



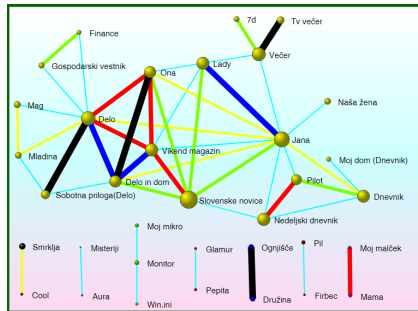
Network weight compatibility normalizations 2

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1311. Sredin seminar
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- 2 Approaches
- 3 Binary projections
- 4 Binary similarity measures
- 5 Fractional approach
- 6 Some other weight normalizations
- 7 Blockmodeling ex-Soviet networks
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Current version of slides (January 19, 2022 at 22:09): [slides PDF](#)

<https://github.com/bavla/NormNet/blob/main/docs/>

Coauthorship between ex-Soviet countries

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Nataliya Matveeva collected from WoS a data set on coauthorships between researchers from ex-Soviet countries for the years 1993, 1998, 2003, 2008, 2013, and 2018. Here is the matrix for the year 2018:

		Aze	Arm	Bel	Est	Geo	Kaz	Kyr	Lat	Lit	Mol	Rus	Tad	Tur	Ukr	Uzb
Motivation	Azerbaijan	255	148	123	4	121	13	3	3	2	7	216	1	0	47	3
Approaches	Armenia	148	245	256	157	274	12	11	146	146	5	375	1	0	199	26
	Belarus	123	256	269	167	269	19	1	158	191	10	642	2	0	211	25
Binary projections	Estonia	4	157	167	587	163	16	13	255	238	6	290	0	0	188	23
	Georgia	121	274	269	163	114	12	7	150	151	10	302	2	0	175	29
Binary similarity measures	Kazakhstan	13	12	19	16	12	357	30	5	10	3	277	5	1	41	15
	Kyrgyzstan	3	11	1	13	7	30	17	3	6	2	40	3	1	13	6
	Latvia	3	146	158	255	150	5	3	242	342	4	302	0	0	289	23
	Lithuania	2	146	191	238	151	10	6	342	1143	5	329	0	0	328	23
Fractional approach	Moldova	7	5	10	6	10	3	2	4	5	37	44	2	0	11	3
	Russia	216	375	642	290	302	277	40	302	329	44	23928	21	0	867	65
	Tadjikistan	1	1	2	0	2	5	3	0	0	2	21	13	0	2	4
	Turkmenistan	0	0	0	0	0	1	1	0	0	0	0	0	1	0	1
Some other weight normalizations	Ukraine	47	199	211	188	175	41	13	289	328	11	867	2	0	1649	32
	Uzbekistan	3	26	25	23	29	15	6	23	23	3	65	4	1	32	72

The matrix contains also the data about coauthorships inside each country. For example, in 2018 for Russia there were 23928 internal coauthorships and only 3770 coauthorships with other countries; compare this with Belarus, inter = 269, other = 2074. This makes Russia a big outlier.



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Because we are interested in strict coauthorship we decided to remove the diagonal values / loops. We denote the corresponding matrix \mathbf{C} . Nonexisting links are represented with the value 0. The matrix \mathbf{C} is symmetric.

The weights in inter-country coauthorship networks depend on the size of the countries and other factors. They span a wide range of values. To compare the countries we have to apply some normalization.



"History"

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- Pat – citations between journals; bistochastic normalization; Preddvor 199?
- Kropivnik: Slovenian political parties 1994 [PDF](#)
- Slovenian journals 2000
- Reuters Terror News 2001 [PDF](#)
- bibliographic networks 2013 [arXiv](#)
- ex-Soviet 2021

Slovenian political parties 1994 / reordered

Compatibility normalizations

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Motivation			1	3	6	8	9	2	4	5	7	10
Approaches	SKD	1	0	114	94	176	117	-215	-89	-77	-170	-210
	SDSS	3	114	0	138	177	180	-217	-203	-80	-109	-174
Binary projections	ZS	6	94	138	0	140	116	-150	-142	-188	-97	-106
	SLS	8	176	177	140	0	235	-253	-241	-120	-184	-132
Binary similarity measures	SPS-SNS	9	117	180	116	235	0	-230	-254	-160	-191	-164
Fractional approach	ZLSD	2	-215	-217	-150	-253	-230	0	134	77	57	49
	LDS	4	-89	-203	-142	-241	-254	134	0	157	173	23
	ZSESS	5	-77	-80	-188	-120	-160	77	157	0	170	-9
Some other weight	DS	7	-170	-109	-97	-184	-191	57	173	170	0	-6
	SNS	10	-210	-174	-106	-132	-164	49	23	-9	-6	0

S. Kropivnik, A. Mrvar: An Analysis of the Slovene Parliamentary Parties Network. in Developments in data analysis, MZ 12, FDV, Ljubljana, 1996, p. 209-216.

