# Blockmodeling temporal networks

an indirect approach

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**1294. Sredin seminar**
on Zoom, February 24, 2021

# Outline







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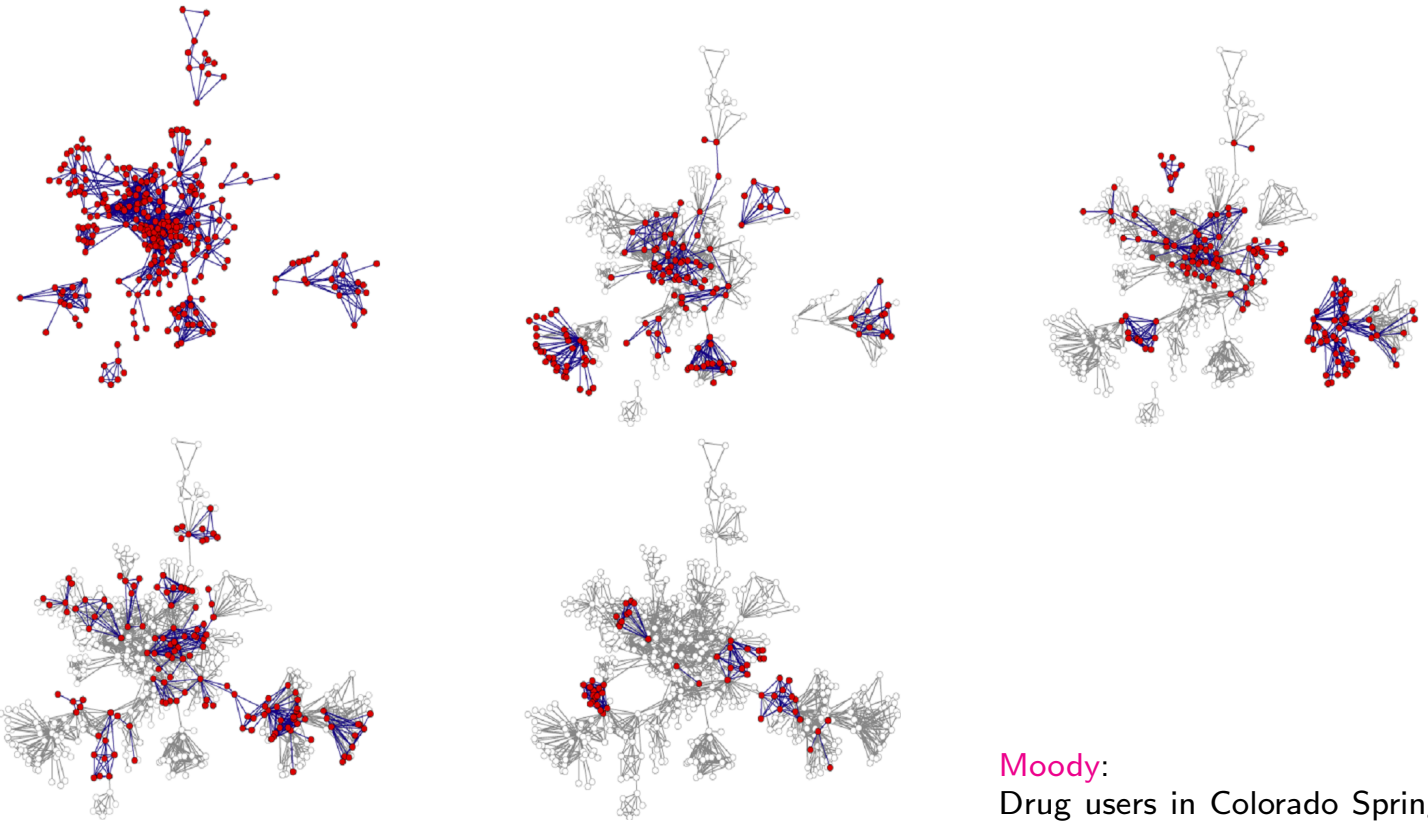
**Current version of slides (February 24, 2021 at 16:07):** slides PDF
https://github.com/bavla/TQ/tree/master/docs
Extended version of Vladimir Batagelj: Toward an indirect approach for blockmodeling temporal networks. Sunbelt (virtual), July 2020

# Temporal networks



In a *temporal network* the presence/activity of node/link can change through time. `Pajek` supports two types of descriptions of temporal networks based on *presence* and on *events*.



Moody:
Drug users in Colorado Springs, 5 years

# Temporal network

*Temporal network*

$$\mathcal{N}_T = (\mathcal{V}, \mathcal{L}, \mathcal{P}, \mathcal{W}, T)$$

is obtained if the *time* $T$ is attached to an ordinary network. $T$ is a set of *time points* $t \in T$.

In temporal network nodes $v \in \mathcal{V}$ and links $l \in \mathcal{L}$ are not necessarily present or active in all time points. If a link $l(u, v)$ is active in time point $t$ then also its endnodes $u$ and $v$ should be active in time $t$.

We will denote the network consisting of links and nodes active in time $t \in T$ by $\mathcal{N}(t)$ and call it a *time slice* in time point $t$. To get time slices (*cross-sectional*) in `Pajek` use

`Network/Temporal Network/Generate in time`

# Temporal networks – presence

```
*Vertices 3
1 "a" [5-10,12-14]
2 "b" [1-3,7]
3 "e" [4-*]
*Edges
1 2 1 [7]
1 3 1 [6-8]
```

Time.net

Node *a* is present in time points 5, 6, 7, 8, 9, 10 and 12, 13, 14.

Edge (1 : 3) is present in time points 6, 7, 8.

$*$ means 'infinity'.

