On projections of two-mode networks

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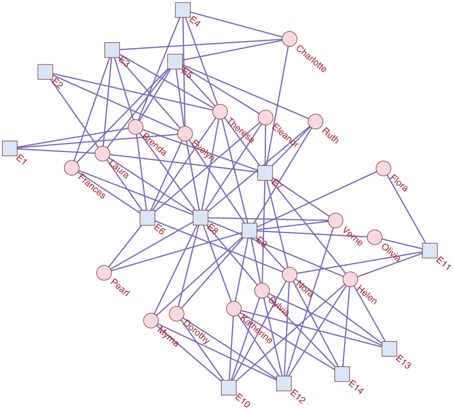
Outline



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Current version of slides (May 23, 2022 at 04 : 07): slides PDF

https://github.com/bavla/NormNet/tree/main/docs

Two-mode networks

A simple directed *two-mode* (affiliation) network $\mathcal{UV} = ((\mathcal{U}, \mathcal{V}), \mathcal{L}, w)$ links the set of *nodes* $\mathcal{U}$ (primary mode) to the set of nodes $\mathcal{V}$ (secondary mode) with the arcs from the set of *links* $\mathcal{L}$. The mapping $w: \mathcal{L} \to W$ assigns to each arc (u, v) its *weight* $w(u, v) \in W$. The network $\mathcal{UV}$ can be represented with the corresponding matrix $\mathbf{UV} = [UV[u, v]]_{u \in \mathcal{U}, v \in \mathcal{V}}$

Names of Participants of Group I	Code Numbers and Dates of Social Events Reported in *Old City Herald*													
	(1) 6/27	(2) 3/2	(3) 4/12	(4) 9/26	(5) 2/25	(6) 5/19	(7) 3/15	(8) 9/16	(9) 4/8	(10) 6/10	(11) 2/23	(12) 4/7	(13) 11/21	(14) 8/3
1. Mrs. Evelyn Jefferson	×	×	×	×	×	×	....	×	×	....	....	....	....	....
2. Miss Laura Mandeville	×	×	×	....	×	×	×	×	....	....	....	....	....	....
3. Miss Theresa Anderson	....	×	×	×	×	×	×	×	×	....	....	....	....	....
4. Miss Brenda Rogers	×	....	×	×	×	×	×	×	....	....	....	....	....	....
5. Miss Charlotte McDowd	....	....	×	×	×	....	×	....	....	....	....	....	....	....
6. Miss Frances Anderson	....	....	×	....	×	×	....	×	....	....	....	....	....	....
7. Miss Eleanor Nye	....	....	....	....	×	×	×	×	....	....	....	....	....	....
8. Miss Pearl Oglethorpe	....	....	....	....	....	×	....	×	×	....	....	....	....	....
9. Miss Ruth DeSand	....	....	....	....	×	....	×	×	×	....	....	....	....	....
10. Miss Verne Sanderson	....	....	....	....	....	....	×	×	×	....	....	×	....	....
11. Miss Myra Liddell	....	....	....	....	....	....	....	×	×	×	....	×	....	....
12. Miss Katherine Rogers	....	....	....	....	....	....	....	×	×	×	....	×	×	×
13. Mrs. Sylvia Avondale	....	....	....	....	....	....	×	×	×	×	....	×	×	×
14. Mrs. Nora Fayette	....	....	....	....	....	×	×	....	×	×	×	×	×	×
15. Mrs. Helen Lloyd	....	....	....	....	....	....	×	×	....	×	×	×	....	....
16. Mrs. Dorothy Murchison	....	....	....	....	....	....	....	×	×	....	....	....	....	....
17. Mrs. Olivia Carleton	....	....	....	....	....	....	....	....	×	....	×	....	....	....
18. Mrs. Flora Price	....	....	....	....	....	....	....	....	×	....	×	....	....	....

Davis: Southern women, 1941 [7]

Two-mode networks data

Many two-mode networks are binary – the weight w has a constant value 1.

In general, the weight can be measured in different measurement scales.

