ENGN0040: Dynamics and Vibrations
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Project 3 Report
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The Shake Shack: Vibration Isolation Project

I. Introduction

For our third ENGN0040 project, we were tasked with creating a device that could isolate a platform from the vibrations of a shake-table. First, we came up with a few ideas for a design, then unanimously decided to build our contraption out of wood and tension springs. We used the formula for natural frequency in a spring-mass system without damping:

$$w_n = \sqrt{\frac{k}{m}}$$

and the fact that we wanted a device with a low natural frequency to determine that we were going to use as large a mass as possible and as stiff springs as possible to minimize the natural frequency of the platform.

We constructed our device, tested it on the shake-table provided, and compared our experimental natural frequency and transmissibilities to the values we calculated in the design process. We were ultimately very satisfied with how little our platform vibrated during testing and with how close our model natural frequency was to our experimental natural frequency.

II. Design Candidates

Before finalizing our design, we had 4 other ideas that we considered.

1. Welded aluminum cube: As we brainstormed initial ideas, welding was one of the first things that we spitballed. We wanted to create a design that stood out, but decided against it for fear that our contraption would weigh more than the 300 gram limit.
2. 3D printing: Our next idea was to 3D print our design. While this seemed like a safe way to get accurate measurements, we feared that the 3D printed model would be too weak. We weren't sure what mass we would be using for our actual test, and did not want to risk having our design snap under the high amplitude vibrations of the shake-tables.
3. Perforated aluminum cube: Our 3rd idea was using perforated aluminum sheet metal to make a cube. This was our 2nd favorite idea, and would have likely worked well, but we did not love the look of the perforated aluminum, and wanted something more aesthetic.
4. Springs below the table: Our fourth idea wasn't related to the type of material being used, but was related to the positioning of the springs on our contraption. At this point we wanted to make something cube shaped for sure, but weren't sure about the ideal spring positioning. We decided against putting our springs below the top of our contraption, because we were concerned about instability, and thought that there were better alternatives for positioning the springs and vibration isolation system.

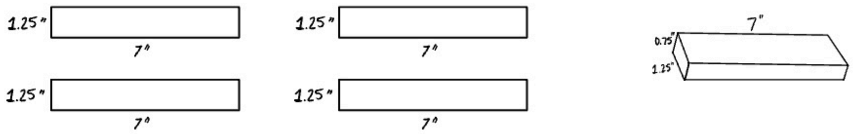
5. **Wooden box:** The design that we landed on was a wooden box with springs on top of the upside-down-table-like contraption. This would meet the project's maximum mass requirement, while also being easy to cut (with a bandsaw), easy to connect (using glue or nails), and sturdy under intense vibrations. Using wood also satisfied our personal requirement of aesthetics, and it was a fun opportunity to use our BDW woodworking skills.

III. Final Design

Our final design was made with 8 pieces of wood plus a platform, secured using nails and a nail gun. The wood base and wood pillars, shaped like a hollow cube without a top, were the foundation that supported our springs and vibration isolation platform. We also screwed 8 pieces of a perforated metal sheet to our contraption, one into the top and bottom of each of the 4 wood pillars. The bottom pieces of metal sheet were used to bolt our contraption to the shake-table. The top pieces of metal sheet were used to attach our tension springs and wooden vibration isolation platform to our contraption. When experimenting with our design, it was important to ensure that the contraption was safely connected to the shake-table, that the springs were correctly attached to the top pieces of perforated metal sheet, and that the mass being used fit comfortably into the platform. The platform, also made of wood, had a hole in each corner for the springs to attach to.

