Toward an indirect approach for blockmodeling temporal networks

Vladimir Batagelj
IMFM Ljubljana, IAM UP Koper and NRU HSE Moscow

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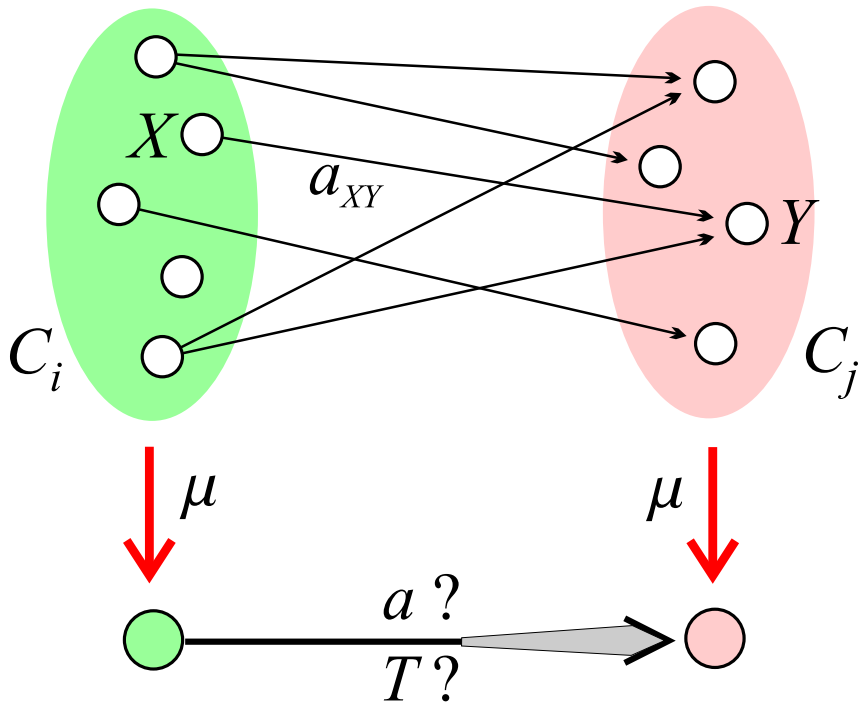
Outline

Vladimir Batagelj: vladimir.batagelj@fmf.uni-lj.si
Current version of slides (July 11, 2020 at 22:08): slides PDF
https://github.com/bavla/SocNet/blob/master/Temp/SNAauthors.pdf

Blockmodeling temporal networks



To the traditional blockmodeling (BM) scheme we add the time dimension. We assume that the network is described using temporal quantities [6] for nodes/links activity/presence, nodes properties and links weights. Then also the BM partition π can be described for each node v with a temporal quantity $\pi(v,t)$ with the meaning:

$\pi(v,t) = i$ means that in time t node v belongs to cluster i.
The clusters $C_i(t) = \{v : \pi(v,t) = i\}$ structure and activity can change through time, but they preserve their identity.

Blockmodeling temporal networks

For the BM μ the clusters are maped into BM nodes $\mu : C_i \rightarrow [i]$. To determine the BM we still have to specify how the links from C_i to C_j are represented in the BM – in general, for the model arc $([i], [j])$, we have to specify two temporal quantities: its value $a_{ij}(t)$ and, in the case of generalized BM, its type $T_{ij}(t)$. In general the value can be an object of different type than the values of the links in the original temporal network.

To develop a BM method we specify a criterion function $P(\mu)$ measuring the "error" of the BM μ. We can introduce additional knowledge by constraining the partitions to a set Φ of feasible partitions. We are searching for a partition $\pi^* \in \Phi$ such that the corresponding BM μ^* minimizes the criterion function $P(\mu)$.

Blockmodeling temporal networks

This is a general framework. In the following years we intend to develop methods case by case.

- constant partition – nodes stay in the same cluster all the time
  - indirect approach based on clustering of temporal quantities
  - temporal version of clustering with relation constraint
  - local optimization of the criterion function P over Φ
- dynamic partition – nodes can move between clusters through time. The details are still to be elaborated.

Clustering of temporal quantities

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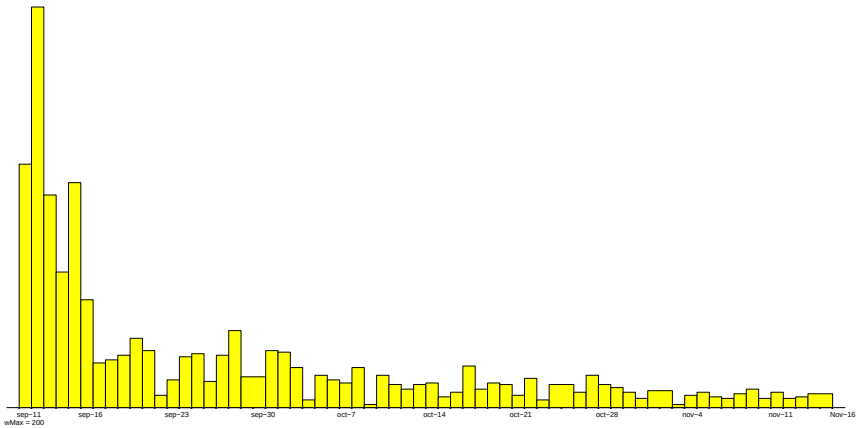
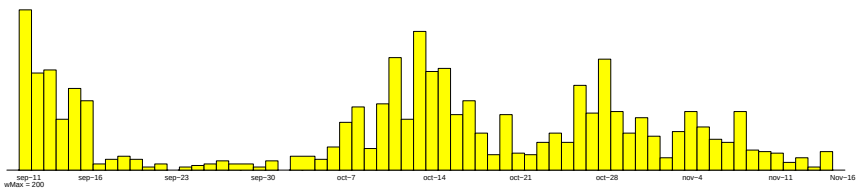


bomb :



A temporal quantity is described by a sequence

$$\mathbf{a} = [(s_r, f_r, v_r) : r = 1, 2, \ldots, k]$$

where $[s_r, f_r)$ determines a time interval and v_r is the value of variable on this interval.