A *signed two-mode network* is described by an affiliation matrix **UV** with rows corresponding to *respondents*/actors U (primary mode) and columns corresponding to different *cases*/choices V (secondary mode).

The matrix entry $UV[u, v]$ contains the decision/opinion of the respondent u about the case v and can take one of the values: p (Yes, Positive, Support), n (No, Negative, Oppose), a (Ambivalent, No opinion), r (Not valid), and z (NA, Not available, Absent). Examples: Bradley Robinson US Senate Voting Records for 1990-2016 / Sessions 101-114, German bundestag, Parlameter, Votes of French deputies (XVth legislature).

. . . Two-mode networks data

Two-mode networks with weights measured on a ratio scale. Shooping on customers $\times$ (types of) articles, weight is the money spent in a month.

Two-mode networks with weights measured on an interval scale.

Two-mode networks with weights measured on an ordinal scale. World City Networks.

Two-mode networks with weights measured on a nominal scale. Travels on persons $\times$ countries, visited $\in$ { tourist, sport, culture, education, science, business, diplomatic, . . . }

Bavla/Two-mode datasets

Two-mode network matrix

The network *matrix* of a two-mode network $\mathcal{UV}$ is defined as

$$UV[u, v] = \begin{cases} w(u, v) & (u, v) \in L \\ \square & \text{otherwise} \end{cases}$$

We represent number 0 with two symbols, 0 (weight 0) and $\square$ (no link) where $\square = 0$ with rules $\square + a = a$ and $\square \cdot a = \square$.

The set $N_{UV}(u)$ of *(out-)neighbors* (successors) of the node $u \in U$

$$N_{UV}(u) = \{v \in V : (u, v) \in L\}$$

and the set $N(v)$ of *in-neighbors* (predecessors) of the node $v \in V$

$$N_{UV}(v) = \{u \in U : (u, v) \in L\}$$

In the following, we will often, when it is obvious from the context, omit the subscript. For example, $N_{UV}(u) = N(u)$.

Some notions

The function $\delta : \{\mathbf{false}, \mathbf{true}\} \to \{0, 1\}$ is determined by $\delta(\mathbf{false}) = 0$ and $\delta(\mathbf{true}) = 1$. We will also use some additional functions:

out/in-degree

$\mathrm{od}_{UV}(u) = \sum_{v \in V} \delta((u,v) \in L) = |N(u)|$ and

$\mathrm{id}_{UV}(v) = \sum_{u \in U} \delta((u,v) \in L) = |N(v)|$

weighted out/in-degree (row/column sums)

$\mathrm{wod}_{UV}(u) = \sum_{v \in V} UV[u,v], \quad \mathrm{wid}_{UV}(v) = \sum_{u \in U} UV[u,v]$ and

$\mathrm{wod}_{UV}(u/t) = \sum_{v \in N(u) \cap N(t)} UV[u,v]$.

It holds $N(u) \cap N(t) \neq \emptyset \Rightarrow \mathrm{wod}_{UV}(u/t) \neq \square$.

We denote $U_{[d]} = \{u \in U : \mathrm{od}(u) \geq d\}$ and

$\mathcal{UV}_{[d]} = ((U_{[d]}, V), L(U_{[d]}), w|U_{[d]})$.

$\hat{U} = \{u \in U : \mathrm{wod}(u) \neq 0\}$

The *total weight* of links in the network $\mathcal{N} = (V, L, w)$

$$T(\mathbf{N}) = \sum_{(u,v) \in L} w(u,v) = \sum_{u,v} N[u,v] = \sum_{u} wod_N(u) = \sum_{v} wid_N(v)$$

Approaches

There are three main approaches to the analysis of two-mode networks:

1. treat the two-mode network as an ordinary one-mode network (degrees, components, etc.) considering a bipartition to sets U and V.
2. apply special methods developed for the analysis of two-mode networks (two-mode hubs and authorities, two-mode cores, 4-ring weights, blockmodeling, etc.).
3. transform (project) the two-mode network to a corresponding one-mode (weighted) network and use the usual methods (link cuts, cores, islands, skeletons, clustering, etc.) to analyze it.

In this paper, we will discuss the last option.

