



Analysis of bibliographic networks

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Outline

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Tip 8 - References: always go back to the original source!

tip 8

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Current version of slides (March 17, 2022 at 02:36): [slides PDF](#)

<http://vladowiki.fmf.uni-lj.si/doku.php?id=notes:vis:nds>



Networks

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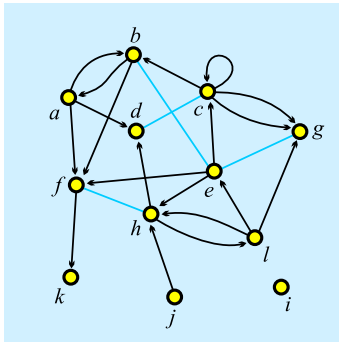
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A *network* is based on two sets – a set of *nodes* (vertices), that represent the selected *units*, and a set of *links* (lines), that represent *ties* between units. They determine a *graph*. A link can be *directed* – an *arc*, or *undirected* – an *edge*. Additional data about nodes or links may be known – their *properties* (attributes). For example: name/label, type, age, value, ...

Network = Graph + Data



Network

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A *network* $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{P}, \mathcal{W})$ consists of:

- a *graph* $\mathcal{G} = (\mathcal{V}, \mathcal{L})$, where \mathcal{V} is the set of nodes, \mathcal{A} is the set of arcs, \mathcal{E} is the set of edges, and $\mathcal{L} = \mathcal{E} \cup \mathcal{A}$ is the set of links.
 $n = |\mathcal{V}|$, $m = |\mathcal{L}|$
- \mathcal{P} *node value functions* / properties: $p: \mathcal{V} \rightarrow A$
- \mathcal{W} *link value functions* / weights: $w: \mathcal{L} \rightarrow B$



Types of networks

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In a *two-mode* network $\mathcal{N} = ((\mathcal{V}_1, \mathcal{V}_2), \mathcal{L}, \mathcal{P}, \mathcal{W})$ its set of nodes is split to two subsets. Each link has its end-nodes in both sets.

In a *multi-relational* network $\mathcal{N} = (\mathcal{V}, (\mathcal{L}_i, i \in I), \mathcal{P}, \mathcal{W})$ the set of its links is partitioned into several mutually disjoint subsets – relations. (Subject Verb Object).

In a *temporal* network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{T}, \mathcal{P}, \mathcal{W})$ the time \mathcal{T} is added. To each node and to each link its *activity* set is assigned. Also properties and weights can change through time – temporal quantities.

In a *linked* or *multimodal* network $\mathcal{N} = ((\mathcal{V}_1, \dots, \mathcal{V}_j), (\mathcal{L}_1, \dots, \mathcal{L}_k), \mathcal{P}, \mathcal{W})$ the set of nodes \mathcal{V} is partitioned into subsets (*modes*) \mathcal{V}_i , $\mathcal{L}_s \subseteq \mathcal{V}_p \times \mathcal{V}_q$, and properties and weights are usually partial functions.

A *collection* of networks consists of some networks with common subsets of nodes.

Types of networks can be combined – for example: a temporal two-mode multi-relational network.



Networks from bibliographies

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For data from the **Web of Science** (Knowledge) we can obtain the corresponding networks using the program **WoS2Pajek**:

- citation network **Ci**: works \times works;
- authorship network **WA**: works \times authors, for works without complete description only the first author is known;
- keywords network **WK**: works \times keywords, only for works with complete description;
- journals network **WJ**: works \times journals;
- partition *year* of works by the publication year;
- partition *CD* of works – complete description (1) / ISI name only (0);

Similar programs exist also for other bibliographic sources/formats: Scopus, BibT_EX, Zentralblatt Math, Google Scholar, DBLP, IMDB, etc.



Record from Web of Science

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PT J
AU Dipple, H
Evans, B
TI The Leicestershire Huntington's disease support group: a social network
analysis
SO HEALTH & SOCIAL CARE IN THE COMMUNITY
LA English
DT Article
C1 Rehabil Serv, Troon Way Business Ctr, Leicester LE4 9HA, Leics, England.
RP Dipple, H, Rehabil Serv, Troon Way Business Ctr, Sandringham
Suite, Humberstone Lane, Leicester LE4 9HA, Leics, England.
CR BORGATTI SP, 1992, UCINET 4 VERSION 1 0
FOLSTEIN S, 1989, HUNTINGTONS DIS DISO
SCOTT J, 1991, SOCIAL NETWORK ANAL
NR 3
TC 3
PU BLACKWELL SCIENCE LTD
PI OXFORD
PA P O BOX 88, OSNEY MEAD, OXFORD OX2 ONE, OXON, ENGLAND
SN 0966-0410
J9 HEALTH SOC CARE COMMUNITY
JI Health Soc. Care Community
PD JUL
PY 1998
VL 6
IS 4
BP 286
EP 289
PG 4
SC Public, Environmental & Occupational Health; Social Work
GA 105UP
UT ISI:000075092200008
ER

WoS2Pajek



Cleaning networks

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The saved records from a bibliographic database can still contain some inconsistencies. Some of them are detected as results of the analyses. The simplest way to deal with them is to correct them in the saved database file and rerun the creation of the network files and analyses.

Check (in Pajek) the obtained networks for multiple links and remove them, if they exist. Remove also the loops from the citation network.



Unit identification problem

Entity Resolution

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The main two problems with units (works, authors, journals, keywords, etc.) are *equivalence* (different words representing the same unit) and *ambiguity* (same word representing different terms).

Synonymy:

Otfried Cheong (formerly Otfried Schwarzkopf): German computational geometer working in South Korea at KAIST.

Michel Marie Deza (formerly Mikhail Efimovich Tytkin): a Soviet and French mathematician, specializing in combinatorics, discrete geometry, and graph theory.

Borštnik, N. S. Mankoč; Mankoč Borštnik, N.; Mankoč-Borštnik, Norma; Mankoč Borštnik, Norma Susana; Mankoc-Borstnik, N.S.; and Mankoč Borštnik, N.S. belong to the same author.

NUCLEIC ACIDS RES, NUCL ACIDS RES, NUCLEIC ACIDS RES S,
NUCLEIC ACIDS RES S2, NUCL ACID RES, NUCL ACIDS RES S2,
NUCL ACIDS S SER, NUCL ACIDS RES S, NUCL AC RES,
NUCLEIC ACIDS RES S1, Nucleic Acids Res, NUCL ACIDS RES S1,



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Homonymy (ambiguity):

Lorenzo Bartolini from **Letters to Juliet**.

Smith, John W. - publications of the author(s) with this name spanned from 1868 to 2007.

Chinese, ≈ 100 surnames – "**three Zhang, four Li**" (there are some hundreds of different mathematicians with the name Zhang, Li in the **MathSciNet** Database).

Kim, Lee, and Park account for nearly one-half of all Koreans.

Nguyen is a surname held by 40% of the Vietnamese population, etc.

Solutions for **names**: **ResearcherID**, **ORCID**, **ISNI**, **AMS**;

for **works**: International Standard Serial Number **ISSN**, Digital

Object Identifier **DOI**, International Standard Book Number **ISBN**;

for **words**: dictionaries, stemming, lemmatization; for **countries**: **ISO**

3166; for **languages**: **ISO 639**



Network boundary problem

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If the obtained collection of networks contains also the citation network it can be used to address the *network boundary problem*. Otherwise we assume that we included in our bibliography (all/most of) the relevant works.

The first option is to limit the network to the works with complete descriptions – records from the WoS file, $DC > 0$. Since for cited-only works only the first author (no keywords, ...) is known this option is used for most analyses.

We can get a richer network if we decide to include also some cited-only works that are cited often – at least k times; we delete nodes for which it holds $(0 < \text{indeg}(v) < k) \wedge (\text{outdeg}(v) = 0)$.

For some (most frequent) of these additional works we can augment the WoS file with their descriptions (without CR data).



Example networks

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SN5: WoS, January 2008, ("social network*" AND SO=(Social networks)) + most frequently cited works + around 100 SNA researchers.
 $|W| = 193376$, $|C| = 7950$, $|A| = 75930$, $|J| = 14651$, $|K| = 29267$.

ZBMath: Data from the Zentralblatt MATH data base for years 1990–2010. $|W| = 1339201$, $|A| = 557104$, $|J| = 3158$, $|K| = 143513$, $|M| = 12390$. [paper](#).

PEERE: WoS, March 2016, using the queries "peer review*" and refereeing + manually prepared descriptions of frequently cited only works. WoS2Pajek produced networks with sets of the following sizes: works $|W| = 721547$, authors $|A| = 295849$, journals $|J| = 39988$, keywords $|K| = 36279$ and $m = 869821$ arcs. 22981 records were collected. [paper](#)

SNA17: WoS, January 2018, $|W| = 1297133/70792$, $|A| = 395971/93011$, $|K| = 32409/32409$, $|J| = 69146/8943$. [paper](#)



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Global/overall properties.

Extreme (minimal/maximal values) units.

Distributions of values.

degree of node v , $\deg(v)$ = number of links with v as an endnode;

indegree of node v , $\text{indeg}(v)$ = number of links with v as a terminal node (endnode is both initial and terminal);

outdegree of node v , $\text{outdeg}(v)$ = number of links with v as an initial node.

source node $v \Leftrightarrow \text{indeg}(v) = 0$

sink node $v \Leftrightarrow \text{outdeg}(v) = 0$

PEERE – most cited works: indegree in Ci

| n | freq | first author | title |
|----|------|-----------------|---|
| 1 | 173 | Cohen, J | Statistical Power Analysis for the Behavioral Sciences. Routledge,1988 |
| 2 | 164 | Peters, DP | Peer-review practices of psychological journals - the fate of ... Behav Brain Sci, 1982 |
| 3 | 151 | Egger, M | Bias in meta-analysis detected by a simple, graphical test. Brit Med J, 1997 |
| 4 | 150 | Stroup, DF | Meta-analysis of observational studies in epidemiology - A proposal for reporting. JAMA, 2000 |
| 5 | 135 | Dersimonian, R | Metaanalysis in clinical-trials. Control Clin Trials, 1986 |
| 6 | 130 | Zuckerman.H | Patterns of evaluation in science - institutionalisation, structure ... Minerva, 1971 |
| 7 | 130 | Higgins, JPT | Cochrane Handbook for Systematic Reviews of Interventions. Cochrane, 2011 |
| 8 | 126 | Moher, D | Preferred Reporting Items for Systematic Reviews and Meta-Analyses. Plos Med, 2009 |
| 9 | 125 | Higgins, JPT | Measuring inconsistency in meta-analyses. Brit Med J, 2003 |
| 10 | 121 | Cicchetti, DV | The reliability of peer-review for manuscript and grant submissions - ... Behav Brain Sci, 1991 |
| 11 | 119 | Hirsch, JE | An index to quantify an individual's scientific research output. P Natl Acad Sci Usa, 2005 |
| 12 | 114 | Mahoney, M | Publication prejudices: An experimental study of confirmatory bias ... Cognitive T& R, 1977 |
| 13 | 114 | van Rooyen, S | Effect of open peer review on quality of reviews and on reviewers' ... Brit Med J, 1999 |
| 14 | 114 | Easterbrook, PJ | Publication bias in clinical research. Lancet, 1991 |
| 15 | 110 | Landis, JR | Measurement Of Observer Agreement For Categorical Data. Biometrics, 1977 |
| 16 | 109 | Godlee, F | Effect on the quality of peer review of blinding reviewers and asking them to sign ... JAMA, 1998 |
| 17 | 108 | Horrobin, DF | The philosophical basis of peer-review and the suppression of innovation. JAMA, 1990 |
| 18 | 107 | Moher, D | Preferred Reporting Items for Systematic Reviews and Meta-Analyses. Ann Intern Med, 2009 |
| 19 | 107 | Jadad, AR | Assessing the quality of reports of randomized clinical trials. Control Clin Trials, 1996 |
| 20 | 105 | McNutt, RA | The effects of blinding on the quality of peer-review - a randomized trial. JAMA, 1990 |
| 21 | 104 | Cole, S | Chance and consensus in peer-review. Science, 1981 |
| 22 | 103 | Moher, D | Improving the quality of reports of meta-analyses of randomised controlled trials. Lancet, 1999 |
| 23 | 98 | Justice, AC | Does masking author identity improve peer review quality? JAMA, 1998 |
| 24 | 97 | Lock, S | A Difficult Balance: Editorial Peer Review in Medicine. Nuffield Trust, 1985 |
| 25 | 95 | van Rooyen, S | Effect of blinding and unmasking on the quality of peer review - A randomized trial. JAMA, 1998 |
| 26 | 92 | Black, N | What makes a good reviewer and a good review for a general medical journal? JAMA, 1998 |
| 27 | 91 | Scherer, RW | Full publication of results initially presented in abstracts - a metaanalysis. JAMA, 1994 |
| 28 | 90 | Higgins, JPT | Quantifying heterogeneity in a meta-analysis. Stat Med, 2002 |
| 29 | 90 | Smith, R | Peer review: a flawed process at the heart of science and journals. J Roy Soc Med, 2006 |
| 30 | 87 | Goodman, SN | Manuscript quality before and after peer-review and editing at ... Ann Intern Med, 1994 |
| 31 | 87 | Chubin, D | Peerless Science: Peer Review and U.S. Science Policy. SUNY Press, 1990 |



Distributions

indegree and outdegree in citation network

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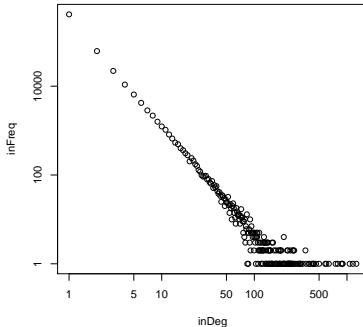
Multiplication

Derived Ns

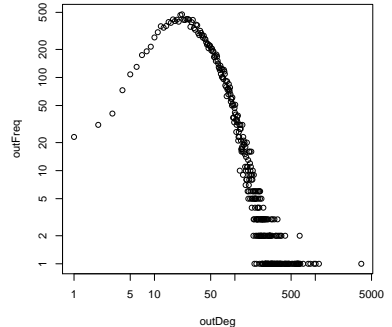
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in-degree distribution



out-degree distribution



The indegree distribution is "scale-free"-like. The parameters can be determined using the package of [Clauset, Shalizi and Newman](#). See also [Stumpf, et al.: Critical Truths About Power Laws](#).