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Weighted network

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A network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, w)$, $w : \mathcal{L} \rightarrow \mathbb{R}$.

Using link cuts, islands, cores, skeletons (spanning tree, Pathfinder, k-neighbors), community detection, hubs and authorities, etc. we can identify the most active subnetworks. We can also apply clustering and blockmodeling methods.



World Trade 1999 spring embedder

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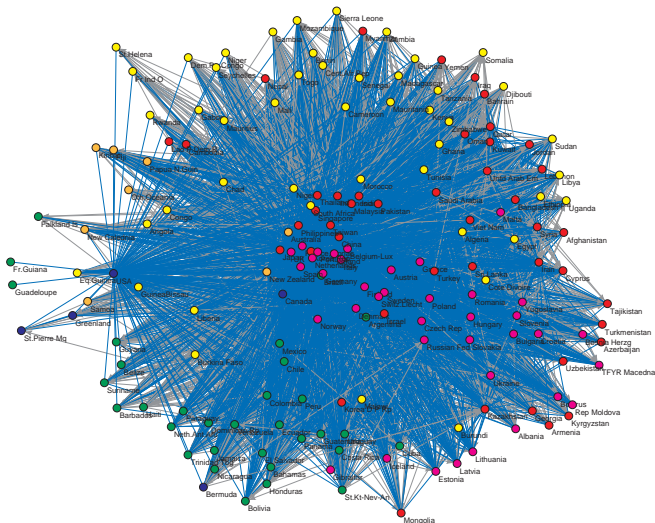
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World Trade 1999 Pathfinder skeleton

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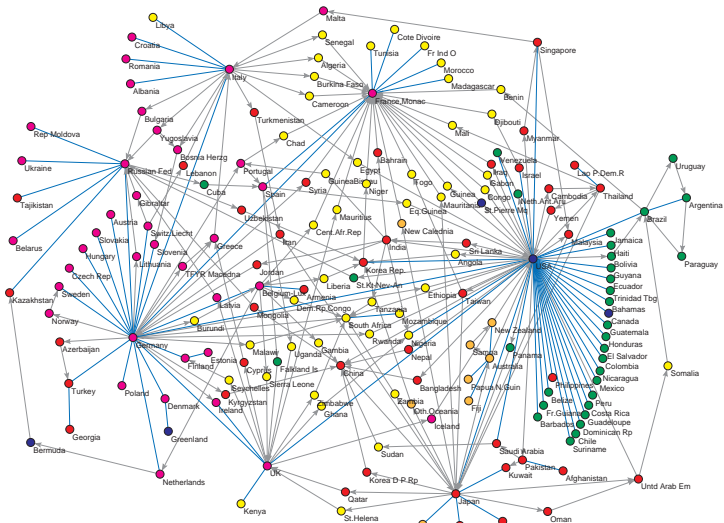
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Normalizations

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Often some nodes/links prevail. How to make weights comparable? [5, 7, p. 94] We assume $w : \mathcal{L} \rightarrow \mathbb{R}_0^+$

$$\text{Geo}_{uv} = \frac{w_{uv}}{\sqrt{w_{uu} w_{vv}}}$$

$$\text{GeoDeg}_{uv} = \frac{w_{uv}}{\sqrt{\deg_u \deg_v}}$$

$$\text{Input}_{uv} = \frac{w_{uv}}{w_{vv}}$$

$$\text{Output}_{uv} = \frac{w_{uv}}{w_{uu}}$$

$$\text{Min}_{uv} = \frac{w_{uv}}{\min(w_{uu}, w_{vv})}$$

$$\text{Max}_{uv} = \frac{w_{uv}}{\max(w_{uu}, w_{vv})}$$

$$\text{MinDir}_{uv} = \begin{cases} \frac{w_{uv}}{w_{uu}} & w_{uu} \leq w_{vv} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{MaxDir}_{uv} = \begin{cases} \frac{w_{uv}}{w_{vv}} & w_{uu} \leq w_{vv} \\ 0 & \text{otherwise} \end{cases}$$