Projections

$\mathcal{UV} = ((U, V), L, w)$ – two-mode network with a network matrix $\mathbf{UV}$.

$p : \mathbf{UV} \to \mathbf{VV}$ *projection*, $\mathbf{VV} = [p(v, z)]$; and $\mathcal{VV}$ the corresponding (ordinary, one-mode) network

1. undirected projection: $p(v, z) = p(z, v)$, resemblance
   - similarity: $p(v, z) \leq \min(p(v, v), p(z, z))$
   - dissimilarity: $p(v, z) \geq \max(p(v, v), p(z, z))$
2. directed projection: $\exists v, z : p(v, z) \neq p(z, v)$ ([5], [9])

Multiplication of networks

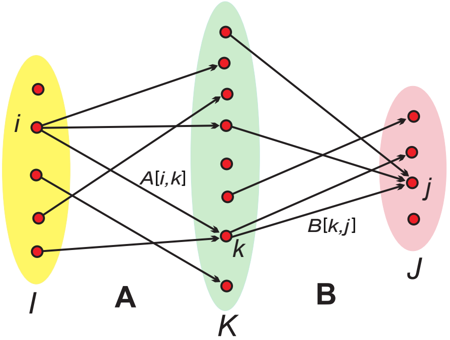
The *product* $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ of two compatible matrices $\mathbf{A}_{I \times K}$ and $\mathbf{B}_{K \times J}$ is defined in the standard way

$$C[i,j] = \sum_{k \in K} A[i,k] \cdot B[k,j]$$

(it can be extended to semirings !!!)

The product of two compatible networks $\mathcal{N}_A = ((I,K), L_A, a)$ and $\mathcal{N}_B = ((K,J), L_B, b)$ is the network $\mathcal{N}_C = ((I,J), L_C, c)$ where $L_C = \{(i,j) : c[i,j] \neq \square\}$ and the weight c is determined by the matrix $\mathbf{C}$, $c(i,j) = C[i,j]$.

Multiplication of networks



In binary networks $\mathcal{N}_A$ and $\mathcal{N}_B$, the value of $C[i,j]$ of $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ counts the number of ways we can go from the node $i \in I$ to the node $j \in J$ passing through K, $C[i,j] = |N_A(i) \cap N_B(j)|$.

$$C[i,j] = \sum_{k \in N_A(i) \cap N_B(j)} A[i,k] \cdot B[k,j]$$

Standard projections

A standard approach to the analysis of a two-mode network $\mathcal{UV}$ is to transform it into the corresponding one-mode networks determined by:

row projection to U: $\mathbf{UU} = \text{row}(\mathbf{UV}) = \mathbf{UV} \cdot \mathbf{UV}^T$, or
column projection to V: $\mathbf{VV} = \text{col}(\mathbf{UV}) = \mathbf{UV}^T \cdot \mathbf{UV}$

and analyze the obtained weighted network.

$$\text{col}(\mathbf{UV}) = \mathbf{UV}^T \cdot \mathbf{UV} = \mathbf{UV}^T \cdot (\mathbf{UV}^T)^T = \text{row}(\mathbf{UV}^T)$$

$$\text{row}(\mathbf{UV}) = \text{col}(\mathbf{UV}^T)$$

We will limit our discussion to column projections.

Binary projection of an authorship network

To support our intuition, let us take an authorship network matrix **WA** linking a work $w \in W$ to its authors from A.

Its column projection $\mathbf{Co} = \mathrm{col}(\mathbf{WA}) = \mathbf{WA}^T \cdot \mathbf{WA}$ has entries

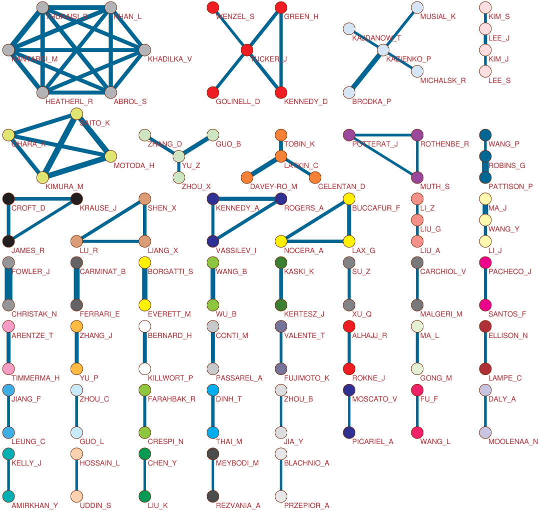
$Co[a, b] = |N(a) \cap N(b)| = Co[b, a] = \#$ of works that authors a and b co-authored

Co is a *co-appearance* / co-authorship matrix.