Distributions

SN5 citation network input degrees – scale-free fit

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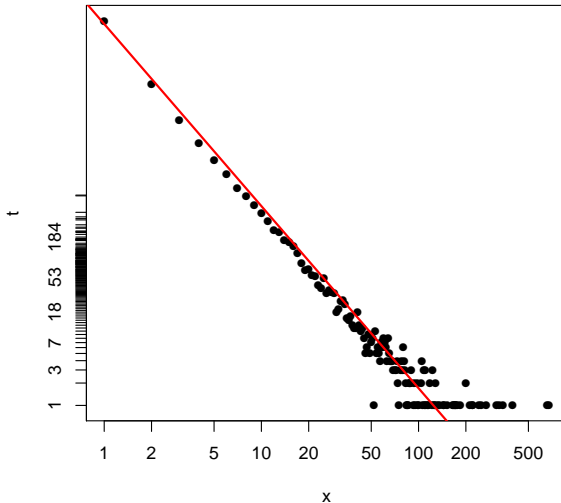
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Distributions

year partition lognormal distribution fit

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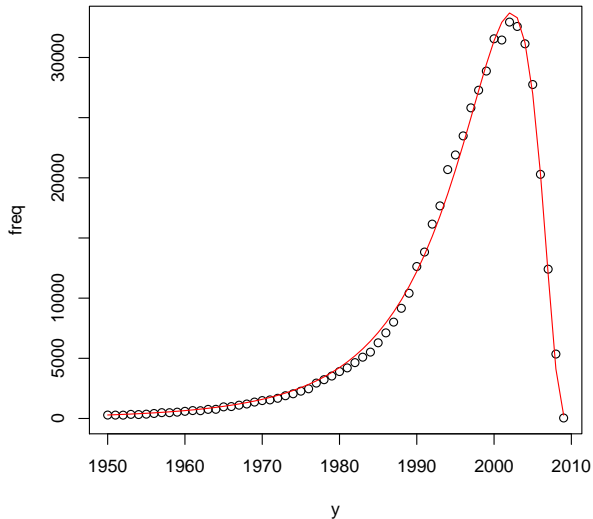
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Distributions

number of keywords in **ZBMath**

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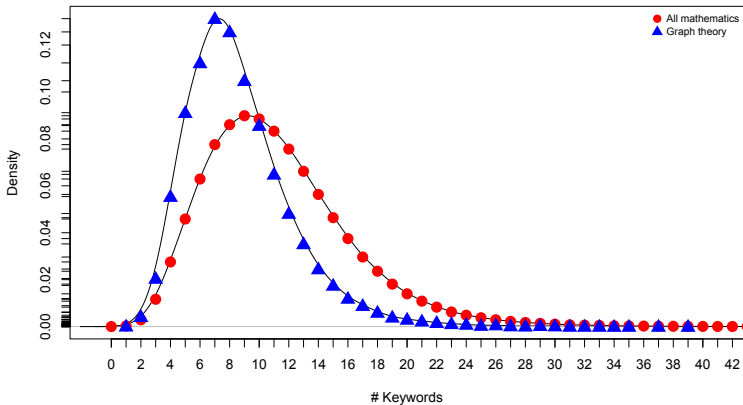
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Distributions

keywords by the number of works using a keyword in their description

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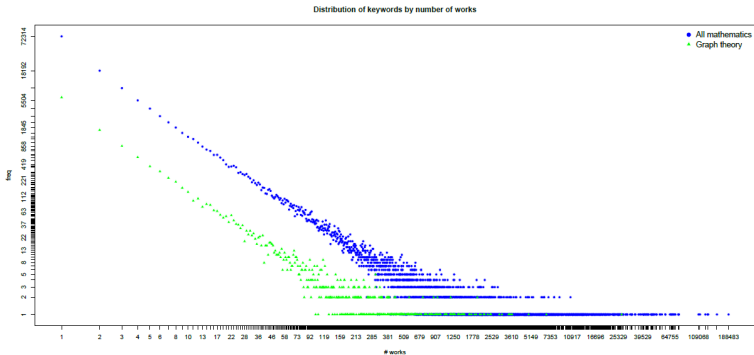
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Temporal distribution

Number of authors SNA 2018

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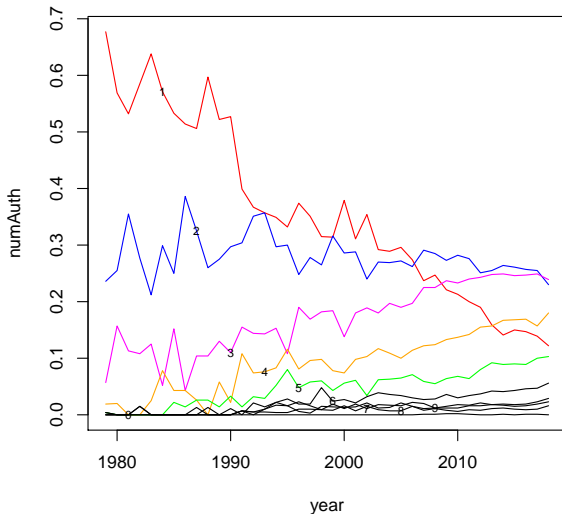
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Citations between years

SNA 2018

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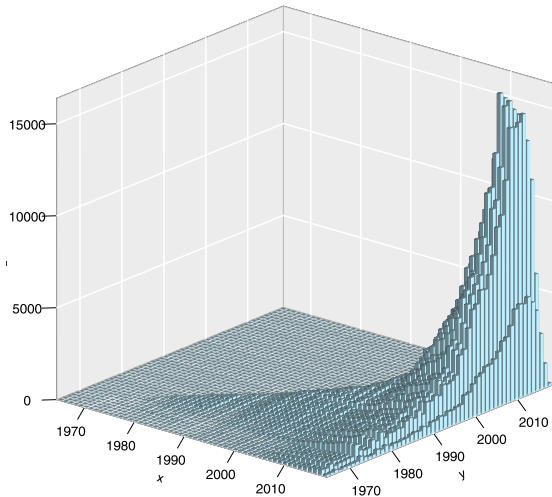
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SocNet



Citations per year

SNA 2018; normalized curves

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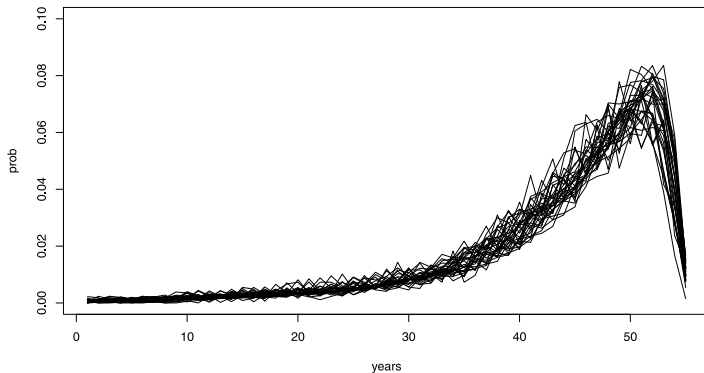
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SocNet



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In a given set of works/nodes W (articles, books, reports, etc.) we introduce a *citing relation*/set of arcs $\mathbf{Ci} \subseteq W \times W$

$$u \mathbf{Ci} v \equiv u \text{ cites } v$$

which determines a *citation network* $\mathcal{N} = (W, \mathbf{Ci})$.

A citing relation is usually *irreflexive* (no loops) and (almost) *acyclic*. We shall assume that it has these two properties. Since in real-life citation networks the strong components are small (usually 2 or 3 nodes) we can transform such network into an acyclic network by shrinking strong components and deleting loops.

A better way is the preprint transformation.



Citation network

Preprint transformation

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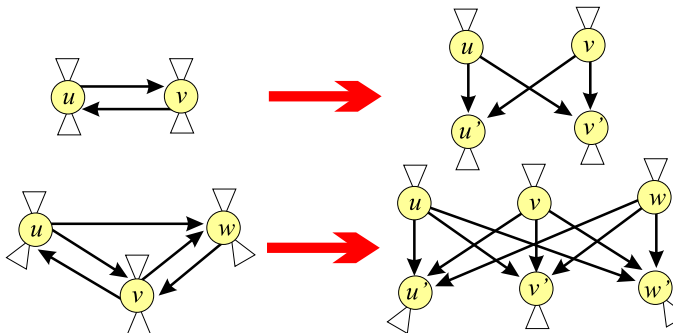
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Transforming a citation network into an acyclic network using the preprint transformation.



Standardized citation network

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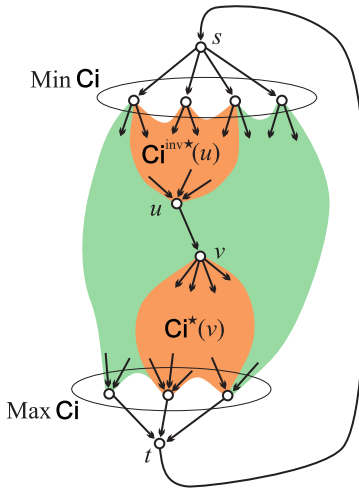
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We assume that the citation relation \mathbf{Ci} is acyclic. It is useful to transform a citation network to its *standardized* form by adding a common *source* node $s \notin W$ and a common *sink* node $t \notin W$. The source s is linked by an arc to all minimal elements of \mathbf{Ci} ; and all maximal elements of \mathbf{Ci} are linked to the sink t . We add also the 'feedback' arc (t, s) .



Search path count method

Hummon and Doreian

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The *search path count* (SPC) method is based on counters $n(u, v)$ that count the number of different paths from s to t through the arc (u, v) .

The values of counters $n(u, v)$ form a flow in the citation network – the *Kirchoff's vertex law* holds: For every node u in a standardized citation network *incoming flow* = *outgoing flow*:

$$\sum_{v: v \text{ Ci } u} n(v, u) = \sum_{v: u \text{ Ci } v} n(u, v)$$

The weight $n(t, s)$ equals to the total flow through network and provides a natural normalization of weights

$$w(u, v) = \frac{n(u, v)}{n(t, s)} \Rightarrow 0 \leq w(u, v) \leq 1$$

and if C is a minimal arc-cut-set $\sum_{(u,v) \in C} w(u, v) = 1$.

The value $w(u, v)$ is equal to the probability that a random s - t path passes through the arc (u, v) .



Cuts

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The standard approach to find interesting groups inside a network is based on properties/weights – they can be *measured* or *computed* from network structure (for example Kleinberg's *hubs and authorities*).

The *node cut* of a network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, p)$, $p : \mathcal{V} \rightarrow \mathbb{R}$, at selected level t is a subnetwork $\mathcal{N}(t) = (\mathcal{V}', \mathcal{L}(\mathcal{V}'), p)$, determined by the set

$$\mathcal{V}' = \{v \in \mathcal{V} : p(v) \geq t\}$$

and $\mathcal{L}(\mathcal{V}')$ is the set of links from \mathcal{L} that have both endnodes in \mathcal{V}' .

The *link cut* of a network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, w)$, $w : \mathcal{L} \rightarrow \mathbb{R}$, at selected level t is a subnetwork $\mathcal{N}(t) = (\mathcal{V}(\mathcal{L}'), \mathcal{L}', w)$, determined by the set

$$\mathcal{L}' = \{e \in \mathcal{L} : w(e) \geq t\}$$

and $\mathcal{V}(\mathcal{L}')$ is the set of all endnodes of the links from \mathcal{L}' .



Citation weights

Link cut for SPC in SOM

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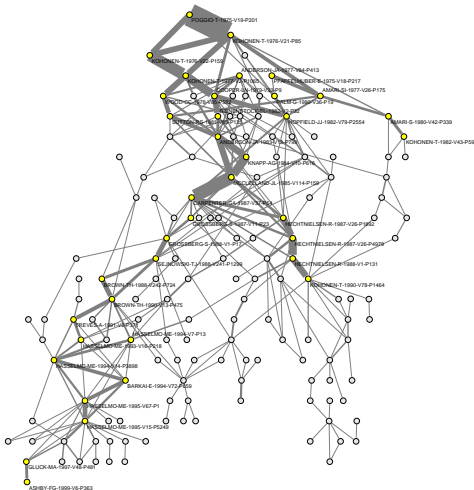
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Main subnetwork (arc cut at level 0.007) of the **SOM** (selforganizing maps) citation network (4470 nodes, 12731 arcs).

For visualization of acyclic networks in Pajek use the macro layers.

See **paper**.



Islands

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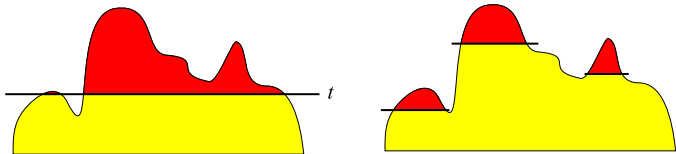
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If we represent a given or computed value of nodes / links as a height of nodes / links and we immerse the network into a water up to selected level we get *islands*. Varying the level we get different islands.



We developed very efficient algorithms to determine the islands hierarchy and to list all the islands of selected sizes.
See [details](#).



... Islands

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Islands are very general and efficient approach to determine the 'important' subnetworks in a given network.

We have to express the goals of our analysis with a related property of the nodes or weight of the links. Using this property we determine the islands of an appropriate size (in the interval k to K).

In large networks we can get many islands which we have to inspect individually and interpret their content.

An important property of the islands is that they identify locally important subnetworks on different levels. Therefore they detect also emerging groups.

The set of nodes $\mathcal{C} \subseteq \mathcal{V}$ is a *local node peak*, if it is a regular node island and all of its nodes have the same value. Node island with a single local node peak is called a *simple node island*. In similar way we define simple link island.



... Islands

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A set of nodes $C \subseteq \mathcal{V}$ is a *regular node island* in a network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, p)$, $p : \mathcal{V} \rightarrow \mathbb{R}$ iff it induces a connected subgraph and the nodes from the island are 'higher' than the neighboring nodes

$$\max_{u \in N(C)} p(u) < \min_{v \in C} p(v)$$

A set of nodes $C \subseteq \mathcal{V}$ is a *regular link island* in a network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, w)$, $w : \mathcal{L} \rightarrow \mathbb{R}$ iff it induces a connected subgraph and the links inside the island are 'stronger related' among them than with the neighboring nodes – in \mathcal{N} there exists a spanning tree \mathcal{T} over C such that

$$\max_{(u,v) \in \mathcal{L}, u \notin C, v \in C} w(u, v) < \min_{(u,v) \in \mathcal{T}} w(u, v)$$

A *simple island* is an island with only one peak.



US patents

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US patents network (**Nber**, **US Patents**) has 3774768 nodes and 16522438 arcs (1 loop). Without the loop it is acyclic. The weight of an arc is the proportion of paths through the arc from some initial node to some terminal node. We determined all (2,90)-link islands. The corresponding subnetwork has 470137 nodes, 307472 arcs and different k : $C_2 = 187610$, $C_5 = 8859$, $C_{30} = 101$, $C_{50} = 30$, ... islands. **Rolex**

| | | | | | | | | | | |
|------|-----|--------|-------|------|------|------|-----|-----|-----|-----|
| [1] | 0 | 139793 | 29670 | 9288 | 3966 | 1827 | 997 | 578 | 362 | 250 |
| [11] | 190 | 125 | 104 | 71 | 47 | 37 | 36 | 33 | 21 | 23 |
| [21] | 17 | 16 | 8 | 7 | 13 | 10 | 10 | 5 | 5 | 5 |
| [31] | 12 | 3 | 7 | 3 | 3 | 3 | 2 | 6 | 6 | 2 |
| [41] | 1 | 3 | 4 | 1 | 5 | 2 | 1 | 1 | 1 | 1 |
| [51] | 2 | 3 | 3 | 2 | 0 | 0 | 0 | 0 | 0 | 1 |
| [61] | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 2 | 0 | 0 |
| [71] | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| [81] | 2 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 0 | 7 |

The **Main path** starts in a link with the largest SPC weight and expands in both directions following the adjacent link with the largest SPC weight.

The **CPM path** is determined using the Critical Path Method from Operations Research (the sum of SPC weights along a path is maximal).



Distribution of island size

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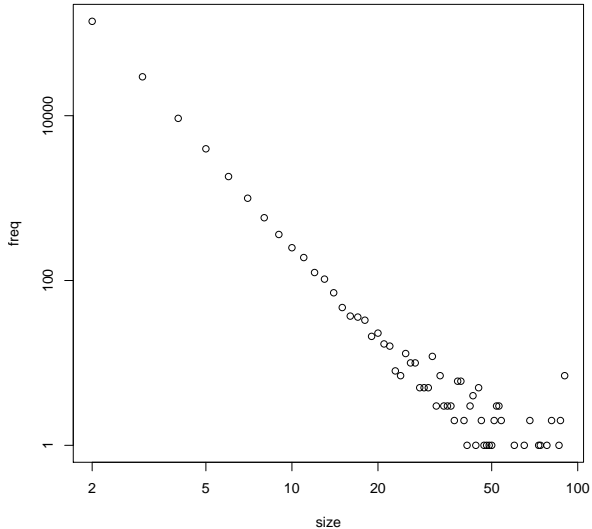
Two-mode Ns

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Main path and main island in US Patents

Nber, US Patents; $n = 3774768$, $m = 16522438$

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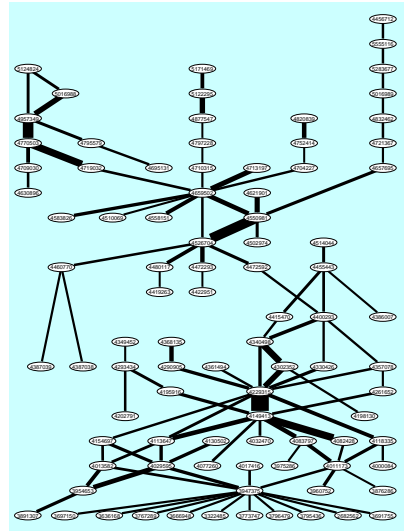
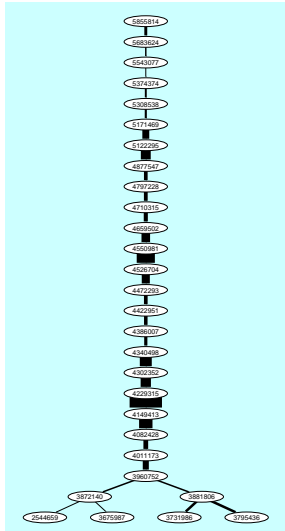
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Main island – Liquid crystal display

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Table 1: Patents on the liquid-crystal display