... Weight normalizations

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In the case of networks without loops we define the diagonal weights for undirected networks as the sum of out-diagonal elements in the row (or column)

$$w_{vv} = \sum_u w_{vu}$$

and for directed networks (for example, trade among world countries) as some mean value of the row and column sum, for example

$$w_{vv} = \frac{1}{2} \left(\sum_u w_{vu} + \sum_u w_{uv} \right)$$

or

$$w_{vv} = \sqrt{\sum_u w_{vu} \cdot \sum_u w_{uv}}$$

Usually we assume that the network does not contain any isolated node.



MinDir of Slovenian journals 2000

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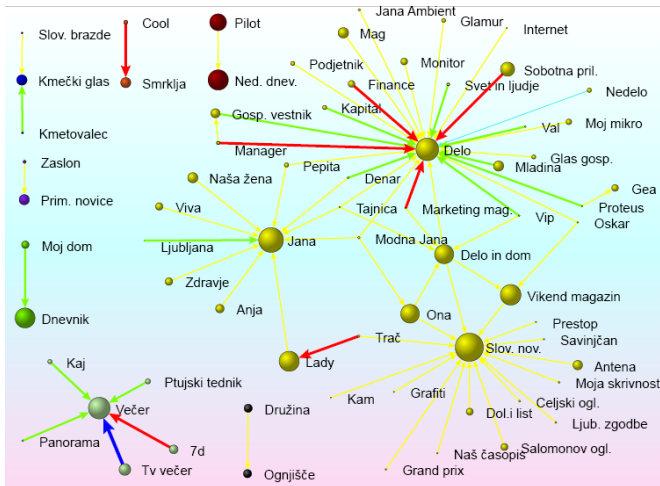
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Over 100000 people were asked in the years 1999 and 2000 about the journals they read. They mentioned 124 different journals. (source Cati)



Nature of weights

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- 1 (directly) measured: road traffic, baboons, ...
- 2 derived – computed from existing data
 - 1 projections of two-mode networks
 - 1 binary
 - 2 binary with NA
 - 3 nonnegative reals
 - 4 general
 - 2 network weight indexes: SPC weights, preferential attachment $w(u : v) = \deg(u) \cdot \deg(v)$, link betweenness, short cycles counts; (dis)similarity between end-nodes
 - 3 ...
- 3 signed networks

many zeros, threshold, only for links of a given network



Binary projections

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2-mode network $((\mathcal{U}, \mathcal{V}), \mathcal{L})$ represented with a matrix $\mathbf{UV} = [UV[u, v]]$. $UV[u, v] = 1$ if $(u, v) \in \mathcal{L}$, otherwise $UV[u, v] = 0$. Neighbors $N_U(v) = \{u \in U : (u, v) \in \mathcal{L}\}$. A weighted network on the set of nodes \mathcal{V} is determined by a projection

$$\mathbf{Co}(\mathbf{UV}) = \mathbf{UV}^T \cdot \mathbf{UV}$$

$$Co[v, z] = \sum_{u \in U} UV[u, v] \cdot UV[u, z] = |N_U(v) \cap N_U(z)|$$

$$Co[v, v] = |N_U(v)| = \deg(v)$$

A weighted network we intend to analyze was often essentially obtained this way.

Multiplication of networks

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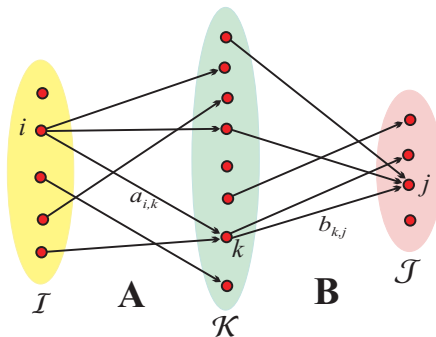
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$$c_{i,j} = \sum_{k \in N_A(i) \cap N_B^-(j)} a_{i,k} \cdot b_{k,j}$$

If all weights in networks \mathcal{N}_A and \mathcal{N}_B are equal to 1 the value of $c_{i,j}$ counts the number of ways we can go from $i \in \mathcal{I}$ to $j \in \mathcal{J}$ passing through \mathcal{K} , $c_{i,j} = |N_A(i) \cap N_B^-(j)|$.

