



Blockmodeling temporal networks

an indirect approach

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on Zoom, February 24, 2021

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- 2 Temporal blockmodeling
- 3 Clustering of TQs
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Current version of slides (February 24, 2021 at 16:25): [slides PDF](#)

<https://github.com/bavla/TQ/tree/master/docs>

Extended version of Vladimir Batagelj: Toward an indirect approach for blockmodeling temporal networks. Sunbelt (virtual), July 2020

Temporal networks

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Temporal blockmodeling

Clustering of TQs

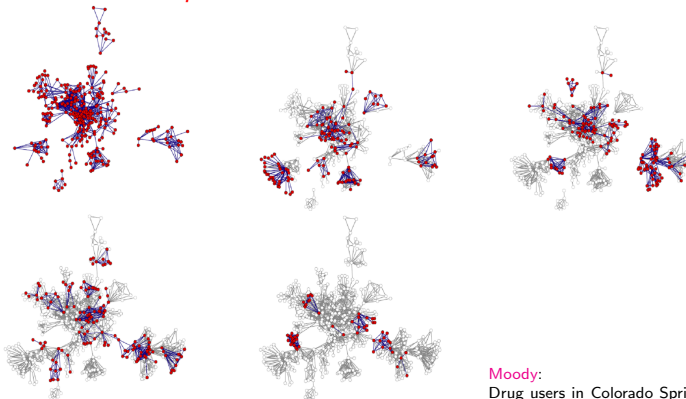
Example:
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In a *temporal network* the presence/activity of node/link can change through time. Pajek supports two types of descriptions of temporal networks based on *presence* and on *events*.



Moody:

Drug users in Colorado Springs, 5 years



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Temporal network

$$\mathcal{N}_T = (\mathcal{V}, \mathcal{L}, \mathcal{P}, \mathcal{W}, T)$$

is obtained if the *time* T is attached to an ordinary network. T is a set of *time points* $t \in T$.

In temporal network nodes $v \in \mathcal{V}$ and links $l \in \mathcal{L}$ are not necessarily present or active in all time points. If a link $l(u, v)$ is active in time point t then also its endnodes u and v should be active in time t .

We will denote the network consisting of links and nodes active in time $t \in T$ by $\mathcal{N}(t)$ and call it a *time slice* in time point t . To get time slices (*cross-sectional*) in Pajek use

