



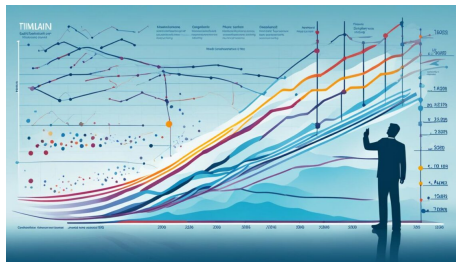
Longitudinal approach to the analysis of temporal networks based on temporal quantities

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Workshop on Innovation in Dynamic Network Modelling
Lugano, September 11-13, 2024

- 1 Semirings
- 2 Temporal quantities
- 3 Activity
- 4 Temporal bibliographic networks
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- 6 References



Epidemiology

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Current version of slides (September 10, 2024 at 00:28): [slides PDF](#)

<https://github.com/bavla/TQ>

Computing with link weights in networks

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Semirings

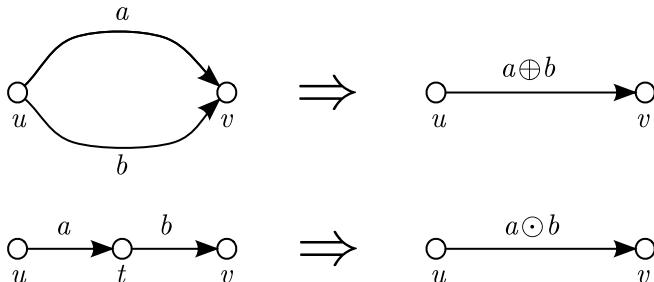
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A semiring is a "natural" algebraic structure to formalize computations with link weights in networks. They allow us to extend the weights from links to nodes, walks (paths) and sets of walks.



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Let \mathbb{A} be a set and a, b, c elements from \mathbb{A} . A *semiring* $[8, 4, 12, 3, 1]$ is an algebraic structure $(\mathbb{A}, \oplus, \odot, 0, 1)$ with two binary operations (addition \oplus and multiplication \odot) where:

$(\mathbb{A}, \oplus, 0)$ is an *abelian monoid* with the neutral element 0 (zero):

$$a \oplus b = b \oplus a \quad - \text{commutativity}$$

$$(a \oplus b) \oplus c = a \oplus (b \oplus c) \quad - \text{associativity}$$

$$a \oplus 0 = a \quad - \text{existence of zero}$$

$(\mathbb{A}, \odot, 1)$ is a *monoid* with the neutral element 1 (unit):

$$(a \odot b) \odot c = a \odot (b \odot c) \quad - \text{associativity}$$

$$a \odot 1 = 1 \odot a = a \quad - \text{existence of a unit}$$

Multiplication \odot *distributes* over addition \oplus :

$$a \odot (b \oplus c) = a \odot b \oplus a \odot c \quad (b \oplus c) \odot a = b \odot a \oplus c \odot a$$

In formulas we assume precedence of the multiplication over the addition.

Some examples of semirings used in network data analysis:

- 1 Combinatorial: $(\mathbb{N}, +, \cdot, 0, 1)$ or $(\mathbb{R}_0^+, +, \cdot, 0, 1)$
- 2 Reachability: $(\{0, 1\}, \vee, \wedge, 0, 1)$
- 3 Shortest paths [13]: $(\mathbb{R}_0^+, \min, +, \infty, 0)$
- 4 MaxMin (capacity): $(\mathbb{R}_0^+, \max, \min, 0, \infty)$
- 5 Pathfinder [18, 17, 19]: $(\overline{\mathbb{R}}_0^+, \min, \boxed{}, \infty, 0)$
where $a \boxed{r} b = \sqrt[r]{a^r + b^r}$ (Minkowski)
- 6 Interval [15, 2]: $[a, A], [b, B] \subset \mathbb{R}_0^+$
 $[a, A] \oplus [b, B] = [a + b, A + B]$ and
 $[a, A] \odot [b, B] = [a \cdot b, A \cdot B]$