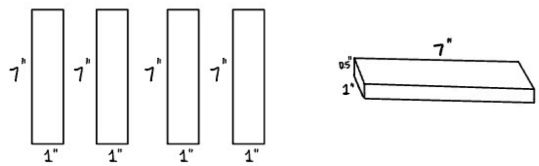
IV. Technical Drawing:

• 4 7"×1.25"×0.75" wood pieces (a)



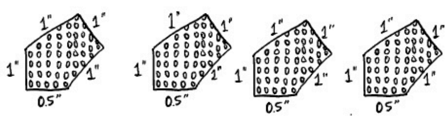
• 8 nails (2 per corner) – nail gun (b)

• 4 7"×1"×½" wood pieces (c)

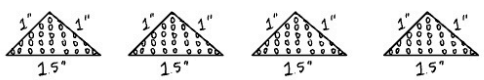


• corrogated aluminum sheet

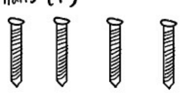
(d)



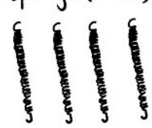
(e)



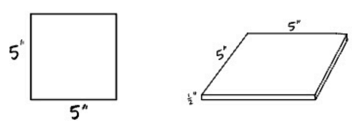
• 4 nails (f)



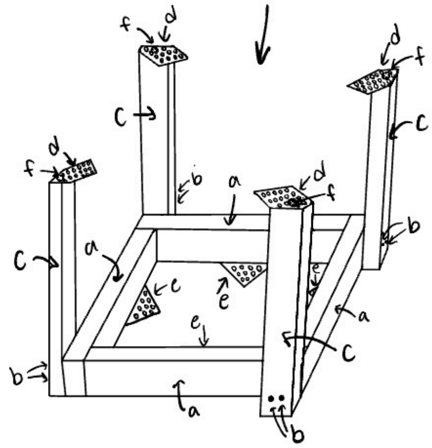
• 4 springs (k=0.2) (g)

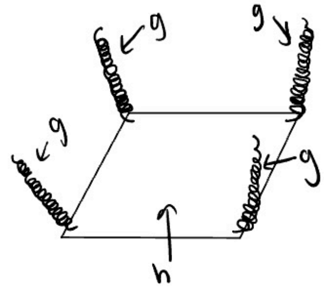


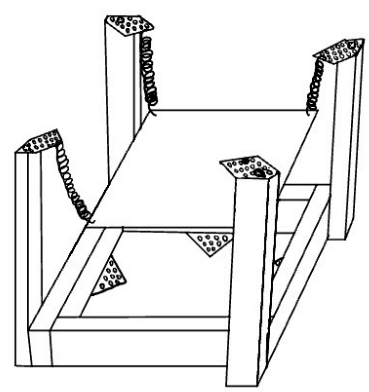
• 1 5"×5"×½" wood piece (h)



base structure







Full construction

V. Calculations for Estimated Natural Frequency

A. Finding the equilibrium height of the platform

Position vectors:

$$\boldsymbol{r_o}^{(1)} = -3.5\boldsymbol{i} + 6\boldsymbol{j}$$
$$\boldsymbol{r_o}^{(2)} = 3.5\boldsymbol{i} + 6\boldsymbol{j}$$
$$\boldsymbol{r_1}^{(1)} = -2.5\boldsymbol{i} + h\boldsymbol{j}$$
$$\boldsymbol{r_1}^{(2)} = 2.5\boldsymbol{i} + h\boldsymbol{j}$$

Spring Lengths:

$$L_1 = ||\boldsymbol{r_1}^{(1)} - \boldsymbol{r_o}^{(1)}||$$
$$L_2 = ||\boldsymbol{r_1}^{(2)} - \boldsymbol{r_o}^{(2)}||$$

Potential Energy of Platform:

$$U = 0.5k\,(L_1 - L_0)^2 + F_0\,(L_1 - L_0) + 0.5k\,(L_2 - L_0)^2 + F_0\,(L_2 - L_1) + mgh$$

To find h: solve $\frac{dU}{dh} = 0$

Matlab calculations for equilibrium height:

Project3 Code

```
% ENGN0040: Dynamics and Vibrations
% Madison Goeke, Baurice Kovatchev, Carter Smith, Henry Zamore
% April 27, 2024

% Setting up problem:
syms h x wn real % Variables
weight = 0.661; % 300 grams in pounds
g = 32.2*12; % gravity (in/s^2)
mass = weight/g; % convert to blobs
k = 0.2; L0 = 1.875; F0 = 0.08; % Spring parameters

% Box and platform dimensions:
width = 7; % inches
box_height = 6; % inches
platform_width = 5; % inches

% Box dimensions for calculations:
r01 = [-width/2,box_height]; r02 = [width/2,box_height]; % top left
r11 = [-platform_width/4,h]; r12 = [platform_width/4,h]; % top right
L1 = sqrt(dot(r11-r01,r11-r01)); % length of spring 1
L2 = sqrt(dot(r12-r02, r12-r02)); % length of spring 2

% Potential energy of platform:
U = (k/2)*(L1-L0)^2 + F0*(L1-L0)...
  + (k/2) *(L2-L0)^2 + F0*(L2-L0)...
```

```
  + weight*h;

% Solve for the height of the platform:
eq = diff(U,h)==0;

h = vpasolve(eq,h) % Height of platform
```

h = 3.1972877050041596820773886398 36

We found the equilibrium height of the platform to be 3.2 inches off the floor.

B. Deriving the equation of motion of the platform, linearizing for a small horizontal displacement

Equation of motion:

$$m\frac{d^2x}{dt^2} - F_x = 0$$

$$F_x = -\frac{\partial U}{\partial x}$$

Linearize for small x:

$$F_x \approx F_x(0) + \frac{\partial F_x}{\partial x}|_{x=0}\, x$$

$$\frac{\partial F_x}{\partial x}|_{x=0} \approx -kx$$

$$\frac{m}{kx}\frac{d^2x}{dt^2} + x = 0$$

$$\frac{d^2x}{dt^2} + \frac{kx}{m}x = 0$$

$$w_n = \sqrt{\frac{kx}{m}}$$

Matlab calculations for natural frequency:

```
% Platform dimensions:
r11 = [-platform_width/4+x,h]; % left side of platform
r12 = [platform_width/4+x,h]; % right side of platform
L1 = sqrt(dot(r11-r01,r11-r01)); % length of spring 1
L2 = sqrt(dot(r12-r02,r12-r02)); % length of spring 2

% Potential energy of platform:
U = (k/2)*(L1-L0)^2 + F0*(L1-L0)...
  + (k/2)*(L2-L0) ^2 + F0*(L2-L0)...
  + weight*h;
```

```
% Natural frequency calculations:
Fx = simplify(diff(U,x));
kx = subs (diff (Fx,x), x, 0)
```

kx = 0.30017680896029191453162642798564

```
wn = sqrt(kx/mass);

wn = vpa(wn/2/pi) % natural frequency
```

wn = 2.108271456795785518327063276288

We estimated our natural frequency to be 2.1Hz.

VI. Calculations for Estimated Transmissibility

A. Find horizontal transmissibility, linearizing for small horizontal and vertical displacements

Equation of motion:

$$\boldsymbol{r_0}^{(1)} = (-3.5 + z)\boldsymbol{i} + 7\boldsymbol{j}$$

$$\boldsymbol{r_0}^{(2)} = (3.5 + z)\boldsymbol{i} + 7\boldsymbol{j}$$

$$m\frac{d^2x}{dt^2} - F_x = 0$$

$$F_x = -\frac{\partial U}{\partial x}$$

Linearize for small x and z:

$$F_x \approx \frac{\partial F_x}{\partial x}x + \frac{\partial F_x}{\partial z}z$$

$$\frac{m}{kx}\frac{d^2x}{dt^2} + x = \frac{k_z}{k_x}z$$

$$k_x = -\frac{\partial F_x}{\partial x}$$

$$k_z = -\frac{\partial F_x}{\partial z}$$

$$\frac{1}{w_n^2}\frac{d^2x}{dt^2} + x = K_z$$

$$w_n = \sqrt{\frac{k_x}{m}}$$

$$K = \frac{k_z}{k_x}$$

Transmissibility for no damping:

$$T(w, w_n) = \frac{1}{\left|1 - \frac{w^2}{w_n^2}\right|}$$

Matlab calculations for transmissibility at 8 and 45 Hz:

```
trans = @(w) 1/norm(1 - w^2/wn^2); % trasmissibility formula (no damping)

trans_at_8 = vpa(trans(8)) % transmissibility at 8 Hz
```

trans_at_8 = 0.074633435410786696266087528251844

```
trans_at_45 = vpa(trans(45)) % transmissibility at 45 Hz
```

trans_at_45 = 0.0021997956573105585833218423770027

We estimated the transmissibility of our contraption to be 0.075 at 8Hz and 0.0022 at 45Hz.