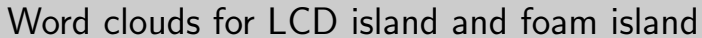
| patent | date | author(s) and title |
|---------|--------------|--|
| 2714869 | Mar 14, 1984 | Shiroi, T. Active matrix liquid-crystal display and the use and the structure and use thereof |
| 2882562 | Jun 23, 1984 | Winder, et al. Reduction of aromatic carbazole |
| 3322480 | May 30, 1987 | Williams. Electro-optical elements utilizing an organic semiconducting |
| 306108 | Jan 18, 1972 | Josephson. Preparation of polycrystalline aromatic compounds having an undistorted image on a distorted background |
| 3099386 | May 30, 1972 | Medvedev, et al. Liquid crystal thermal imaging system having a undistorted image on a distorted background |
| 367088 | Jul 11, 1972 | Rafael. Check with digital display |
| 3691755 | Oct 10, 1972 | Girard. Liquid crystal composition and device |
| 3691755 | Oct 10, 1972 | Wyszski. Electro-optic system in which an electrochromatic or di-dye material is disposed throughout a liquid crystal to reduce the turn-off time |
| 3731996 | May 8, 1973 | Ferguson. Display device utilizing liquid crystal light modulation |
| 3767280 | Oct 23, 1973 | Astruc, et al. Class of small triazene-ethylene compounds, some displaying nematic mesophase at or near room temperature and others in a range up to 100°C |
| 3773747 | Nov 20, 1973 | Schneiderbauer. Substituted azine benzene compounds |
| 3795436 | Mar 5, 1974 | Buller, et al. Nematic liquid crystal which exhibits the Kerr effect at isotropic temperature |
| 3796473 | Mar 12, 1974 | Helfrich, et al. Electro-optical light modulation cell utilizing a nematic liquid crystal which exhibits the Kerr effect at isotropic temperature |
| 382140 | Mar 18, 1975 | Kleinman, et al. Liquid crystalline compositions and method |
| 3826296 | Apr 8, 1975 | Deutscher, et al. Use of nematic liquid crystalline substances |
| 3861866 | Jun 6, 1975 | Sanku. Electro-optical display device |
| 3901387 | Jun 24, 1975 | Tokumoto, et al. Phase control of the voltage applied to opposite electrodes for a cholesteric to nematic phase transition display |
| 3947375 | Mar 30, 1976 | Gray, et al. Liquid crystal materials and devices |
| 3954653 | May 4, 1976 | Vismale. Liquid crystal composition having high dielectric anisotropy and display device incorporating same |
| 3967572 | Jun 1, 1976 | Kleinman, et al. Liquid crystal compositions |
| 3977286 | Aug 17, 1976 | Oh. Low voltage actuated field effect liquid crystals composition and method of synthesis |
| 4000084 | Dec 28, 1976 | Black, et al. Liquid crystal materials for electro-optical display devices |
| 4011373 | Mar 8, 1977 | Stroobant. Modified nematic mixtures with positive dielectric anisotropy |
| 4013362 | Mar 22, 1977 | Gierthoefer. Liquid crystal compounds and electro-optic display device incorporating them |
| 4017416 | Apr 12, 1977 | Shibata, et al. Liquid crystalline 4-alkyl-4'-hydroxybenzoates, method for preparing same and liquid crystal compositions |
| 4025995 | Jun 14, 1977 | Ross, et al. Novel liquid crystal compounds and electro-optic display device incorporating them |
| 4032470 | Aug 28, 1977 | Bliss, et al. Electro-optic device |
| 4077289 | Mar 7, 1978 | Gray, et al. Optically active cyano-biphenyl compounds and liquid crystal materials containing them |
| 4083428 | Apr 4, 1978 | Hsu. Liquid crystal composition and method |

Table 2: Patents on the liquid-crystal display

| patent | date | author(s) and title |
|---------|--------------|---|
| 4083499 | Apr 11, 1978 | U.S. Nematic liquid crystal compositions |
| 4113847 | Sep 12, 1978 | Coutts, et al. Liquid crystalline materials |
| 4113855 | Oct 3, 1978 | Krause, et al. Liquid crystalline materials of reduced viscosity |
| 4120562 | Dec 10, 1978 | Edelshchik, et al. Liquid crystalline cyclohexanone derivatives |
| 4144113 | Apr 17, 1979 | Gray, et al. Optically active liquid crystal mixtures and liquid crystal devices containing them |
| 4154467 | May 15, 1979 | Edelshchik, et al. Liquid crystalline benzylidenebiphenyl derivatives |
| 4159216 | May 1, 1979 | Coutts, et al. Liquid crystal compounds |
| 4195120 | Apr 15, 1980 | Bellor, et al. Liquid crystal mixtures |
| 4202770 | May 13, 1980 | Sato, et al. Nematic liquid crystalline materials |
| 4229315 | Oct 21, 1980 | Krause, et al. Liquid crystalline cyclohexanone derivatives |
| 4265652 | Apr 14, 1981 | Gray, et al. Liquid crystal compounds and materials and devices containing them |
| 4298095 | Sep 22, 1981 | Kashe, Ester compound |
| 4295434 | Oct 6, 1981 | Deutscher, et al. Liquid crystal compounds |
| 4323352 | Nov 24, 1981 | Edelshchik, et al. Fluorophenylcyclohexanones, the preparation thereof and their use as components of liquid crystal dielectrics |
| 4330426 | Nov 18, 1982 | Edelshchik, et al. Cyclohexylbiphenyls, their preparation and use in dielectric and electrophotical display elements |
| 4340498 | Jul 20, 1982 | Sagimoto. Halogenated opter derivatives |
| 4349452 | Jul 13, 1982 | Oman, et al. Cyclohexylbiphenyl compounds |
| 4352378 | Nov 2, 1982 | Carr, et al. Liquid crystal compounds containing an alkylic ring and exhibiting a low dielectric anisotropy and liquid crystalline materials and devices incorporating such compounds |
| 4361494 | Nov 30, 1982 | Oman, et al. Anisotropic cyclohexyl cyclohexylbiphenyl ethers |
| 4360135 | Jul 11, 1983 | Oman, et al. Anisotropic compounds with negative or positive DC-anisotropy and low optical anisotropy |
| 4386807 | May 31, 1983 | Krause, et al. Liquid crystalline cyclohexanone derivatives |
| 4387038 | Jun 7, 1983 | Fukui, et al. 4-(Trans-4'-alkylcyclohexyl) benzoic acid 4'-c-Trans-4'-biphenyl esters |
| 4387039 | Jun 7, 1983 | Sagimoto, et al. Trans-4-(trans-4'-alkylcyclohexyl)-cyclohexanone derivatives and 4'-cyclohexylbiphenyl derivatives |
| 4400253 | Aug 23, 1983 | Rowner, et al. Liquid crystalline cyclohexylbiphenyl derivatives |
| 4415470 | Nov 15, 1983 | Edelshchik, et al. Liquid crystalline fluorine-containing cyclohexylbiphenyls and dielectric and electro-optical display elements based thereon |
| 4419363 | Dec 6, 1983 | Prud'homme, et al. Liquid crystalline cyclohexylcyclohexanone derivatives |
| 4422951 | Dec 27, 1983 | Sagimoto, et al. Liquid crystal benzene derivatives |
| 4452443 | Jun 19, 1984 | Taketsu, et al. Nematic liquid compound |
| 4456712 | Jun 26, 1984 | Chen, et al. High birefringence liquid crystal composition |
| 4460778 | Jul 17, 1984 | Petrizka, et al. Liquid crystal mixture |
| 4472283 | Sep 18, 1984 | Sagimoto, et al. High birefringence liquid crystal compositions of low viscosity and liquid crystal compositions containing the same |
| 4472295 | Sep 18, 1984 | Taketsu, et al. Nematic liquid crystalline compounds |
| 4500274 | Oct 30, 1984 | Edelshchik, et al. High temperature liquid-crystalline ester compounds |
| 4510809 | Apr 9, 1985 | Edelshchik, et al. Cyclohexanone derivatives |

Table 3: Patents on the liquid-crystal display

| patent | date | author(s) and title |
|---------|--------------|---|
| 4511891 | Apr 30, 1985 | Petrizka, et al. 11-trimethyl-4-alkylcyclohexyl-2-(trimethyl-4-(p-methylphenyl)cyclohexyl)benzoate and liquid crystal mixture |
| 4526794 | Jul 2, 1985 | Petrizka, et al. Multilayer liquid crystal mixture |
| 4530981 | Nov 5, 1985 | Petrizka, et al. Liquid crystalline esters and mixtures |
| 4531511 | Dec 10, 1985 | Taketsu, et al. Nematic liquid crystalline compounds |
| 4533828 | Apr 22, 1986 | Petrizka, et al. Phenylbenzoates |
| 4621391 | Nov 11, 1986 | Petrizka, et al. Novel liquid crystal mixtures |
| 4638988 | Dec 23, 1986 | Petrizka, et al. Benzocyclobutenes |
| 4651703 | Apr 14, 1987 | Sato, et al. Substituted pyridines |
| 4650562 | Apr 21, 1987 | Furuta, et al. Ethane derivatives |
| 4697015 | Sep 22, 1987 | Balwitke, et al. Disubstituted ethanes and their use in liquid crystal materials and devices |
| 4704227 | Nov 3, 1987 | Krause, et al. Liquid crystal compounds |
| 4709038 | Nov 24, 1987 | Petrizka, et al. Novel liquid crystalline materials |
| 4710311 | Dec 1, 1987 | Schick, et al. Anisotropic compounds and liquid crystal mixtures therefrom |
| 4711397 | Dec 15, 1987 | Edelshchik, et al. Nitrogen-containing heterocyclic compounds |
| 4719032 | Jan 12, 1988 | Wachler, et al. Cyclohexanone derivatives |
| 4721367 | Jan 26, 1988 | Voshagin, et al. Liquid crystal device |
| 4723414 | Feb 24, 1988 | Edelshchik, et al. Nitrogen-containing heterocyclic compounds |
| 4727051 | Sep 13, 1988 | Rudischer, et al. Liquid crystalline compounds |
| 4750579 | Jun 3, 1989 | Vanderhoff, et al. 2,2'-difluoro-4-biphenyl-4'-hydroxybiphenyls and their derivatives, their production process and their use in liquid crystal display devices |
| 4752228 | Jun 10, 1989 | Goto, et al. Cyclohexanone derivatives and liquid crystal compositions containing same |
| 4820829 | Apr 11, 1989 | Krause, et al. Nitrogen-containing heterocyclic esters |
| 4824842 | May 23, 1989 | Clark, et al. Liquid crystal devices |
| 4877447 | Oct 17, 1989 | Weber, et al. Liquid crystal display element |
| 4877549 | Sep 18, 1989 | Chen, et al. Active matrix screen for the color display of television pictures, control system and process for producing said screen |
| 5005989 | May 21, 1991 | Bauer. Liquid crystal display device with a liquidcrystal compensator |
| 5005989 | May 21, 1991 | Chen. Liquid crystal element with improved contrast and color |
| 5122295 | Jun 16, 1992 | Weber, et al. Matrix liquid crystal display |
| 5124924 | Jun 23, 1992 | Kozaki, et al. Liquid crystal display device comprising a retardation compensation layer having a maximum principal refractive index in the thickness direction |
| 5137140 | Dec 15, 1992 | Hirak, et al. Liquid-crystal matrix displays |
| 5137677 | Feb 1, 1993 | Sagimoto, et al. Liquid crystal display with great regions between terminal groups |
| 5206738 | Mar 3, 1994 | Weber, et al. Superliquid liquid-crystal display |
| 5244774 | Dec 20, 1994 | Weber, et al. Superliquid liquid-crystal display |
| 5343877 | Apr 6, 1996 | Rayer, et al. Nematic liquid-crystal composition |
| 5551138 | Sep 10, 1996 | Hirak, et al. Liquid crystal display having adjacent dielectric terminals set equal in length |
| 5606246 | Nov 19, 1996 | Schick, et al. Liquid crystal composition |
| 5655541 | Nov 5, 1999 | Mazard, et al. Liquid crystal compositions and liquid crystal display elements |



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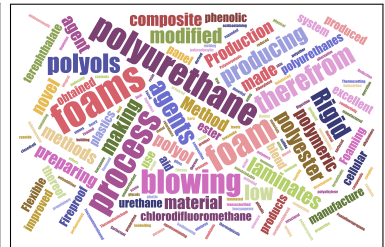
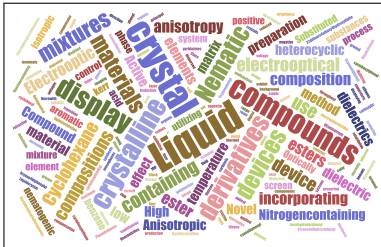
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Main SPC island in SN5

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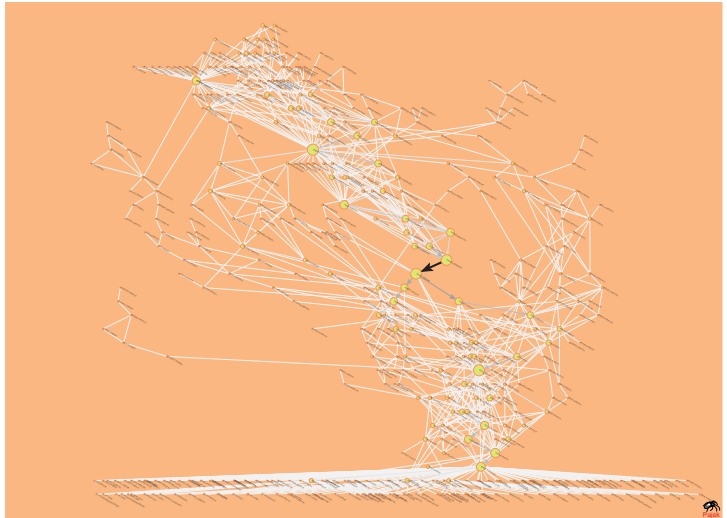
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PEERE – Main path

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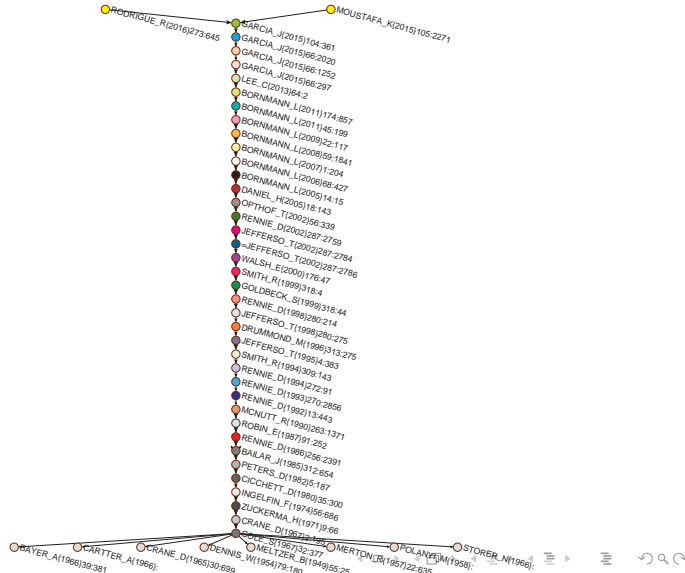
Two-mode Ns

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PEERE – List of publications on main path

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Citation

Two-mode Ns

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Derived Ns

Temporal Ns

References

| year | first author | title | journal |
|------|----------------|---|-------------------|
| 1949 | Meltzer BN | The productivity of social scientists | AmJSociol |
| 1954 | Dennis W | Bibliographies of eminent scientists | ScientificM |
| 1957 | Merton RK | Priorities in scientific discovery - a chapter in the sociology of sci... | AmSociolRev |
| 1958 | Polanyi M | Personal Knowledge: Towards a Post-Critical Philosophy | UPChicago |
| 1965 | Crane D | Scientists at major and minor universities | AmSociolRev |
| 1966 | Bayer AE | Some correlates of citation measure of productivity in science | SociolEduc |
| 1966 | Storer NW | The Social System of Science | HRW |
| 1966 | Cartter A | An Assessment of Quality in Graduate Education | ACE |
| 1967 | Crane D | Gatekeepers of science - some factors affecting selection... | AmSociol |
| 1967 | Cole S | Scientific output and recognition - study in operation of reward... | AmSociolRev |
| 1971 | Zuckerman H | Patterns of evaluation in science - institutionalisation, struct... | Minerva |
| 1974 | Ingelfinger FJ | Peer review in biomedical publication | AmJMed |
| 1980 | Cicchetti DV | Reliability of reviews for the american-psychologist | AmPsychol |
| 1982 | Peters DP | Peer-review practices of psychological journals - the fate... | BehavBrainSci |
| 1985 | Bailar JC | Journal peer-review - the need for a research agenda | NewEnglJMed |
| 1986 | Rennie D | Guarding the guardians - a conference on editorial peer-review | Jama |
| 1987 | Robin ED | Peer-review in medical journals | Chest |
| 1990 | McNutt RA | The effects of blinding on the quality of peer-review | Jama |
| 1992 | Rennie D | Suspended judgment - editorial peer-review - let us put it on trial | ControlClinTrials |
| 1993 | Rennie D | More peering into editorial peer-review | Jama |
| 1994 | Rennie D | The 2nd international-congress on peer-review in biomedical... | Jama |
| 1994 | Smith R | Promoting research into peer-review | BritMedJ |
| 1995 | Jefferson T | Are guidelines for peer-reviewing economic evaluations necessary | HealthEcon |
| 1996 | Drummond M | Guidelines for authors and peer reviewers of economic submis... | BritMedJ |
| 1998 | Jefferson T | Evaluating the BMJ guidelines for economic submissions | Jama |
| 1998 | Rennie D | Peer review in Prague | Jama |



