THEORETICAL AND METHODOLOGICAL FOUNDATIONS OF VISUAL COMPUTING

ASSIGNMENT 04

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Exercise 4.1:

$\vec{f}(x, y, z) = (2x\cos y\,, -x^2\sin y, 2z)^T$

(a) $curl(\vec{f}) = \begin{pmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x\cos y & -x^2\sin y & 2z \end{pmatrix}$

[**i, j** and **k** are the unit vectors in the x, y and z directions respectively]

$$= \left\{\frac{\partial}{\partial y}(2z) - \frac{\partial}{\partial z}(-x^2\sin y)\right\}\boldsymbol{i} - \{\frac{\partial}{\partial x}(2z) - \frac{\partial}{\partial z}(2x\cos y)\}\boldsymbol{j} + \{\frac{\partial}{\partial x}(-x^2\sin y - \frac{\partial}{\partial y}(2x\cos y)\}\boldsymbol{k}$$

$$= (0)\boldsymbol{i} - (0)\,\boldsymbol{j} + (0)\boldsymbol{k}$$
$$= 0$$

(b) A function $\psi(x, y, z)$ such that $\vec{f} = (\partial_x\psi, \partial_y\psi, \partial_z\psi)^T$ is:
$\psi(x, y, z) = x^2\cos y + z^2$

Exercise 4.2:

$z = 2 - x^2 - y^2$

$$\iint_{\boldsymbol{D}} (2 - x^2 - y^2)d(x, y) = \int_{-1}^{1}\left(\int_{-1}^{1}(2 - x^2 - y^2)dx\right)dy$$

$$= \int_{-1}^{1}\left[2x - \frac{x^3}{3} - y^2x\right]_{x=-1}^{x=1} dy$$

$$= 2\int_{-1}^{1}\left(\frac{5}{3} - y^2\right) dy$$

$$= 2\left[\frac{5}{3}y - \frac{y^3}{3}\right]_{y=-1}^{y=1}$$

$$= 2 \times 2 \times \frac{4}{3}$$

$$= \frac{16}{3}$$

$$= 5.33$$

Exercise 4.3:

$$\vec{H}(x, y, z) = \frac{1}{x^2 + y^2}\begin{pmatrix} -y \\ x \\ 0 \end{pmatrix}$$

Let each point on the circular line be represented by $\alpha(t) = \begin{pmatrix} r\cos t \\ r\sin t \\ 0 \end{pmatrix}$

$$\therefore \vec{H}\big(\vec{\alpha}(t)\big) = \vec{H}(r\cos t\,, r\sin t\,, 0) = \begin{pmatrix} \dfrac{-r\sin t}{r^2(\sin^2 t + \cos^2 t)} \\ \dfrac{r\cos t}{r^2(\sin^2 t + \cos^2 t)} \\ 0 \end{pmatrix} = \begin{pmatrix} \dfrac{-\sin t}{r} \\ \dfrac{\cos t}{r} \\ 0 \end{pmatrix}$$

$$\therefore \int_{\vec{\alpha}} \vec{H} \cdot d\vec{x} = \int_0^{2\pi} \langle \vec{H}\big(\vec{\alpha}(t)\big), \dot{\vec{\alpha}}(t) \rangle \, dt = \int_0^{2\pi} (\sin^2 t + \cos^2 t + 0)\, dt = [t]_0^{2\pi} = 2\pi$$

Exercise 4.4:

$$A_{\boldsymbol{n}} = \iint_{\Omega_n} e^{-\frac{1}{2}(x^2+y^2)} dxdy$$

Converting *x*,*y* into polar coordinates, $x = r\cos\theta, y = r\sin\theta$

$$\therefore A_{\boldsymbol{n}} = \iint_{\Omega_n} e^{-\frac{1}{2}(x^2+y^2)} dxdy = \iint_{\Omega_n} e^{-\frac{1}{2}(r^2\cos^2\theta + r^2\sin^2\theta)}\, r\, d\theta\, dr = \iint_{\Omega_n} e^{-\frac{1}{2}(r^2)}\, r\, d\theta\, dr$$

$$= \iint_{\Omega_n} e^{-\frac{1}{2}(r^2)}\, r\, dr\, d\theta = \iint_{\frac{1}{2n^2}}^{\frac{n^2}{2}} e^{-\frac{1}{2}(r^2)}\, d\left\{\frac{1}{2}(r^2)\right\} d\theta$$

$$= \int_0^{2\pi} \left[-e^{-\frac{1}{2}(r^2)}\right]_{\frac{1}{2n^2}}^{\frac{n^2}{2}} d\theta = \int_0^{2\pi} \left[-e^{-\frac{n^2}{2}} + e^{-\frac{1}{2n^2}}\right] d\theta$$

$$= \left[-e^{-\frac{n^2}{2}} + e^{-\frac{1}{2n^2}}\right] \int_0^{2\pi} d\theta = 2\pi \left[-e^{-\frac{n^2}{2}} + e^{-\frac{1}{2n^2}}\right]$$

$$\lim_{n\to\infty} A_{\boldsymbol{n}} = 2\pi[0+1] = 2\pi$$