$Co[a, a] = |N(a)| = \deg(a) = \#$ of works (co-)authored by the author a.

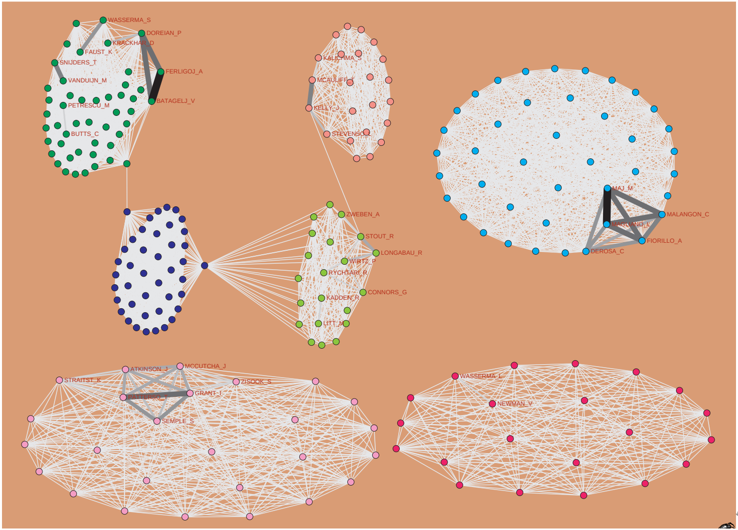
Example: SNA18[10] /edge cut at level 15

WA: $|W| = 70792$, $|A| = 93011$



Cores of orders 20–47 in **Co**(SN5)

Network SN5 (2008): for "social network*" + most frequent references + around 100 social networkers; $|W| = 193376$, $|C| = 7950$, $|A| = 75930$, $|J| = 14651$, $|K| = 29267$



p_S-core at level 20 of **Co**(SN5)

2-mode network projections

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Two-mode networks

Projections

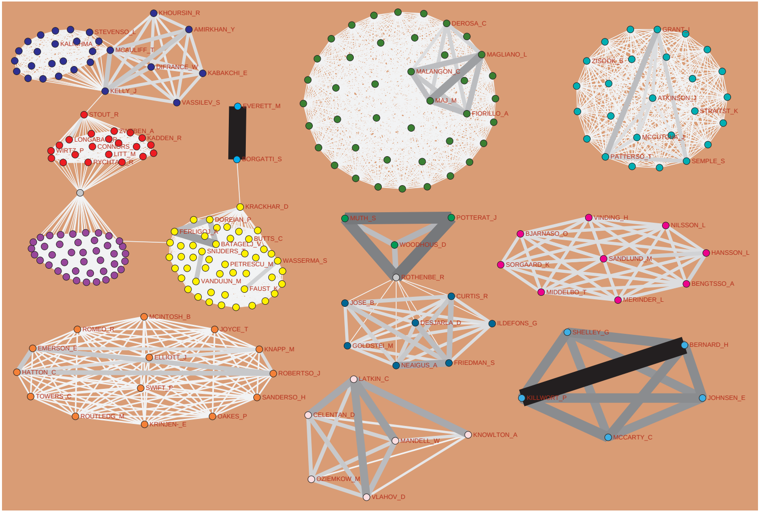
Outer product decomposition

Fractional approach

Inner product

Additional options

References



Outer product decomposition

For vectors $x = [x_1, x_2, \ldots, x_n]$ and $y = [y_1, y_2, \ldots, y_m]$ their *outer product* $x \circ y$ is defined as a matrix

$$x \circ y = [x_i \cdot y_j]_{n \times m}$$

then we can express the product $\mathbf{C}$ of two compatible matrices $\mathbf{A}$ and $\mathbf{B}$ as the *outer product decomposition*

$$\mathbf{C} = \mathbf{A} \cdot \mathbf{B} = \sum_k \mathbf{H}_k \quad \text{where} \quad \mathbf{H}_k = \mathbf{A}[\cdot, k] \circ \mathbf{B}[k, \cdot],$$