We adapted traditional leaders and agglomerative hierarchical clustering methods for clustering units described by variables that have for their values temporal quantities [2, 5].

ClusTQ at https://github.com/bavla/TQ .

Clustering of temporal quantities

For a unit X_i, each variable V_j is described with a weight h_{ij} and a temporal quantity $\mathbf{x}_{ij}$

$$X_{ij} = (h_{ij}, \mathbf{x}_{ij})$$

In our algorithms we use *normalized* values of temporal variables $Y' = (h, \mathbf{p})$ where

$$\mathbf{p} = [(s_r, f_r, p_r) : r = 1, 2, \ldots, k] \qquad \text{and} \qquad p_r = \frac{v_r}{h}$$

In the case, when $h = \mathrm{tot}(\mathbf{x}) = \sum v_r$, the normalized TQ is essentially a probability distribution.

Both methods create cluster representatives that are represented in the same way.

September 11th Reuters terror news

The Reuters terror news network was obtained from the CRA (Centering Resonance Analysis) networks produced by Steve Corman and Kevin Dooley at Arizona State University. The network is based on all the stories released during 66 consecutive days by the news agency Reuters concerning the September 11 attack on the U.S., beginning at 9:00 AM EST 9/11/01.

The nodes of this network are important words (terms). There is an edge between two words iff they appear in the same utterance (for details see the paper [**?**]). The weight of an edge is its frequency. The network has $n = 13332$ nodes (different words in the news) and $m = 243447$ edges, 50859 with value larger than 1. There are no loops in the network.

The Reuters terror news network was used as a case network for the Viszards visualization session on the Sunbelt XXII International Sunbelt Social Network Conference, New Orleans, USA, 13-17. February 2002.

Details

We transformed the Pajek version of the network into NetsJSON format used in TQ and Nets.

For a temporal description of each node/word for clustering we took its activity = sum of all TQs on edges adjacent to a given node.

Our leaders and hierarchical clustering methods are compatible. They are based on the same clustering error criterion function.

Usually the leaders method is used to reduce the problem to up to some hundred units. With hierarchical clustering of the leaders of the obtained clusters we afterward determine the "right" number of clusters and their representatives.

Leaders method

To cluster all 13332 words (nodes) in Terror news we used an adapted leaders method searching for 100 clusters:
After 50 steps we stopped the search. $P_1 = 41012.94$, $P_{50} = 23897.74$

