately to obtain

$$\begin{bmatrix} \mathbf{I}_n \\ \cdots \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{nn} & \vdots & \mathbf{Y}_{ns} \\ \cdots & \cdots & \cdots \\ \mathbf{Y}_{sn} & \vdots & \mathbf{Y}_{ss} \end{bmatrix} \begin{bmatrix} \mathbf{E}_n \\ \cdots \\ \mathbf{V}_s \end{bmatrix} \tag{14.74}$$

The subscript n is used to denote the internal generator nodes, and the subscript s is used for all the remaining nodes. Note that the voltage at the internal generator nodes are given by the internal emf's. Expanding (14.74), we get

$$\mathbf{I}_n = \mathbf{Y}_{nn}\mathbf{E}_n + \mathbf{Y}_{ns}\mathbf{V}_s \qquad \mathbf{0} = \mathbf{Y}_{sn}\mathbf{E}_n + \mathbf{Y}_{ss}\mathbf{V}_s$$

from which we eliminate $\mathbf{V}_s$ to determine

$$\mathbf{I}_n = (\mathbf{Y}_{nn} - \mathbf{Y}_{ns}\mathbf{Y}_{ss}^{-1}\mathbf{Y}_{sn})\mathbf{E}_n = \hat{\mathbf{Y}}\mathbf{E}_n \tag{14.75}$$

The matrix $\hat{\mathbf{Y}}$ is the desired reduced admittance matrix. It has dimensions $(n \times n)$, where n is the number of the generators. From (14.75) we also observe that the reduced admittance matrix provides us a complete description of all the injected currents in terms of the internal generator bus voltages. We will now use this relationship to derive an expression for the (active) electrical power output of each generator and hence obtain the differential equations governing the dynamics of the system.

The power injected into the network at node i, which is the electrical power output of machine i, is given by $P_{Gi} = Re\, E_i I_i^*$ The expression for the injected current at each generator bus I_i in terms of the reduced admittance matrix parameters is given by (14.75). From (14.75), (10.3), and (10.5), we have

$$P_{Gi} = |E_i|^2\hat{G}_{ii} + \sum_{\substack{j=1\\ j\neq i}}^{n} |E_i||E_j|[\hat{B}_{ij}\sin(\delta_i - \delta_j) + \hat{G}_{ij}\cos(\delta_i - \delta_j)]$$

$$i = 1, 2, \ldots, n \tag{14.76}$$

Substituting the preceding expression for P_{Gi} into the differential equation (14.19) governing the dynamics of the synchronous machine, and neglecting the damping coefficient, we have for a multimachine system

$$M_i\ddot{\delta}_i = P_{Mi}^0 - P_{Gi} \qquad i = 1, 2, \ldots, n \tag{14.77}$$

Substituting (14.76) for P_{Gi} in (14.77), we obtain

$$M_i\ddot{\delta}_i = P_{Mi}^0 - |E_i|^2\hat{G}_{ii} - \sum_{\substack{j=1\\ j\neq i}}^{n} |E_i||E_j|[\hat{B}_{ij}\sin(\delta_i - \delta_j) + \hat{G}_{ij}\cos(\delta_i - \delta_j)]$$

$$i = 1, 2, \ldots, n \tag{14.78}$$

In the preceding equations, the value of the mechanical power for each machine is determined from the prefault conditions. The mechanical power is set equal to the (active) electrical power output of each generator at the prefault conditions. This provides the equilibrium conditions and the initial angles for each generator as given by δ_i^0 calculated in step 3 in the preceding calculations. The equations given in (14.78) are second-order differential equations. In order to perform numerical integration, we convert these equations into a set of coupled first-order differential equations given by

$$M_i\dot{\omega}_i = P_{Mi}^0 - |E_i|^2\hat{G}_{ii} - \sum_{\substack{j=1\\ j\neq i}}^{n} |E_i||E_j|[\hat{B}_{ij}\sin(\delta_i - \delta_j) + \hat{G}_{ij}\cos(\delta_i - \delta_j)]$$

$$\dot{\delta}_i = \omega_i \qquad i = 1, 2, \ldots, n \tag{14.79}$$

Several numerical techniques are available to solve the preceding set of differential equations. For a detailed description of the numerical techniques, students are referred to Kundur, and Sauer and Pai. Standard techniques to solve these equations are available in numerical packages like MATLAB. An example that describes the application of such a package is provided next.