$\mathbf{A}[\cdot, k]$ is the k-th column of matrix $\mathbf{A}$, and $\mathbf{B}[k, \cdot]$ is the k-th row of matrix $\mathbf{B}$.

On the basis of outer product decomposition we have

$$T(\mathbf{C}) = T(\sum_k \mathbf{H}_k) = \sum_k T(\mathbf{H}_k) \ \text{ and } \ T(\mathbf{H}_k) = \mathrm{wid}_A(k) \cdot \mathrm{wod}_B(k)$$

Structure of projection

Therefore for **Co** we have $\mathbf{H}_w = \mathbf{K}_{N(w)}$ and

$$T(\mathbf{Co}) = \sum_{w \in W} \deg(w)^2$$

In words

1. a derived network is a sum of complete subgraphs;
2. the contribution of a node $w \in W$ to the total T is $\deg(w)^2$.

This means that the nodes with a large degree in W are over-represented in the projection.

A real-life network $\mathcal{WA}$ can contain nodes $w \in W$ of degree 0 (works with no author) and 1 (single author works). Works with no author do not contribute to the matrix **Co**. Single author works contribute only to the author's diagonal entry in the matrix **Co**.

Fractional approach / "stochastic" normalization

$$n(\mathbf{UV}) = [n(UV)[u,v]]$$

$$n(UV)[u,v] = \begin{cases} \dfrac{UV[u,v]}{\mathrm{wod}_{UV}(u)} & u \in \hat{U} \\ 0 & u \notin \hat{U} \end{cases}$$

$$T(n(\mathbf{UV})) = \sum_{u \in U} \mathrm{wod}_{n(UV)}(u) = \sum_{u \in \hat{U}} 1 = |\hat{U}|$$

Interpretation: probabilistic co-linkage.

Fractional co-appearance: $\mathbf{Cn} = n(\mathbf{UV})^T \cdot n(\mathbf{UV})$,
$Cn[v,z] = \sum_{u \in \hat{U}} \frac{UV[u,v] \cdot UV[u,z]}{\mathrm{wod}(u)^2}$
$Cn[v,z] = Cn[z,v], \quad T(u) = T(\mathbf{H}_u) = \mathrm{wod}_{n(UV)}(u)^2$

$$T(\mathbf{Cn}) = \sum_{u \in U} \mathrm{wod}_{n(UV)}(u)^2 = \sum_{u \in \hat{U}} 1 = |\hat{U}|$$

Newman's normalization

Mark Newman proposed an alternative normalization that considers only co-authorship between different authors – single-author works and self co-authorship are excluded.

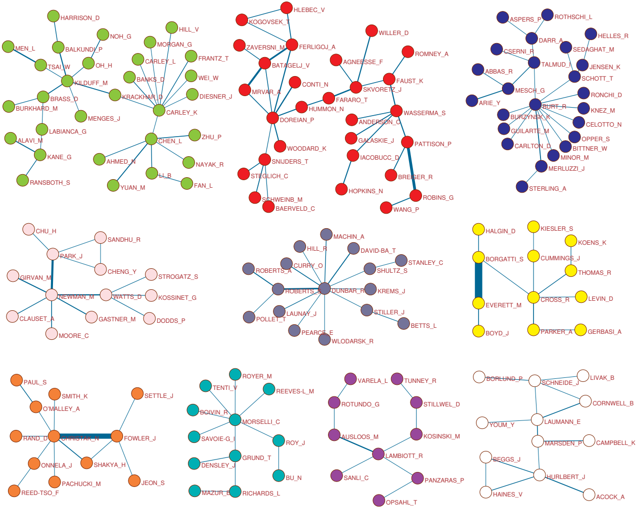
The *Newman's normalized* 2-mode network $n'(\mathbf{WA})$ has weights

$$n'(WA)[w,a] = \begin{cases} \dfrac{WA[w,a]}{\deg(w)-1} & w \in W_{[2]} \\ 0 & \text{otherwise} \end{cases}$$

Newman's projection $\mathbf{Cn}' = n(\mathbf{WA})^T \cdot n'(\mathbf{WA})$.
Network: symmetrize with sum and remove loops.

