# ASSIGNMENT 08

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**Q 8.2**:

According to the question, $A, B, C, D \in \mathbb{E}^3$ are the 4 corner points of a tetrahedron, having position vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d} \in \mathbb{R}^3$. A point $P$ with position vector $\vec{p}$ can be represented as

$$\vec{p} = \alpha_1\vec{a} + \alpha_2\vec{b} + \alpha_3\vec{c} + \alpha_4\vec{d}$$

where $\sum_{i=1}^{i=4} \alpha_i = 1$. From this, we can write,

$$\begin{gathered}
\vec{p} - \vec{d} = \alpha_1\vec{a} + \alpha_2\vec{b} + \alpha_3\vec{c} + \alpha_4\vec{d} - \vec{d} \\
\Rightarrow \vec{p} - \vec{d} = \alpha_1\vec{a} + \alpha_2\vec{b} + \alpha_3\vec{c} + \vec{d}(\alpha_4 - 1) \\
\Rightarrow \vec{p} - \vec{d} = \alpha_1\vec{a} + \alpha_2\vec{b} + \alpha_3\vec{c} + \vec{d}(-\alpha_1 - \alpha_2 - \alpha_3) \\
\Rightarrow \vec{p} - \vec{d} = \alpha_1(\vec{a} - \vec{d}) + \alpha_2(\vec{b} - \vec{d}) + \alpha_3(\vec{c} - \vec{d})
\end{gathered}$$

In matrix form, this can be written as,

$$\vec{p} - \vec{d} = \begin{pmatrix} a_x - d_x & b_x - d_x & c_x - d_x \\ a_y - d_y & b_y - d_y & c_y - d_y \\ a_z - d_z & b_z - d_z & c_z - d_z \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

$$\therefore \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = A^{-1}(\vec{p} - \vec{d})$$

where $A = \begin{pmatrix} a_x - d_x & b_x - d_x & c_x - d_x \\ a_y - d_y & b_y - d_y & c_y - d_y \\ a_z - d_z & b_z - d_z & c_z - d_z \end{pmatrix}$

The above calculation will fail if $A$ is not invertible, that is if $\det A$ is 0.