Hamming distance

Co is a similarity measure.

A related dissimilarity between sets is the Hamming distance

$$d_H(A, B) = |A \oplus B| = |A \cup B \setminus A \cap B| = |A| + |B| - 2|A \cap B|$$

d_H is a distance

- 1 $d_H(A, B) = 0 \implies A = B$
- 2 $d_H(A, B) = d_H(B, A)$
- 3 $d_H(A, B) + d_H(B, C) \geq d_H(A, C)$

or in our case

$$\begin{aligned} D_H(v, z) &= d_H(N_U(v), N_U(z)) = |N_U(v) \oplus N_U(z)| \\ &= |Co[v, v]| + |Co[z, z]| - |Co[v, z]| \end{aligned}$$

Note: $D_H(v, z) = 0$ implies $N_U(v) = N_U(z)$, but not $v = z$; v and z are structurally equivalent in the 2-mode network.

Binary similarity measures

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The use of Jaccard similarity and some other similarities between binary vectors for analysis of two-mode networks was already proposed in SNA literature [11, p.420-424]. In principle, we could consider any similarity measure between binary vectors [2].

For all these similarities the corresponding matrices can be computed from the events co-affiliation matrix (ordinary column projection) **Co**. The similarities between vectors e and f are expressed in terms of the quantities a , b , c , and d

		z		
		1	0	
v	1	a	b	$a + b$
	0	c	d	$c + d$
		$a + c$	$b + d$	$ U $

The quantity a counts the number of cases (indices) for which both vectors v and z have value 1, etc.



Association coefficients

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Association Coefficients

measure		definition	range	class
Russel and Rao (1940)	s_1	$\frac{a}{m}$	[1, 0]	
Kendall, Sokal-Michener (1958)	s_2	$\frac{a+d}{m}$	[1, 0]	S
Rogers and Tanimoto (1960)	s_3	$\frac{a+d}{m+b+c}$	[1, 0]	S
Hamann (1961)	s_4	$\frac{a+d-b-c}{m}$	[1, -1]	S
Sokal & Sneath (1963), un_3^{-1} , S	s_5	$\frac{b+c}{a+d}$	[0, ∞]	S
Jaccard (1900)	s_6	$\frac{a}{a+b+c}$	[1, 0]	T
Kulczynski (1927), T^{-1}	s_7	$\frac{a}{b+c}$	[∞ , 0]	T
Dice (1945), Czekanowski (1913)	s_8	$\frac{a}{a+\frac{1}{2}(b+c)}$	[1, 0]	T
Sokal and Sneath	s_9	$\frac{a}{a+2(b+c)}$	[1, 0]	T
Kulczynski	s_{10}	$\frac{1}{2}(\frac{a}{a+b} + \frac{a}{a+c})$	[1, 0]	
Sokal & Sneath (1963), un_4	s_{11}	$\frac{1}{4}(\frac{a}{a+b} + \frac{a}{a+c} + \frac{d}{d+b} + \frac{d}{d+c})$	[1, 0]	
Q_0	s_{12}	$\frac{bc}{ad}$	[0, ∞]	Q
Yule (1912), ω	s_{13}	$\frac{\sqrt{ad}-\sqrt{bc}}{\sqrt{ad}+\sqrt{bc}}$	[1, -1]	Q
Yule (1927), Q	s_{14}	$\frac{ad-bc}{ad+bc}$	[1, -1]	Q
$-bc -$	s_{15}	$\frac{4bc}{m^2}$	[0, 1]	
Driver & Kroeber (1932), Ochiai (1957)	s_{16}	$\frac{a}{\sqrt{(a+b)(a+c)}}$	[1, 0]	
Sokal & Sneath (1963), un_5	s_{17}	$\frac{ad}{\sqrt{(a+b)(a+c)(d+b)(d+c)}}$	[1, 0]	
Pearson, ϕ	s_{18}	$\frac{ad-bc}{\sqrt{(a+b)(a+c)(d+b)(d+c)}}$	[1, -1]	
Baroni-Urbani, Buser (1976), S^{**}	s_{19}	$\frac{a+\sqrt{ad}}{a+b+c+\sqrt{ad}}$	[1, 0]	
Braun-Blanquet (1932)	s_{20}	$\frac{a}{\max(a+b, a+c)}$	[1, 0]	
Simpson (1943)	s_{21}	$\frac{a}{\min(a+b, a+c)}$	[1, 0]	
Michael (1920)	s_{22}	$\frac{4(ad-bc)}{(a+d)^2+(b+c)^2}$	[1, -1]	