Example: SNA18[10] /selected Newman islands 10-30

$|W| = 70792$, $|A| = 93011$



Total weight preserving normalization

$$s(UV)[u,v] = \begin{cases} \dfrac{UV[u,v]}{\sqrt{\mathrm{wod}_{UV}(u)}} & u \in \hat{U} \\ 0 & u \notin \hat{U} \end{cases}$$

$$\mathrm{wod}_{n(UV)}(u) = \begin{cases} 1 & u \in \hat{U} \\ 0 & u \notin \hat{U} \end{cases}, \quad wod_{s(UV)}(u) = \begin{cases} \sqrt{\mathrm{wod}_{UV}(u)} & u \in \hat{U} \\ 0 & u \notin \hat{U} \end{cases}$$

Total weight preserving projection: $\mathbf{Cs} = s(\mathbf{UV})^T \cdot s(\mathbf{UV})$

$$Cs[v,z] = \sum_{u \in \hat{U}} \frac{UV[u,v] \cdot UV[u,z]}{\mathrm{wod}(u)}$$

$$Cs[v,z] = Cs[z,v], \; T(u) = \mathrm{wod}_{s(UV)}(u)^2 = \mathrm{wod}_{UV}(u)$$

$$T(\mathbf{Cs}) = \sum_{u \in U} \mathrm{wod}_{s(UV)}(u)^2 = \sum_{u \in \hat{U}} \mathrm{wod}_{UV}(u) = T(\mathbf{UV})$$

Embedding primary node values

Node values $c : U \to \mathbb{R}_0^+$ – impact factor, number of citations,...

$$x(UV)[u,v] = \begin{cases} \dfrac{\sqrt{c(u)}}{\mathrm{wod}_{UV}(u)} UV[u,v] & u \in \hat{U} \\ 0 & u \notin \hat{U} \end{cases}$$

$$\mathrm{wod}_{x(UV)}(u) = \begin{cases} \sqrt{c(u)} & u \in \hat{U} \\ 0 & u \notin \hat{U} \end{cases}$$

Embedded primary node values: $\mathbf{Cx} = x(\mathbf{UV})^T \cdot x(\mathbf{UV})$

$$Cx[v,z] = \sum_{u \in \hat{U}} \frac{c(u)}{\mathrm{wod}(u)^2} UV[u,v] \cdot UV[u,z], \quad Cx[v,z] = Cx[z,v]$$

$$T(u) = \sum_{v \in V} \sum_{z \in V} Cx[v,z] = \frac{c(u)}{\mathrm{wod}(u)^2} \sum_{v \in V} UV[u,v] \cdot \sum_{z \in V} UV[u,z] = c(u)$$

$$T(\mathbf{Cx}) = \sum_{u \in U} \mathrm{wod}_{x(UV)}(u)^2 = \sum_{u \in \hat{U}} c(u)$$

Binarization and left and right (fractional) contribution

Binarization: $b(\mathbf{UV})$: $b(UV)[u,v] = \delta(UV[u,v] \neq \square)$
Left contribution: $L(\mathbf{UV}) = \mathbf{UV} \cdot b(\mathbf{UV})^T$

$$L(UV)[u,t] = \sum_{v \in V} UV[u,v] \cdot b(UV)[t,v] = \mathrm{wod}_{UV}(u/t)$$

Left fractional contribution: $\ell(\mathbf{UV}) = n(\mathbf{UV}) \cdot b(\mathbf{UV})^T$

$$\ell(UV)[u,t] = \frac{1}{\mathrm{wod}_{UV}(u)} \sum_{v \in V} UV[u,v] \cdot b(UV)[t,v] = \frac{\mathrm{wod}_{UV}(u/t)}{\mathrm{wod}_{UV}(u)} \leq 1$$