**A link is present, if both its endnodes are present.**

# Temporal networks – events



| Event | Explanation |
|---|---|
| TI $t$ | initial events – following events happen when time point $t$ starts |
| TE $t$ | end events – following events happen when time point $t$ is finished |
| AV $v$ $n$ $s$ | add vertex $v$ with label $n$ and properties $s$ |
| HV $v$ | hide node $v$ |
| SV $v$ | show node $v$ |
| DV $v$ | delete node $v$ |
| AA $u$ $v$ $s$ | add arc $(u, v)$ with properties $s$ |
| HA $u$ $v$ | hide arc $(u, v)$ |
| SA $u$ $v$ | show arc $(u, v)$ |
| DA $u$ $v$ | delete arc $(u, v)$ |
| AE $u$ $v$ $s$ | add edge $(u : v)$ with properties $s$ |
| HE $u$ $v$ | hide edge $(u : v)$ |
| SE $u$ $v$ | show edge $(u : v)$ |
| DE $u$ $v$ | delete edge $(u : v)$ |
| CV $v$ $s$ | change property of node $v$ to $s$ |
| CA $u$ $v$ $s$ | change property of arc $(u, v)$ to $s$ |
| CE $u$ $v$ $s$ | change property of edge $(u : v)$ to $s$ |
| CT $u$ $v$ | change (un)directedness of link $(u, v)$ |
| CD $u$ $v$ | change direction of arc $(u, v)$ |
| PE $u$ $v$ $s$ | replace pair of arcs $(u, v)$ and $(v, u)$ by single edge $(u : v)$ with properties $s$ |
| AP $u$ $v$ $s$ | add pair of arcs $(u, v)$ and $(v, u)$ with properties $s$ |
| DP $u$ $v$ | delete pair of arcs $(u, v)$ and $(v, u)$ |
| EP $u$ $v$ $s$ | replace edge $(u : v)$ by pair of arcs $(u, v)$ and $(v, u)$ with properties $s$ |

$s$ can be empty.
In case of parallel links : $k$ denotes the $k$-th link – `HE:3 14 37` hides the third edge linking nodes 14 and 37.

```
*Vertices 3
*Events
TI 1
AV 2 "b"
TE 3
HV 2
TI 4
AV 3 "e"
TI 5
AV 1 "a"
TI 6
AE 1 3 1
TI 7
SV 2
AE 1 2 1
TE 7
DE 1 2
DV 2
TE 8
DE 1 3
TE 10
HV 1
TI 12
SV 1
TE 14
DV 1
```

Time.tim. Friends.tim.

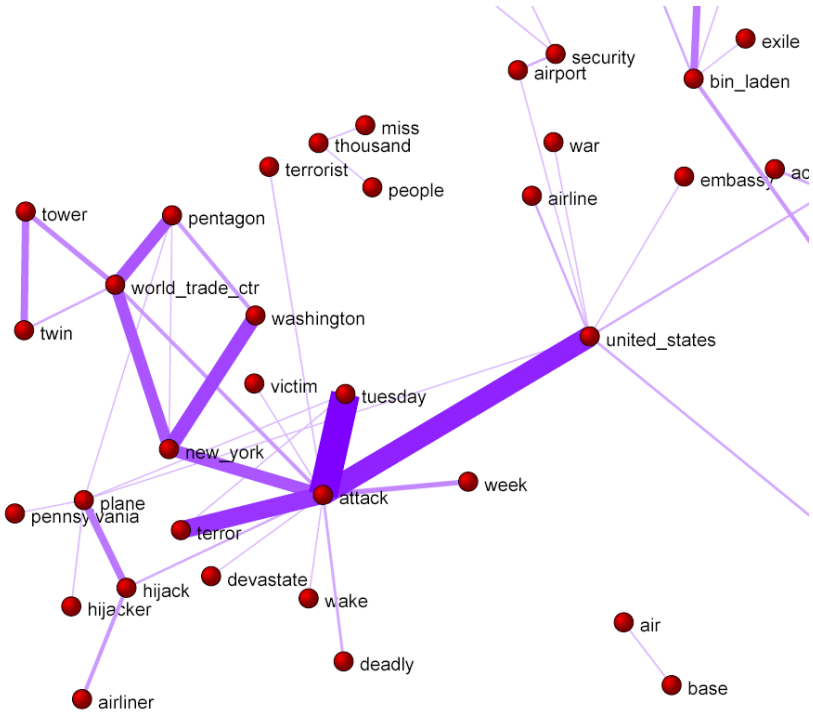
File/Network/Read Time Events

# Temporal network datasets

- Linden Strasse (GD99): paj; html
- KEDS / WEIS: *KEDS*, *Tabari*, KEDS / Gulf.
- Reuters terror news network: net
- Violence (Roberto Franzosi): github
- Bibliographic networks: paper
- ICON:timestamps ON
- Network Repository: Dynamic networks

# Temporal networks / September 11






Pictures in SVG: **66 days**.

Steve Corman with collaborators from Arizona State University transformed, using his Centering Resonance Analysis (**CRA**), daily Reuters news (66 days) about September 11th into a temporal network of words coappearance.