- 7 Let the set of bins $\mathbf{B} = \{B_1, B_2, \dots, B_k\}$ be a partition of the set B such that (\mathbf{B}, \circ) is a semigroup. A *histogram* $h : \mathbf{B} \rightarrow \mathbb{N}$
 $h_i = h(B_i) = |\{X : v(X) \in B_i\}|$

$$h \oplus g = h + g \quad (h \oplus g)(i) = h(i) + g(i)$$

$$h \odot g = h * g \quad \text{convolution [10, 7]}$$

$$(h * g)(i) = \sum_{p \circ q = i} h(p) \cdot g(q)$$

- 8 A temporal quantity (TQ) a is a function $a : \mathcal{T} \rightarrow A \cup \{\mathbb{H}\}$ where \mathbb{H} denotes the value *undefined*. $(A, +, \cdot, 0, 1)$ is a semiring. The *activity time set* T_a of a consists of instants $t \in \mathcal{T}$ in which a is defined $T_a = \{t \in \mathcal{T} : a(t) \in A\}$.

We can extend both operations to the set $A_{\mathbb{H}} = A \cup \{\mathbb{H}\}$ by requiring that for all $a \in A_{\mathbb{H}}$ it holds $a + \mathbb{H} = \mathbb{H} + a = a$ and $a \cdot \mathbb{H} = \mathbb{H} \cdot a = \mathbb{H}$.

The structure $(A_{\mathbb{H}}, +, \cdot, \mathbb{H}, 1)$ is also a semiring.

Let $A_{\mathbb{H}}(\mathcal{T})$ denote the set of all TQs over $A_{\mathbb{H}}$ in time \mathcal{T} . To extend the operations to networks and their matrices we first define the *sum* (parallel links) $a + b$ as

$$(a + b)(t) = a(t) + b(t) \quad \text{and} \quad T_{a+b} = T_a \cup T_b.$$

The *product* (sequential links) $a \cdot b$ is defined as

$$(a \cdot b)(t) = a(t) \cdot b(t) \quad \text{and} \quad T_{a \cdot b} = T_a \cap T_b.$$

Let us define TQs $\mathbf{0}$ and $\mathbf{1}$ with requirements $\mathbf{0}(t) = \mathbb{K}$ and $\mathbf{1}(t) = 1$ for all $t \in \mathcal{T}$. Again, the structure $(A_{\mathbb{K}}(\mathcal{T}), +, \cdot, \mathbf{0}, \mathbf{1})$ is a semiring.

To produce a software support for computation with TQs we limit it to TQs that can be described as a sequence of disjoint time intervals with a constant value

$$a = [(s_i, f_i, v_i)]_{i \in 1..k}$$

where s_i is the starting time and f_i the finishing time of the i -th time interval $[s_i, f_i)$, $s_i < f_i$ and $f_i \leq s_{i+1}$, and v_i is the value of a on this interval (over combinatorial semiring). Outside the intervals the value of TQ a is undefined, \mathbb{K} .

See also [16].

Another approach based on semirings [14, 11]

Sum and product of temporal quantities

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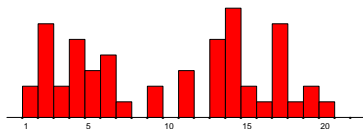
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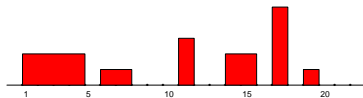
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$a + b :$

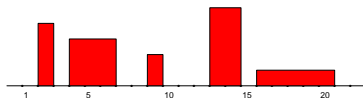


$a :$



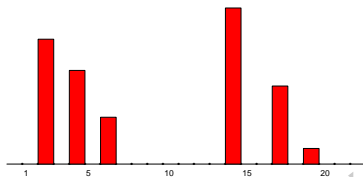
$a = [(1, 5, 2), (6, 8, 1),$
 $(11, 12, 3), (14, 16, 2),$
 $(17, 18, 5), (19, 20, 1)]$