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{74: 716, 43: 535, 82: 378, 2: 372, 9: 338, 69: 325, 96: 307, 46: 307, 100: 291, 26: 275,
 62: 257, 13: 241, 85: 238, 81: 237, 34: 233, 98: 229, 27: 228, 29: 222, 19: 203, 22: 199,
 10: 196, 88: 195, 37: 192, 12: 191, 30: 186, 54: 183, 72: 180, 66: 177, 14: 175, 33: 172,
 25: 162, 23: 162, 92: 156, 71: 155, 93: 153, 87: 152, 51: 151, 24: 150, 16: 148, 58: 146,
 59: 146, 47: 145, 63: 143, 61: 141, 4: 139, 45: 132, 89: 130, 8: 128, 17: 126, 56: 114,
 50: 105, 77: 102, 42: 101, 1: 96, 55: 96, 97: 95, 53: 91, 94: 89, 5: 87, 70: 83, 90: 81,
 44: 76, 3: 75, 60: 74, 40: 73, 52: 64, 15: 63, 84: 56, 20: 51, 57: 45, 64: 44, 67: 44,
 65: 43, 21: 39, 31: 37, 11: 37, 39: 36, 79: 35, 38: 35, 36: 34, 76: 32, 91: 32, 99: 32,
 80: 31, 73: 31, 68: 31, 7: 29, 6: 29, 35: 29, 28: 28, 49: 27, 86: 26, 78: 23, 83: 18,
 41: 18, 32: 18, 95: 15, 18: 14, 48: 13, 75: 12}
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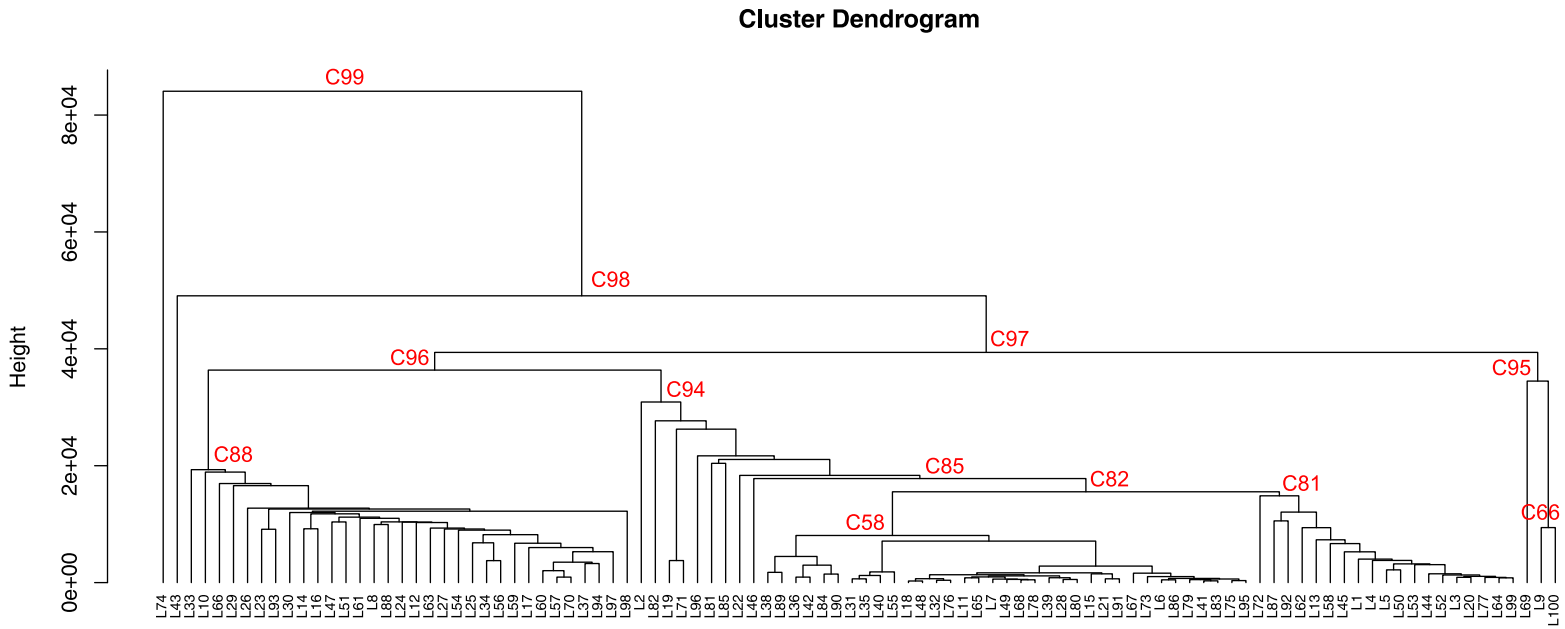
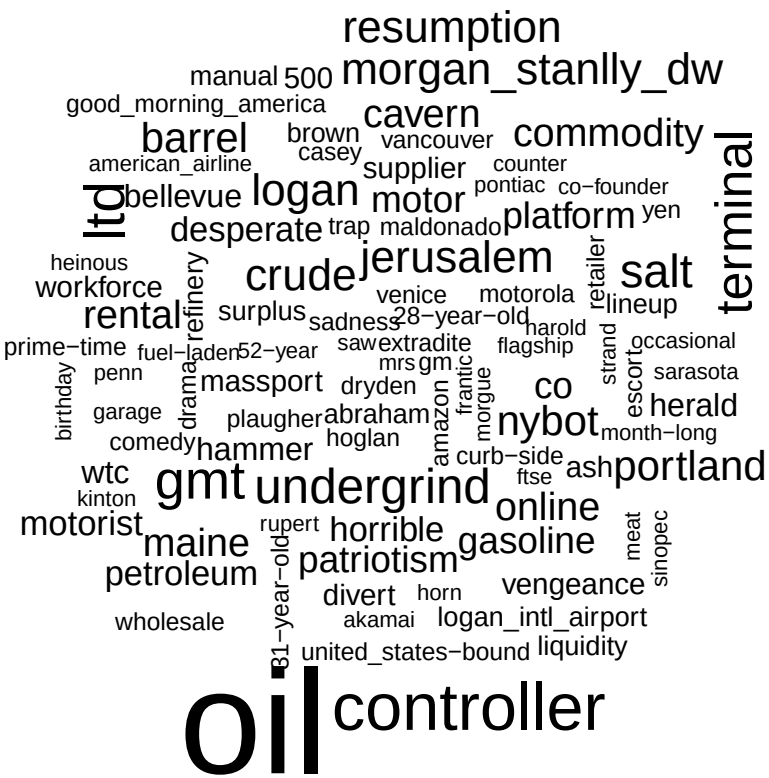
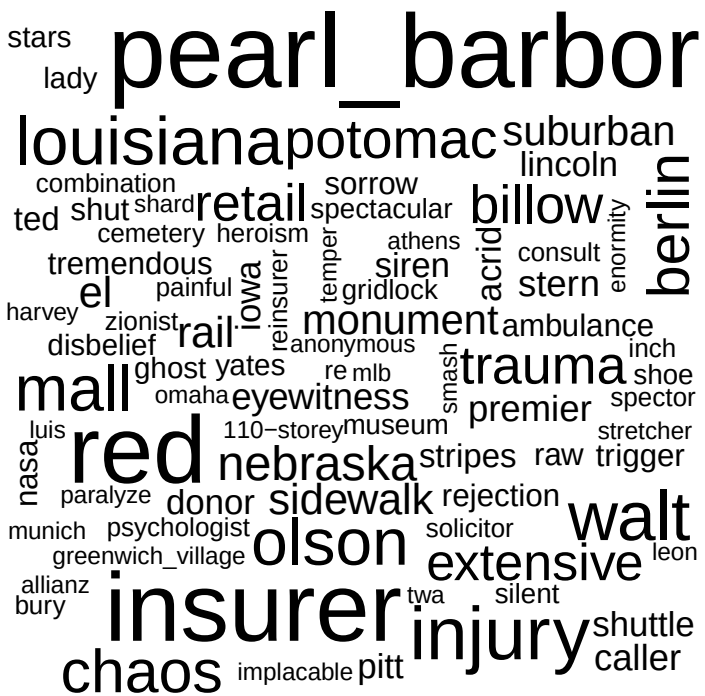
Hierarchical clustering