For example, the Jaccard similarity

$$J = \frac{|N_U(v) \cap N_U(z)|}{|N_U(v) \cup N_U(z)|} = \frac{a}{a + b + c}.$$

The following equalities hold

$$a = |N_U(v) \cap N_U(z)| = Co[v, z]$$

$$a + b = |N_U(v)| = \deg(v) = Co[v, v]$$

$$a + c = |N_U(z)| = \deg(z) = Co[z, z]$$

$$a + b + c + d = |U|.$$

From them we get

$$b = |N_U(v) \setminus N_U(z)| = Co[v, v] - Co[v, z]$$

$$c = |N_U(z) \setminus N_U(v)| = Co[z, z] - Co[v, z]$$

$$d = |U| + Co[v, z] - Co[v, v] - Co[z, z]$$

$$a + b + c = |N_U(v) \cup N_U(z)| = Co[v, v] + Co[z, z] - Co[v, z]$$



Jaccard

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For Jaccard similarity we get [10]

$$Jcol[v, z] = \frac{Co[v, z]}{Co[v, v] + Co[z, z] - Co[v, z]}$$

In this sense, the similarity measures (matrices) can be seen as a kind of compatibility normalization of the weights obtained with the standard projection [5].

The corresponding Jaccard network $\mathcal{J} = (V, L_J, Jcol)$ is undirected with loops removed.

The Jaccard dissimilarity

$$d_J(A, B) = 1 - J(A, B) = \frac{|A \oplus B|}{|A \cup B|}$$

is also a distance.



Coappearances and similarities

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- in creating the coappearances network **Co** it is important to include the loops.
- the (a,b,c,d) (dis)similarities can be computed from the coappearances network **Co**.



Fractional approach

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To make in a projection the contributions of all nodes from the set \mathcal{U} equal we apply the *fractional approach* by normalizing the weights [8].

The normalized 2-mode network $n(\mathbf{UV})$ has weights

$$n(UV)[u, v] = \begin{cases} \frac{UV[u, v]}{\text{outdeg}(u)} & u \in \mathcal{U}^+ \\ 0 & u \notin \mathcal{U}^+ \end{cases}$$

where $\mathcal{U}^+ = \{u \in \mathcal{U} : \text{outdeg}(u) > 0\}$.

Normalized projection $\mathbf{Cn} = n(\mathbf{UV})^T \cdot n(\mathbf{UV})$

In bibliometric applications on network **WA**, the derived network **Co** describes the co-authorship (collaboration) among authors. The network **Cn** is the corresponding fractional version.

Mark Newman proposed an alternative normalization that considers only co-authorship between different authors – single-author works and self co-authorship are excluded.

The Newman's normalized 2-mode network $n'(\mathbf{UV})$ has weights

$$n'(\mathbf{UV})[u, v] = \begin{cases} \frac{UV[u, v]}{\text{outdeg}_{UV}(u) - 1} & \text{outdeg}_{UV}(u) > 1 \\ 0 & \text{otherwise} \end{cases}$$

Newman's projection $\mathbf{Cn}' = n(\mathbf{WA})^T \cdot n'(\mathbf{WA})$.

Network: symmetrize with sum and remove loops.



Stochastic or Markov normalization

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Let $R(v) = \sum_z w[v, z]$ denote the row sum for a node v ; and $Q(z) = \sum_v w[v, z]$ the column sum for a node z . For $R(v) > 0$, we define a stochastic normalization with

$$\mathbf{S}[v, z] = \frac{w[v, z]}{R(v)}$$

In real life networks it can happen that for some node v we have $R(v) = 0$. In such a case all entries in the row corresponding to v are 0, $w[v, z] = 0$. Then also $\mathbf{S}[v, z] = 0$.

For $R(v) > 0$ we have $\sum_z \mathbf{S}[v, z] = 1$ – the matrix entries can be interpreted as probabilities that a node v coappears with a node z . Usually the matrix \mathbf{S} is not symmetric.