$b :$



$b = [(2, 3, 4), (4, 7, 3),$
 $(9, 10, 2), (13, 15, 5),$
 $(16, 21, 1)]$

$a \cdot b :$



Let $T_V(v) \subseteq \mathcal{T}$, $T_V \in \mathcal{P}$, be the activity set for a node $v \in \mathcal{V}$ and $T_L(\ell) \subseteq \mathcal{T}$, $T_L \in \mathcal{W}$, the activity set for a link $\ell \in \mathcal{L}$. The following *consistency condition* must be fulfilled for activity sets:

If a link $\ell(u, v)$ is active at the time point t then its end-nodes u and v should be active at the time point t :
 $T_L(\ell(u, v)) \subseteq T_V(u) \cap T_V(v)$.

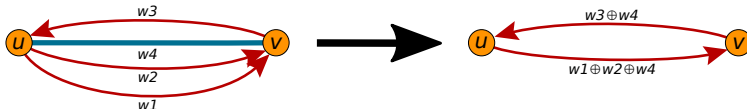
In the following, we will need

- ① *Total*: $\text{total}(a) = \sum_i (f_i - s_i) \cdot v_i$
- ② *Average*: $\text{average}(a) = \frac{\text{total}(a)}{|T_a|}$ where $|T_a| = \sum_i (f_i - s_i)$
- ③ *Maximum*: $\max(a) = \max_i v_i$

To support the computations with TQs we developed in Python the libraries TQ and Nets, see <https://github.com/bavla/TQ>.

Simplification of weighted networks

For a subset $S \subseteq L$ of links, we define its *total* weight as $\Sigma(S) = \bigoplus_{\ell \in S} w(\ell)$.



A *simplification* of a weighted network $\mathcal{N} = (V, L, w)$ over an abelian monoid $(\mathbb{A}, \oplus, 0)$ is its transformation into a *simple* directed weighted network $\hat{\mathcal{N}} = (V, A, \hat{w})$ such that for $L(u, v) = \{\ell \in L : \text{init}(\ell) = u \wedge \text{term}(\ell) = v\}$ it holds $L(u, v) \neq \emptyset \Rightarrow (u, v) \in A \wedge \hat{w}(u, v) = \Sigma(L(u, v))$.

For undirected weighted networks, a special undirected simplification can be defined similarly.



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$$\text{inStar}(v) = \{\ell \in L : \text{term}(\ell) = v\}, \quad \text{outStar}(v) = \{\ell \in L : \text{init}(\ell) = v\}$$

Using the link weight w of the network \mathcal{N} we can compute some interesting node properties such as *in-sum* (weighted indegree):

$$\text{inS}(\mathcal{N}, v) = \Sigma(\text{inStar}(v))$$

and *out-sum* (weighted outdegree):

$$\text{outS}(\mathcal{N}, v) = \Sigma(\text{outStar}(v))$$

of a node v . They are describing the *activity* of the node v .

For a simple weighted network $\mathcal{N} = (V, A, w)$ and a subset of nodes $C \subseteq V$ we can define the *boundaries* of C :

$$\text{inBd}(C) = \{\ell \in A : \text{init}(\ell) \notin C \wedge \text{term}(\ell) \in C\}$$

$$\text{outBd}(C) = \{\ell \in A : \text{init}(\ell) \in C \wedge \text{term}(\ell) \notin C\}$$

$$\text{Bd}(C) = \text{inBd}(C) \cup \text{outBd}(C)$$

the *interior* of C :

$$\text{Int}(C) = \{\ell \in A : \text{init}(\ell) \in C \wedge \text{term}(\ell) \in C\}$$

and the corresponding *activities*:

$$\text{inPut}(C) = \Sigma(\text{inBd}(C)) \quad \text{and} \quad \text{outPut}(C) = \Sigma(\text{outBd}(C))$$

$$\text{Act}(C) = \Sigma(\text{Int}(C))$$

Let the binary *affiliation* matrix $\mathbf{A} = [a_{ep}]$ describe a two-mode network on the set of events E and the set of participants P :

$$a_{ep} = \begin{cases} 1 & p \text{ participated at the event } e \\ 0 & \text{otherwise} \end{cases}$$