Example 14.6

The one-line diagram of the system being analyzed is shown in Figure E14.6(a). For the transmission lines in the system, all the shunt elements are capacitors with an admittance of $y_c = j\,0.01$ p.u., while all the series elements are inductors with an impedance $z_L = j\,0.1$ p.u. The desired voltages, generation levels, and load levels are indicated in the figure.

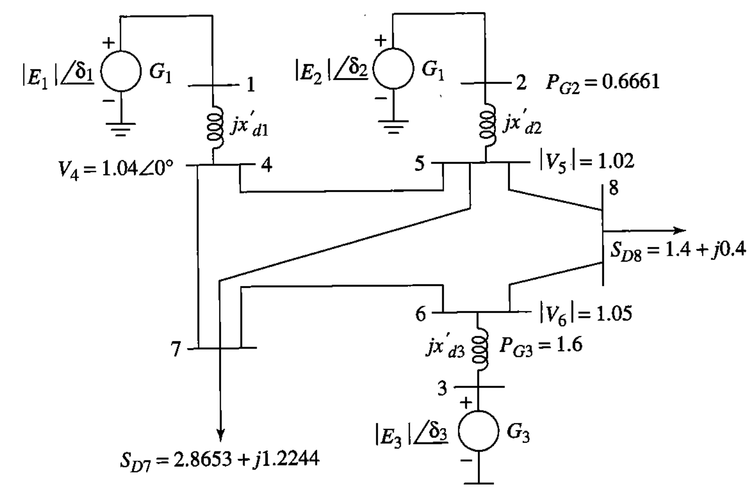

Figure E14.6(a) System one-line diagram.

The generator dynamic data on a 100-MVA base are given in Table E14.6(a).

TABLE E14.6(A) GENERATOR DATA

Gen. #	x'_d	H
1	$j\,0.08$	10 s
2	$j\,0.18$	3.01 s
3	$j\,0.12$	6.4 s

The power flow solution for the sample system is as follows:

$$\begin{bmatrix} V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 1.04\angle 0° \\ 1.02\angle -3.55° \\ 1.05\angle -2.90° \\ 0.9911\angle -7.48° \\ 1.0135\angle -7.05° \end{bmatrix}, \quad \begin{bmatrix} P_{G1} \\ P_{G2} \\ P_{G3} \end{bmatrix} = \begin{bmatrix} 1.9991 \\ 0.6661 \\ 1.6000 \end{bmatrix}, \quad \begin{bmatrix} Q_{G1} \\ Q_{G2} \\ Q_{G3} \end{bmatrix} = \begin{bmatrix} 0.8134 \\ 0.2049 \\ 1.051 \end{bmatrix}$$

For a three-phase to ground fault at Bus 7, with line 7–6 removed at 0.10 s, simulate the differential equations for the classical model of the system. Plot all the rotor angles versus time. Also, obtain the relative rotor angles with respect to Generator 1.

Solution *Step 1:* In this example, all the load data are provided on a 100-MVA base; as a result, no conversion of base is required.

Step 2: Knowing the voltages at the load buses, and the complex power demand, the admittances corresponding to the loads are given by

$$y_{77} = \frac{P_{L7}}{|V_7|^2} - j\frac{Q_{L7}}{|V_7|^2} = \frac{2.8653}{(0.9911)^2} - j\frac{1.2244}{(0.9911)^2} = 2.9170 - j1.2465$$

Similarly, $y_{88} = 1.3630 - j0.3894$.

Step 3: The internal generator voltages are calculated. The details of the calculation are shown for Generator 2.

$$P_2 + jQ_2 = 0.6661 + j0.2049$$

$$V_5 = 1.02\angle -3.55°, \qquad x'_{d2} = j0.18$$

From (14.71), we have

$$\begin{aligned} |E_2|\angle\delta'_2 &= (1.02 + 0.2049 \times 0.18/1.02) + j(0.6661 \times 0.18/1.02) \\ &= 1.0562 + j0.1175 = 1.0627\angle 6.3507° \end{aligned}$$

From (14.72),

$$\delta_2^0 = 6.3507 - 3.55 = 2.8006°$$

We can similarly calculate the other two internal voltages to be

$$|E_1|\angle\delta_1^0 = 1.1132\angle 7.9399°, \quad |E_3|\angle\delta_3^0 \doteq 1.1844\angle 5.9813°$$

Step 4: We now form the prefault, faulted, and postfault admittance matrices, with the augmented internal generator buses 1, 2, and 3.