Let $T = \sum_{v,z} w[v,z]$ the total sum of weights in the network. If the network is undirected/symmetric then $R(v) = Q(v)$.

Then $R(v)/T$ is the proportion/probability of activity of the node v , and the expected activity $\hat{w}[v,z]$ from v to z is equal to

$$\hat{w}[v,z] = \frac{R(v)}{T} \cdot Q(z)$$

The measured weight $w[v,z]$ may deviate for a factor $a[v,z]$ from the expected value $w[v,z] = a[v,z] \cdot \hat{w}[v,z]$ or

$$a[v,z] = \frac{w[v,z] \cdot T}{R(v) \cdot Q(z)}$$

If $a[v,z] > 1$ the measured weight is larger than expected, ...

The deviation measure a is called the Balassa index or the "revealed comparative advantage" [1].

The Balassa index is not 'symmetric' – the larger values have range $(1, \infty)$, the smaller values have range $(1, 0)$.

To 'symmetrize' it we apply, as suggested by [19], a logarithmic function to it. For easier interpretation, we selected base 2 logarithms

$$b[v, z] = \log_2 a[v, z], \quad \text{for } a[v, z] > 0$$

In our analysis of ex-Soviet countries we used the index b .



Salton or cosine similarity

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In the case of general weights in the two-mode network **UV** the Salton index is often used in construction of the network on the set of nodes \mathcal{V}

$$s[v, z] = \frac{\sum_u UV[u, v] \cdot UV[u, z]}{\sqrt{\sum_u UV[u, v]^2 \cdot \sum_u UV[u, z]^2}}$$

In general $s \in [-1, 1]$. If $UV \geq 0$ then $s \in [0, 1]$.

Using s we can define the angular distance θ **wp**

$$\theta[v, z] = \frac{\arccos(s[v, z])}{\pi}$$

[15]

We could use any (dis)similarity measure between appropriately normalized columns of **UV** [13].

Dissimilarities on $\mathbb{R}^m / 1$

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n	measure	definition	range	note
1	Euclidean	$\sqrt{\sum_{i=1}^m (x_i - y_i)^2}$	$[0, \infty)$	$M(2)$
2	Sq. Euclidean	$\sum_{i=1}^m (x_i - y_i)^2$	$[0, \infty)$	$M(2)^2$
3	Manhattan	$\sum_{i=1}^m x_i - y_i $	$[0, \infty)$	$M(1)$
4	rook	$\max_{i=1}^m x_i - y_i $	$[0, \infty)$	$M(\infty)$
5	Minkowski	$\sqrt[p]{\sum_{i=1}^m (x_i - y_i)^p}$	$[0, \infty)$	$M(p)$

Dissimilarities on \mathbb{R}^m / 2

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n	measure	definition	range	note
6	Canberra	$\sum_{i=1}^m \frac{ x_i - y_i }{ x_i + y_i }$	$[0, \infty)$	\mathbb{R}_0^+
7	Heincke	$\sqrt{\sum_{i=1}^m \left(\frac{ x_i - y_i }{ x_i + y_i } \right)^2}$	$[0, \infty)$	\mathbb{R}_0^+
8	Self-balanced	$\sum_{i=1}^m \frac{ x_i - y_i }{\max(x_i, y_i)}$	$[0, \infty)$	\mathbb{R}_0^+
9	Lance-Williams	$\frac{\sum_{i=1}^m x_i - y_i }{\sum_{i=1}^m x_i + y_i}$	$[0, \infty)$	\mathbb{R}_0^+
10	Correlation c.	$\frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X)\text{var}(Y)}}$	$[1, -1]$	



General projections

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The binarization $b(\mathbf{UV})$ of the network \mathbf{UV} is obtained by setting weights of all its links to 1.

$$b(UV)[u, v] = 1 \Leftrightarrow (u, v) \in \mathcal{L}$$

We define, generalizing [8], the left and the right contribution $l(\mathbf{UV}) = \mathbf{UV}^T \cdot b(\mathbf{UV})$ and $r(\mathbf{UV}) = b(\mathbf{UV})^T \cdot \mathbf{UV}$. It holds

$$r(UV)[z, v] = \sum_{u \in \mathcal{U}} b(UV)^T[z, u] \cdot UV[u, v] =$$

$$\sum_{u \in \mathcal{U}} b(UV)[u, z] \cdot UV[u, v] = \sum_{u \in \mathcal{U}} UV^T[v, u] \cdot b(UV)[u, z] = l(UV)[v, z]$$

The joint contribution can be defined as some mean of both contributions

$$\begin{aligned} biCnX[v, z] &= meanX(l(UV)[v, z], r(UV)[v, z]) = \\ &= meanX(l(UV)[v, z], l(UV)[z, v]) \end{aligned}$$

We need to compute only the left contributions.