Network/Temporal Network/Generate in time



Temporal networks – presence

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```
*Vertices 3
1 "a" [5-10,12-14]
2 "b" [1-3,7]
3 "e" [4-*]
*Edges
1 2 1 [7]
1 3 1 [6-8]
```

Time.net

Node *a* is present in time points 5, 6, 7, 8, 9, 10 and 12, 13, 14.

Edge (1 : 3) is present in time points 6, 7, 8.

* means 'infinity'.

A link is present, if both its endnodes are present.



Temporal networks – events

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Event	Explanation
TI t	initial events – following events happen when time point t starts
TE t	end events – following events happen when time point t is finished
AV $v \ n \ s$	add vertex v with label n and properties s
HV v	hide node v
SV v	show node v
DV v	delete node v
AA $u \ v \ s$	add arc (u, v) with properties s
HA $u \ v$	hide arc (u, v)
SA $u \ v$	show arc (u, v)
DA $u \ v$	delete arc (u, v)
AE $u \ v \ s$	add edge $(u : v)$ with properties s
HE $u \ v$	hide edge $(u : v)$
SE $u \ v$	show edge $(u : v)$
DE $u \ v$	delete edge $(u : v)$
CV $v \ s$	change property of node v to s
CA $u \ v \ s$	change property of arc (u, v) to s
CE $u \ v \ s$	change property of edge $(u : v)$ to s
CT $u \ v$	change (un)directedness of link (u, v)
CD $u \ v$	change direction of arc (u, v)
PE $u \ v \ s$	replace pair of arcs (u, v) and (v, u) by single edge $(u : v)$ with properties s
AP $u \ v \ s$	add pair of arcs (u, v) and (v, u) with properties s
DP $u \ v$	delete pair of arcs (u, v) and (v, u)
EP $u \ v \ s$	replace edge $(u : v)$ by pair of arcs (u, v) and (v, u) with properties s

s can be empty.

In case of parallel links : k denotes the k -th link – HE:3 14 37 hides the third edge linking nodes 14 and 37.

*Vertices 3

*Events

TI 1
AV 2 "b"
TE 3
HV 2
TI 4
AV 3 "e"
TI 5
AV 1 "a"
TI 6
AE 1 3 1
TI 7
SV 2
AE 1 2 1
TE 7
DE 1 2
DV 2
TE 8
DE 1 3
TE 10
HV 1
TI 12
SV 1
TE 14
DV 1

Time.tim. Friends.tim.

File/Network/Read Time Events



Temporal network datasets

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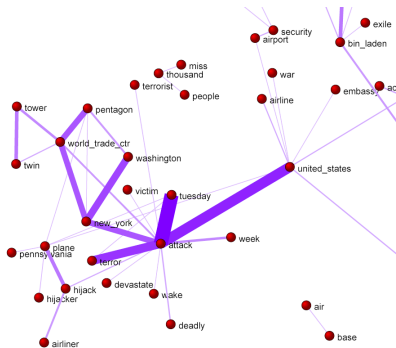
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References

- Linden Strasse (GD99): [paj](#); [html](#)
- KEDS / WEIS: [KEDS](#), [Tabari](#), [KEDS / Gulf](#).
- Reuters terror news network: [net](#)
- Violence ([Roberto Franzosi](#)): [github](#)
- Bibliographic networks: [paper](#)
- ICON: [timestamps](#) [ON](#)
- Network Repository: [Dynamic networks](#)



Steve Corman with collaborators from Arizona State University transformed, using his Centering Resonance Analysis (**CRA**), daily Reuters news (66 days) about September 11th into a temporal network of words coappearance.

Pictures in SVG: **66 days**.



Multi-relational temporal network – KEDS/WEIS

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```
% Recoded by WEISmonths, Sun Nov 28 21:57:00 2004
% from http://www.ku.edu/~keds/data.dir/balk.html
```

```
*vertices 325
```

```
1 "AFG" [1-*
```

```
2 "AFR" [1-*
```

```
3 "ALB" [1-*
```

```
4 "ALBMED" [1-*
```

```
5 "ALG" [1-*
```

```
...
```

```
318 "YUGGOV" [1-*
```

```
319 "YUGMAC" [1-*
```

```
320 "YUGMED" [1-*
```

```
321 "YUGMTN" [1-*
```

```
322 "YUGSER" [1-*
```

```
323 "ZAI" [1-*
```

```
324 "ZAM" [1-*
```

```
325 "ZIM" [1-*
```

```
*arcs :0 "*** ABANDONED"
```

```
*arcs :10 "YIELD"
```

```
*arcs :11 "SURRENDER"
```

```
*arcs :12 "RETREAT"
```

```
...
```

```
*arcs :223 "MIL ENGAGEMENT"
```

```
*arcs :224 "RIOT"
```

```
*arcs :225 "ASSASSINATE TORTURE"
```

```
*arcs
```

```
224: 314 153 1 [4]
```

```
212: 314 83 1 [4]
```

```
224: 3 83 1 [4]
```

```
123: 83 153 1 [4]
```

```
...
```

```
42: 105 63 1 [175]
```

```
212: 295 35 1 [175]
```

```
43: 306 87 1 [175]
```

```
13: 295 35 1 [175]
```

```
121: 295 22 1 [175]
```

```
122: 246 295 1 [175]
```

```
121: 35 295 1 [175]
```