# Multi-relational temporal network – KEDS/WEIS



```
% Recoded by WEISmonths, Sun Nov 28 21:57:00 2004
% from http://www.ku.edu/~keds/data.dir/balk.html
*vertices 325
1 "AFG" [1-*]
2 "AFR" [1-*]
3 "ALB" [1-*]
4 "ALBMED" [1-*]
5 "ALG" [1-*]
 ...
318 "YUGGOV" [1-*]
319 "YUGMAC" [1-*]
320 "YUGMED" [1-*]
321 "YUGMTN" [1-*]
322 "YUGSER" [1-*]
323 "ZAI" [1-*]
324 "ZAM" [1-*]
325 "ZIM" [1-*]
*arcs :0 "*** ABANDONED"
*arcs :10 "YIELD"
*arcs :11 "SURRENDER"
*arcs :12 "RETREAT"
 ...
*arcs :223 "MIL ENGAGEMENT"
*arcs :224 "RIOT"
*arcs :225 "ASSASSINATE TORTURE"
*arcs
224: 314 153 1 [4]                  890402  YUG     KSV     224  (RIOT)  RIOT-TORN
212: 314 83 1 [4]                   890404  YUG     ETHALB  212  (ARREST PERSON) ALB ETHNIC JAILED
224: 3 83 1 [4]                     890407  ALB     ETHALB  224  (RIOT)  RIOTS
123: 83 153 1 [4]                   890408  ETHALB  KSV     123  (INVESTIGATE)   PROBING
 ...
42: 105 63 1 [175]                  030731  GER     CYP     042  (ENDORSE)       GAVE SUPPORT
212: 295 35 1 [175]                 030731  UNWCT   BOSSER  212  (ARREST PERSON) SENTENCED TO PRIS
43: 306 87 1 [175]                  030731  VAT     EUR     043  (RALLY) RALLIED
13: 295 35 1 [175]                  030731  UNWCT   BOSSER  013  (RETRACT)       CLEARED
121: 295 22 1 [175]                 030731  UNWCT   BAL     121  (CRITICIZE)     CHARGES
122: 246 295 1 [175]                030731  SER     UNWCT   122  (DENIGRATE)     TESTIFIED
121: 35 295 1 [175]                 030731  BOSSER  UNWCT   121  (CRITICIZE)     ACCUSED
```

Kansas Event Data System *KEDS*

# Temporal quantities



Besides activity, also properties of nodes and weights on links can change through time. To describe them (*longitudinal*), we introduced *temporal quantities*

$$a(t) = \begin{cases} a'(t) & t \in T_a \\ \text{⌘} & t \in \mathcal{T} \setminus T_a \end{cases}$$

where $T_a$ is the *activity time set* of $a$ and $a'(t)$ is the value of $a$ in an instant $t \in T_a$, and ⌘ denotes the value *undefined*.
A temporal quantity (TQ) is described by a sequence

$$\mathbf{a} = [(s_r, f_r, v_r) : r = 1, 2, \ldots, k]$$

where $[s_r, f_r)$ determines a time interval and $v_r$ is the value of variable on this interval.

Semirings [6].

# Temporal quantities

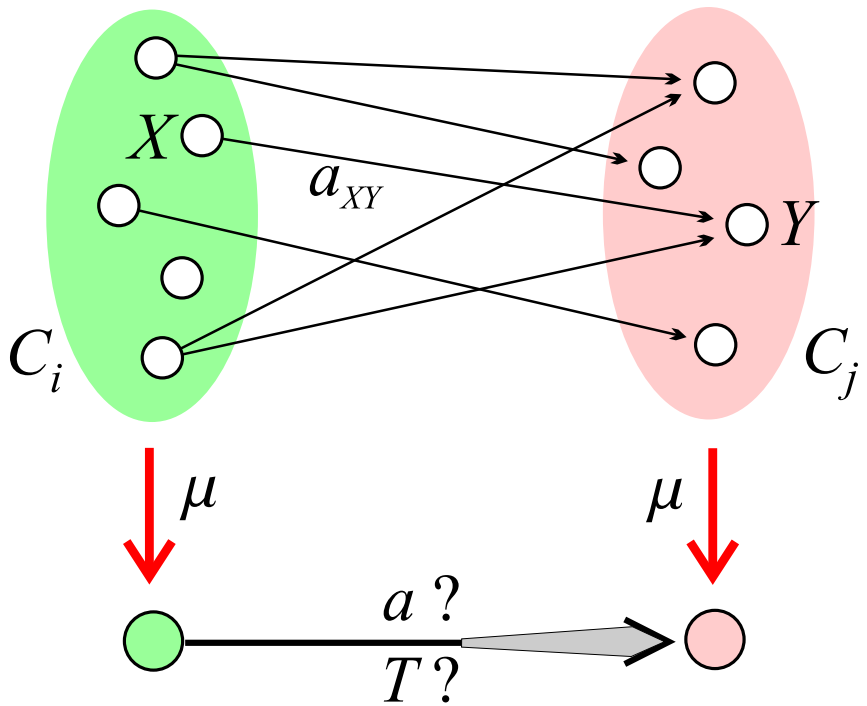
TQs have the following advantages:

- TQs work for both discrete and continuous time
- TQs internally (inside operations) adapt to the granularity of data
- the result of a method is usually again a temporal network or a list of TQs.

For describing (temporal) networks with structured values we defined a new format netsJSON.

# Blockmodeling temporal networks






To the traditional blockmodeling (BM) scheme [10] we add the time dimension. We assume that the network is described using temporal quantities [6] for nodes/links activity/presence, nodes properties and links weights. Then also the BM partition $\pi$ can be described for each node $v$ with a temporal quantity $\pi(v,t)$ with the meaning:

$\pi(v,t) = i$ means that in time $t$ node $v$ belongs to cluster $i$.

The clusters $C_i(t) = \{v : \pi(v,t) = i\}$ structure and activity can change through time, but they preserve their identity.

# Blockmodeling temporal networks



For the BM $\mu$ the clusters are maped into BM nodes $\mu : C_i \rightarrow [i]$. To determine the BM we still have to specify how the links from $C_i$ to $C_j$ are represented in the BM – in general, for the model arc $([i], [j])$, we have to specify two temporal quantities: its value $a_{ij}(t)$ and, in the case of generalized BM, its type $T_{ij}(t)$. In general the value can be an object of different type than the values of the links in the original temporal network.