$$\mathbf{Y}_{\text{bus}}^{\text{prefault}} = \begin{bmatrix} -j12.5 & 0 & 0 & j12.5 & 0 & 0 & 0 & 0 \\ 0 & -j5.556 & 0 & 0 & j5.556 & 0 & 0 & 0 \\ 0 & 0 & -j8.333 & 0 & 0 & j8.333 & 0 & 0 \\ j12.5 & 0 & 0 & -j32.48 & j10.0 & 0 & j10.0 & 0 \\ 0 & j5.556 & 0 & j10.0 & -j35.526 & 0 & j10.0 & j10.0 \\ 0 & 0 & j8.333 & 0 & 0 & -j28 & j10.0 & j10.0 \\ 0 & 0 & 0 & j10.0 & j10.0 & j10.0 & 2.917 - j31.217 & 0 \\ 0 & 0 & 0 & 0 & j10.0 & j10.0 & 0 & 1.363 - j20.369 \end{bmatrix}$$

$$\mathbf{Y}_{\text{bus}}^{\text{faulted}} = \begin{bmatrix} -j12.5 & 0 & 0 & j12.5 & 0 & 0 & 0 & 0 \\ 0 & -j5.556 & 0 & 0 & j5.556 & 0 & 0 & 0 \\ 0 & 0 & -j8.333 & 0 & 0 & j8.333 & 0 & 0 \\ j12.5 & 0 & 0 & -j32.48 & j10.0 & 0 & 0 & 0 \\ 0 & j5.556 & 0 & j10.0 & -j35.526 & 0 & 0 & j10.0 \\ 0 & 0 & j8.333 & 0 & 0 & -j28.313 & 0 & j10.0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & j10.0 & j10.0 & 0 & 1.363 - j20.369 \end{bmatrix}$$

Observe that in the faulted admittance matrix the row and column corresponding to the faulted Bus 7 contain only zeroes.

$$\mathbf{Y}_{\text{bus}}^{\text{postfault}} = \begin{bmatrix} -j12.5 & 0 & 0 & j12.5 & 0 & 0 & 0 & 0 \\ 0 & -j5.556 & 0 & 0 & j5.556 & 0 & 0 & 0 \\ 0 & 0 & -j8.333 & 0 & 0 & j8.333 & 0 & 0 \\ j12.5 & 0 & 0 & -j32.48 & j10.0 & 0 & j10.0 & 0 \\ 0 & j5.556 & 0 & j10.0 & -j35.526 & 0 & j10.0 & j10.0 \\ 0 & 0 & j8.333 & 0 & 0 & -j18.313 & 0 & j10.0 \\ 0 & 0 & 0 & j10.0 & j10.0 & 0 & 2.917-j21.217 & 0 \\ 0 & 0 & 0 & 0 & j10.0 & j10.0 & 0 & 1.363-j20.369 \end{bmatrix}$$

In the postfault admittance matrix the line between buses 7–6 is removed. Hence we suitably alter element $Y_{6,6}$, $Y_{7,7}$, $Y_{6,7}$, and $Y_{7,6}$.

Step 5: We now perform Kron reduction on the three admittance matrices given previously and obtain the reduced admittance matrices at the internal generator buses. For the example considered, these matrices are given by

$$\mathbf{Y}_{\text{reduced}}^{\text{prefault}} = \begin{bmatrix} 0.5595 - j4.8499 & 0.3250 + j1.997 & 0.4799 + j1.9573 \\ 0.3250 + j1.997 & 0.1954 - j3.7709 & 0.2913 + j1.2535 \\ 0.4799 + j1.9573 & 0.2913 + j1.2535 & 0.4352 - j3.9822 \end{bmatrix}$$

$$\mathbf{Y}_{\text{reduced}}^{\text{faulted}} = \begin{bmatrix} 0.01 - j7.1316 & 0.0145 + j0.8052 & 0.0249 + j0.2513 \\ 0.0145 + j0.8052 & 0.0209 - j4.3933 & 0.0359 + j0.3628 \\ 0.0249 + j0.2513 & 0.0359 + j0.3628 & 0.0618 - j5.257 \end{bmatrix}$$

$$\mathbf{Y}_{\text{reduced}}^{\text{postfault}} = \begin{bmatrix} 0.7849 - j4.4002 & 0.4147 + j2.1410 & 0.3326 + j1.1458 \\ 0.4147 + j2.1410 & 0.2300 - j3.7254 & 0.2165 + j0.9857 \\ 0.3326 + j1.1458 & 0.2165 + j0.9857 & 0.2930 - j2.6377 \end{bmatrix}$$