PEERE – ... List of works on main path

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References

| year | first author | title | journal |
|------|----------------|--|-----------------|
| 1999 | Smith R | Opening up BMJ peer review - A beginning that should lead to... | BritMedJ |
| 1999 | Goldbeck-W. S | Evidence on peer review - scientific quality control or smokescreen? | BritMedJ |
| 2000 | Walsh E | Open peer review: a randomised controlled trial | BritJPsychiat |
| 2002 | Jefferson T | Effects of editorial peer review - A systematic review | Jama |
| 2002 | Rennie D | Fourth International Congress on Peer Review in Biomedical Pub... | Jama |
| 2002 | Opthof T | The significance of the peer review process against ... bias | CardiovascRes |
| 2002 | Jefferson T | Measuring the quality of editorial peer review | Jama |
| 2005 | Bornmann L | Committee peer review at an international research foundation | ResEvaluat |
| 2005 | Daniel HD | Publications as a measure of scientific advancement and of... | LearnPubl |
| 2006 | Bornmann L | Selecting scientific excellence through committee peer review | Scientometrics |
| 2007 | Bornmann L | Convergent validation of peer review decisions using the h index | JInformetr |
| 2008 | Bornmann L | Selecting manuscripts for a high-impact journal through peer review | JAmSocInfSciTe |
| 2009 | Bornmann L | The luck of the referee draw: the effect of exchanging reviews | LearnPubl |
| 2011 | Bornmann L | Scientific Peer Review | AnnuRevInform |
| 2011 | Bornmann L | A multilevel modelling approach to investigating the predictive... | JRStatSocASTa |
| 2013 | Lee CJ | Bias in peer review | JAmSocInfSciTe |
| 2015 | Garcia JA | The Principal-Agent Problem in Peer Review | JAssocInfSciTec |
| 2015 | Garcia JA | Adverse selection of reviewers | JAssocInfSciTec |
| 2015 | Garcia JA | Bias and effort in peer review | JAssocInfSciTec |
| 2015 | Garcia JA | The author-editor game | Scientometrics |
| 2015 | Moustafa K | Don't infer anything from unavailable data | Scientometrics |
| 2016 | Rodriguez-S. R | Evolutionary games between authors and their editors | ApplMathComp |



The main path publications

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- before 1982: social science journals;
- from 1982 to 2002: biomedical journals;
- after 2002: specialized journals on science studies.

Journals, Main authors, Topics.



PEERE – Main paths for 100 largest weights

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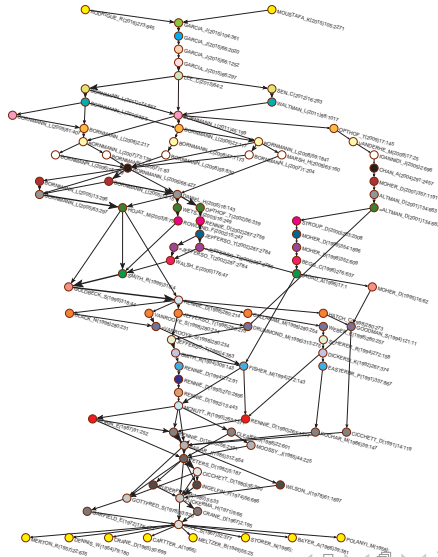
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PEERE – SPC link islands [20 200]

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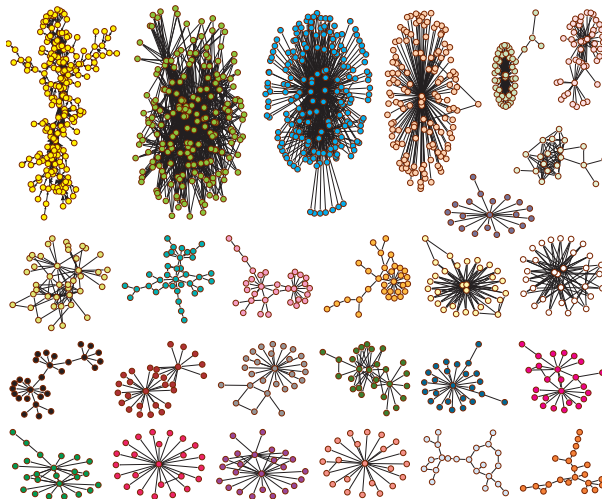
Two-mode Ns

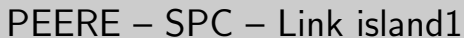
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$$w_{max} = 0.297$$

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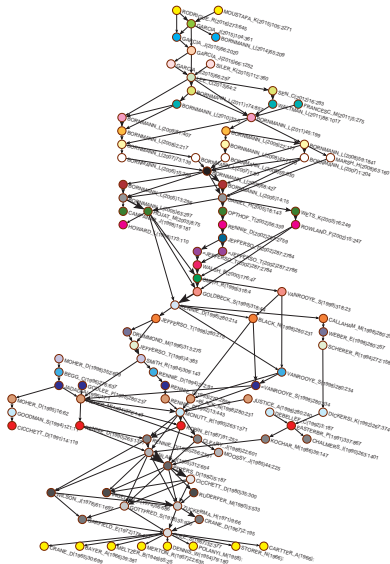
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This island is very similar to the main paths for 100 largest weights and includes main path.



Islands

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A recent extension of the main paths approach enables a researcher to determine main paths through a given set of nodes (works) in a citation network. This can be used to position a given set of nodes in a citation network – it can, either attach to the principal main paths, or form a separate stream.

In the literature on network clustering we considered works on valued networks, { ZIBERNA_A(2007)29:105, NORDLUND_C(2007)29:59, ZIBERNA_A (2008)32:57, ZIBERNA_A(2009)6:99, ZIBERNA_A(2013):, ZIBERNA_A (2013)10:99, ZIBERNA_A(2014)39:46, ZIBERNA_A(2016)12:137, NORDLUND_C(2016)44:160 }, as shown in figure in the following slide.



Islands

positioning in **NetClus** – valued networks in clustering

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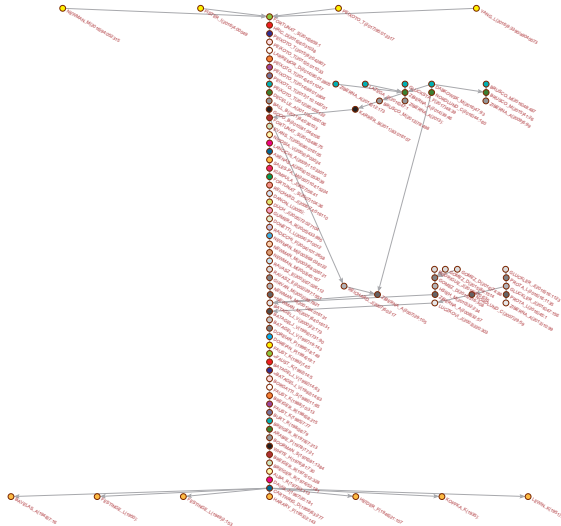
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In a *two-mode* network $\mathcal{N} = ((\mathcal{U}, \mathcal{V}), \mathcal{L}, \mathcal{P}, \mathcal{W})$ the set of nodes consists of two disjoint sets of nodes \mathcal{U} and \mathcal{V} , and all the links from \mathcal{L} have one end-node in \mathcal{U} and the other in \mathcal{V} . Often also a *weight* $w : \mathcal{L} \rightarrow \mathbb{R} \in \mathcal{W}$ is given; if not, we assume $w(u, v) = 1$ for all $(u, v) \in \mathcal{L}$.

A two-mode network can also be described by a rectangular matrix $\mathbf{A} = [a_{uv}]_{\mathcal{U} \times \mathcal{V}}$.

$$a_{uv} = \begin{cases} w_{uv} & (u, v) \in \mathcal{L} \\ 0 & \text{otherwise} \end{cases}$$

Examples: (persons, societies, years of membership),
(buyers/consumers, goods, quantity),
(parliamentarians, problems, positive vote),
(persons, journals, reading),
(papers, keywords, is described by), etc.

The usual approach to analyze a two-mode network is to transform it to a one-mode network and use standard methods on it.



Multiplication of networks

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To a simple (no parallel arcs) two-mode *network* $\mathcal{N} = (\mathcal{I}, \mathcal{J}, \mathcal{A}, w)$; where \mathcal{I} and \mathcal{J} are sets of *nodes*, \mathcal{A} is a set of *arcs* linking \mathcal{I} and \mathcal{J} , and $w : \mathcal{A} \rightarrow \mathbb{R}$ (or some other semiring) is a *weight*; we can assign a *network matrix* $\mathbf{W} = [w_{i,j}]$ with elements: $w_{i,j} = w(i,j)$ for $(i,j) \in \mathcal{A}$ and $w_{i,j} = 0$ otherwise.

Given a pair of compatible networks $\mathcal{N}_A = (\mathcal{I}, \mathcal{K}, \mathcal{A}_A, w_A)$ and $\mathcal{N}_B = (\mathcal{K}, \mathcal{J}, \mathcal{A}_B, w_B)$ with corresponding matrices $\mathbf{A}_{\mathcal{I} \times \mathcal{K}}$ and $\mathbf{B}_{\mathcal{K} \times \mathcal{J}}$ we call a *product of networks* \mathcal{N}_A and \mathcal{N}_B a network $\mathcal{N}_C = (\mathcal{I}, \mathcal{J}, \mathcal{A}_C, w_C)$, where $\mathcal{A}_C = \{(i,j) : i \in \mathcal{I}, j \in \mathcal{J}, c_{i,j} \neq 0\}$ and $w_C(i,j) = c_{i,j}$ for $(i,j) \in \mathcal{A}_C$. The product matrix $\mathbf{C} = [c_{i,j}]_{\mathcal{I} \times \mathcal{J}} = \mathbf{A} * \mathbf{B}$ is defined in the standard way

$$c_{i,j} = \sum_{k \in \mathcal{K}} a_{i,k} \cdot b_{k,j}$$

In the case when $\mathcal{I} = \mathcal{K} = \mathcal{J}$ we are dealing with ordinary one-mode networks (with square matrices).



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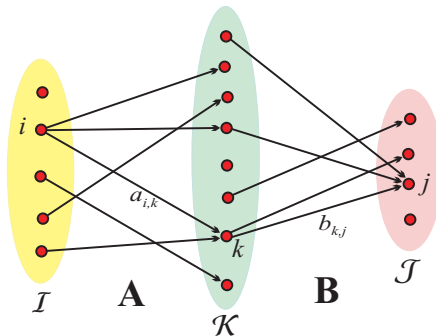
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$$c_{i,j} = \sum_{k \in N_A(i) \cap N_B^-(j)} a_{i,k} \cdot b_{k,j}$$

If all weights in networks \mathcal{N}_A and \mathcal{N}_B are equal to 1 the value of $c_{i,j}$ counts the number of ways we can go from $i \in \mathcal{I}$ to $j \in \mathcal{J}$ passing through \mathcal{K} .



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The standard matrix multiplication has the complexity $O(|\mathcal{I}| \cdot |\mathcal{K}| \cdot |\mathcal{J}|)$ – it is too slow to be used for large networks. For sparse large networks we can multiply much faster considering only nonzero elements.

In general the multiplication of large sparse networks is a 'dangerous' operation since the result can 'explode' – it is not sparse.

If for the sparse networks \mathcal{N}_A and \mathcal{N}_B there are in \mathcal{K} only few nodes with large degree and no one among them with large degree in both networks then also the resulting product network \mathcal{N}_C is sparse.

For more details see the [paper](#).



Two-mode network analysis

by conversion to one-mode network – projections

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Often we transform a two-mode network $\mathcal{N} = ((\mathcal{U}, \mathcal{V}), \mathcal{E}, w)$ into an ordinary (one-mode) network $\mathcal{N}_1 = (\mathcal{U}, \mathcal{E}_1, w_1)$ or/and $\mathcal{N}_2 = (\mathcal{V}, \mathcal{E}_2, w_2)$, where \mathcal{E}_1 and w_1 are determined by the matrix $\mathbf{W}^{(1)} = \mathbf{W}\mathbf{W}^T$, $w_{uv}^{(1)} = \sum_{z \in \mathcal{V}} w_{uz} \cdot w_{zv}^T$. Evidently $w_{uv}^{(1)} = w_{vu}^{(1)}$. There is an edge $(u : v) \in \mathcal{E}_1$ in \mathcal{N}_1 iff $N(u) \cap N(v) \neq \emptyset$. Its weight is $w_1(u, v) = w_{uv}^{(1)}$.

The network \mathcal{N}_2 is determined in a similar way by the matrix $\mathbf{W}^{(2)} = \mathbf{W}^T \mathbf{W}$.

The networks \mathcal{N}_1 and \mathcal{N}_2 are analyzed using standard methods.



Authorship networks

Bibliographic networks

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Let **WA** be the works \times authors two mode authorship network; $wa_{pi} \in \{0, 1\}$ is describing the authorship of author i of work p .

$$\forall p \in W : \sum_{i \in A} wa_{pi} = \text{outdeg}_{WA}(p) = \# \text{ authors of work } p$$

Let **N** be its normalized version

$$\forall p \in W : \sum_{i \in A} n_{pi} \in \{0, 1\}$$

obtained from **WA** by $n_{pi} = wa_{pi} / \max(1, \text{outdeg}_{WA}(p))$, or by some other rule determining the author's contribution – the *fractional* approach.



Some transformations of networks

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Binarization $b(\mathcal{N})$ is a network obtained from the \mathcal{N} in which all weights are set to 1.

Transposition \mathcal{N}^T or $t(\mathcal{N})$ is a network obtained from \mathcal{N} in which to all arcs their direction is reversed. $\mathbf{AW} = \mathbf{WA}^T$, $\mathbf{KW} = \mathbf{WK}^T$, ...

(Out) normalization $n(\mathcal{N})$ is a network obtained from \mathcal{N} in which the weight of each arc a is divided by the sum of weights of all arcs having the same initial node as the arc a . For binary networks

$$n(\mathbf{A}) = \text{diag}\left(\frac{1}{\max(1, \text{outdeg}_{\mathbf{WA}}(i))}\right)_{i \in \mathcal{I}} * \mathbf{A}$$

$$\mathbf{N} = n(\mathbf{WA}), \mathbf{WA} = b(\mathbf{N})$$



First co-authorship network

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$$\mathbf{Co} = \mathbf{AW} * \mathbf{WA}$$

$$co_{ij} = \sum_{p \in W} wa_{pi} wa_{pj} = \sum_{p \in N^-(i) \cap N^-(j)} 1$$

co_{ij} = the number of works that authors i and j wrote together

co_{ii} = the total number of works that author i wrote

It holds: $co_{ij} = co_{ji}$.

Using the weights co_{ij} we can determine the Salton's cosine similarity or Ochiai coefficient between authors i and j as

$$\cos(i, j) = \frac{co_{ij}}{\sqrt{co_{ii} co_{jj}}}, \quad \text{for } co_{ij} > 0$$



Cores and generalized cores

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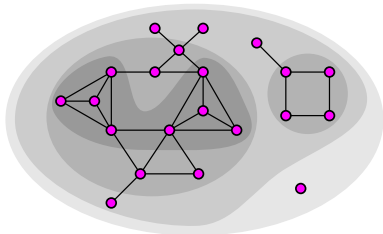
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The notion of core was introduced by Seidman in 1983. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a graph. A subgraph $\mathcal{H} = (C, \mathcal{E}|_C)$ induced by the set C is a *k-core* or a *core of order k* iff $\forall v \in C : \deg_{\mathcal{H}}(v) \geq k$, and \mathcal{H} is a maximal subgraph with this property. The core of maximum order is also called the *main* core.

The *core number* of a node v is the highest order of a core that contains this node. The degree $\deg(v)$ can be: in-degree, out-degree, in-degree + out-degree, etc., determining different types of cores.

p_S-cores

$$p_S(v, U) = \sum_{u \in N_U(v)} w(v, u), \text{ where } w : \mathcal{E} \rightarrow \mathbb{R}_0^+$$

The cores can be determined very efficiently. [paper 1](#), [paper 2](#).