Word clouds for L74 and L43

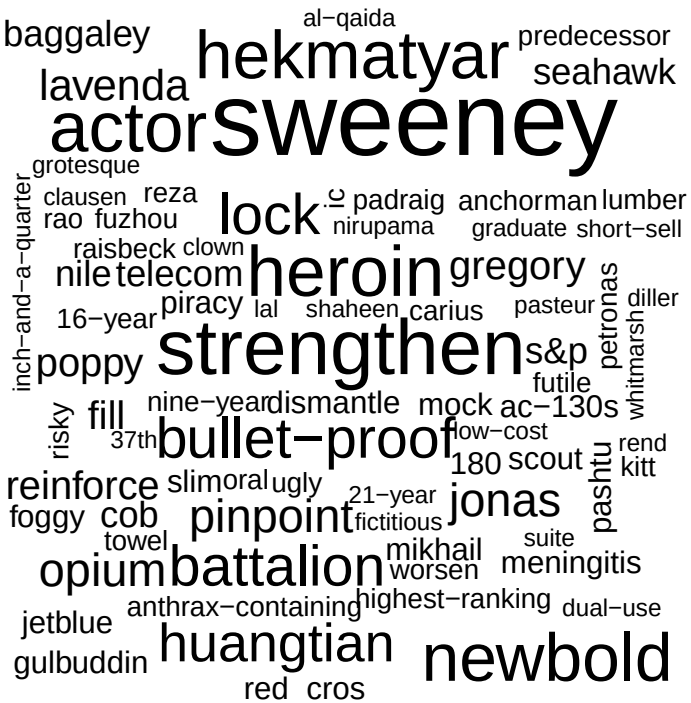
$|L74| = 716, \quad |L43| = 535$

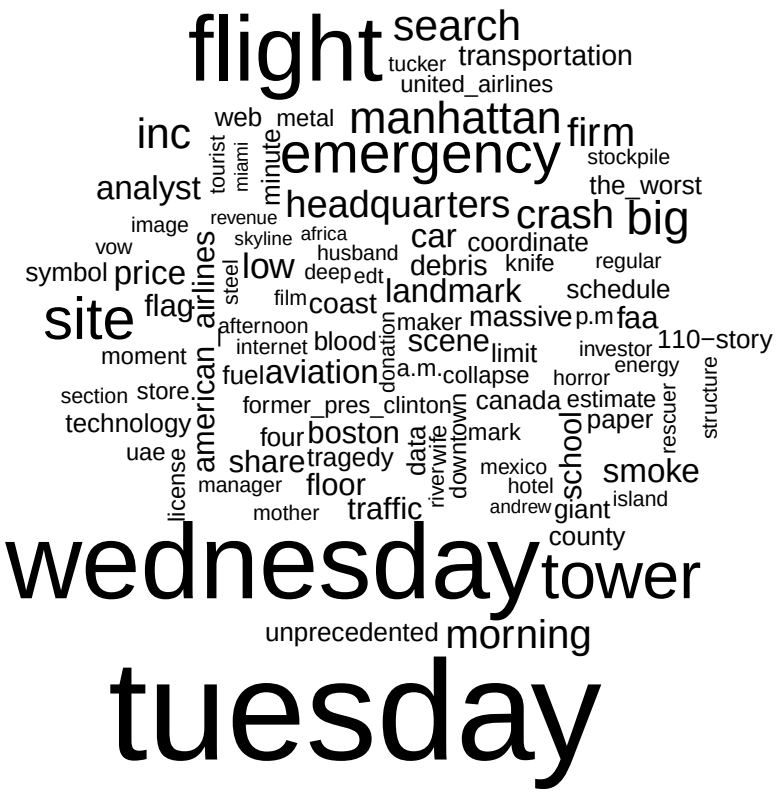




Word clouds for L46 and C46

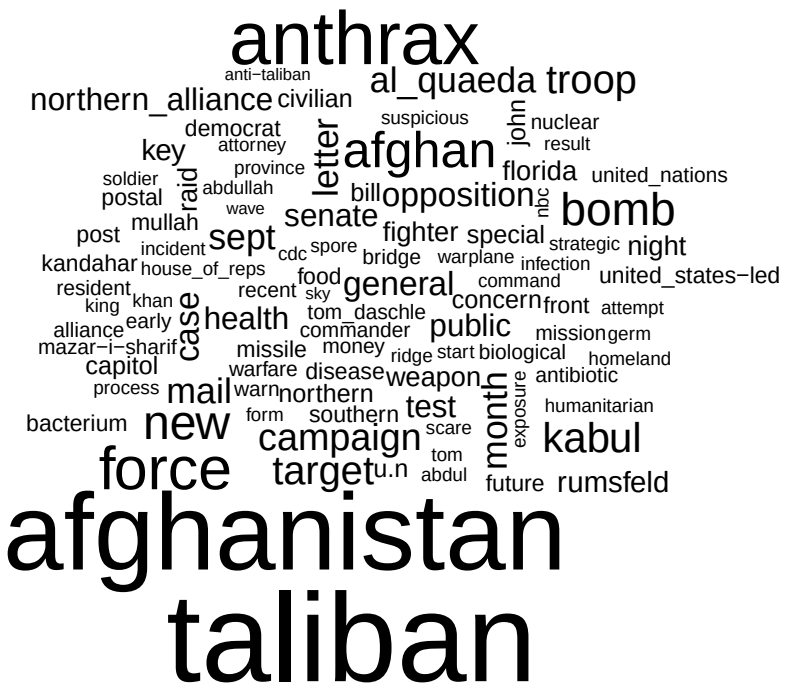
$|L46| = 307, \quad |C46| = 358$

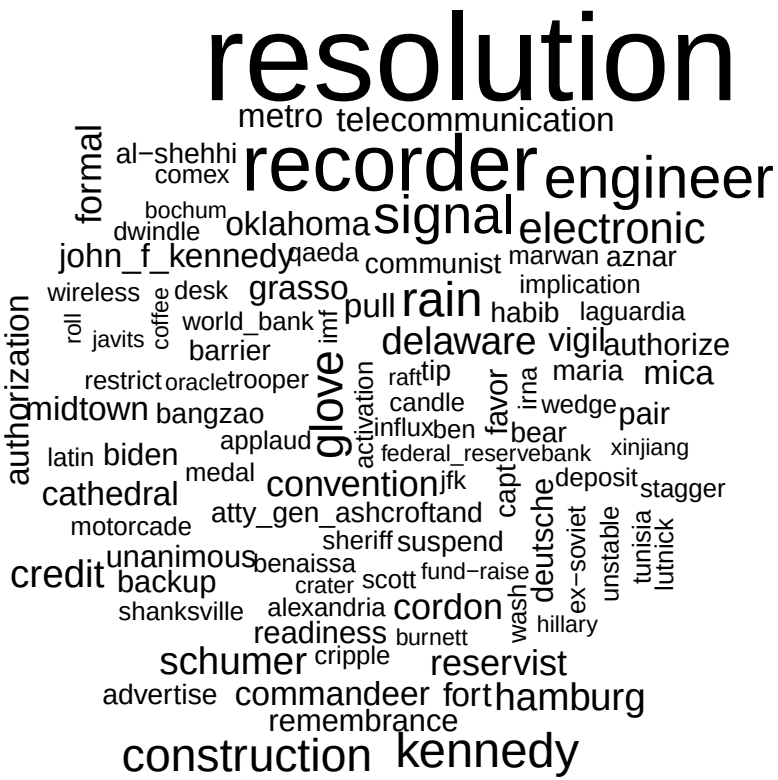




Word clouds for C88, C95