To develop a BM method we specify a criterion function $P(\mu)$ measuring the "error" of the BM $\mu$. We can consider additional knowledge by constraining the partitions to a set $\Phi$ of feasible partitions. We are searching for a partition $\pi^* \in \Phi$ such that the corresponding BM $\mu^*$ minimizes the criterion function $P(\mu)$.

$$\mu^* = \text{argmin}_{\mu:\pi(\mu)\in\Phi} P(\mu)$$

# BM time slices

- $\mathcal{N} \xrightarrow{slice} \mathcal{N}(t), t = 1, \ldots, s$
- $\mathcal{N}(t) \xrightarrow{BM} \mu^*(t), t = 1, \ldots, s$
- $\mu^*(t), t = 1, \ldots, s \xrightarrow{TQ} \mu^*$

$\mu^*$ is a TQ. Identification of clusters !!!

Longitudinal constraints!?

# Blockmodeling temporal networks

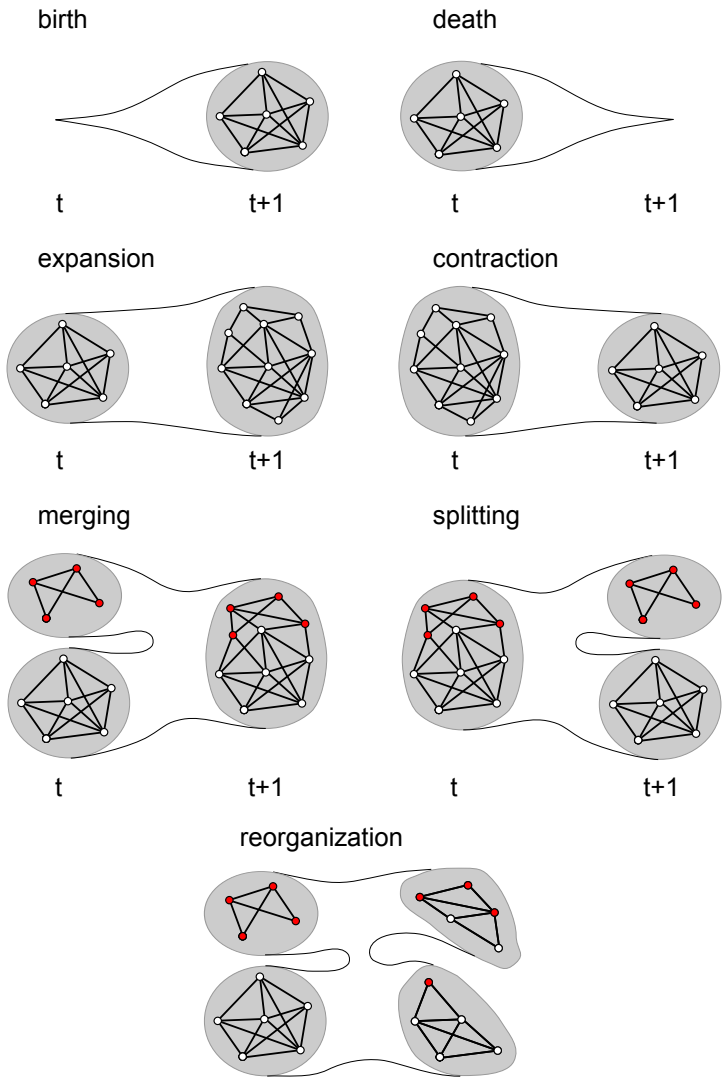


This is a general framework. In the following years we intend to develop methods case by case.

- constant partition – nodes stay in the same cluster all the time
  - indirect approach based on clustering of temporal quantities
  - temporal version of clustering with relation constraint
  - local optimization of the criterion function $P$ over $\Phi$
    - binary network
    - weighted network
- dynamic partition / types – nodes can move between clusters through time. The details are still to be elaborated.

# Clusters evolution



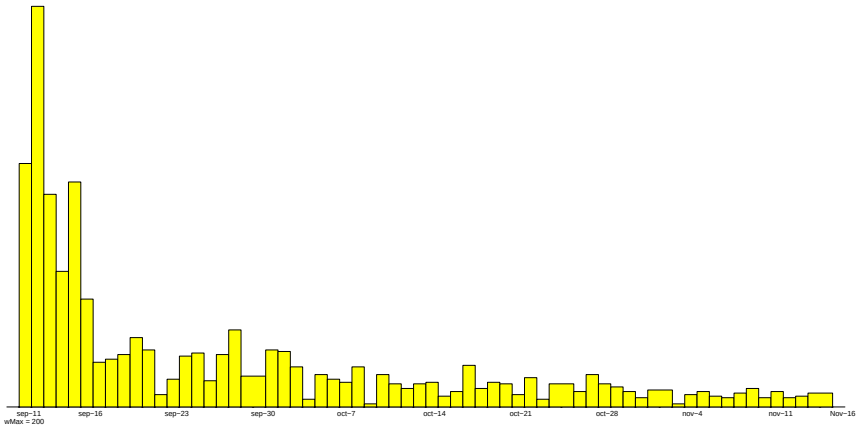



Temporal cores (V. Batagelj, M. Cerinšek, AS 2016)

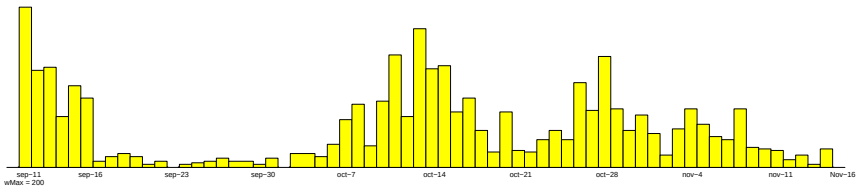
# Clustering of temporal quantities



hijack:




bomb :




We adapted traditional leaders [11, 9] and agglomerative hierarchical [15, 1] clustering methods for clustering units described by variables that have for their values temporal quantities [13, 5].

ClusTQ at https://github.com/bavla/TQ .

Dissimilarities between TQs – many options !!!