We can now formulate the differential equation given by (14.79) and numerically integrate them. The fault is applied at $t = 0$ sec. Hence, for the period from 0 sec to 0.10 sec, we use the faulted reduced admittance matrix elements with the initial angles at δ_i^0, $i = 1, 2, 3$. The initial speeds $\omega_i^0 = 0$, $i = 1, 2, 3$. Since the system is at equilibrium prior to the fault, the angles and speeds cannot change instantaneously. We solve this initial value problem, until $t = 0.1$ sec. At this time the fault is cleared, and we integrate the system beyond this time with the reduced postfault admittance matrix.

The plots of the absolute rotor angles obtained by integrating the equations using the numerical integration technique available in MATLAB are shown in Figure E14.6(b).

We note from the plot of the absolute rotor angles that we cannot make any judgment regarding transient stability. We observe that all the machines angles remain together but increase monotonically. As a result, we plot the relative rotor angles with respect to Generator 1. Typically, we choose the generator with the largest inertia to obtain the relative rotor angle plot. This plot is shown in Figure E14.6(c). We observe from the figure that the generators remain in synchronism, and the relative rotor angles oscillate. We assume here that the inherent damping in the system will bring the machine back to its equilibrium position eventually.

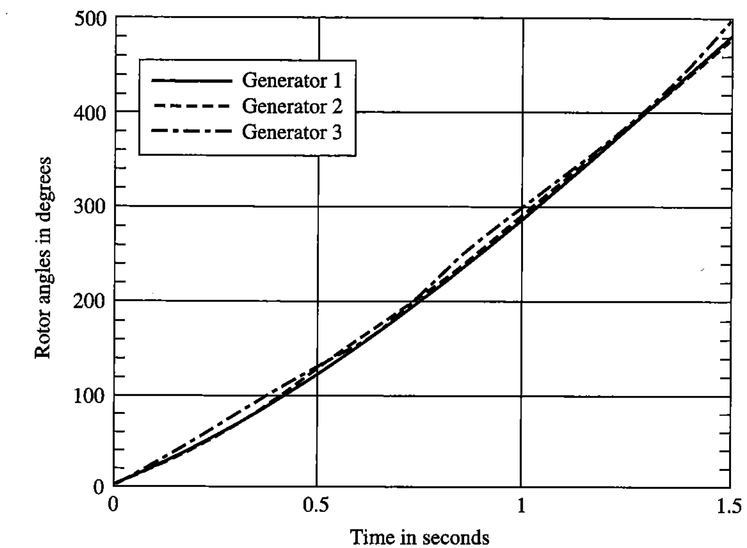

Figure E14.6(b) Plot of absolute rotor angles.

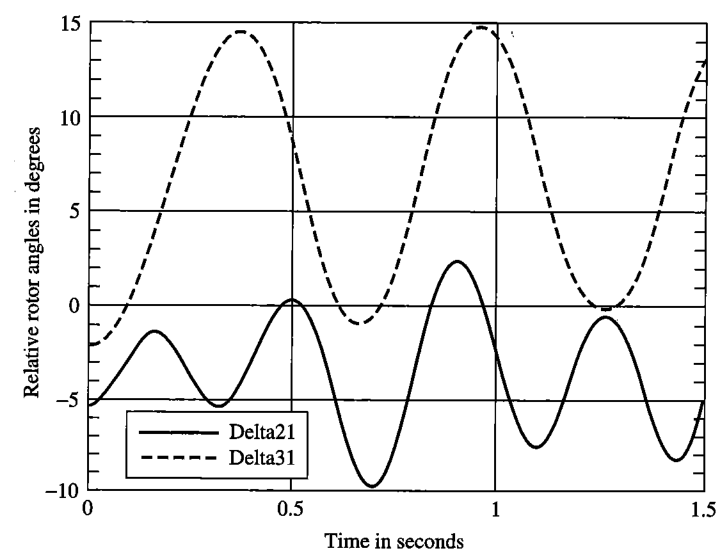

Figure E14.6(c) Plot of relative rotor angles.

We finally show a plot of the rotor speeds in per unit in Figure E14.6(d).