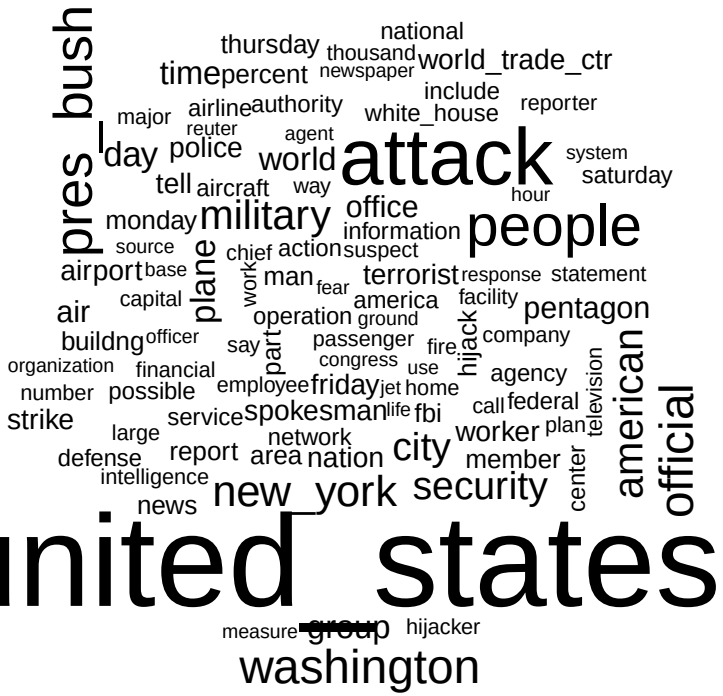
$|C88| = 5109, \quad |C95| = 954$





Word clouds for C58 and C81

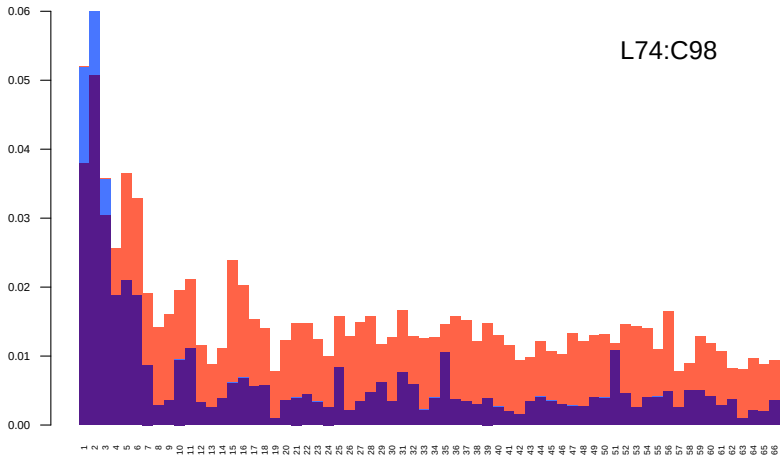
$|C58| = 1396, \quad |C81| = 2226$

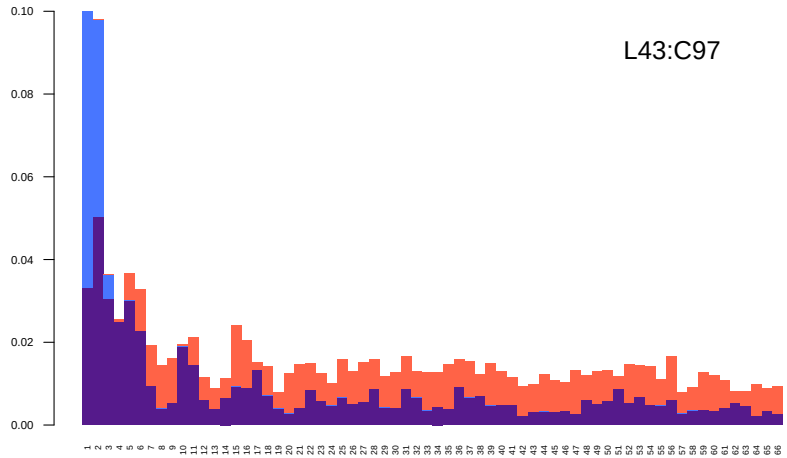


Word cloud for C94

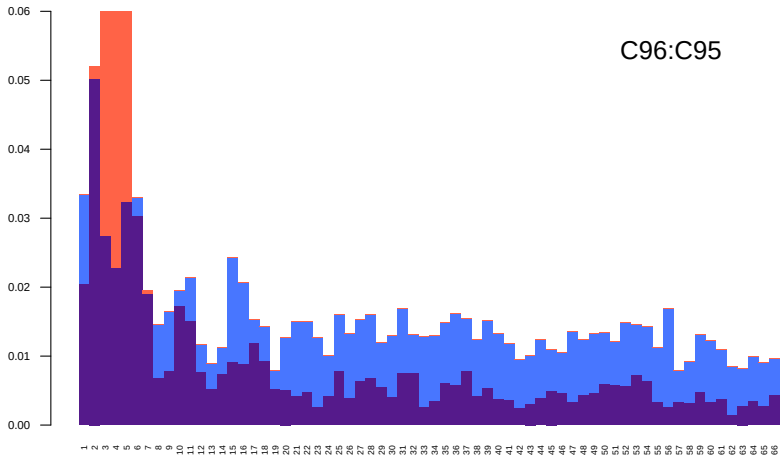
$|C94| = 6018\ , \quad C58, C81 \subset C94$