890402	YUG	KSV	224	(RIOT)	RIOT-TORN
890404	YUG	ETHALB	212	(ARREST PERSON)	ALB ETHNIC JAILED
890407	ALB	ETHALB	224	(RIOT)	RIOTS
890408	ETHALB	KSV	123	(INVESTIGATE)	PROBING
030731	GER	CYP	042	(ENDORSE)	GAVE SUPPORT
030731	UNWCT	BOSSER	212	(ARREST PERSON)	SENTENCED TO PRIS
030731	VAT	EUR	043	(RALLY)	RALLIED
030731	UNWCT	BOSSER	013	(RETRACT)	CLEARED
030731	UNWCT	BAL	121	(CRITICIZE)	CHARGES
030731	SER	UNWCT	122	(DENIGRATE)	TESTIFIED
030731	BOSSER	UNWCT	121	(CRITICIZE)	ACCUSED

Kansas Event Data System *KEDS*



Besides activity, also properties of nodes and weights on links can change through time. To describe them (*longitudinal*), we introduced *temporal quantities*

$$a(t) = \begin{cases} a'(t) & t \in T_a \\ \mathbb{X} & t \in \mathcal{T} \setminus T_a \end{cases}$$

where T_a is the *activity time set* of a and $a'(t)$ is the value of a in an instant $t \in T_a$, and \mathbb{X} denotes the value *undefined*.

A temporal quantity (TQ) is described by a sequence

$$\mathbf{a} = [(s_r, f_r, v_r) : r = 1, 2, \dots, k]$$

where $[s_r, f_r)$ determines a time interval and v_r is the value of variable on this interval.

Semirings [6].



Temporal quantities

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TQs have the following advantages:

- TQs work for both discrete and continuous time
- TQs internally (inside operations) adapt to the granularity of data
- the result of a method is usually again a temporal network or a list of TQs.

For describing (temporal) networks with structured values we defined a new format **netsJSON**.

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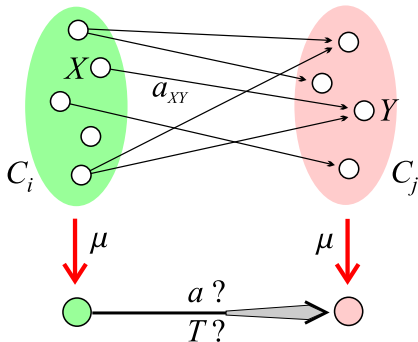
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To the traditional blockmodeling (BM) scheme [10] we add the time dimension. We assume that the network is described using temporal quantities [6] for nodes/links activity/presence, nodes properties and links weights. Then also the BM partition π can be described for each node v with a temporal quantity $\pi(v, t)$ with the meaning:

$\pi(v, t) = i$ means that in time t node v belongs to cluster i .

The clusters $C_i(t) = \{v : \pi(v, t) = i\}$ structure and activity can change through time, but they preserve their identity.



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For the BM μ the clusters are mapped into BM nodes $\mu : C_i \rightarrow [i]$. To determine the BM we still have to specify how the links from C_i to C_j are represented in the BM – in general, for the model arc $([i], [j])$, we have to specify two temporal quantities: its value $a_{ij}(t)$ and, in the case of generalized BM, its type $T_{ij}(t)$. In general the value can be an object of different type than the values of the links in the original temporal network.

To develop a BM method we specify a criterion function $P(\mu)$ measuring the "error" of the BM μ . We can consider additional knowledge by constraining the partitions to a set Φ of feasible partitions. We are searching for a partition $\pi^* \in \Phi$ such that the corresponding BM μ^* minimizes the criterion function $P(\mu)$.

$$\mu^* = \operatorname{argmin}_{\mu: \pi(\mu) \in \Phi} P(\mu)$$



BM time slices

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- $\mathcal{N} \xrightarrow{\text{slice}} \mathcal{N}(t), t = 1, \dots, s$
- $\mathcal{N}(t) \xrightarrow{BM} \mu^*(t), t = 1, \dots, s$
- $\mu^*(t), t = 1, \dots, s \xrightarrow{TQ} \mu^*$

μ^* is a TQ. Identification of clusters !!!

Longitudinal constraints!?



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This is a general framework. In the following years we intend to develop methods case by case.

- constant partition – nodes stay in the same cluster all the time
 - indirect approach based on clustering of temporal quantities
 - temporal version of clustering with relation constraint
 - local optimization of the criterion function P over Φ
 - binary network
 - weighted network
- dynamic partition / types – nodes can move between clusters through time. The details are still to be elaborated.