Cores of orders 10–21 in Computational Geometry

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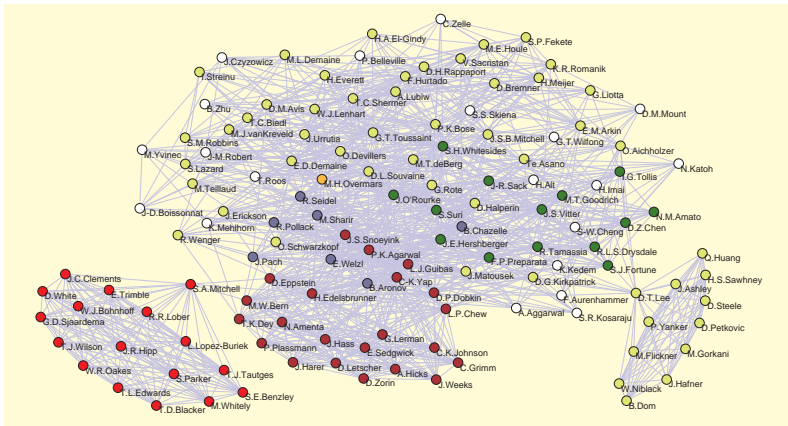
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p_5 -core at level 46 in Computational Geometry network

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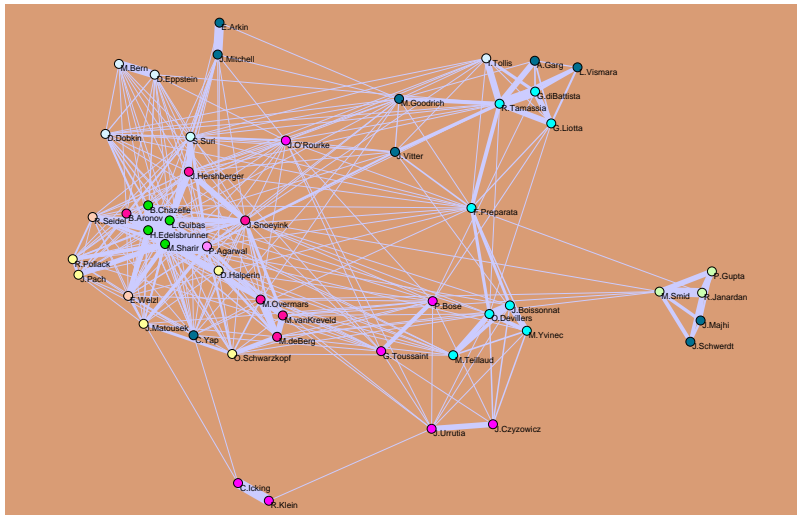
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Cores of orders 20–47 in **Co(SN5)**

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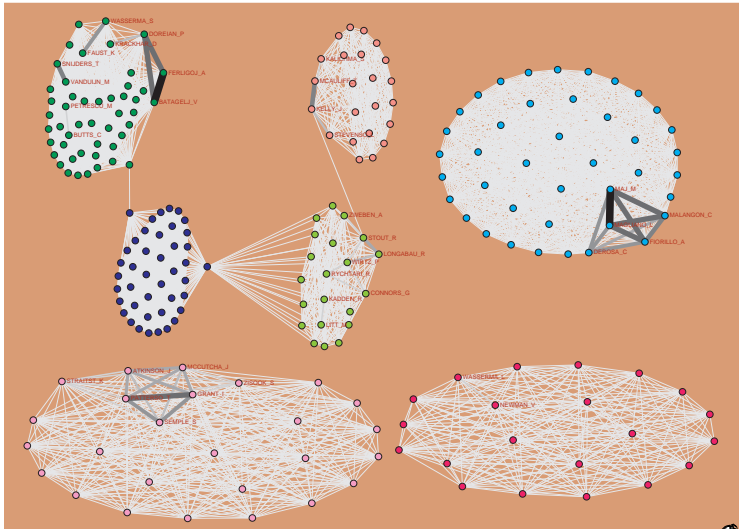
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Papers by number of authors

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Problem: The **Co** network is composed of complete graphs on the set of work's authors. Works with many authors produce large complete subgraphs and are over-represented, thus blurring the collaboration structure.

| outdeg _{WA} | frequency | outdeg _{WA} | frequency | paper |
|----------------------|-----------|----------------------|-----------|------------------------|
| 1 | 2637 | 12 | 8 | |
| 2 | 2143 | 13 | 4 | |
| 3 | 1333 | 14 | 3 | |
| 4 | 713 | 15 | 2 | |
| 5 | 396 | 21 | 1 | Pierce et al. (2007) |
| 6 | 206 | 22 | 1 | Allen et al. (1998) |
| 7 | 114 | 23 | 1 | Kelly et al. (1997) |
| 8 | 65 | 26 | 1 | Semple et al. (1993) |
| 9 | 43 | 41 | 1 | Magliano et al. (2006) |
| 10 | 24 | 42 | 1 | Doll et al. (1992) |
| 11 | 10 | 48 | 1 | Snijders et al. (2007) |



p_5 -core at level 20 of Co(SN5)

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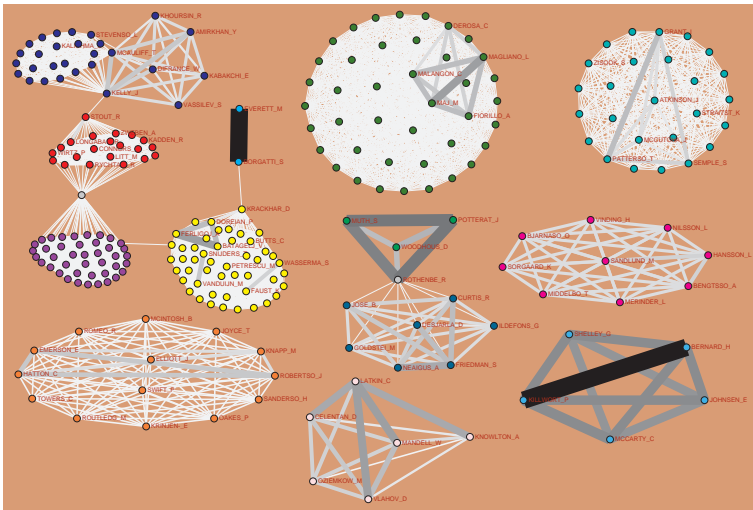
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Second co-authorship network

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$$\mathbf{Cn} = \mathbf{AW} * \mathbf{N}$$

$$cn_{ij} = \sum_{p \in W} wa_{pi} n_{pj} = \sum_{p \in N^-(i) \cap N^-(j)} n_{pj}$$

cn_{ij} = contribution of author j to works, that (s)he wrote together with the author i .

It holds $\sum_{j \in A} \sum_{j \in A} wa_{pi} n_{pj} = \text{outdeg}_{WA}(p)$ and $\sum_{j \in A} cn_{ij} = \text{indeg}_{WA}(i)$

$cn_{ii} = \sum_{p \in N(i)} n_{pi}$ is the contribution of author i to his/her works.

Self-sufficiency: $S_i = \frac{cn_{ii}}{\text{indeg}_{WA}(i)}$

Collaborativeness: $K_i = 1 - S_i$

$$\sum_{i \in A} \sum_{j \in A} cn_{ij} = \sum_{i \in A} \text{indeg}_{WA}(i) = m_{WA}$$



The "best" authors in Social Networks

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| i | author | cn_{ij} | total | K_j | i | author | cn_{ij} | total | K_j |
|----|--------------|-----------|-------|-------|----|--------------|-----------|-------|-------|
| 1 | Burt,R | 43.83 | 53 | 0.173 | 26 | Latkin,C | 10.14 | 37 | 0.726 |
| 2 | Newman,M | 36.77 | 60 | 0.387 | 27 | Morris,M | 9.98 | 20 | 0.501 |
| 3 | Doreian,P | 34.44 | 47 | 0.267 | 28 | Rothenberg,R | 9.82 | 28 | 0.649 |
| 4 | Bonacich,P | 30.17 | 41 | 0.264 | 29 | Kadushin,C | 9.75 | 11 | 0.114 |
| 5 | Marsden,P | 29.42 | 37 | 0.205 | 30 | Faust,K | 9.72 | 18 | 0.460 |
| 6 | Wellman,B | 26.87 | 41 | 0.345 | 31 | Batagelj,V | 9.69 | 20 | 0.516 |
| 7 | Leydesdorf,L | 24.37 | 35 | 0.304 | 32 | Mizruchi,M | 9.67 | 15 | 0.356 |
| 8 | White,H | 23.50 | 33 | 0.288 | 33 | [Anon] | 9.00 | 9 | 0.000 |
| 9 | Friedkin,N | 20.00 | 23 | 0.130 | 34 | Johnson,J | 8.89 | 21 | 0.577 |
| 10 | Borgatti,S | 19.20 | 41 | 0.532 | 35 | Fararo,T | 8.83 | 16 | 0.448 |
| 11 | Everett,M | 16.92 | 31 | 0.454 | 36 | Lazega,E | 8.50 | 12 | 0.292 |
| 12 | Litwin,H | 16.00 | 21 | 0.238 | 37 | Knoke,D | 8.33 | 11 | 0.242 |
| 13 | Freeman,L | 15.53 | 20 | 0.223 | 38 | Ferligoj,A | 8.19 | 19 | 0.569 |
| 14 | Barabasi,A | 14.99 | 35 | 0.572 | 39 | Brewer,D | 8.03 | 11 | 0.270 |
| 15 | Snijders,T | 14.99 | 30 | 0.500 | 40 | Klov Dahl,A | 7.96 | 17 | 0.532 |
| 16 | Valente,T | 14.80 | 34 | 0.565 | 41 | Hammer,M | 7.92 | 10 | 0.208 |
| 17 | Breiger,R | 14.44 | 20 | 0.278 | 42 | White,D | 7.83 | 15 | 0.478 |
| 18 | Skvoretz,J | 14.43 | 27 | 0.466 | 43 | Holme,P | 7.42 | 14 | 0.470 |
| 19 | Krackhardt,D | 13.65 | 25 | 0.454 | 44 | Boyd,J | 7.37 | 13 | 0.433 |
| 20 | Carley,K | 12.93 | 28 | 0.538 | 45 | Kilduff,M | 7.25 | 16 | 0.547 |
| 21 | Pattison,P | 12.10 | 27 | 0.552 | 46 | Small,H | 7.00 | 7 | 0.000 |
| 22 | Wasserman,S | 11.72 | 26 | 0.549 | 47 | Iacobucci,D | 7.00 | 12 | 0.417 |
| 23 | Berkman,L | 11.21 | 30 | 0.626 | 48 | Pappi,F | 6.83 | 10 | 0.317 |
| 24 | Moody,J | 10.83 | 15 | 0.278 | 49 | Chen,C | 6.78 | 12 | 0.435 |
| 25 | Scott,J | 10.47 | 15 | 0.302 | 50 | Seidman,S | 6.75 | 9 | 0.250 |



Outer product decomposition

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For vectors $x = [x_1, x_2, \dots, x_n]$ and $y = [y_1, y_2, \dots, y_m]$ their *outer product* $x \circ y$ is defined as

$$x \circ y = [x_i \cdot y_j]_{n \times m}$$

Let $S_x = \sum_i x_i$ and $S_y = \sum_j y_j$ then

$$S = \sum_{i,j} (x \circ y)_{ij} = \sum_i \sum_j x_i \cdot y_j = \sum_i x_i \cdot \sum_j y_j = S_x \cdot S_y$$

Therefore, if $S_x = S_y = 1$ then also $S = 1$.

It is easy to verify that the *outer product decomposition* holds

$$\mathbf{AK} = \mathbf{AW} \cdot \mathbf{WK} = \sum_w \mathbf{H}_w \quad \text{where} \quad \mathbf{H}_w = \mathbf{WA}[w, \cdot] \circ \mathbf{WK}[w, \cdot]$$

For binary (weights) networks we have $\mathbf{H}_w = K_{N_{WA}(w), N_{WK}(w)}$.



$WA^T \cdot WK$

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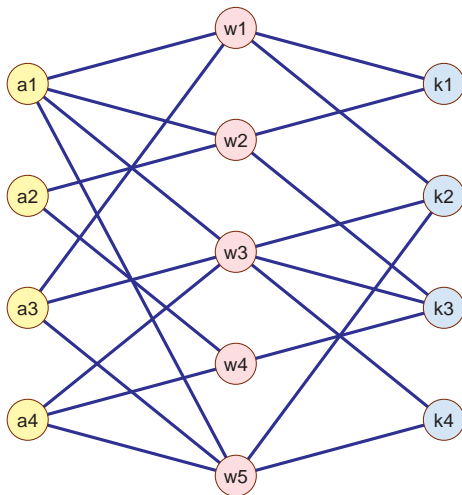
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$$\mathbf{H} = \mathbf{A}\mathbf{W} \cdot \mathbf{W}\mathbf{K}, \quad \mathbf{W}\mathbf{A} = \begin{matrix} & a_1 & a_2 & a_3 & a_4 \\ w_1 & \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \\ w_2 & \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \\ w_3 & \begin{bmatrix} 1 & 0 & 1 & 1 \end{bmatrix} \\ w_4 & \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix} \\ w_5 & \begin{bmatrix} 1 & 0 & 1 & 1 \end{bmatrix} \end{matrix}, \quad \mathbf{W}\mathbf{K} = \begin{matrix} & k_1 & k_2 & k_3 & k_4 \\ w_1 & \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \\ w_2 & \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix} \\ w_3 & \begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix} \\ w_4 & \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} \\ w_5 & \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$\begin{matrix} \mathbf{H} & & \mathbf{H}_1 & & \mathbf{H}_2 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ 2 & 3 & 2 & 2 \\ 1 & 0 & 2 & 0 \\ 1 & 3 & 1 & 2 \\ 0 & 2 & 2 & 2 \end{bmatrix} & = & \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & + & \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & + \end{matrix}$$

$$\begin{matrix} \mathbf{H}_3 & & \mathbf{H}_4 & & \mathbf{H}_5 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} & + & \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} & + & \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{matrix}$$



Fractional approach

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In general each term \mathbf{H}_w in the outer product decomposition has different value $S(\mathbf{H}_w) = \sum_{a,k} (\mathbf{H}_w)_{ak}$ leading to over-representation of works with large values.

To make the contributions of all works equal we can apply the *fractional* approach by normalizing the weights: setting $x' = x/S_x$ we get $S_{x'} = 1$ and therefore $S(\mathbf{H}'_w) = 1$ for all works w .

In the case of two-mode networks **WA** and **WK** we denote

$$S_w^{\mathbf{WA}} = \begin{cases} \sum_a \mathbf{WA}[w, a] & \text{outdeg}_{\mathbf{WA}}(w) > 0 \\ 1 & \text{outdeg}_{\mathbf{WA}}(w) = 0 \end{cases}$$

(and similarly $S_w^{\mathbf{WK}}$)



... fractional approach

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and define the *normalized* matrices

$$\mathbf{WAn} = \text{diag}\left(\frac{1}{S^{WA}}\right)\mathbf{WA}, \quad \mathbf{WKn} = \text{diag}\left(\frac{1}{S^{WK}}\right)\mathbf{WK}$$

Then the *normalized product* matrix is

$$\mathbf{AKt} = \mathbf{WAn}^T \cdot \mathbf{WKn}$$

Denoting $\mathbf{F}_w = \frac{1}{S_w^{WA} S_w^{WK}} \mathbf{H}_w$ the outer product decomposition gets form

$$\mathbf{AKt} = \sum_w \mathbf{F}_w$$



... fractional approach

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Since

$$T(\mathbf{F}_w) = \begin{cases} 1 & (\text{outdeg}_{\mathbf{WA}}(w) > 0) \wedge (\text{outdeg}_{\mathbf{WK}}(w) > 0) \\ 0 & \text{otherwise} \end{cases}$$

we have further

$$\sum_{a,k} \mathbf{F}[a, k] = \sum_{a,k} \sum_w \mathbf{F}_w[a, k] = \sum_w T(\mathbf{F}_w) = |W^+|$$

where $W^+ = \{w \in W : (\text{outdeg}_{\mathbf{WA}}(w) > 0) \wedge (\text{outdeg}_{\mathbf{WK}}(w) > 0)\}$.

In the network **AKt** the contribution of each work to the bibliography is 1. These contributions are redistributed to arcs from authors to keywords.