# Clustering of temporal quantities

For a unit $X_i$, each variable $V_j$ is described with a size $h_{ij}$ and a temporal quantity $\mathbf{x}_{ij}$

$$X_{ij} = (h_{ij}, \mathbf{x}_{ij})$$

In our algorithms we use *normalized* values of temporal variables $V' = (h, \mathbf{p})$ where

$$\mathbf{p} = [(s_r, f_r, p_r) : r = 1, 2, \ldots, k] \qquad \text{and} \qquad p_r = \frac{v_r}{h}$$

In the case, when $h = \mathrm{tot}(\mathbf{x}) = \sum v_r$, the normalized TQ $\mathbf{p}$ is essentially a probability distribution.

We introduce a *dissimilarity measure* between SOs and $T$ with $d(X, T) = \sum_j \alpha_j d_j(\mathbf{x}_j, \mathbf{t}_j), \quad \alpha_j \geq 0, \quad \sum_j \alpha_j = 1$ where $\alpha_j$ are weights for variables and $d(\mathbf{x}, \mathbf{t}) = u_{\mathbf{x}} \mathrm{tot}(\delta(\mathbf{p}_{\mathbf{x}}, \mathbf{t}))$ where $u_{\mathbf{x}}$ is the weights and $\delta$ is a *basic dissimilarity*. We will use $\delta(p, t) = (p - t)^2$. This is a kind of a generalization of the *squared Euclidean distance* [13].
Both methods create cluster representatives that are represented in the same way.

# September 11th Reuters terror news

The Reuters terror news network was obtained from the CRA (Centering Resonance Analysis) networks produced by Steve Corman and Kevin Dooley at Arizona State University. The network is based on all the stories released during 66 consecutive days by the news agency Reuters concerning the September 11 attack on the U.S., beginning at 9:00 AM EST 9/11/01.

The nodes of this network are important words (terms). There is an edge between two words iff they appear in the same utterance (for details see the paper [8]). The weight of an edge is its frequency. The network has $n = 13332$ nodes (different words in the news) and $m = 243447$ edges, 50859 with value larger than 1. There are no loops in the network.

The Reuters terror news network was used as a case network for the Viszards visualization session on the Sunbelt XXII International Sunbelt Social Network Conference, New Orleans, USA, 13-17. February 2002.

# Details

We transformed the Pajek version of the network into NetsJSON format used in TQ and Nets.

For a temporal description of each node/word for clustering we took its activity = sum of all TQs on edges adjacent to a given node.

Our leaders and hierarchical clustering methods are compatible. They are based on the same clustering error criterion function.

Usually the leaders method is used to reduce a large clustering problem to up to some hundred units. With hierarchical clustering of the leaders of the obtained clusters we afterward determine the "right" number of clusters and their representatives.

# Clustering

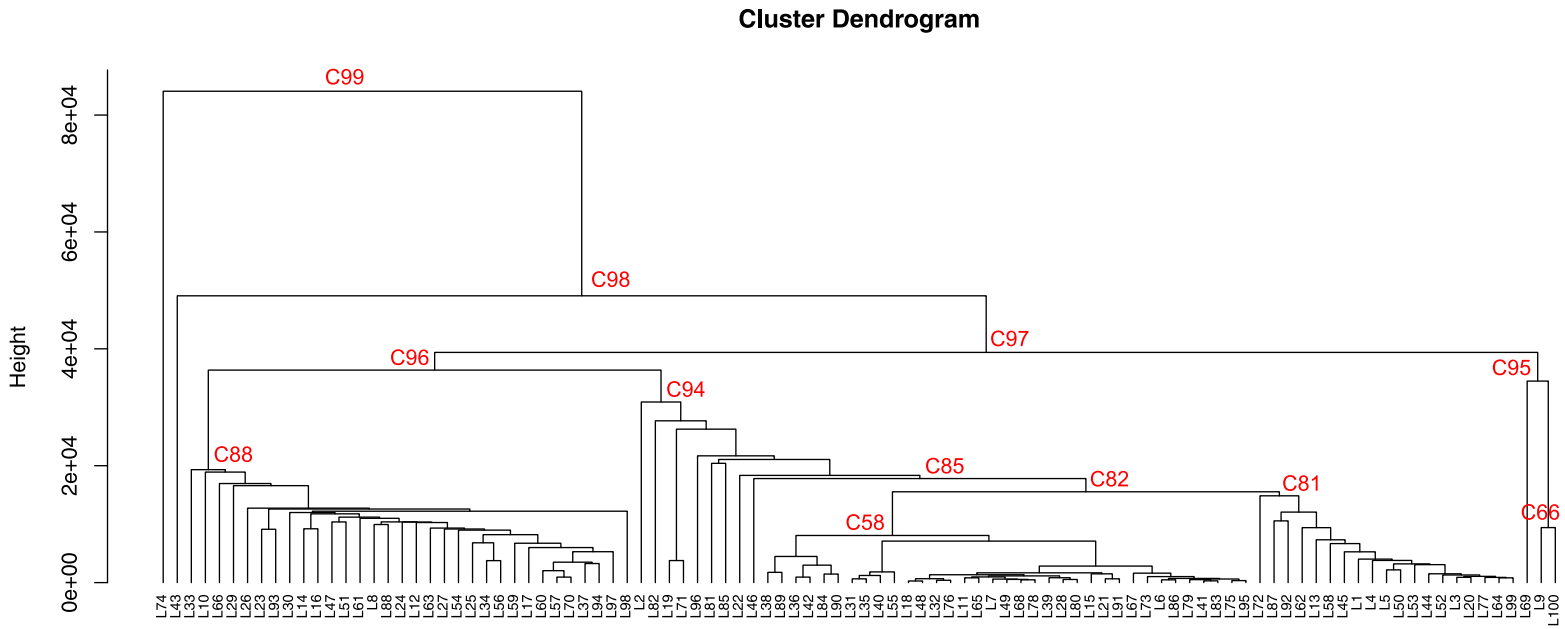
To cluster all 13332 words (nodes) in Terror news we used the adapted leaders method searching for 100 clusters.
After 50 steps we stopped the search.
$P_1 = 41012.94$, $P_{50} = 23897.74$

```
{74: 716, 43: 535, 82: 378, 2: 372, 9: 338, 69: 325, 96: 307, 46: 307, 100: 291, 26: 275,
 62: 257, 13: 241, 85: 238, 81: 237, 34: 233, 98: 229, 27: 228, 29: 222, 19: 203, 22: 199,
 10: 196, 88: 195, 37: 192, 12: 191, 30: 186, 54: 183, 72: 180, 66: 177, 14: 175, 33: 172,
 25: 162, 23: 162, 92: 156, 71: 155, 93: 153, 87: 152, 51: 151, 24: 150, 16: 148, 58: 146,
 59: 146, 47: 145, 63: 143, 61: 141, 4: 139, 45: 132, 89: 130, 8: 128, 17: 126, 56: 114,
 50: 105, 77: 102, 42: 101, 1: 96, 55: 96, 97: 95, 53: 91, 94: 89, 5: 87, 70: 83, 90: 81,
 44: 76, 3: 75, 60: 74, 40: 73, 52: 64, 15: 63, 84: 56, 20: 51, 57: 45, 64: 44, 67: 44,
 65: 43, 21: 39, 31: 37, 11: 37, 39: 36, 79: 35, 38: 35, 36: 34, 76: 32, 91: 32, 99: 32,
 80: 31, 73: 31, 68: 31, 7: 29, 6: 29, 35: 29, 28: 28, 49: 27, 86: 26, 78: 23, 83: 18,
 41: 18, 32: 18, 95: 15, 18: 14, 48: 13, 75: 12}
```