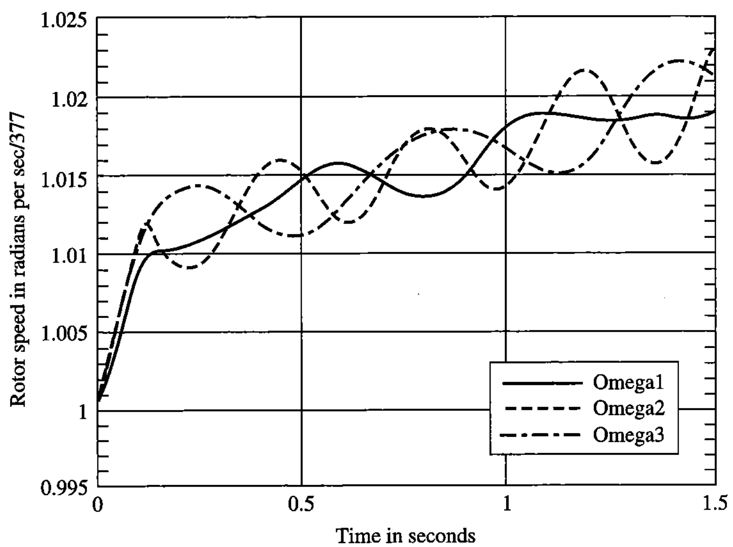

Figure E14.6(d) Plot of rotor speeds.

14.9 SUMMARY

In this chapter we consider the problem of power system stability, in particular the problem of transient stability. For this purpose a dynamic model of the turbine-generator unit is required. Since it is observed in practice that the rotor angle (measured with respect to a synchronous rotating reference frame) undergoes relatively slow variations during these transients, we can derive a very simple generator model. In this model the internal voltage magnitude is assumed to be constant while the phase angle δ is variable. Using the method of pseudo-steady-state analysis, the power output of the generator is found by using the steady-state formula derived in Chapter 6; in this formula we allow δ to be slowly varying. For the case of a turbine-generator unit connected through a lossless transmission link to an infinite bus, the end result is (14.19), a second-order nonlinear differential equation in the power angle δ. We note that the same equation describes an analogous spring-mass system with a nonlinear spring characteristic.

For the transient stability problem, linearization is not usually suitable; we can use an energy method, however, as suggested by the spring-mass analogy. We find that

the sum of kinetic energy and potential energy decreases with time and use this property to infer the system behavior.

Once we understand the principles involved, we may simplify the determination of stability by using the equal-area criterion. If the system is stable, we can also use the equal-area construction to determine the extent of the swings of the power angle. We note that the question of stability is settled in the first swing; if the power angle is initially increasing but reaches a maximum value and then starts to decrease, stability is assured.

The analysis can also be extended to the case of two machines connected through a lossless transmission line. Another extension to the case of two groupings of generators is possible if the machines are closely coupled within each group and weakly coupled between groups.

Although the equal-area criterion is based on a model too simple to give accurate quantitative results, its simplicity enhances understanding and helps explain the measures taken in industry practice to improve stability.

Multimachine transient stability studies are routinely carried out to design systems and also to obtain operating limits to satisfy reliability criteria. In these studies the synchronous generators with the associated controls are modeled appropriately using a system of differential equations. The transmission network that interconnects the synchronous generators is modeled by a system of algebraic equations. As a result, the system is represented by a coupled set of differential and algebraic equations. For the *classical model* representation of the system, the loads are represented as constant impedances. This allows the algebraic equations to be eliminated, and we obtain a set of coupled differential equations that represent the dynamics of the system. Knowing the initial operating condition, the states of the system can be evaluated, and for a given fault, the differential equations can be numerically integrated to obtain the time evolution of the state variables. The plots of the relative rotor angles can then be used to determine the transient stability of the system.

Problems

14.1. Given a turbine-generator unit rated 100 MVA with a per unit inertia constant, $H = 5$ sec.
(a) Calculate the kinetic energy stored at synchronous speed (3600 rpm).
(b) Compare this figure with the kinetic energy of a 10-ton truck going at 60 mph.
(c) Suppose that the generator is delivering 100 MW and then, at $t = 0$, the line circuit breakers open. Calculate the shaft acceleration in rad/sec^2.
(d) At this rate, how long does it take for δ to increase from δ^0 to $\delta^0 + 2\pi$ radians?

14.2. In the model used to study the stability of the single generator connected to an infinite bus, we assumed that $P_M = P_M^0$ was constant. Consider steam governor action such that the mechanical power input to the generator is given by (Figure P14.2)

$$P_M = P_M^0 - k(\omega - \omega_0)$$