The function $d : E \rightarrow \mathcal{T}$ assigns to each event e the date $d(e)$ when it happened. Assume $\mathcal{T} = [first, last] \subset \mathbb{N}$. Using these data we can construct two temporal affiliation matrices [6]:

- **instantaneous** $\mathbf{A}^i = [a^i_{ep}]$, where

$$a^i_{ep} = \begin{cases} [(d(e), d(e) + 1, 1)] & a_{ep} = 1 \\ [] & \text{otherwise} \end{cases}$$

- **cumulative** $\mathbf{A}^c = [a^c_{ep}]$, where

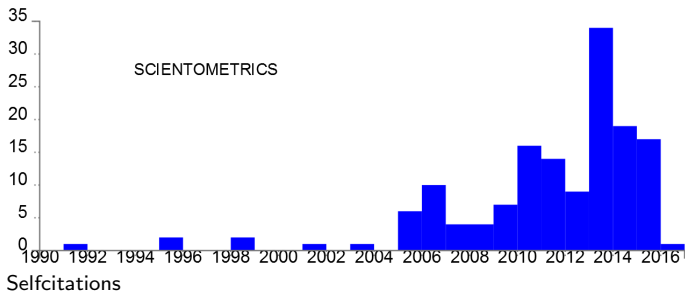
$$a^c_{ep} = \begin{cases} [(d(e), last + 1, 1)] & a_{ep} = 1 \\ [] & \text{otherwise} \end{cases}$$

Networks are from the collection of bibliographic networks on peer review from WoS till 2017.

The derived network describing citations between journals is obtained as

$$JCJ = WJi^T \cdot CiI \cdot WJc$$

Note that the third network in the product is cumulative.



Application – temporal bibliographic networks

Temporal citations between journals

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Semirings

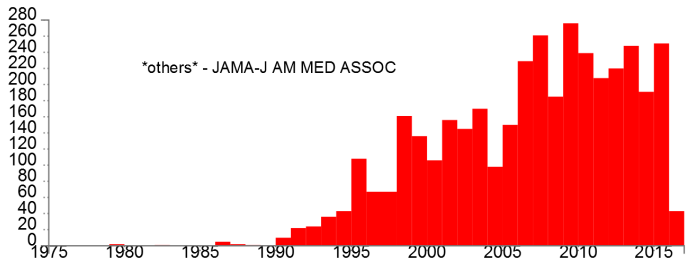
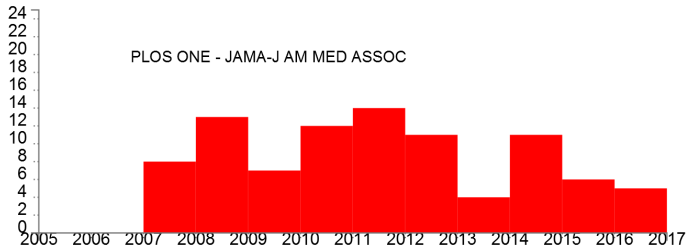
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Citations between journals



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Work in progress.

Most of the approaches to temporal network analysis are cross-sectional – based on time slices. The longitudinal approach is an alternative to be considered.

Collections of interesting, understandable, and well-documented datasets (bikes, world trade, bibliographic data, etc.). Sources of problems and ideas.



Acknowledgments

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The computational work reported in this presentation was performed in Python using libraries TQ and Nets. The code and data are available at <https://github.com/bavla/TQ>.

This work is supported in part by the Slovenian Research Agency (research program P1-0294 and research projects J5-2557, J1-2481, and J5-4596), and prepared within the framework of the COST action CA21163 (HiTEc).



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