We continued with hierarchical clustering of the obtained leaders.

# Hierarchical clustering

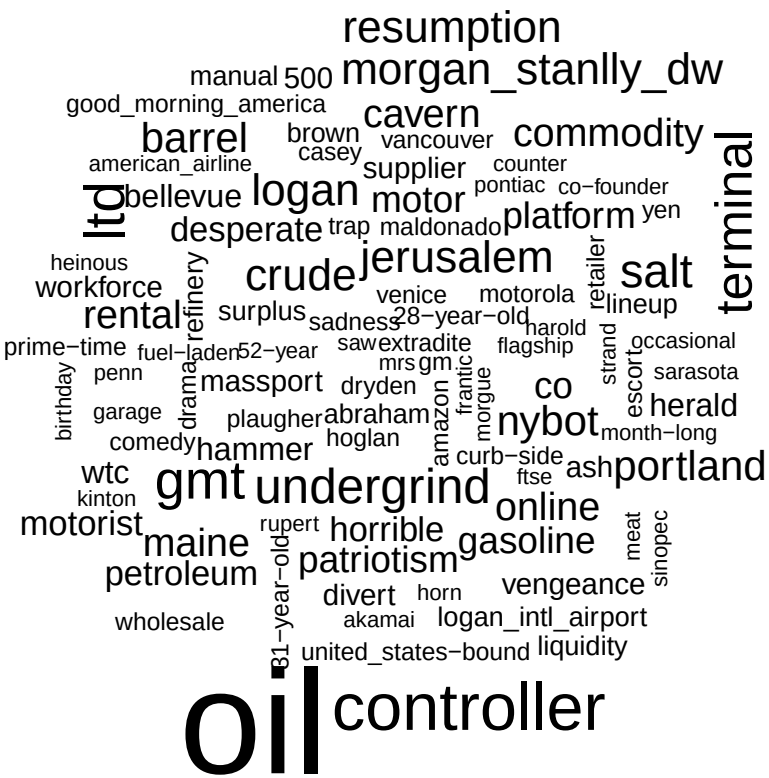


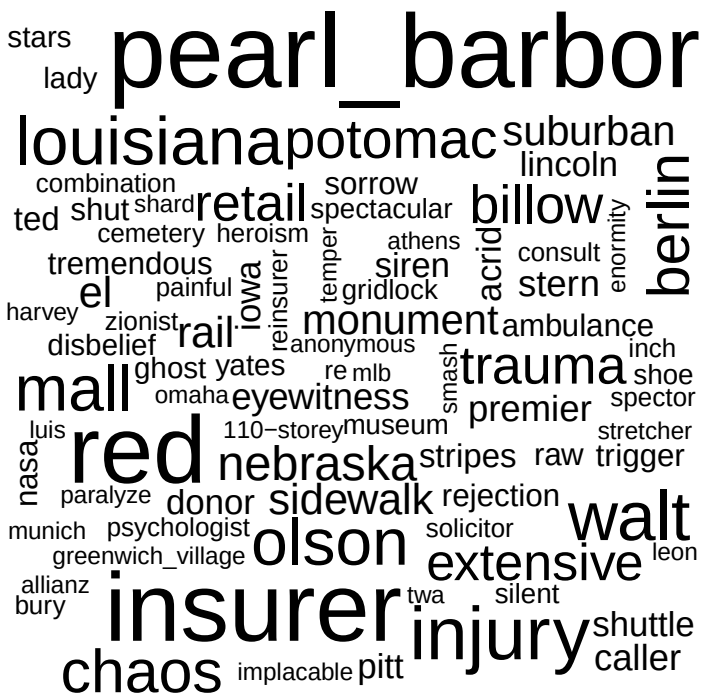


# Word clouds for L74 and L43

$|L74| = 716, \quad |L43| = 535$