Clusters evolution

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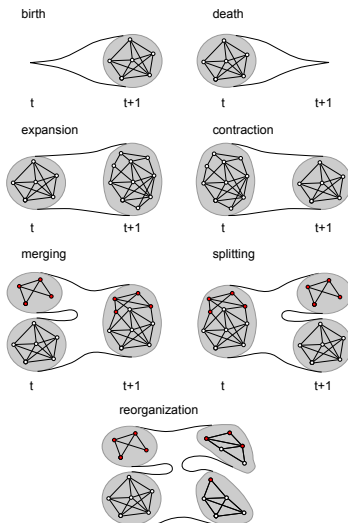
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Temporal cores (V. Batagelj, M. Cerinšek, [AS 2016](#))

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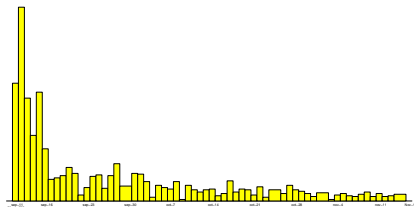
Example:
Terror news

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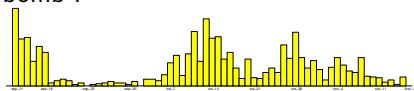
Conclusions

References

hijack:



bomb :



We adapted traditional leaders [11, 9] and agglomerative hierarchical [15, 1] clustering methods for clustering units described by variables that have for their values temporal quantities [13, 5].

ClusTQ at <https://github.com/bavla/TQ> .

Dissimilarities between TQs – many options !!!



Clustering of temporal quantities

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For a unit X_i , each variable V_j is described with a size h_{ij} and a temporal quantity \mathbf{x}_{ij}

$$X_{ij} = (h_{ij}, \mathbf{x}_{ij})$$

In our algorithms we use *normalized* values of temporal variables $V' = (h, \mathbf{p})$ where

$$\mathbf{p} = [(s_r, f_r, p_r) : r = 1, 2, \dots, k] \quad \text{and} \quad p_r = \frac{v_r}{h}$$

In the case, when $h = \text{tot}(\mathbf{x}) = \sum (f_r - s_r) \cdot v_r$, the normalized TQ \mathbf{p} is essentially a probability distribution.

We introduce a *dissimilarity measure* between SOs and T with

$$d(X, T) = \sum_j \alpha_j d_j(\mathbf{x}_j, \mathbf{t}_j), \quad \alpha_j \geq 0, \quad \sum_j \alpha_j = 1$$

where α_j are weights for variables and $d(\mathbf{x}, \mathbf{t}) = u_{\mathbf{x}} \text{tot}(\delta(\mathbf{p}_{\mathbf{x}}, \mathbf{t}))$ where $u_{\mathbf{x}}$ is the weights and δ is a *basic dissimilarity*. We will use $\delta(p, t) = (p - t)^2$. This is a kind of a generalization of the *squared Euclidean distance* [13].

Both methods create cluster representatives that are represented in the same way.



September 11th Reuters terror news

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The Reuters terror news network was obtained from the CRA (Centering Resonance Analysis) networks produced by Steve Corman and Kevin Dooley at Arizona State University. The network is based on all the stories released during 66 consecutive days by the news agency Reuters concerning the September 11 attack on the U.S., beginning at 9:00 AM EST 9/11/01.

The nodes of this network are important words (terms). There is an edge between two words iff they appear in the same utterance (for details see the paper [8]). The weight of an edge is its frequency. The network has $n = 13332$ nodes (different words in the news) and $m = 243447$ edges, 50859 with value larger than 1. There are no loops in the network.

The Reuters terror news network was used as a case network for the Vizards visualization session on the Sunbelt XXII International Sunbelt Social Network Conference, New Orleans, USA, 13-17. February 2002.



Details

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We transformed the Pajek version of the network into NetsJSON format used in TQ and Nets.

For a temporal description of each node/word for clustering we took its activity = sum of all TQs on edges adjacent to a given node.