Example

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$$\text{diag}\left(\frac{1}{S^{WA}}\right) = \begin{matrix} & \begin{matrix} w_1 & w_2 & w_3 & w_4 & w_5 \end{matrix} \\ \begin{matrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{matrix} & \begin{bmatrix} 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/3 \end{bmatrix} \end{matrix}$$

$$\text{diag}\left(\frac{1}{S^{WK}}\right) = \begin{matrix} & \begin{matrix} w_1 & w_2 & w_3 & w_4 & w_5 \end{matrix} \\ \begin{matrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{matrix} & \begin{bmatrix} 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1/2 \end{bmatrix} \end{matrix}$$

$$WAn = \begin{matrix} & \begin{matrix} a_1 & a_2 & a_3 & a_4 \end{matrix} \\ \begin{matrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{matrix} & \begin{bmatrix} 1/2 & 0 & 1/2 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 \\ 0 & 1/2 & 0 & 1/2 \\ 1/3 & 0 & 1/3 & 1/3 \end{bmatrix} \end{matrix}, \quad WKn = \begin{matrix} & \begin{matrix} k_1 & k_2 & k_3 & k_4 \end{matrix} \\ \begin{matrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{matrix} & \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 1 & 0 \\ 0 & 1/2 & 0 & 1/2 \end{bmatrix} \end{matrix}$$



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$$\mathbf{F}_1 = \begin{matrix} & k_1 & k_2 & k_3 & k_4 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} 1/4 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/4 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$\mathbf{F}_5 = \begin{matrix} & k_1 & k_2 & k_3 & k_4 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} 0 & 1/6 & 0 & 1/6 \\ 0 & 0 & 0 & 0 \\ 0 & 1/6 & 0 & 1/6 \\ 0 & 1/6 & 0 & 1/6 \end{bmatrix} \end{matrix}$$

$$\mathbf{AKt} = \mathbf{F} = \begin{matrix} & k_1 & k_2 & k_3 & k_4 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} 0.50000 & 0.52778 & 0.36111 & 0.27778 \\ 0.25000 & 0.00000 & 0.75000 & 0.00000 \\ 0.25000 & 0.52778 & 0.11111 & 0.27778 \\ 0.00000 & 0.27778 & 0.61111 & 0.27778 \end{bmatrix} \end{matrix}$$



Third co-authorship network

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$$\mathbf{Ct} = \mathbf{N}^T * \mathbf{N}$$

ct_{ij} = the total contribution of 'collaboration' of authors i and j to works.

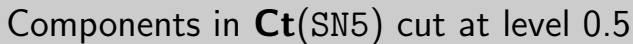
It holds $ct_{ij} = ct_{ji}$ and

$$\sum_{i \in A} \sum_{j \in A} n_{pi} n_{pj} = 1$$

The total contribution of a complete subgraph corresponding to the authors of a work p is 1.

$\sum_{j \in A} ct_{ij} = \sum_{p \in W} n_{pi}$ = the total contribution of author i to works from W .

$$\sum_{i \in A} \sum_{j \in A} ct_{ij} = |W|$$



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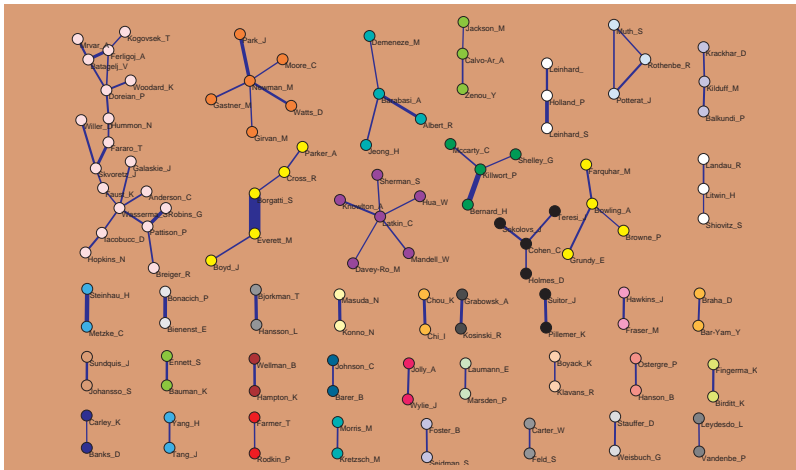
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p_S -core at level 0.75 in **Ct(SN5)**

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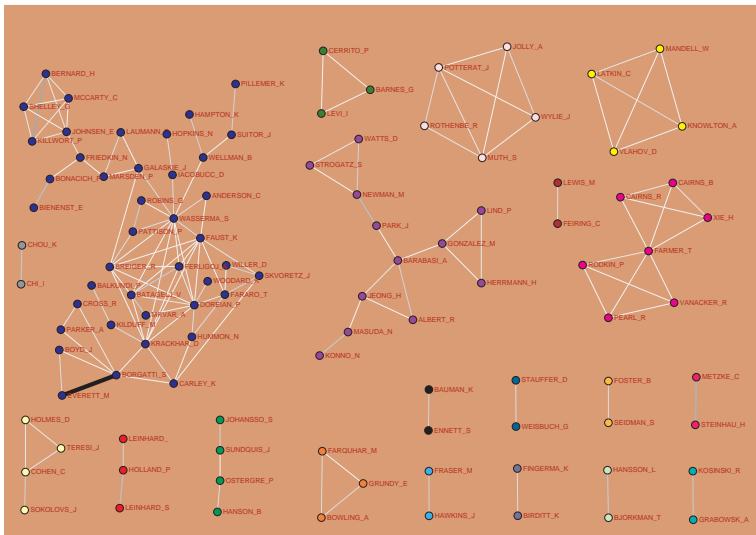
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Some link islands [5,20] in **Ct**(SN5)

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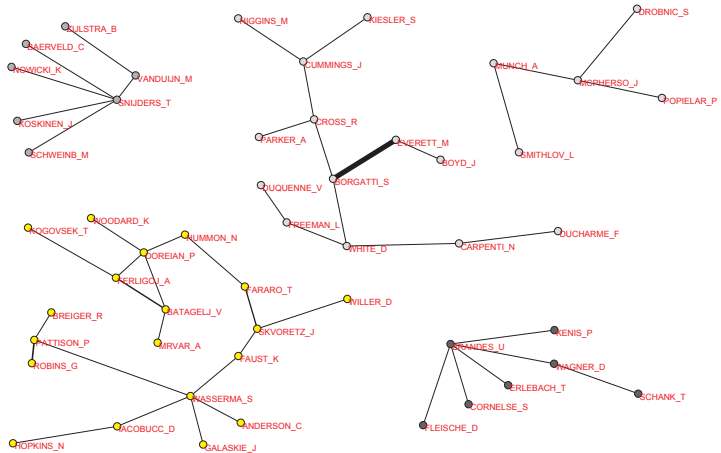
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Fourth co-authorship network

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$\mathbf{Ct}' = \mathbf{N}^T * \mathbf{N}'$, where $n'_{pi} = wa_{pi} / \max(1, \text{outdeg}_{WA}(p) - 1)$ is the Newman's normalization.

ct'_{ij} = the total contribution of 'strict collaboration' of authors i and j to works.

The final result is returned as an undirected simple network with weights (for $i \neq j$)

$$ct'_{ij} = \sum_p \frac{2 \cdot wa_{pi} \cdot wa_{pj}}{\max(1, \text{outdeg}_{WA}(p)) \cdot \max(1, \text{outdeg}_{WA}(p) - 1)}$$



Authors' citations network

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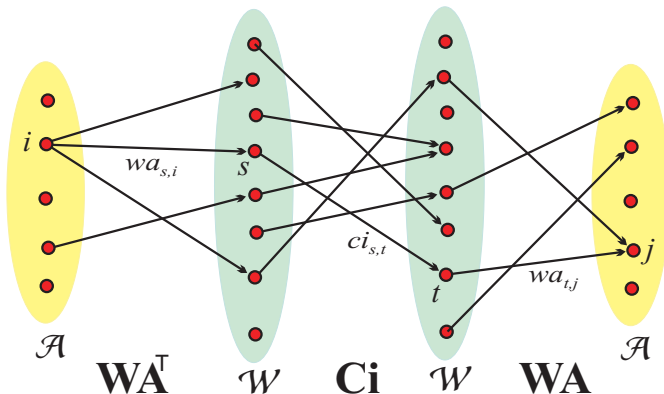
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$\mathbf{Ca} = \mathbf{AW} * \mathbf{Ci} * \mathbf{WA}$ is a network of citations between authors. The weight $w(i,j)$ counts the number of times a work authored by i is citing a work authored by j .



Islands in SN5 authors citation network

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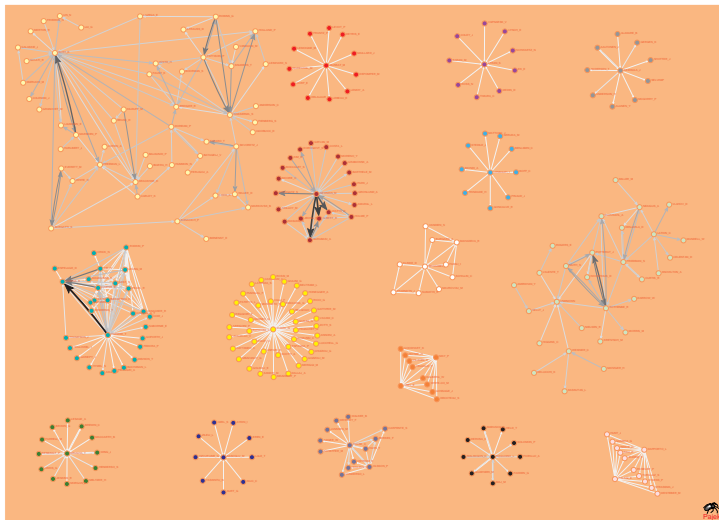
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Linking through a network

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Let a network \mathbf{S} links works to works. The derived network $\mathbf{WA}^T \cdot \mathbf{S} \cdot \mathbf{WA}$ links authors to authors *through* \mathbf{S} . Again, the normalization question has to be addressed. Among different options let us consider the derived networks defined as:

$$\mathbf{C} = \mathbf{WAn}^T \cdot \mathbf{S} \cdot \mathbf{WAn}$$

It is easy to verify that:

- if \mathbf{S} is symmetric, $\mathbf{S}^T = \mathbf{S}$, then also \mathbf{C} is symmetric, $\mathbf{C}^T = \mathbf{C}$;
- if $W^+ = \{w \in W : \text{outdeg}_{\mathbf{WA}}(w) > 0\} = W$, the total of weights of \mathbf{S} is redistributed in \mathbf{C} :

$$T(\mathbf{C}) = \sum_{e \in L(\mathbf{C})} c(e) = \sum_{e \in L(\mathbf{S})} s(e) = T(\mathbf{S})$$



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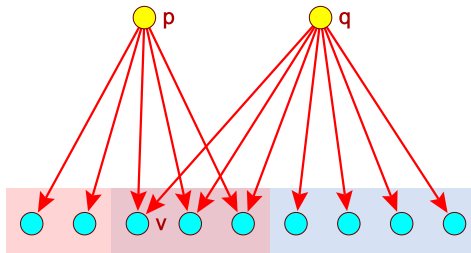
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Bibliographic coupling occurs when two works each cite a third work in their bibliographies, see figure. The idea was introduced by Kessler (1963) and has been used extensively since then. See figure where two citing works, p and q , are shown. Work p cites five works and q cites seven works. The key idea is that there are three works cited by both p and q . This suggests some content communality for the three works cited by both p and q . Having more works citing pairs of prior works increases the likelihood of them sharing content.





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We assume that the citation relation means $p \mathbf{Ci} q \equiv$ work p cites work q . Then the *bibliographic coupling* network **biCo** can be determined as

$$\mathbf{biCo} = \mathbf{Ci} * \mathbf{Ci}^T$$

The weight $bico_{pq}$ is equal to the number of works cited by both works p and q ; $bico_{pq} = |\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|$. Bibliographic coupling weights are symmetric: $bico_{pq} = bico_{qp}$:

$$\mathbf{biCo}^T = (\mathbf{Ci} \cdot \mathbf{Ci}^T)^T = \mathbf{Ci} \cdot \mathbf{Ci}^T = \mathbf{biCo}$$



Co-citation

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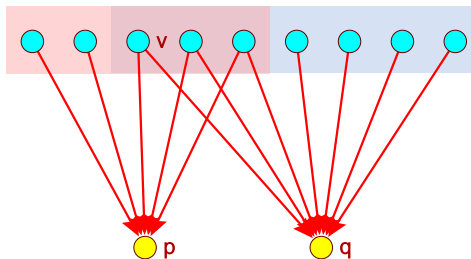
Multiplication

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Co-citation is a concept with strong parallels with bibliographic coupling (Small and Marshakova 1973). The focus is on the extent to which works are co-cited by later works. The basic intuition is that the more earlier works are cited, the higher the likelihood that they have common content.





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The *co-citation* network **coCi** can be determined as

$$\mathbf{coCi} = \mathbf{Ci}^T \cdot \mathbf{Ci}.$$

The weight $coci_{pq}$ is equal to the number of works citing both works p and q . The network **coCi** is symmetric $coci_{pq} = coci_{qp}$:

$$\mathbf{coCi}^T = (\mathbf{Ci}^T \cdot \mathbf{Ci})^T = \mathbf{Ci}^T \cdot \mathbf{Ci} = \mathbf{coCi}$$

An important property of co-citation is that $\mathbf{coCi}(\mathbf{Ci}) = \mathbf{biCo}(\mathbf{Ci}^T)$:

$$\mathbf{biCo}(\mathbf{Ci}^T) = \mathbf{Ci}^T \cdot (\mathbf{Ci}^T)^T = \mathbf{Ci}^T \cdot \mathbf{Ci} = \mathbf{coCi}(\mathbf{Ci})$$

Therefore the constructions proposed for bibliographic coupling can be applied also for co-citation.



Normalization

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What about normalizations? Searching for the most coupled works we have again problems with works with many citations, especially with review papers. To neutralize their impact we can introduce normalized measures. The fractional approach works fine for normalized co-citation

$$\mathbf{CoCit} = \mathbf{Cin}^T \cdot \mathbf{Cin}$$

where $\mathbf{Cin} = \mathbf{D} \cdot \mathbf{Ci}$ and $\mathbf{D} = \text{diag}(\frac{1}{\max(1, \text{outdeg}(p))})$. $\mathbf{D}^T = \mathbf{D}$. In the normalized network every work has value 1 and it is equally distributed to all cited works.



Normalization of Bibliographic Coupling

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The fractional approach can not be directly applied to bibliographic coupling – to get the outer product decomposition work we would need to normalize $\mathbf{C_i}$ by columns – a cited work has value 1 which is distributed equally to the citing works – the most cited works give the least. This is against our intuition. To construct a reasonable measure we can proceed as follows. Let us first look at

$$\mathbf{biC} = \mathbf{C_{in}} \cdot \mathbf{C_i}^T$$

we have

$$\mathbf{biC} = (\mathbf{D} \cdot \mathbf{C_i}) \cdot \mathbf{C_i}^T = \mathbf{D} \cdot \mathbf{biCo}$$

$$\mathbf{biC}^T = (\mathbf{D} \cdot \mathbf{biCo})^T = \mathbf{biCo}^T \cdot \mathbf{D}^T = \mathbf{biCo} \cdot \mathbf{D}$$



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For $\mathbf{Ci}(p) \neq \emptyset$ and $\mathbf{Ci}(q) \neq \emptyset$ it holds

$$\mathbf{biC}_{pq} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(p)|} \quad \text{and} \quad \mathbf{biC}_{qp} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(q)|} = \mathbf{biC}_{pq}^T$$

and $\mathbf{biC}_{pq} \in [0, 1]$. \mathbf{biC}_{pq} is the proportion of its references that the work p shares with the work q . The network \mathbf{biC} is not symmetric.

We have different options to construct normalized symmetric measures such as

$$\mathbf{biCoa}_{pq} = \frac{1}{2}(\mathbf{biC}_{pq} + \mathbf{biC}_{qp}) \quad \text{Average}$$

$$\mathbf{biCom}_{pq} = \min(\mathbf{biC}_{pq}, \mathbf{biC}_{qp}) \quad \text{Minimum}$$

$$\mathbf{biCoM}_{pq} = \max(\mathbf{biC}_{pq}, \mathbf{biC}_{qp}) \quad \text{Maximum}$$



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or, may be more interesting

$$\mathbf{biCog}_{pq} = \sqrt{\mathbf{biC}_{pq} \cdot \mathbf{biC}_{qp}} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{\sqrt{|\mathbf{Ci}(p)| \cdot |\mathbf{Ci}(q)|}}$$

Geometric mean
Salton cosinus

$$\mathbf{biCoh}_{pq} = 2 \cdot (\mathbf{biC}_{pq}^{-1} + \mathbf{biC}_{qp}^{-1})^{-1} = \frac{2|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(p)| + |\mathbf{Ci}(q)|}$$

Harmonic mean

$$\mathbf{biCoj}_{pq} = (\mathbf{biC}_{pq}^{-1} + \mathbf{biC}_{qp}^{-1} - 1)^{-1} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(p) \cup \mathbf{Ci}(q)|}$$

Jaccard index

All these measures are similarities.

It is easy to verify that $\mathbf{biCoX}_{pq} \in [0, 1]$ and: $\mathbf{biCoX}_{pq} = 1$ iff the works p and q are referencing the same works, $\mathbf{Ci}(p) = \mathbf{Ci}(q)$.