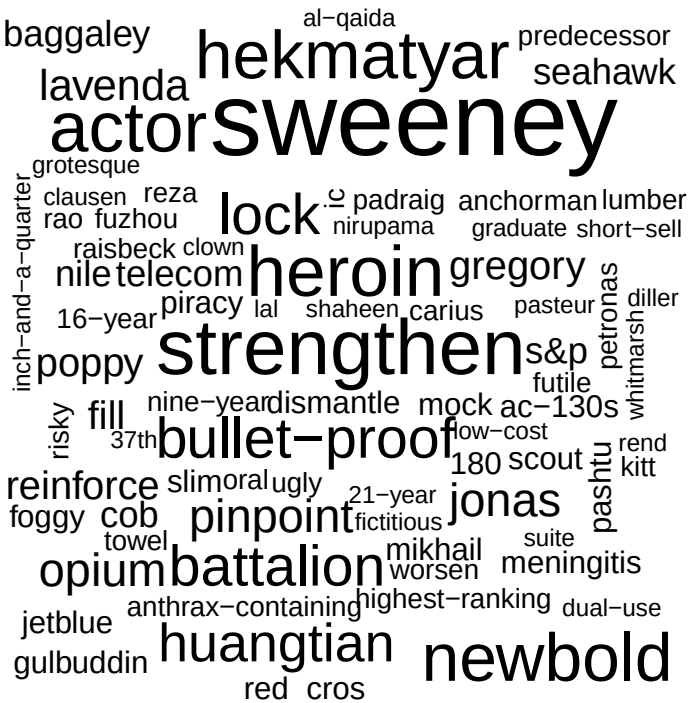


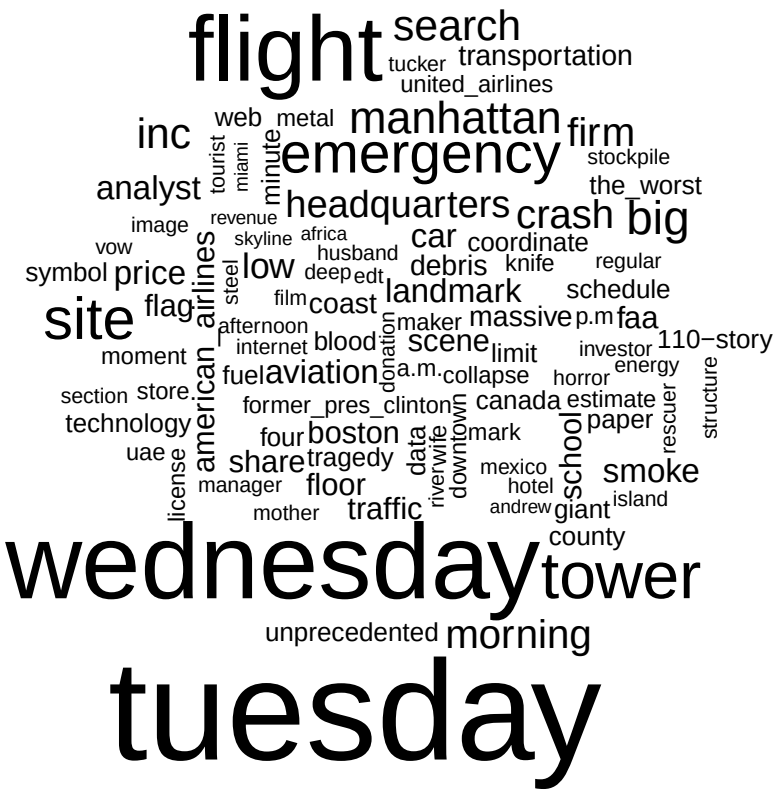



# Word clouds for L46 and C46

$|L46| = 307, \quad |C46| = 358$



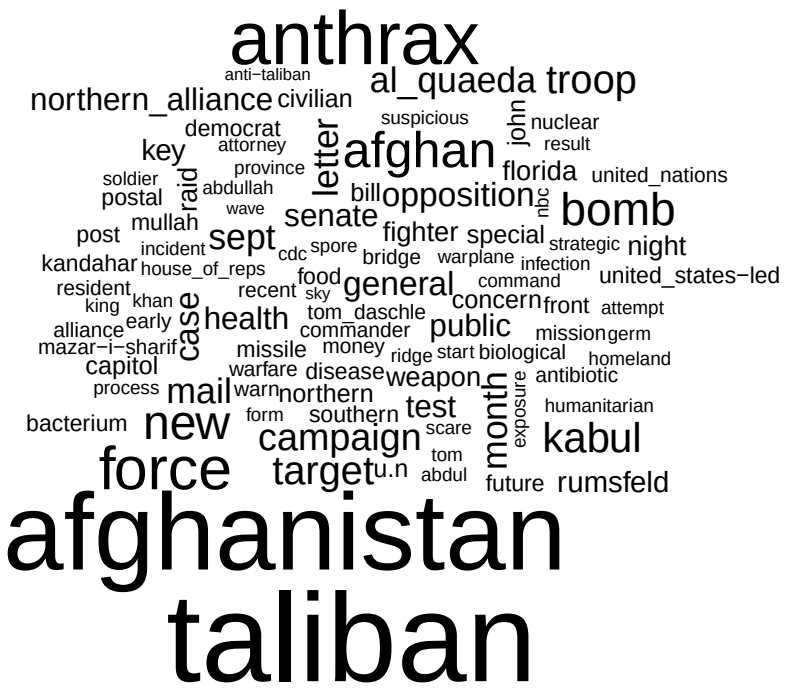


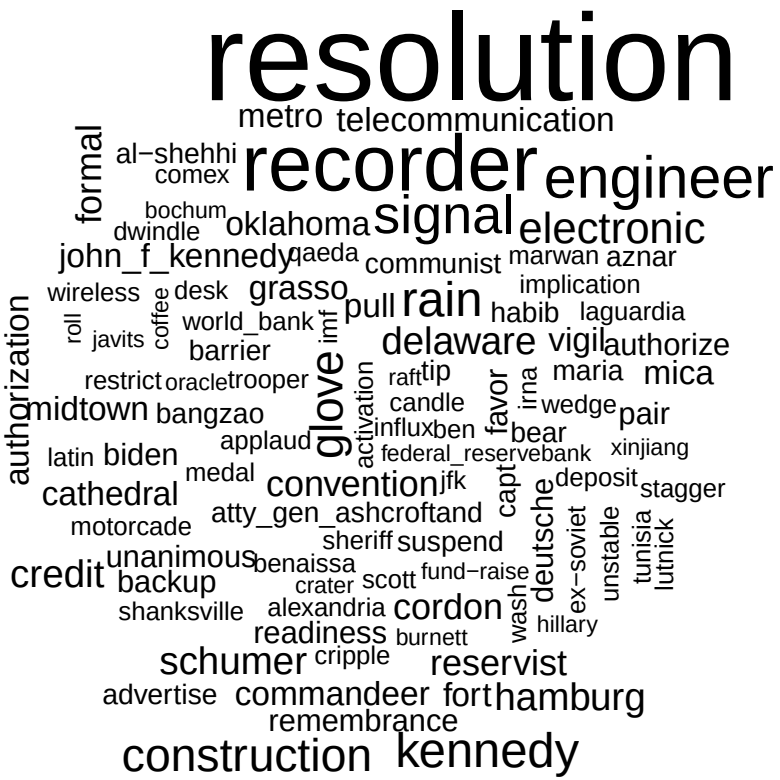



