ASSIGNMENT 5

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Q5.1:

The given spheroid equation is: $\frac{x^2+y^2}{a^2}+\frac{z^2}{c^2}=1$

Let us consider a disc centred on the z-axis connecting the poles of the spheroid. Its radius is thus on the y-axis and height is *dx*. The volume of the disc will be $\pi r^2 dx$, where r is the radius of the disc.

∴ Replacing x = 0:

$\frac{y^2}{a^2}+\frac{z^2}{c^2}=1$

or, $\frac{y^2}{a^2}=1-\frac{z^2}{c^2}$

or, $y^2 = a^2\left(1-\frac{z^2}{c^2}\right)$

∴ Volume of the spheroid = $\int_{-c}^{c}\pi a^2\left(1-\frac{z^2}{c^2}\right)dz=\pi\int_{-c}^{c}\left(a^2-\frac{a^2z^2}{c^2}\right)dz=\pi\left[a^2z-\frac{a^2z^3}{3c^2}\right]_{-c}^{c}$

$$=\pi\left\{a^2c-\frac{a^2c^3}{3c^2}-\left(-a^2c+\frac{a^2c^3}{3c^2}\right)\right\}$$

$$=\pi\left(2a^2c-\frac{2a^2c^3}{3c^2}\right)$$

$$=\pi\left(\frac{4a^2c^3}{3c^2}\right)$$

$$=\frac{4}{3}\pi a^2c.$$

Q5.2:

Monte-Carlo Integration:

```
trials = 10
res = zeros(trials,1);
N = 10000;
a=0;b=1;
for i=1:trials
% N random points in the
interval[0,1]
x_i =rand(N,1);
integral=0;
for k=1:N
    integral=integral+f(x_i(k));
end
res(i)= ((b-a)/N)*integral;
end

%Error
SD = zeros(trials,1);
for i=1:trials
    SD(i) = std(res(1:i));
end
res
SD

function func = f(x)
func = 1/(1+sinh(2*x)*(log(x)^2));
end
```

```
res =

    0.6900
    0.6927
    0.6893
    0.6907
    0.6890
    0.6884
    0.6903
    0.6920
    0.6890
    0.6930
```

```
SD =

         0
    0.0019
    0.0018
    0.0014
    0.0015
    0.0015
    0.0014
    0.0015
    0.0014
    0.0016
```

Simpson's Method:

```
f = @(x) 1/(1+(sinh(2*x))*((log(x))^2));
N = 100000;
b = 1;
a = 0;
del_x = (b - a)/N;
fsum = f(a+del_x) + f(b);
for i = 2:N-1
    xj = a + i*del_x;
    fsum = fsum + (3 - (-1)^i)*f(xj);
end
result = fsum*(del_x/3)
```

```
>> Simpson

result =

    0.6905
```