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From $m \leq H \leq G \leq A \leq M$ and $J \leq m$,
 $(\frac{|P \cap Q|}{|P \cup Q|} \leq \min(\frac{|P \cap Q|}{|P|}, \frac{|P \cap Q|}{|Q|}))$ we get

$$\mathbf{biCoj}_{pq} \leq \mathbf{biCom}_{pq} \leq \mathbf{biCoh}_{pq} \leq \mathbf{biCog}_{pq} \leq \mathbf{biCoa}_{pq} \leq \mathbf{biCoM}_{pq}$$

The equalities hold iff $\mathbf{Ci}(p) = \mathbf{Ci}(q)$.

To get a dissimilarity we can use transformations $dis = 1 - sim$ or $dis = \frac{1}{sim} - 1$ or $dis = -\log sim$. For example

$$\mathbf{biCod}_{pq} = 1 - \mathbf{biCoj}_{pq} = \frac{|\mathbf{Ci}(p) \oplus \mathbf{Ci}(q)|}{|\mathbf{Ci}(p) \cup \mathbf{Ci}(q)|} \quad \text{Jaccard distance}$$

where \oplus denotes the symmetric difference of sets.



Bibliographic Coupling, Co-citation and Linking through a network

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Bibliographic coupling and co-citation networks are linking works to works. To get linking between authors, journals or keywords considering citation similarity we can apply the construction from *Linking through a network* to the normalized co-citation or bibliographic coupling network.



Temporal quantities

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We introduce a notion of a *temporal quantity*

$$a(t) = \begin{cases} a'(t) & t \in T_a \\ \mathbb{X} & t \in \mathcal{T} \setminus T_a \end{cases}$$

where T_a is the *activity time set* of a and $a'(t)$ is the value of a in an instant $t \in T_a$, and \mathbb{X} denotes the value *undefined*.

We assume that the values of temporal quantities belong to a set A which is a *semiring* $(A, +, \cdot, 0, 1)$ for binary operations $+: A \times A \rightarrow A$ and $\cdot: A \times A \rightarrow A$.

Let $A_{\mathbb{X}}(\mathcal{T})$ denote the set of all temporal quantities over $A_{\mathbb{X}}$ in time \mathcal{T} . To extend the operations to networks and their matrices we first define the *sum* (parallel links) $a + b$ as

$$(a + b)(t) = a(t) + b(t) \quad \text{and} \quad T_{a+b} = T_a \cup T_b.$$

The *product* (sequential links) $a \cdot b$ is defined as

$$(a \cdot b)(t) = a(t) \cdot b(t) \quad \text{and} \quad T_{a \cdot b} = T_a \cap T_b.$$



Sum and product of temporal quantities

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$$a = [(1, 5, 2), (6, 8, 1), (11, 12, 3), (14, 16, 2), \\ (17, 18, 5), (19, 20, 1)]$$

$$b = [(2, 3, 4), (4, 7, 3), (9, 10, 2), (13, 15, 5), (16, 21, 1)]$$

The following are the sum $s = a + b$ and the product $p = a \cdot b$ of temporal quantities a and b over combinatorial semiring.

$$s = [(1, 2, 2), (2, 3, 6), (3, 4, 2), (4, 5, 5), (5, 6, 3), \\ (6, 7, 4), (7, 8, 1), (9, 10, 2), (11, 12, 3), \\ (13, 14, 5), (14, 15, 7), (15, 16, 2), (16, 17, 1), \\ (17, 18, 6), (18, 19, 1), (19, 20, 2), (20, 21, 1)]$$

$$p = [(2, 3, 8), (4, 5, 6), (6, 7, 3), (14, 15, 10), \\ (17, 18, 5), (19, 20, 1)]$$

They are visually displayed at the bottom half of figures on the following slides.



Addition of temporal quantities.

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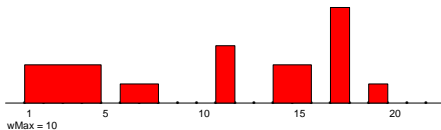
Multiplication

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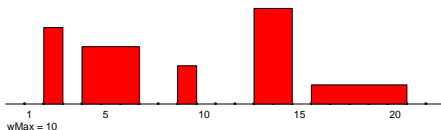
Temporal Ns

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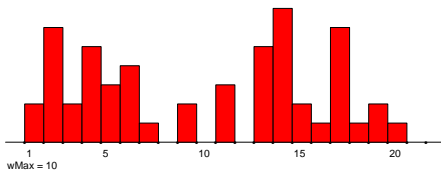
$a :$



$b :$



$a + b :$





Multiplication of temporal quantities.

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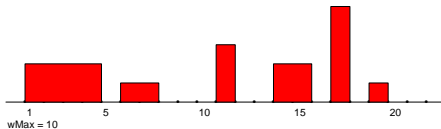
Multiplication

Derived Ns

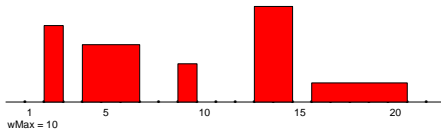
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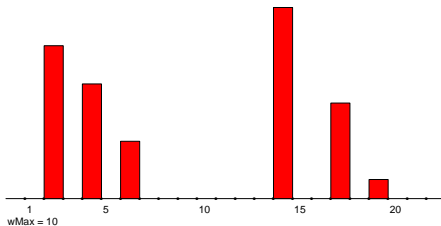
$a :$



$b :$



$a \cdot b :$





Temporal affiliation networks

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Let the binary **affiliation** matrix $\mathbf{A} = [a_{ep}]$ describe a two-mode network on the set of events E and the set of participants P :

$$a_{ep} = \begin{cases} 1 & p \text{ participated in the event } e \\ 0 & \text{otherwise} \end{cases}$$

The function $d : E \rightarrow \mathcal{T}$ assigns to each event e the date $d(e)$ when it happened. $\mathcal{T} = [first, last] \subset \mathbb{N}$. Using these data we can construct two temporal affiliation matrices:

- **instantaneous** $\mathbf{A}i = [ai_{ep}]$, where

$$ai_{ep} = \begin{cases} [(d(e), d(e) + 1, 1)] & a_{ep} = 1 \\ [] & \text{otherwise} \end{cases}$$

- **cumulative** $\mathbf{A}c = [ac_{ep}]$, where

$$ac_{ep} = \begin{cases} [(d(e), last + 1, 1)] & a_{ep} = 1 \\ [] & \text{otherwise} \end{cases}$$



Cumulative properties

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In general a temporal quantity a is called *cumulative* iff it has for $t, t' \in \mathcal{T}$ the property

$$t \in T_a \wedge t' > t \Rightarrow t' \in T_a \wedge a(t') \geq a(t)$$

A sum and product (over combinatorial semiring) of cumulative temporal quantities are cumulative temporal quantities.

For a temporal quantity $a = [(s_i, f_i, v_i)]_{i \in 1..k}$ its *cumulative* $cum(a)$ is defined as

$$cum(a) = [(s_i, s_{i+1}, V_i)]_{i \in 1..k}$$

where $s_{k+1} = last$ and $V_i = \sum_{j=1}^i v_j$.

A temporal network is cumulative for a weight w iff all its values are cumulative.



Temporal properties

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Let \mathbf{N} be a temporal network on $E \times P$. On it we can define some interesting temporal quantities such as *in-sum*:

$$iS(\mathbf{N}, p) = \sum_{e \in E} n_{ep}$$

and *out-sum*:

$$oS(\mathbf{N}, e) = \sum_{p \in P} n_{ep}$$

In a special case where $\mathbf{N} \equiv \mathbf{WAI}$ we get the *productivity of an author a*

$$pr(a) = iS(\mathbf{WAI}, a) = \text{number of publications of the author } a \text{ by year}$$

and for $\mathbf{N} \equiv \mathbf{WAc}$ we get the *cumulative productivity of an author a*

$$cpr(a) = iS(\mathbf{WAc}, a) = \text{cumulative number of publications of } a \text{ by year.}$$

It holds $cpr(a) = cum(pr(a))$.



Group productivity

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The productivity of an author can be extended to the *productivity of a group of authors* C

$$pr(C) = \sum_{a \in C} pr(a) = \sum_{a \in C} iS(\mathbf{W}\mathbf{A}\mathbf{i}, a)$$

There is a problem with the productivity of a group. In the case when two authors from a group co-authored the same paper it is counted twice. To account for a “real” contribution of each author the fractional approach is used. It is based on normalized networks (matrices) – in the case of co-authorship on $n(\mathbf{W}\mathbf{A}) = \mathbf{W}\mathbf{A}\mathbf{n} = [wan_{wa}]$

$$wan_{wa} = \frac{wa_{wa}}{\max(1, \text{outdeg}_{\mathbf{W}\mathbf{A}}(w))}.$$

This leads to the *fractional productivity of an author* a

$fpr(a) = iS(\mathbf{W}\mathbf{A}\mathbf{n}\mathbf{i}, a)$ = fractional contribution of publications of a by year



Temporal properties in networks on peer review

Productivity and cumulative productivity of Lutz Bornmann

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In the analysis of the ordinary authorship network **WA** we get that Lutz Bornmann is the author who wrote the largest number, 61, of works on peer review. To see the dynamics of his publishing we compute his productivity

$$pr = [(2005, 2006, 4), (2006, 2007, 3), (2007, 2008, 4), (2008, 2009, 9), (2009, 2010, 4), (2010, 2011, 14), (2011, 2012, 5), (2012, 2013, 7), (2013, 2014, 2), (2014, 2015, 3), (2015, 2016, 6)]$$

see the top of the figure. The corresponding cumulative productivity is

$$cpr = [(2005, 2006, 4), (2006, 2007, 7), (2007, 2008, 11), (2008, 2009, 20), (2009, 2010, 24), (2010, 2011, 38), (2011, 2012, 43), (2012, 2013, 50), (2013, 2014, 52), (2014, 2015, 55), (2015, 2017, 61)]$$

see the bottom of the figure. Note that $cpr = cum(pr)$: $7 = 4 + 3$, $11 = 4 + 3 + 4$, ...



Temporal properties in networks on peer review

Productivity and cumulative productivity of Lutz Bornmann

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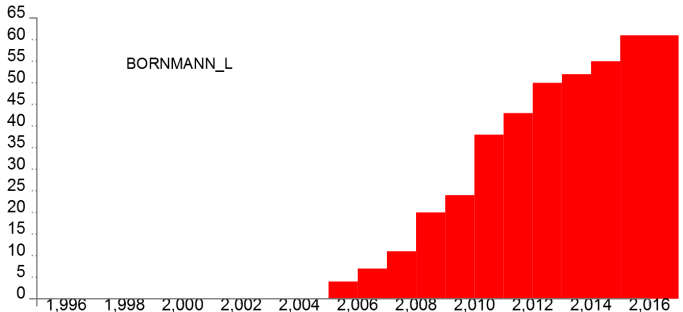
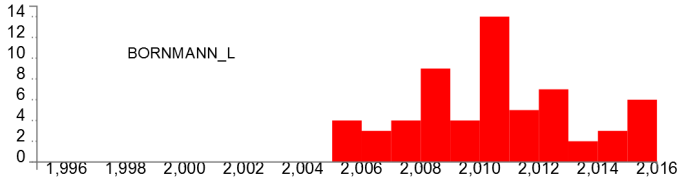
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Fractional productivity of Lutz Bornmann

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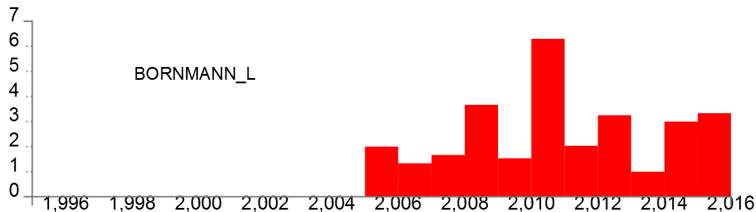
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The fractional productivity of Lutz Bornmann is

$$\begin{aligned} fpr = & [(2005, 2006, 2.0), (2006, 2007, 1.333), (2007, 2008, 1.667), \\ & (2008, 2009, 3.667), (2009, 2010, 1.533), (2010, 2011, 6.3), \\ & (2011, 2012, 2.033), (2012, 2013, 3.25), (2013, 2014, 1.0), \\ & (2014, 2015, 3.0), (2015, 2016, 3.333)] \end{aligned}$$

see the figure





Citations to a paper through years

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In the citation network **Cite** for the peer review bibliography the most cited, 164, paper is Peters, D. P., Ceci, S. J. (1982). Peer-review practices of psychological journals: The fate of published articles, submitted again. *Behavioral and Brain Sciences*, 5(2), 187-255. The temporal quantity $citP = iS(\mathbf{C}_{ii}, \text{PETERS_D}(1982)5:187)$ describes the number of citations to this paper through years.

$$citP = [(1982, 1983, 1), (1983, 1984, 4), (1984, 1986, 3), (1986, 1987, 2), (1987, 1988, 3), (1988, 1989, 5), (1989, 1990, 2), (1990, 1991, 4), (1991, 1992, 5), (1992, 1993, 3), (1993, 1994, 8), (1994, 1996, 5), (1996, 1997, 6), (1997, 1998, 1), (1998, 1999, 5), (1999, 2000, 2), (2000, 2001, 1), (2001, 2002, 2), (2002, 2003, 4), (2003, 2004, 5), (2004, 2005, 4), (2005, 2006, 6), (2006, 2008, 5), (2008, 2009, 3), (2009, 2010, 9), (2010, 2011, 7), (2011, 2012, 10), (2012, 2013, 11), (2013, 2014, 4), (2014, 2015, 5), (2015, 2016, 14), (2016, 2017, 2)]$$

See the figure at the next slide.



Peer review

Citations to Peters

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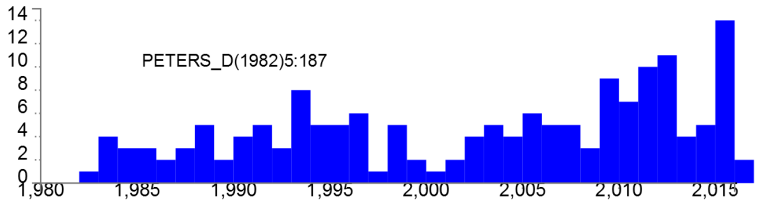
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Similarly we could look at the number of works by year

$wy = \sum_{j \in J} iS(\mathbf{WJi}, j)$, the popularity of a keyword k :

$pop(k) = iS(\mathbf{WKi}, k)$, etc.



Peer review

Citations to Hirsch

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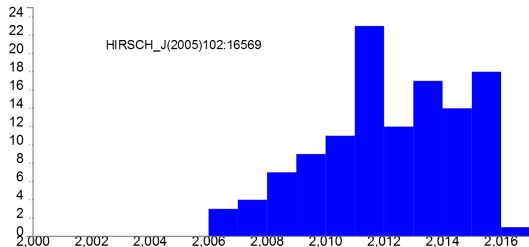
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Another well known paper is Hirsch, J.E. (2005). An index to quantify an individual's scientific research output. Proc Natl Acad Sci U S A. 2005 Nov 15;102(46):16569-72 with 119 citations and $citH = iS(\mathbf{C}_{ii}, \text{HIRSCH_J}(2005)102:16569)$

$citH = [(2006, 2007, 3), (2007, 2008, 4), (2008, 2009, 7), (2009, 2010, 9), (2010, 2011, 11), (2011, 2012, 23), (2012, 2013, 12), (2013, 2014, 17), (2014, 2015, 14), (2015, 2016, 18), (2016, 2017, 1)]$





Main journals publishing on peer review

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To identify the main journals publishing on peer review we determined first the temporal inSums J_t in the network **JWi** for all journals. An entry $J_t(j) = iS(\mathbf{JWi}, j)$ contains the temporal quantity counting the number of papers on peer review published in the journal j in each year. Because most of frequencies are small (one digit numbers) we decided to change the time scale (granularity) to time intervals: 1: 1900-1970, 2: 1971-1980, 3: 1981-1990, 4: 1991-2000, 5: 2001-2005, 6: 2006-2010, 7: 2011-2015. The recoded table J_t is labeled J_r . For the table J_r we determined for each time interval three the most frequently used journals – they are listed on the right side of the figure. The corresponding data were exported as `journals.csv` and visualized using R. The picture on the left side presents the trajectories of relative importance (journal's frequency divided with the maximum frequency on the interval) for the selected journals.



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The papers on peer review (refereeing) published till 1970 appeared most often in J ASSOC OFF AGR CHEM. Till 2005 the dominant journals were JAMA, SCIENCE, NATURE, BRIT MED J, and LANCET (general medical and science journals). In the period 2006-2010 the leading role was overtaken by a specialized journal SCIENTOMETRICS. In the last period 2011-2015 the primate is shifted to the mega-journals BMJ OPEN and PLOS ONE. Note that the frequencies for SCIENTOMETRICS are 3: 6, 4: 25, 5:18, 6: 44, 7: 78 and in the period 2011-2015 489 papers on peer review were published in BMJ OPEN.



