THEORETICAL AND METHODOLOGICAL FOUNDATIONS OF VISUAL COMPUTING

ASSIGNMENT 3

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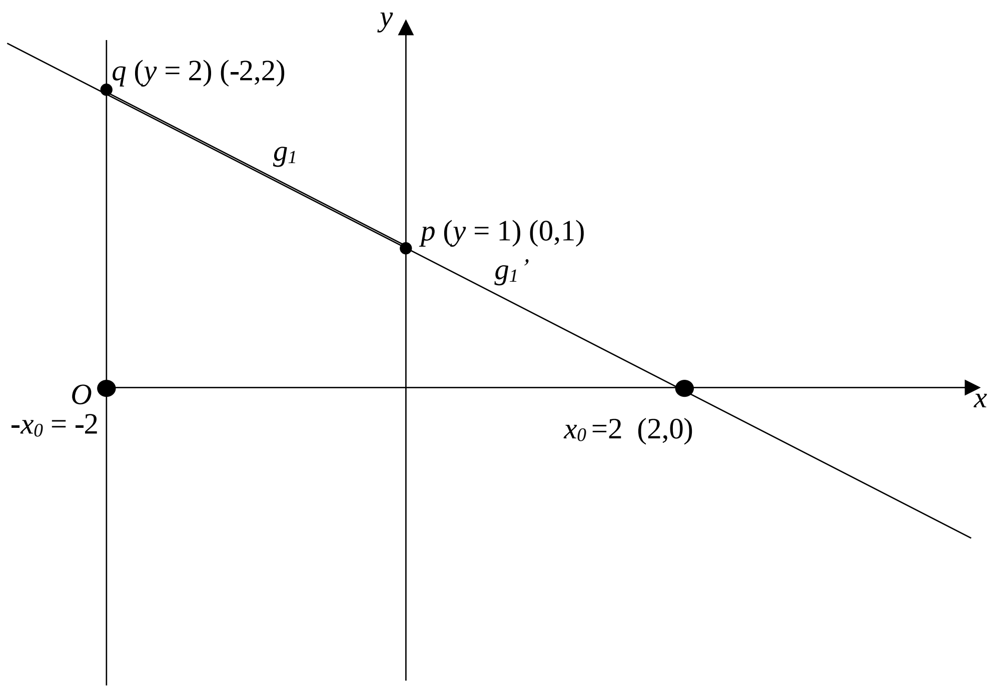
Exercise 3.1:

(a) The diagram corresponding to the given problem:



Point p is at (0, y_p) and point q is at (-x_0:y_q:1). The observer O is at (-x_0:0:1). As is evident from the figure, since Oq || y-axis, it will not have any projection on the Y-axis that is the projection will be at infinity.

For the line segment *pq:*

- Slope $m = \frac{y_q - y_p}{-x_0 - 0} = \frac{y_p - y_q}{x_0}$
- The intercept $y_0 = y_p$
- The normal to the line segment pq will be along the line segment with slope $\frac{x_0}{y_q - y_p}$

Applying the perspective transformation on the representations of *pq*:

- $g'_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{x_0} & 0 & 1 \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{pmatrix} = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \frac{\xi_1}{x_0} + \xi_3 \end{pmatrix}$

- $g'_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{x_0} & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ mx + y_0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ mx + y_0 \\ \frac{x}{x_0} + 1 \end{pmatrix} = \begin{pmatrix} x \\ mx + y_0 \\ \frac{x}{x_0} + 1 \end{pmatrix}$

Applying the perspective transformation to the line g_1 for the input values $y_p = 1$, $y_q = 2$, $x_o = 2$, we get,

$m = \frac{y_p - y_q}{x_0} = \frac{1-2}{2} = -\frac{1}{2}$

$g_1' = \begin{pmatrix} x \\ -\frac{1}{2}x + 1 \\ \frac{x}{2} + 1 \end{pmatrix}$

The equation of g_1' will be $y = -\frac{1}{2}x + 1$. Since point p lies on the projection line, it is unchanged after transformation and g_1' will pass through p. However, since the point q does not have any projection on the y-axis, it will be transformed to the point at infinity.

$q' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{x_0} & 0 & 1 \end{pmatrix} \begin{pmatrix} -x_0 \\ y_q \\ 1 \end{pmatrix} = \begin{pmatrix} -x_0 \\ y_q \\ 0 \end{pmatrix}$, which indicates a point at infinity.

(b) The point at infinity for g_1 will be the transformed point q', since point q is the image of that point on g_1. The point in homogeneous coordinates is $(-x_0 : y_q : 0)$.
(c) The vanishing point for g_1' will be point q, since q' is at infinity.

Exercise 3.2:

$$f(x, y) = e^{-2x^2 - 2y^2 + 4x + 2y - 3}$$

Total derivative of $f(x, y)$ at a point (a, b) is given by:

$$g(x, y) = f(a, b) + \frac{\partial f}{\partial x}|_{(a,b)}.(x - a) + \frac{\partial f}{\partial y}|_{(a,b)}.(y - b)$$

$$g(x, y) = e^{-2a^2 - 2b^2 + 4a + 2b - 3} + (-4x + 4)e^{-2a^2 - 2b^2 + 4a + 2b - 3}.(x - a) + (-4y + 2)e^{-2a^2 - 2b^2 + 4a + 2b - 3}.(y - b)$$

$$= e^{-2a^2 - 2b^2 + 4a + 2b - 3}[(-4x + 4).(x - a) + (-4y + 2).(y - b)]$$

(Remaining part in Matlab Code)

Exercise 3.3:

$f(x,y) = x^2 - 6x^2y + 2y^3$

$f_x = 2x - 12xy$

$f_y = -6x^2 + 6y^2$

At extrema positions, $f_x = 0$, $f_y = 0$

$$2x - 12xy = 0 \quad => x(1-6y) = 0 \quad \therefore x = 0, y = 1/6$$

$$-6x^2 + 6y^2 = 0 \quad => x^2 = y^2 \quad => x = \pm y$$

∴The extremum points are (0,0), $\left(\frac{1}{6},\frac{1}{6}\right),\left(-\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{6},-\frac{1}{6}\right),(-\frac{1}{6},-\frac{1}{6})$

To find the saddle points and the local extrema, we have to perform the second-derivative test:

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

$f_{xx} = 2 - 12y$

$f_{yy} = 12y$

$f_{xy} = -12x$

$D = (2-12y)(12y) - (-12x)^2$

$D_{(0,0)} = 0$

$D_{\left(\frac{1}{6},\frac{1}{6}\right)} = -4 \quad < 0 \quad \therefore saddle\ point$

$D_{\left(\frac{1}{6},-\frac{1}{6}\right)} = -12 \quad < 0 \quad \therefore saddle\ point$

$D_{\left(-\frac{1}{6},\frac{1}{6}\right)} = -4 \quad < 0 \quad \therefore saddle\ point$