Instead of the network **UV** we can use its generalized normalized version

$$n(UV)[u, v] = \begin{cases} \frac{UV[u, v]}{R(u)} & u \in \mathcal{U}^+ \\ 0 & u \notin \mathcal{U}^+ \end{cases}$$



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$$biCnA[v, z] = \frac{1}{2}(I(UV)[v, z] + I(UV)[z, v]) - \text{arithmetic}$$

$$biCnm[v, z] = \min(I(UV)[v, z], I(UV)[z, v]) - \text{minimum}$$

$$biCnM[v, z] = \max(I(UV)[v, z], I(UV)[z, v]) - \text{maximum}$$

$$biCnG[v, z] = \sqrt{I(UV)[v, z] \cdot I(UV)[z, v]} - \text{geometric, Salton}$$

$$biCnH[v, z] = 2(I(UV)[v, z]^{-1} + I(UV)[z, v]^{-1})^{-1} - \text{harmonic, Dice}$$

$$biCnJ[v, z] = (I(UV)[v, z]^{-1} + I(UV)[z, v]^{-1} - 1)^{-1} - \text{Jaccard}$$

$$biCnJ[v, z] \leq biCnm[v, z] \leq biCnH[v, z] \leq biCnG[v, z] \leq biCnA[v, z] \leq biCnM[v, z]$$

$$biCnX[v, z] = biCnX[z, v]$$

$$I \in [0, 1] \Rightarrow biCnX \in [0, 1]$$



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$$Cn_A[v, z] = \frac{|N_U(v) \cap N_U(z)|}{2} \left(\frac{1}{|N_U(v)|} + \frac{1}{|N_U(z)|} \right)$$

$$Cn_m[v, z] = \frac{|N_U(v) \cap N_U(z)|}{\max(|N_U(v)|, |N_U(z)|)}$$

$$Cn_M[v, z] = \frac{|N_U(v) \cap N_U(z)|}{\min(|N_U(v)|, |N_U(z)|)}$$

$$Cn_G[v, z] = \frac{|N_U(v) \cap N_U(z)|}{\sqrt{|N_U(v)| \cdot |N_U(z)|}}$$

$$Cn_H[v, z] = \frac{2|N_U(v) \cap N_U(z)|}{|N_U(v)| + |N_U(z)|}$$

$$Cn_J[v, z] = \frac{|N_U(v) \cap N_U(z)|}{|N_U(v) \cup N_U(z)|}$$

For clustering the nodes we used the normalized matrices with zero diagonals. For the index b we set the values of non-links to 0. When computing dissimilarity $\mathbf{D}[v, z]$ between nodes v and z in a network it is important to use the corrected dissimilarities that compare the entry $w_{vz} = w[v, z]$ with the entry $w_{zv} = w[z, v]$ and the entry $w[v, v]$ with the entry $w[z, z]$. We selected the corrected Euclidean distance [14, p. 181]

$$\mathbf{D}[v, z] = \sqrt{(w_{vz} - w_{zv})^2 + (w_{vv} - w_{zz})^2 + \sum_{y: y \neq v, y \neq z} (w_{vy} - w_{zy})^2}$$