Right fractional contribution: $r(\mathbf{UV}) = b(\mathbf{UV}) \cdot n(\mathbf{UV})^T$

$$r(UV)[u,t] = \frac{1}{\mathrm{wod}_{UV}(t)} \sum_{v \in V} b(UV)[u,v] \cdot UV[t,v] = \frac{\mathrm{wod}_{UV}(t/u)}{\mathrm{wod}_{UV}(t)}$$

$$r(UV)[u,t] = \ell(UV)[t,u]$$

Mean value similarities

$$
\begin{aligned}
VV_X[u,t] &= meanX(\ell(UV)[u,t], r(UV)[u,t]) \\
&= meanX(\ell(UV)[u,t], \ell(UV)[t,u])
\end{aligned}
$$

$VV_A[u,t] = \frac{1}{2}(\ell(UV)[u,t] + \ell(UV)[t,u])$ – arithmetic mean
$VV_m[u,t] = \min(\ell(UV)[u,t], \ell(UV)[t,u])$ – minimum
$VV_M[u,t] = \max(\ell(UV)[u,t], \ell(UV)[t,u])$ – maximum
$VV_G[u,t] = \sqrt{\ell(UV)[u,t] \cdot \ell(UV)[t,u]}$ – geometric, Salton
$VV_H[u,t] = 2(\ell(UV)[u,t]^{-1} + \ell(UV)[t,u]^{-1})^{-1}$ – harmonic, Dice
$VV_J[u,t] = (\ell(UV)[u,t]^{-1} + \ell(UV)[t,u]^{-1} - 1)^{-1}$ – Jaccard

Note: $\ell(\mathbf{UV})$ can be computed from $L(\mathbf{UV})$
$L(UV)[u,u] = \mathrm{wod}_{UV}(u/u) = \mathrm{wod}_{UV}(u)$.

It holds: $VV_X[u,t] = VV_X[t,u]$, $VV_X[u,t] \in [0,1]$ and $VV_J[u,t] \leq VV_m[u,t] \leq VV_H[u,t] \leq VV_G[u,t] \leq VV_A[u,t] \leq VV_M[u,t]$.

Inner product

The *inner product* of vectors $x, y \in \mathbb{R}^n$ is defined as

$$\langle x, y \rangle = \sum_{i=1}^{n} x_i \cdot y_i$$

Using the inner product we can write $C[i,j] = \langle A^T[i,\cdot], B[\cdot,j] \rangle$.

The following four properties hold for all $x, y, z \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$:

1. $\langle x, x \rangle \geq 0$ and $\langle x, x \rangle = 0$ if and only if $x = 0$,
2. $\langle x, y + z \rangle = \langle x, y \rangle + \langle x, z \rangle$,
3. $\langle x, \alpha y \rangle = \alpha \langle x, y \rangle$,
4. $\langle x, y \rangle = \langle y, x \rangle$.

An inner product $\langle ., . \rangle$ induces the *norm* of x

$$\|x\| = \sqrt{\langle x, x \rangle}$$

Inner product and measurement scales

1. binary $x, y \in \{0, 1\}^n$: $\langle x, y\rangle = |X \cap Y|$, where $X = \{i : x_i = 1\}$
2. integer $x, y \in \mathbb{N}^n$: number of paths – basic rules of combinatorics
3. positive or nonnegative real numbers – similarity measure $x \leq y \Rightarrow \langle x, z\rangle \leq \langle y, z\rangle$
4. positive and negative real numbers – similarity measure
5. finite ordinal set W, ord : $W \to 1..|W|$, $UV[u, v] = \text{ord}(w(u, v))$ for $(u, v) \in L$
6. finite nominal set W, $\langle x, y\rangle = \sum_{i=1}^{n} \delta(x_i = y_i)$