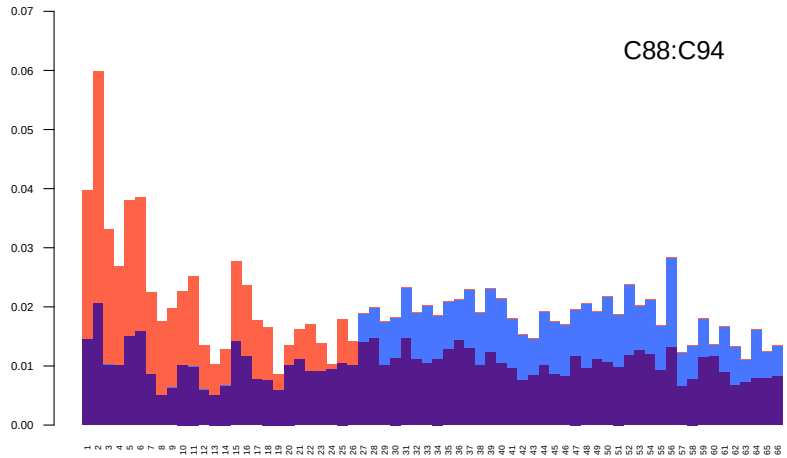
Comparisons of leaders and cluster representatives L74:C98, L43:C97, C96:C95, C88:C94





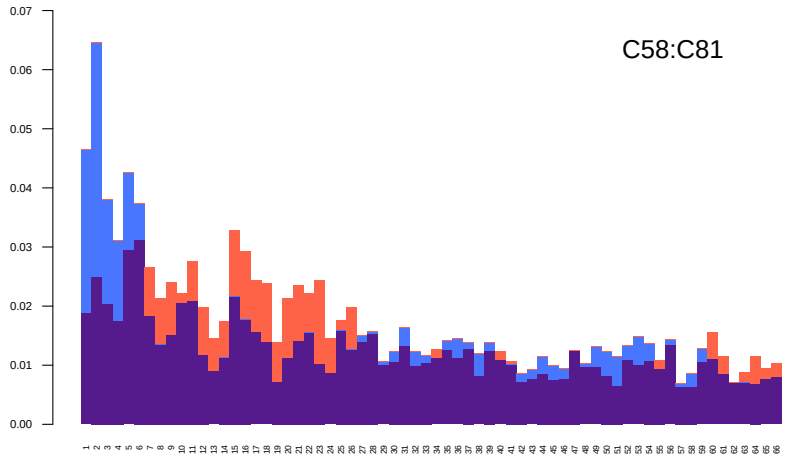
$L74[2] = 0.5894, \quad L43[1] = 0.4393, \quad L43[2] = 0.0981$

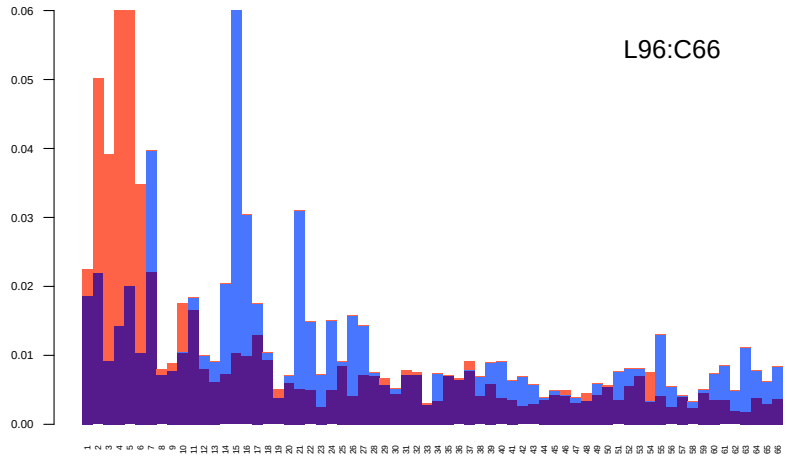




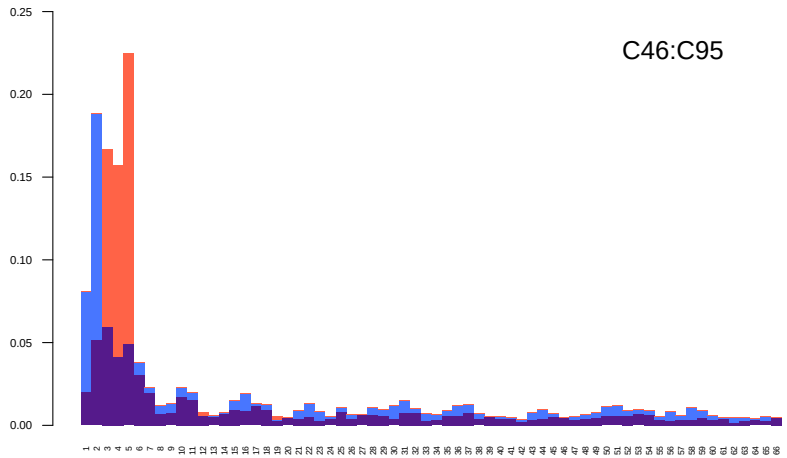
$C95[3] = 0.1665, \quad C95[4] = 0.1570, \quad C95[5] = 0.2250$