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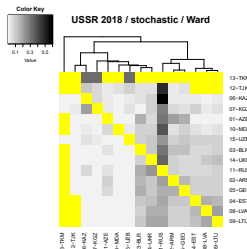
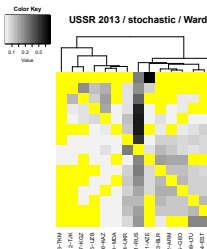
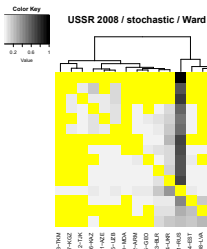
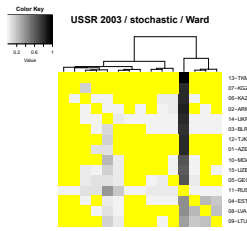
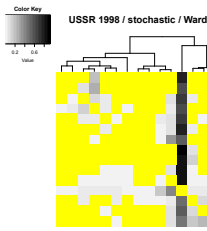
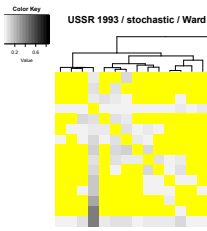
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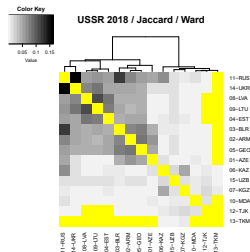
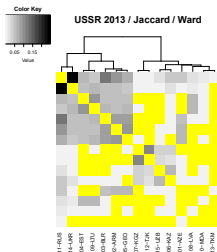
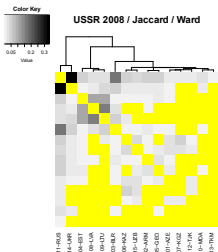
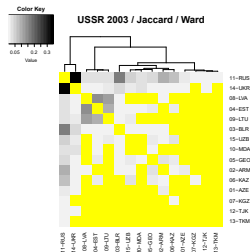
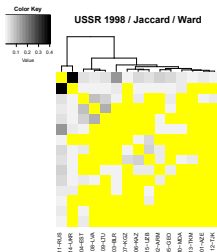
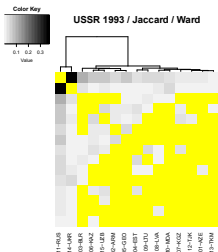
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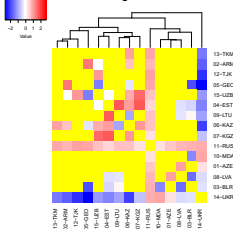
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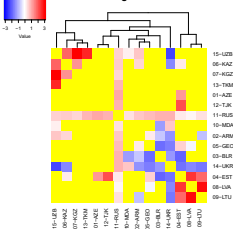
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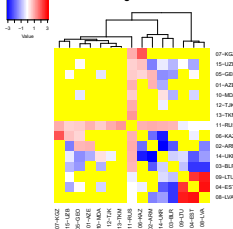
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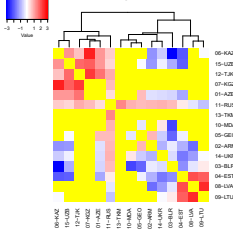
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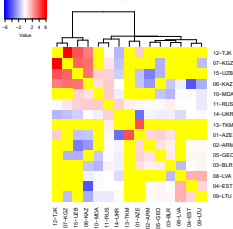
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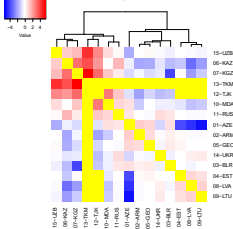
Color Key USSR 2008 / log deviation / Ward



Color Key USSR 2013 / log deviation / Ward



Color Key USSR 2018 / log deviation / Ward



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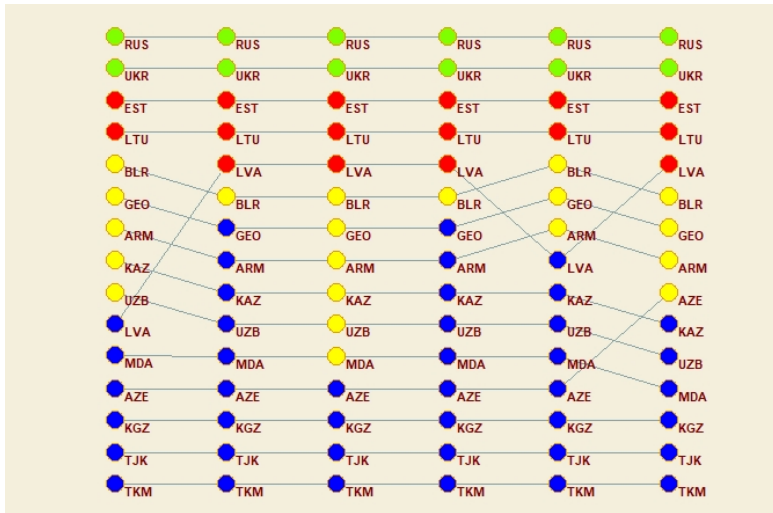
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