Our leaders and hierarchical clustering methods are compatible. They are based on the same clustering error criterion function.

Usually the leaders method is used to reduce a large clustering problem to up to some hundred units. With hierarchical clustering of the leaders of the obtained clusters we afterward determine the "right" number of clusters and their representatives.



Clustering

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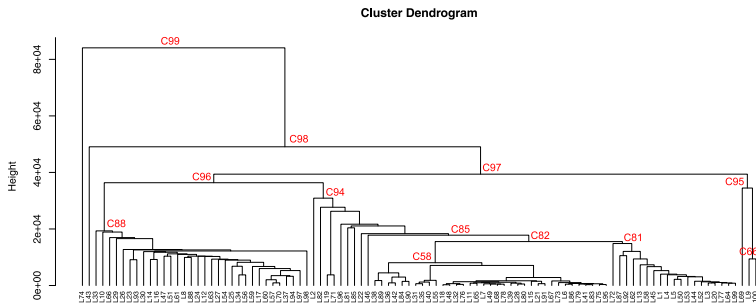
To cluster all 13332 words (nodes) in Terror news we used the adapted leaders method searching for 100 clusters.

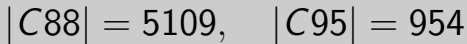
After 50 steps we stopped the search.

$$P_1 = 41012.94, P_{50} = 23897.74$$

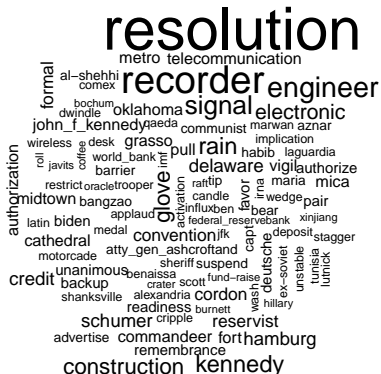
{74: 716, 43: 535, 82: 378, 2: 372, 9: 338, 69: 325, 96: 307, 46: 307, 100: 291, 26: 275, 62: 257, 13: 241, 85: 238, 81: 237, 34: 233, 98: 229, 27: 228, 29: 222, 19: 203, 22: 199, 10: 196, 88: 195, 37: 192, 12: 191, 30: 186, 54: 183, 72: 180, 66: 177, 14: 175, 33: 172, 25: 162, 23: 162, 92: 156, 71: 155, 93: 153, 87: 152, 51: 151, 24: 150, 16: 148, 58: 146, 59: 146, 47: 145, 63: 143, 61: 141, 4: 139, 45: 132, 89: 130, 8: 128, 17: 126, 56: 114, 50: 105, 77: 102, 42: 101, 1: 96, 55: 96, 97: 95, 53: 91, 94: 89, 5: 87, 70: 83, 90: 81, 44: 76, 3: 75, 60: 74, 40: 73, 52: 64, 15: 63, 84: 56, 20: 51, 57: 45, 64: 44, 67: 44, 65: 43, 21: 39, 31: 37, 11: 37, 39: 36, 79: 35, 38: 35, 36: 34, 76: 32, 91: 32, 99: 32, 80: 31, 73: 31, 68: 31, 7: 29, 6: 29, 35: 29, 28: 28, 49: 27, 86: 26, 78: 23, 83: 18, 41: 18, 32: 18, 95: 15, 18: 14, 48: 13, 75: 12}

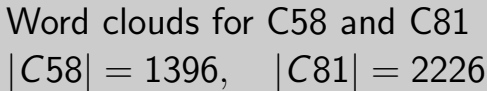
We continued with hierarchical clustering of the obtained leaders.