VII. Experimental Natural Frequency

During testing, we found the natural frequency to be 2.2Hz. At this frequency, the movement of the platform and oscillations of the springs would line up leading to a very high amplitude of oscillations that ended in the mass flying off the platform. Once the mass was no longer on the platform, the resonance ended as the mass of the platform had decreased significantly, increasing its natural frequency to higher than the 2.2Hz the table was shaking at. After this test, we no longer questioned the eye protection mandate.

Our experimental natural frequency was only 0.1Hz off of our estimated natural frequency, 2.1Hz, which was the smallest increment on the shake-table. It had a 4.76% error with respect to our estimated natural frequency. It is notable that even when we set the table to oscillate at 2.1Hz, this coincided with the natural frequency of the platform enough to aggressively throw the mass off its platform. 2.2Hz just did this slightly faster.

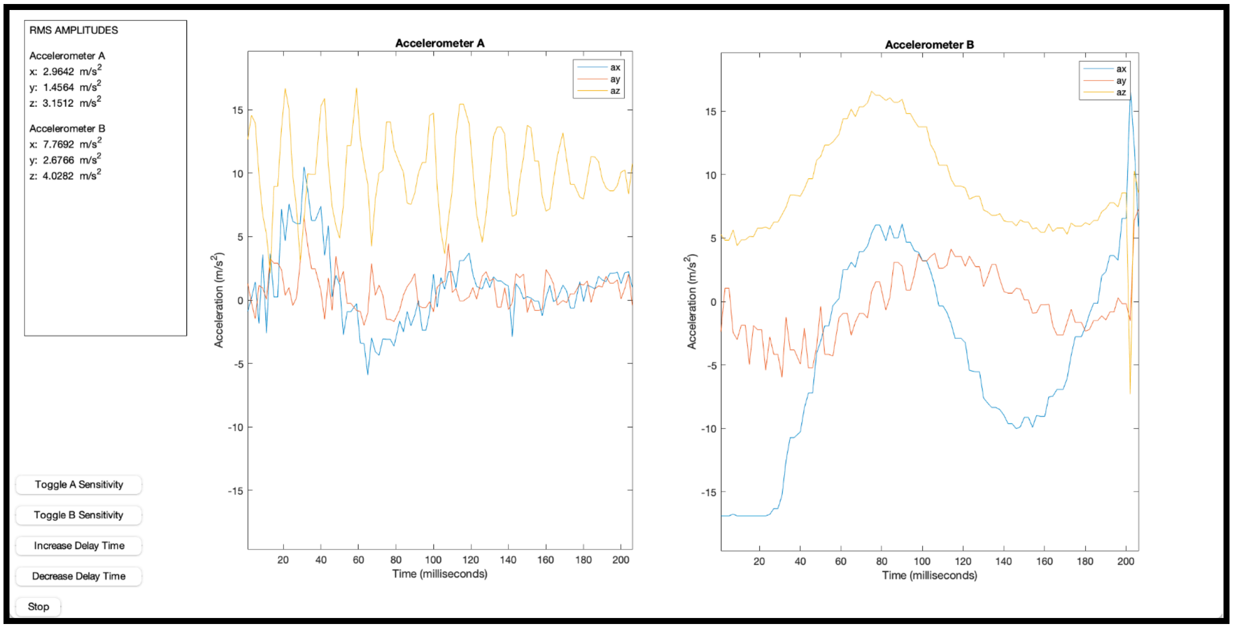


Figure 1: Accelerometer Data at Resonant Frequency 2.2Hz

As seen in Figure 1, both the accelerometer attached to the table and platform detected large and unpredictable oscillations at 2.2Hz.