Some inner product inequalities

Cauchy-Schwarz inequality

$$|\langle x, y\rangle| \leq \|x\| \cdot \|y\|$$

Salton index, cosine

$$S(x, y) = \frac{\langle x, y\rangle}{\|x\| \cdot \|y\|} \in [-1, 1]$$

$\mathrm{incr}(x)$ = vector of elements of vector x ordered in increasing order
$\mathrm{decr}(x)$ = vector of elements of vector x ordered in decreasing order

$$m(x, y) = \langle \mathrm{incr}(x), \mathrm{decr}(y)\rangle \leq \langle x, y\rangle \leq \langle \mathrm{incr}(x), \mathrm{incr}(y)\rangle = M(x, y)$$

$$N(x, y) = \frac{\langle x, y\rangle - m(x, y)}{M(x, y) - m(x, y)} \in [0, 1]$$

$N(x, x) = 1,\ N(x, 0) = 1,\ N(x, y) = N(y, x),\ N(\alpha x, y) = N(x, y),$
$\alpha > 0,\ N(e, x) = 1$

Salton and ordering

From the column projection matrix $\mathbf{VV} = \mathrm{col}(\mathbf{VV})$ we can compute the corresponding Salton similarity matrix $S(\mathbf{UV})$

$$S(\mathbf{UV})[v,z] = \frac{VV[v,z]}{\sqrt{VV[v,v] \cdot VV[z,z]}}$$

For computing the ordering similarity matrix $N(\mathbf{UV})$ we additionally need matrices $m(\mathbf{UV})$ and $M(\mathbf{UV})$

$$m(\mathbf{UV})[v,z] = m(UV[\cdot,v], UV[\cdot,z])$$

$$M(\mathbf{UV})[v,z] = M(UV[\cdot,v], UV[\cdot,z])$$

Then

$$N(\mathbf{UV})[v,z] = \frac{VV[v,z] - m(UV)[v,z]}{M(UV)[v,z] - m(UV)[v,z]}$$

(Dis)similarity based projections

$$VV[v,z] = r(UV[\cdot,v], UV[\cdot,z])$$

where r is a selected resemblance ((dis)similarity) measure compatible with the weight measurement scale [8].
Often the matrix **UV** is first normalized in an appropriate way.

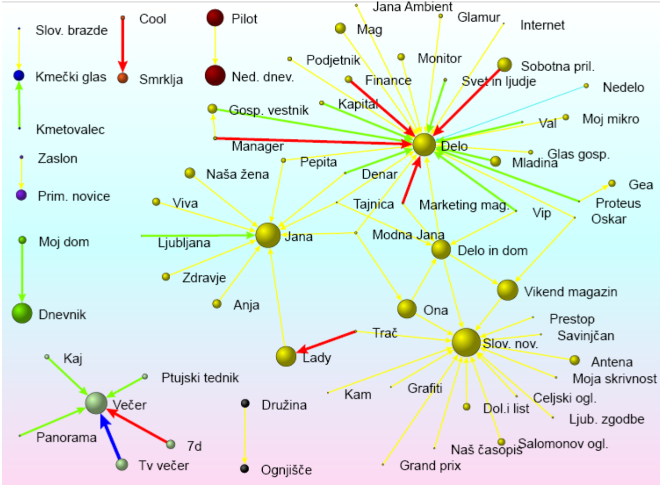
Asymmetric projections

The column projection matrix **VV** can be further transformed into an asymmetric matrix/network For example [5, 4, p. 94]

$$\mathrm{MinDir}[v,z] = \begin{cases} \frac{VV[v,z]}{VV[v,v]} & VV[v,v] \leq VV[z,z] \\ \square & \textit{otherwise} \end{cases}$$

$$\mathrm{MaxDir}[v,z] = \begin{cases} \frac{VV[v,z]}{VV[z,z]} & VV[v,v] \leq VV[z,z] \\ \square & \textit{otherwise} \end{cases}$$

MinDir of Slovenian journals 2000



Over 100000 people were asked in the years 1999 and 2000 about the journals they read. They mentioned 124 different journals. (source Cati)

Acknowledgments

The computational work reported on in this presentation was performed using R and Pajek. The code and data are available at https://github.com/bavla/NormNet/

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