References




$$|C58| = 1396, \quad |C81| = 2226$$

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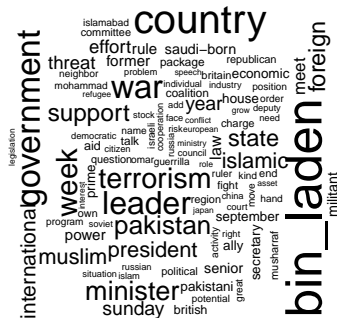
Clustering of TQs

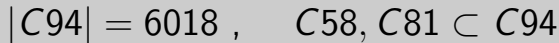
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[illegible]

pres_bush



Comparisons of leaders and cluster representatives

L74:C98, L43:C97, C96:C95, C88:C94

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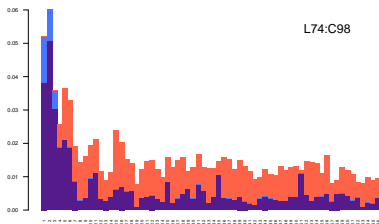
Clustering of
TQs

Example:
Terror news

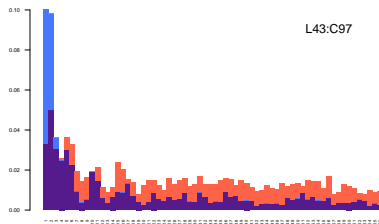
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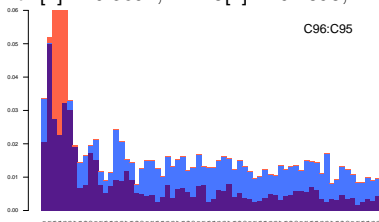


L74:C98

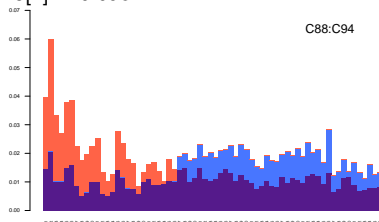


L43:C97

$$L74[2] = 0.5894, \quad L43[1] = 0.4393, \quad L43[2] = 0.0981$$



C96:C95



C88:C94

$$C95[3] = 0.1665, \quad C95[4] = 0.1570, \quad C95[5] = 0.2250$$



Comparisons of leaders and cluster representatives

C58:C81, L96:C66, C46:C955, C46

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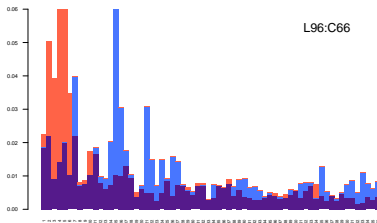
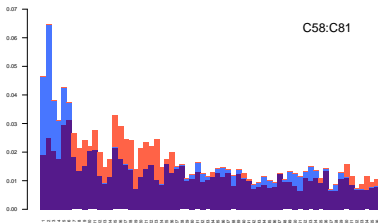
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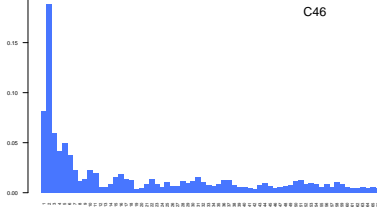
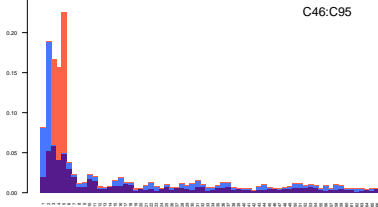
Conclusions

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$L96[15] = 0.3524$, $C66[4] = 0.1961$,

$C66[5] = 0.2917$



To produce a block model we have to specify the values on its links. Because the original network is undirected, so is the BM. There are different options.

Temporal quantities:

$$\mathbf{a}([i] : [j]) = \text{activity}(C_i, C_j) = \sum_{u \in C_i, v \in C_j} \mathbf{a}(u : v), \quad \text{for } i \neq j$$

and $\mathbf{a}([i] : [i]) = \frac{1}{2} \text{activity}(C_i, C_i) .$

Total intensities: $a([i] : [j]) = \text{tot}(\mathbf{a}([i] : [j])) .$

Geometric average intensities: $g([i] : [j]) = \frac{a([i] : [j])}{\sqrt{|C_i| \cdot |C_j|}} .$