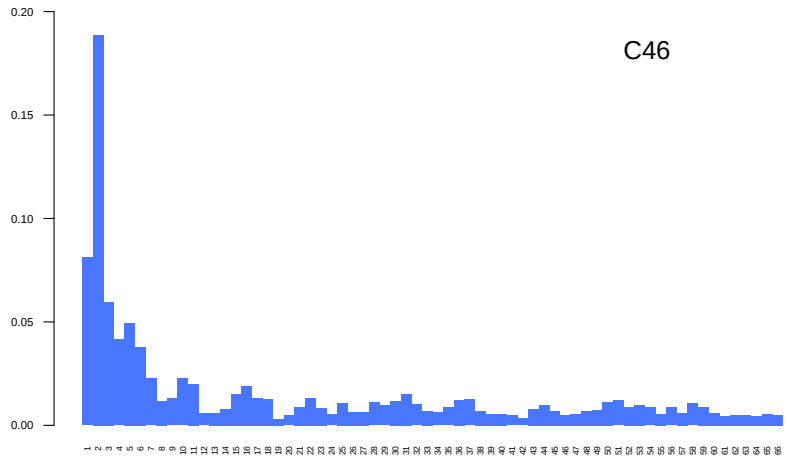
Comparisons of leaders and cluster representatives C58:C81, L96:C66, C46:C955, C46





$L96[15] = 0.3524, \quad C66[4] = 0.1961, \quad C66[5] = 0.2917$





Block model

To produce a block model we have to specify the values on its links. Because the original network is undirected, so is the BM. There are different options.

Temporal quantities:

$$\mathbf{a}(([i]:[j])) = \text{activity}(C_i, C_j) = \sum_{u \in C_i, v \in C_j} \mathbf{a}(u:v), \quad \text{for } i \neq j$$

and $\mathbf{a}(([i]:[i])) = \frac{1}{2}\text{activity}(C_i, C_i)$.

Total intensities: $a(([i]:[j])) = \text{tot}(\mathbf{a}(([i]:[j])))$.

Geometric average intensities: $g(([i]:[j])) = \dfrac{a(([i]:[j]))}{\sqrt{|C_i| \cdot |C_j|}}$.

Block model

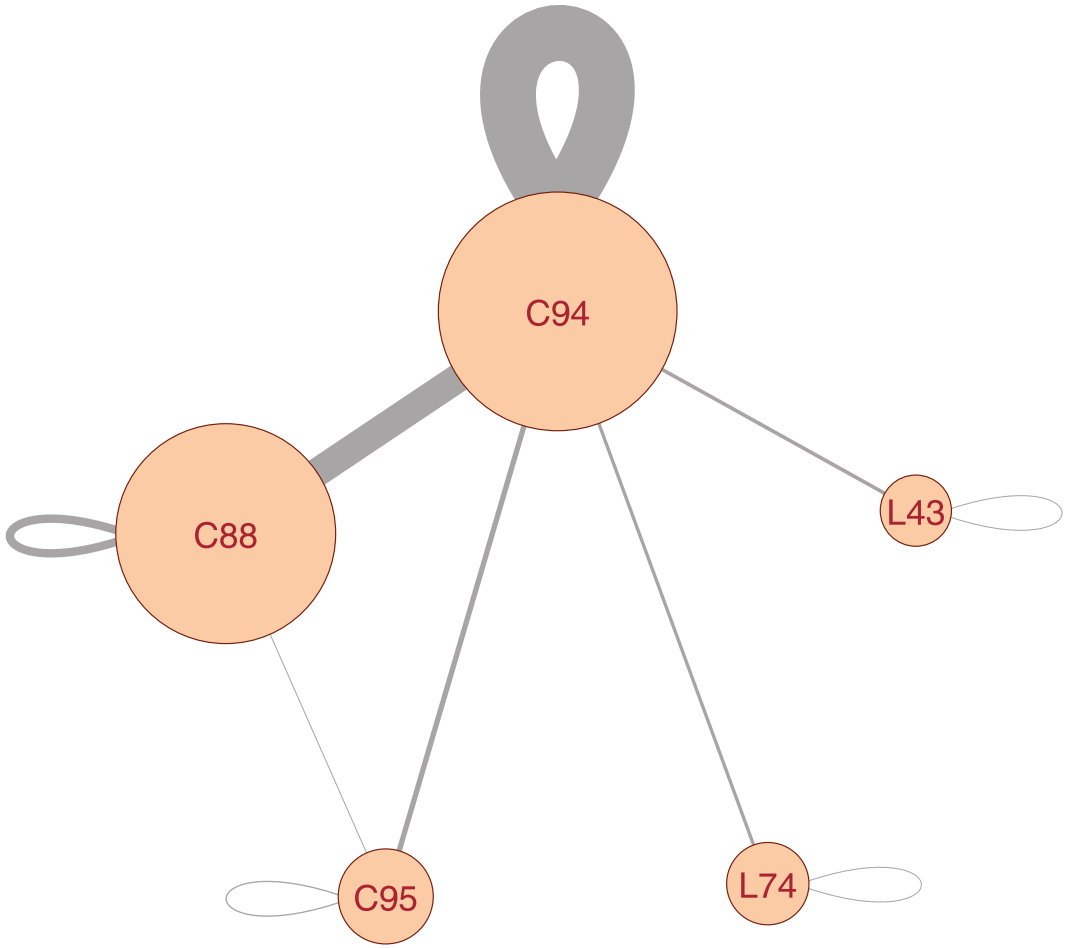
Total intensities:

i	*cluster*	*size*	1	2	3	4	5
1	*C*94	6018	143549	67801	5422	2816	2939
2	*C*88	5109	0	18288	739	357	357
3	*C*95	954	0	0	535	53	54
4	*L*43	535	0	0	0	205	51
5	*L*74	716	0	0	0	0	281