VIII. Experimental Transmissibility

At 8Hz, the accelerometers measured our transmissibility to be:

$$T = \frac{0.96597}{7.2183}$$

$$T = 0.13388$$

This had a 78.5% error with respect to our calculated value.

At 45Hz, the accelerometers measured our transmissibility to be:

$$T = \frac{0.36785}{3.712}$$

$$T = 0.099098$$

This had a 4,400% error with respect to our calculated value.

Despite these large percent errors compared to the seeming accuracy of the rest of our calculations, we did not expect our calculated values for transmissibility to be very accurate. For example, when deriving the equation of motion of the platform when the shake-table is set to 45Hz, assuming zero damping or other interference with the platform, the oscillations should be so small that they could not be seen or felt, leading to an impressive transmissibility. However in the lab, the damping, while it is small, is enough to raise the experimental transmissibility, slightly in absolute value but very noticeably in percentage change.

In addition to measuring the natural frequency and transmissibility at 8- and 45Hz, we also observed the accelerometer data of the table and platform at 8-, 15-, 30-, and 45Hz.

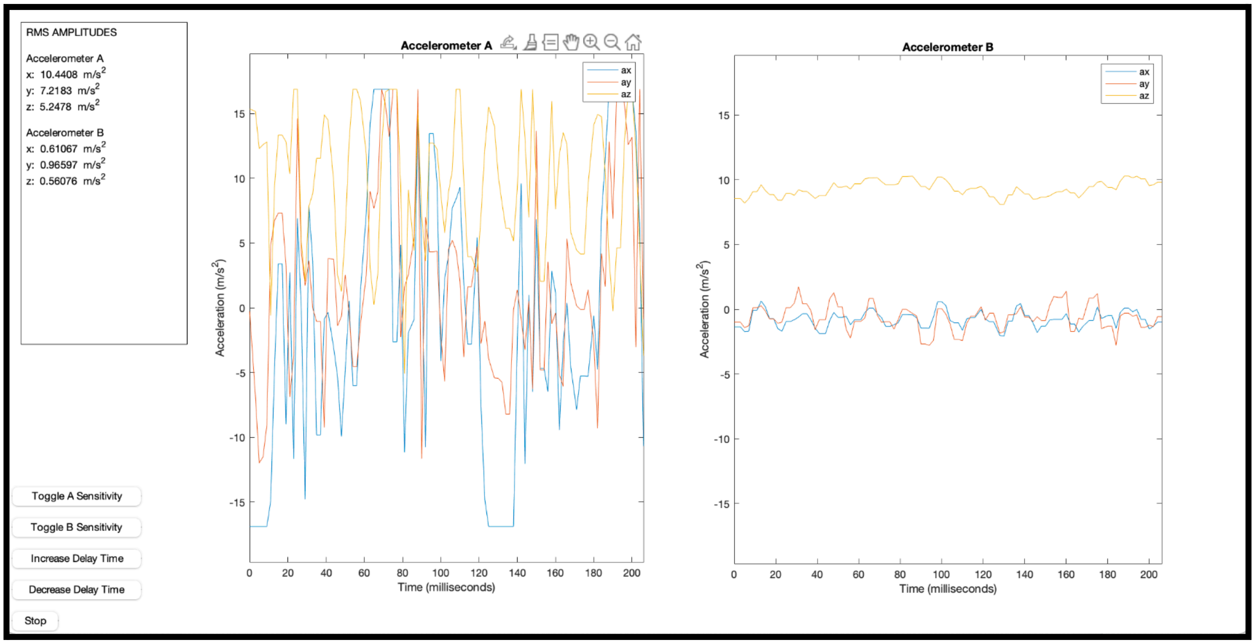


Figure 2: Accelerometer Data at 8Hz

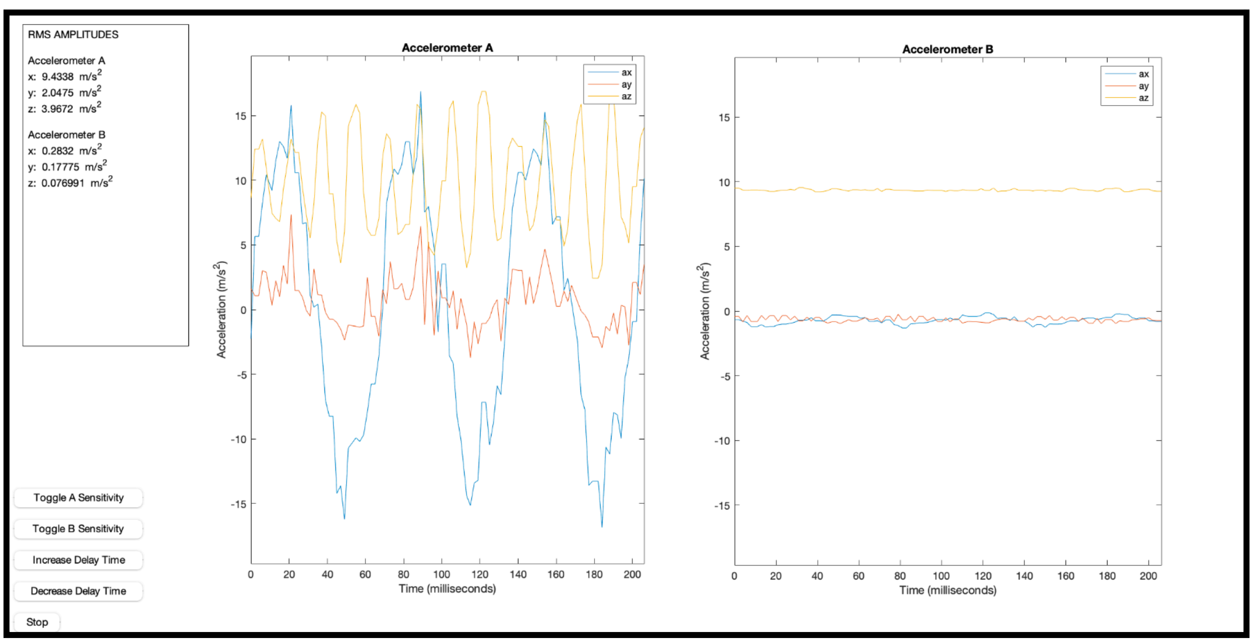


Figure 3: Accelerometer Data at 15Hz

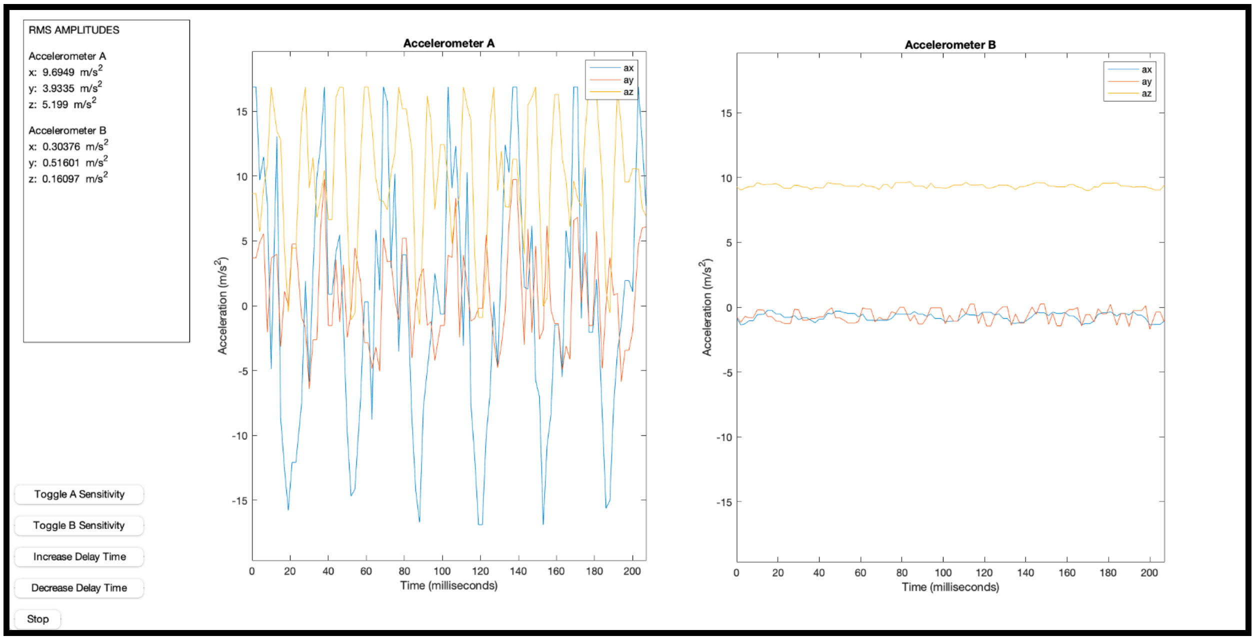


Figure 4: Accelerometer Data at 30Hz

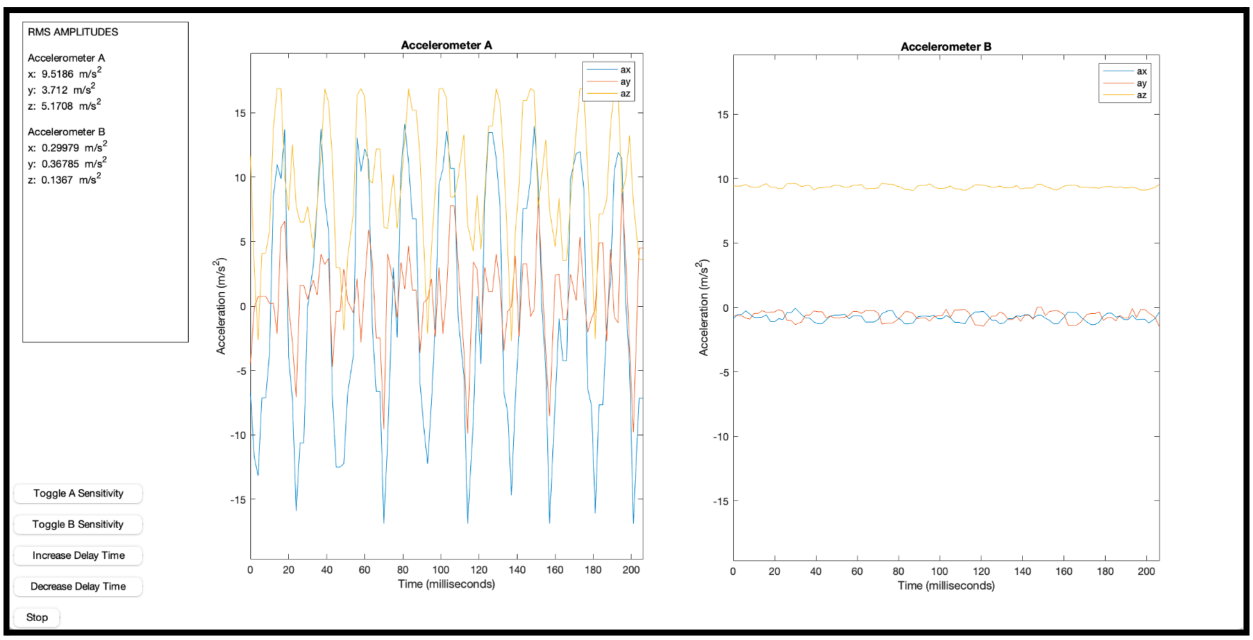


Figure 5: Accelerometer Data at 45Hz

While the amplitudes of vibrations of the platform under 15-, 30-, and 45Hz were about the same and very minimal, we did notice a slight increase in the amplitude of the platform at 8Hz, which we attributed to the fact that this was closer to the natural frequency of the platform, 2.2Hz.