Block model

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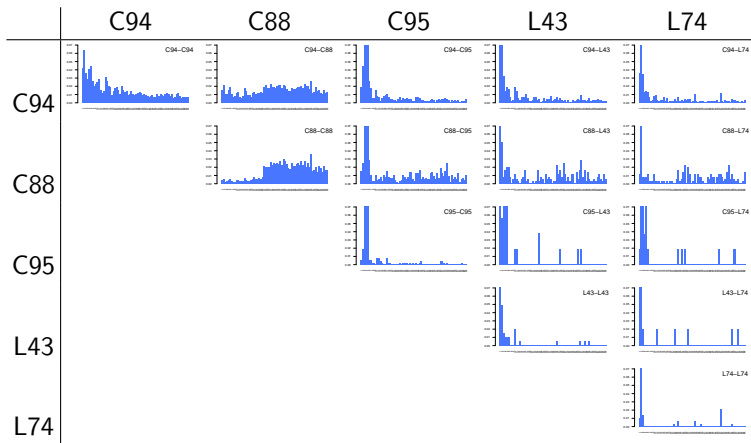
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Temporal quantities:





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Total intensities:

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<i>i</i>	<i>cluster</i>	<i>size</i>	1	2	3	4	5
1	C94	6018	143549	67801	5422	2816	2939
2	C88	5109	0	18288	739	357	357
3	C95	954	0	0	535	53	54
4	L43	535	0	0	0	205	51
5	L74	716	0	0	0	0	281

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Example:
Terror news

Geometric average intensities:

Block model

<i>i</i>	<i>cluster</i>	1	2	3	4	5
1	C94	23.85	12.23	2.26	1.57	1.42
2	C88	0	3.58	0.33	0.22	0.19
3	C95	0	0	0.56	0.07	0.07
4	L43	0	0	0	0.38	0.08
5	L74	0	0	0	0	0.39

Conclusions

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Block model

Geometric average intensities, cut at level 0.3

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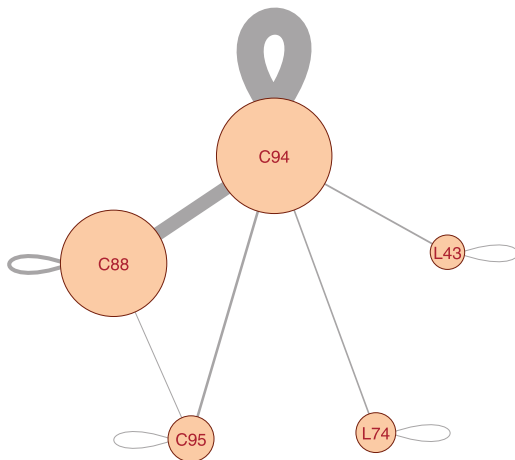
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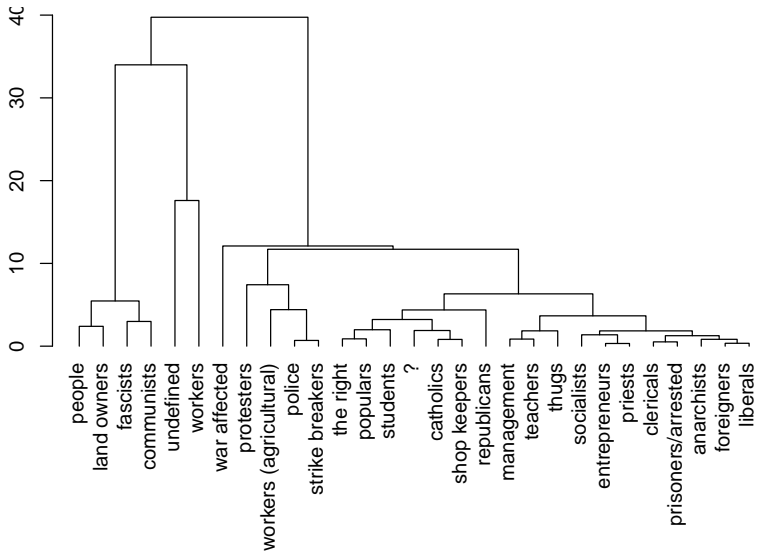
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References

- only started
- how to present/display the temporal BMs ?
- compatibility of clustering criterion function P and BM μ



Acknowledgments

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