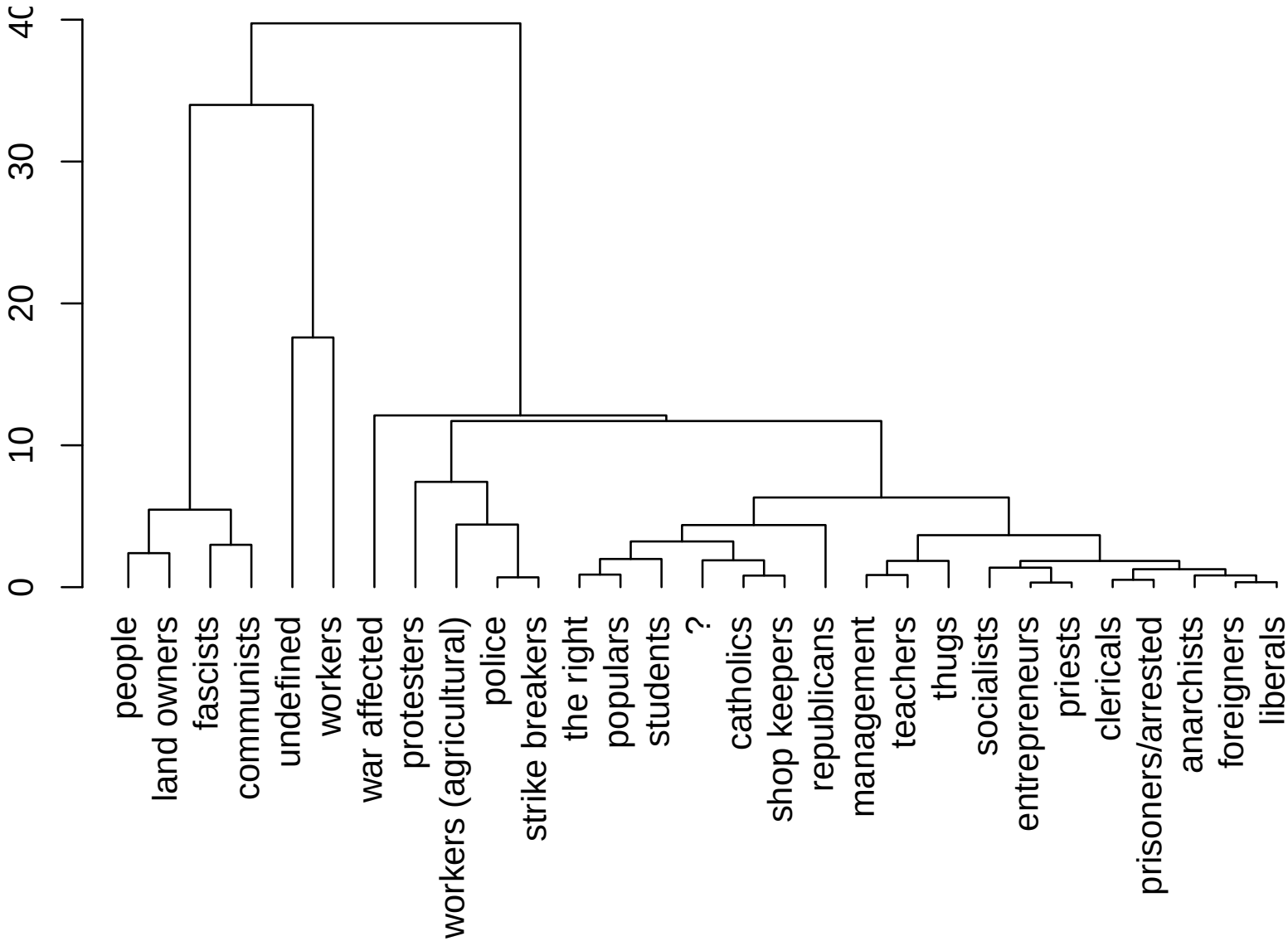
Geometric average intensities:

i	*cluster*	1	2	3	4	5
1	*C*94	23.85	12.23	2.26	1.57	1.42
2	*C*88	0	3.58	0.33	0.22	0.19
3	*C*95	0	0	0.56	0.07	0.07
4	*L*43	0	0	0	0.38	0.08
5	*L*74	0	0	0	0	0.39

Block model
Geometric average intensities, cut at level 0.3



Hierarchical clustering of Franzosi's violence network



Conclusions

Acknowledgments

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References I

Anderberg, M.R. (1973) Cluster analysis for applications. Academic Press, New York

Batagelj, V., Kejžar, N., Korenjak-Černe, S. (2015) Clustering of Modal Valued Symbolic Data. arXiv:1507.06683

Batagelj, V., Kejžar, N. (2010) Clamix – Clustering Symbolic Objects. Program in R
https://r-forge.r-project.org/projects/clamix/

Batagelj, V. (1988) Generalized Ward and related clustering problems. In: Bock HH (ed) Classification and related methods of data analysis, North-Holland, Amsterdam, pp 67-74

Batagelj, V. (2020) TQ / clusTQ – clustering temporal quantities. https://github.com/bavla/TQ

Batagelj, V., Praprotnik, S. (2016) An algebraic approach to temporal network analysis based on temporal quantities. Social Network Analysis and Mining, 6(1), 1-22.

Diday, E. (1979) Optimisation en classification automatique. Tome 1.,2. INRIA, Rocquencourt (in French)

References II

 Hartigan, J.A. (1975) Clustering algorithms. Wiley-Interscience, New York

 Ward, J.H. (1963) Hierarchical grouping to optimize an objective function. J Am Stat Assoc 58:236-244