Main journals publishing on peer review

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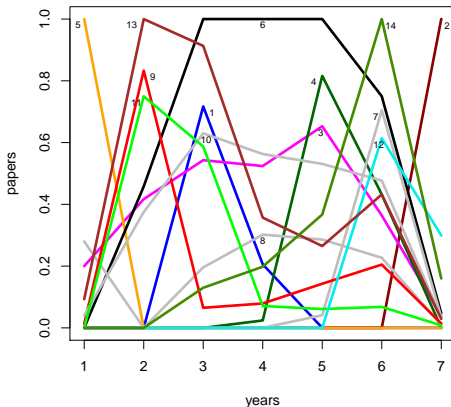
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Proportions of papers



| i | journal |
|----|----------------------|
| 1 | BEHAV BRAIN SCI |
| 2 | BMJ OPEN |
| 3 | BRIT MED J |
| 4 | CUTIS |
| 5 | J ASSOC OFF AGR CHEM |
| 6 | JAMA-J AM MED ASSOC |
| 7 | J SEX MED |
| 8 | LANCET |
| 9 | MED J AUSTRALIA |
| 10 | NATURE |
| 11 | NEW ENGL J MED |
| 12 | PLOS ONE |
| 13 | SCIENCE |
| 14 | SCIENTOMETRICS |



Multiplication of temporal networks

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Let \mathbf{A} on $A \times P$ and \mathbf{B} on $P \times B$ be (matrices of) co-occurrence networks. Then $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ is a temporal network on $A \times B$. What is its meaning? Consider the value of its item in an instant t

$$c_{ij}(t) = \sum_{p \in P} a_{ip}(t)^T \cdot b_{pj}(t) = \sum_{p \in P} a_{pi}(t) \cdot b_{pj}(t)$$

For $c_{ij}(t)$ to be defined (different from \mathbb{X}) there should be at least one $p \in P$ such that $a_{pi}(t)$ and $b_{pj}(t)$ are both defined, i.e. $t \in T_{a_{pi}} \cap T_{b_{pj}}$. Then there exists g_{pi} such that $(s_{g_{pi}}, f_{g_{pi}}, v_{g_{pi}}) \in a_{pi}$, $t \in [s_{g_{pi}}, f_{g_{pi}})$, and $a_{pi}(t) = v_{g_{pi}}$. Similarly $b_{pj}(t) = v_{h_{pj}}$. Therefore

$$c_{ij}(t) = \sum_{p: t \in T_{a_{pi}} \cap T_{b_{pj}}} v_{g_{pi}} \cdot v_{h_{pj}}$$



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For binary instantaneous two-mode networks **A** and **B** the value $c_{ij}(t)$ of the product $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ is equal to the number of different members of P with which both i and j have contact in the instant t .

The product of cumulative networks is cumulative itself. For binary cumulative two-mode networks **A** and **B** the value $c_{ij}(t)$ of the product $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ is equal to the number of different members of P with which both i and j had contact in instants up to including the instant t .



Temporal co-occurrence networks

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Using the multiplication of temporal affiliation networks over the combinatorial semiring we get the corresponding instantaneous and cumulative co-occurrence networks

$$\mathbf{Ci} = \mathbf{Ai}^T \cdot \mathbf{Ai} \quad \text{and} \quad \mathbf{Cc} = \mathbf{Ac}^T \cdot \mathbf{Ac}$$

The triple (s, f, v) in a temporal quantity ci_{pq} tells that in the time interval $[s, f)$ there were v events in which both p and q took part.

The triple (s, f, v) in a temporal quantity cc_{pq} tells that in the time interval $[s, f)$ there were in total v accumulated events in which both p and q took part.

The diagonal (loop) weights ci_{pp} and cc_{pp} contain the temporal quantities counting the number of events in the time intervals in which the participant p took part.

A typical example of such a network is the works authorship network **WA** where E is the set of papers W , P is the set of authors A , and d is the publication year.



Example: Temporal coauthorship network

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The instantaneous coauthorship network **Coi** is obtained as

$$\mathbf{Coi} = \mathbf{W}\mathbf{Ai}^T \cdot \mathbf{W}\mathbf{Ai}$$

Bibliographic networks are usually sparse. Often also the product of sparse networks is sparse itself. Considering in computation only non zero elements it can be computed fast. In our example, the network **WA** has 22104 works, 62106 authors and 80021 arcs. The derived network **Coi** has 633977 edges and was computed on a laptop in 12.7 seconds.



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For the peer review data we get the largest values

$$co[BORNMANN_L, DANIEL_H] = 42,$$

$$co[MOHER_D, ALTMAN_D] = 24,$$

$$co[REYES_H, ANDRESEN_M] = 17.$$

The corresponding temporal quantities

$$bd = tq(BORNMANN_L, DANIEL_H) \text{ and}$$

$$ra = tq(REYES_H, ANDRESEN_M) \text{ are}$$

$$bd = [(2005, 2006, 4), (2006, 2007, 3), (2007, 2008, 4), (2008, 2009, 7), \\ (2009, 2010, 4), (2010, 2011, 11), (2011, 2013, 4), (2015, 2016, 1)]$$

$$ra = [(1997, 1998, 3), (1998, 1999, 1), (2000, 2002, 1), (2004, 2005, 1), \\ (2005, 2006, 2), (2006, 2008, 1), (2009, 2010, 2), (2011, 2012, 1), \\ (2013, 2016, 1)]$$

Both temporal quantities are presented on the next slide.



Example: Temporal coauthorship network

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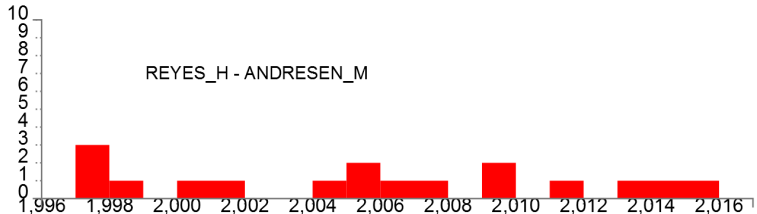
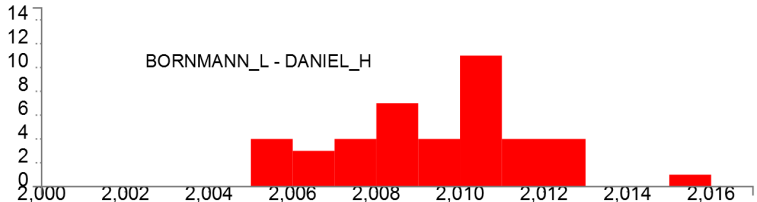
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Example: Temporal citations between journals

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The derived network describing citations between journals is obtained as

$$JCJ = WJi^T \cdot CiI \cdot WJc$$

Note that the third network in the product is cumulative.

The weight of the element jc_{ij} is equal to the number of citations per year from works published in journal i to works published in journal j . In a special case when $i = j$ we get a temporal quantity describing selfcitations of journal i . In the peer review data the largest number of selfcitations are 320 in JAMA and 148 in Scientometrics.



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The corresponding temporal quantities $jm = jcj[\text{JAMA}, \text{JAMA}]$ and $sm = jcj[\text{SCIENTOMETRICS}, \text{SCIENTOMETRICS}]$ are:

$$\begin{aligned} jm &= [(1973, 1976, 1), (1988, 1989, 1), (1989, 1990, 2), (1990, 1991, 16), (1991, 1992, 1), \\ &\quad (1992, 1993, 11), (1993, 1994, 4), (1994, 1995, 44), (1995, 1996, 9), (1996, 1997, 2), \\ &\quad (1997, 1998, 3), (1998, 1999, 68), (1999, 2000, 14), (2000, 2001, 10), (2001, 2002, 7), \\ &\quad (2002, 2003, 60), (2003, 2004, 11), (2004, 2005, 4), (2005, 2006, 1), (2006, 2007, 16), \\ &\quad (2007, 2008, 8), (2008, 2009, 2), (2009, 2010, 4), (2012, 2013, 4), (2013, 2014, 3), \\ &\quad (2014, 2015, 7), (2015, 2016, 5)] \\ sm &= [(1991, 1992, 1), (1995, 1996, 2), (1998, 1999, 2), (2001, 2002, 1), (2003, 2004, 1), \\ &\quad (2005, 2006, 6), (2006, 2007, 10), (2007, 2009, 4), (2009, 2010, 7), (2010, 2011, 16), \\ &\quad (2011, 2012, 14), (2012, 2013, 9), (2013, 2014, 34), (2014, 2015, 19), (2015, 2016, 17), \\ &\quad (2016, 2017, 1)] \end{aligned}$$

and are presented on the next slide.



Example: Journals Selfcitations

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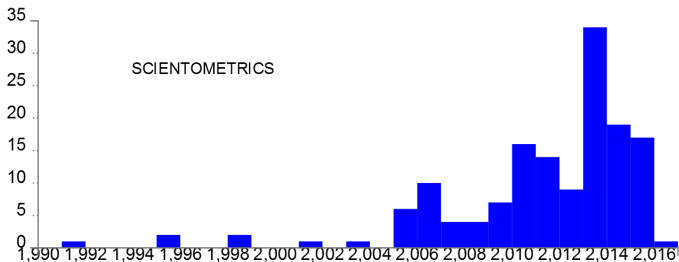
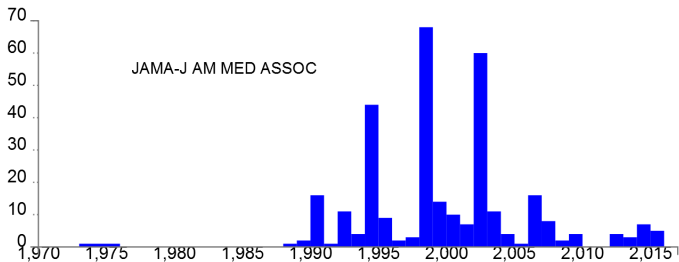
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Example: Temporal citations between journals

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The largest number of citations are from journals BMJ Open (142) and Scientometrics (108) to the unknown journal *****, followed by $bj = jcj[\text{BRIT MED J, JAMA-J AM MED ASSOC}]$ and $pj = jcj[\text{PLOS ONE, JAMA-J AM MED ASSOC}]$ with totals 96 and 91.

$$\begin{aligned}bj &= [(1994, 1996, 8), (1996, 1997, 4), (1997, 1998, 6), (1998, 1999, 2), (1999, 2000, 24), \\&\quad (2000, 2001, 1), (2001, 2002, 3), (2002, 2003, 6), (2003, 2004, 11), (2004, 2005, 6), \\&\quad (2005, 2006, 1), (2008, 2009, 2), (2009, 2010, 1), (2010, 2011, 4), (2011, 2012, 1), \\&\quad (2012, 2013, 8)] \\pj &= [(2007, 2008, 8), (2008, 2009, 13), (2009, 2010, 7), (2010, 2011, 12), (2011, 2012, 14), \\&\quad (2012, 2013, 11), (2013, 2014, 4), (2014, 2015, 11), (2015, 2016, 6), (2016, 2017, 5)]\end{aligned}$$



Example: Temporal citations between journals

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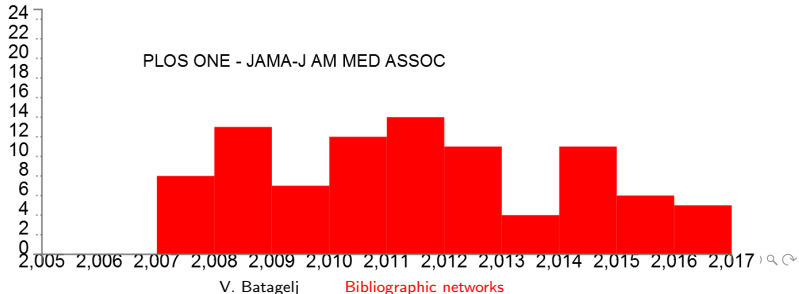
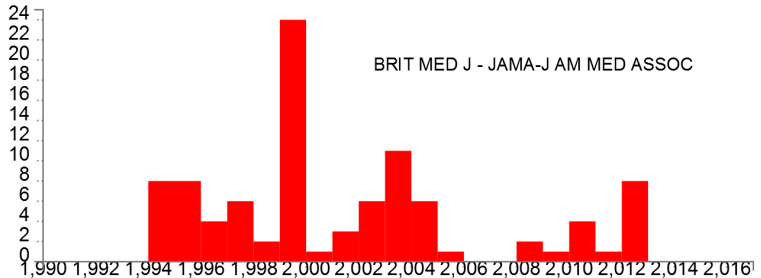
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Example: Temporal citations between journals

In the peer review data the journal JAMA is the most prominent. To get the temporal quantity describing citations of others to JAMA we compute $jci = iS(\mathbf{JCJ}, \text{JAMA-J AM MED ASSOC})$:

$$jci = [(1979, 1980, 2), (1982, 1983, 1), (1986, 1987, 5), (1987, 1988, 2), (1988, 1989, 1), (1990, 1991, 10), (1991, 1992, 22), (1992, 1993, 24), (1993, 1994, 36), (1994, 1995, 43), (1995, 1996, 108), (1996, 1998, 67), (1998, 1999, 161), (1999, 2000, 136), (2000, 2001, 106), (2001, 2002, 156), (2002, 2003, 145), (2003, 2004, 170), (2004, 2005, 98), (2005, 2006, 150), (2006, 2007, 229), (2007, 2008, 261), (2008, 2009, 185), (2009, 2010, 276), (2010, 2011, 239), (2011, 2012, 208), (2012, 2013, 220), (2013, 2014, 248), (2014, 2015, 191), (2015, 2016, 251), (2016, 2017, 43)]$$

See the figure on the next slide.



Example: Temporal citations between journals

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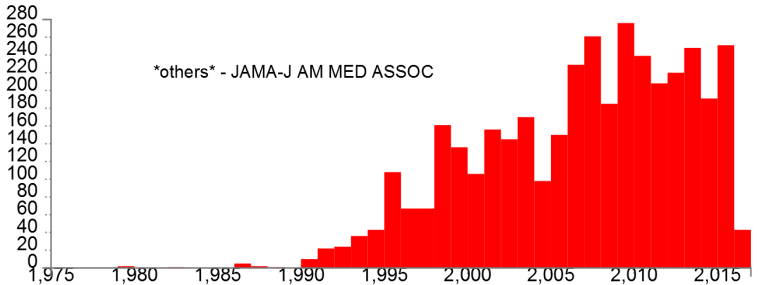
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Similarly we get the temporal network describing citations between authors

$$\mathbf{ACA} = \mathbf{WAI}^T \cdot \mathbf{CiI} \cdot \mathbf{WAc}$$

The weight of the element aca_{ab} is equal to the number of citations per year from works coauthored by author a to works coauthored by author b .



ESNA Pajek

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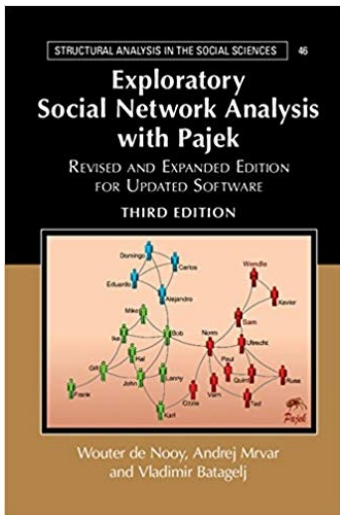
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An introduction to social network analysis with Pajek is available in the book **ESNA 3** (de Nooy, Mrvar, Batagelj, CUP 2005, 2011, 2018).

ESNA in Japanese was published by Tokyo Denki University Press in 2010; and in Chinese by Beijing World Publishing in November 2012.

Pajek – program for analysis and visualization of large networks is freely available, for noncommercial use, at its web site.

<http://mrvar.fdv.uni-lj.si/pajek/>



Understanding large networks

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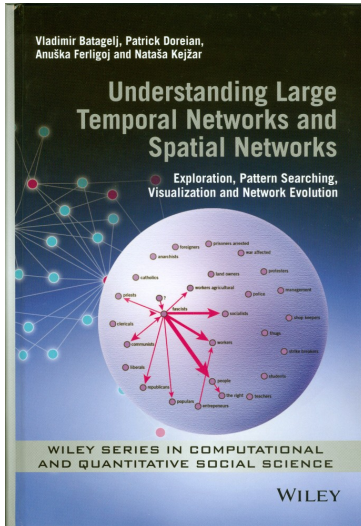
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This lecture is closely related to chapters 2 and 3 in the book:

Vladimir Batagelj, Patrick Doreian, Anuška Ferligoj and Nataša Kejžar: Understanding Large Temporal Networks and Spatial Networks: Exploration, Pattern Searching, Visualization and Network Evolution. Wiley Series in Computational and Quantitative Social Science. **Wiley**, October 2014.



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