# Word clouds for C88, C95

$|C88| = 5109, \quad |C95| = 954$



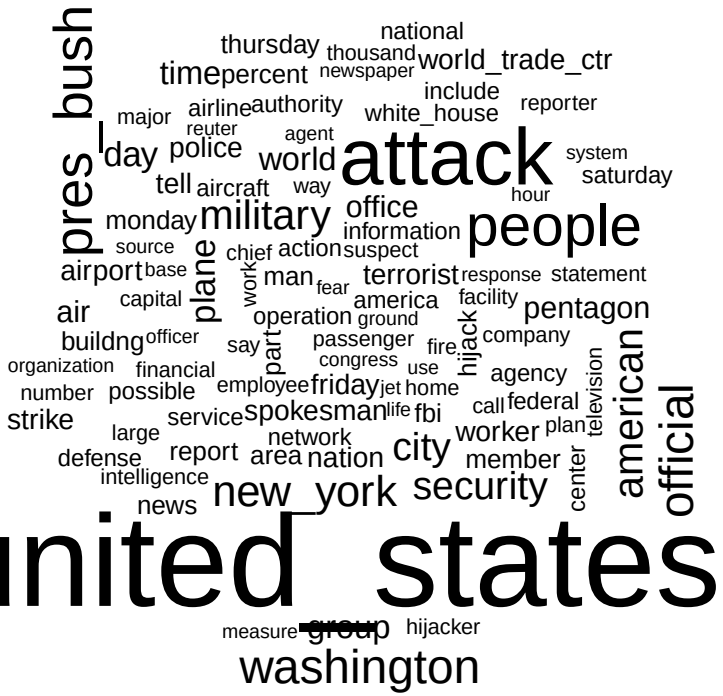


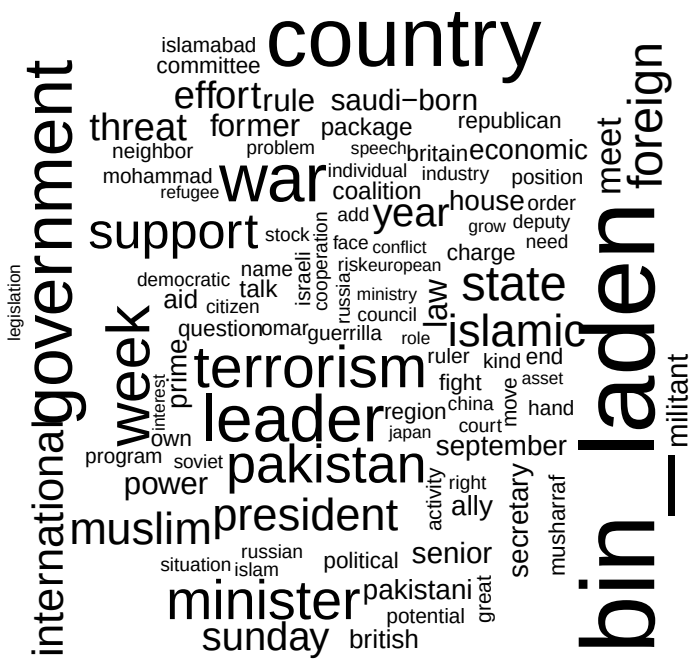



# Word clouds for C58 and C81

$|C58| = 1396, \quad |C81| = 2226$





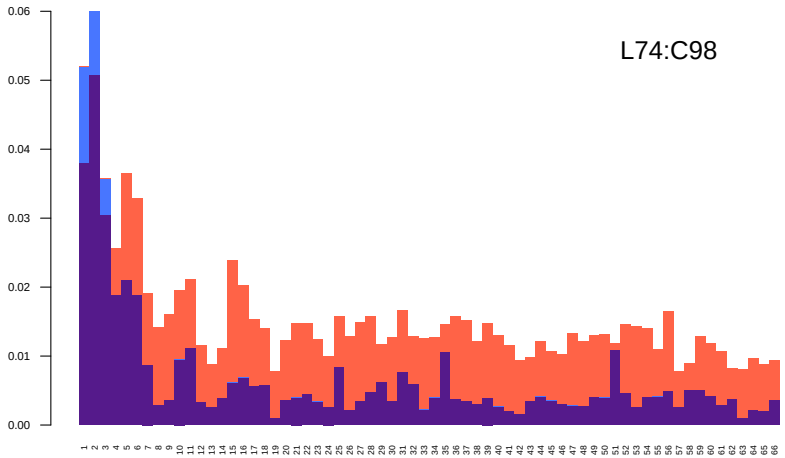
# Word cloud for C94

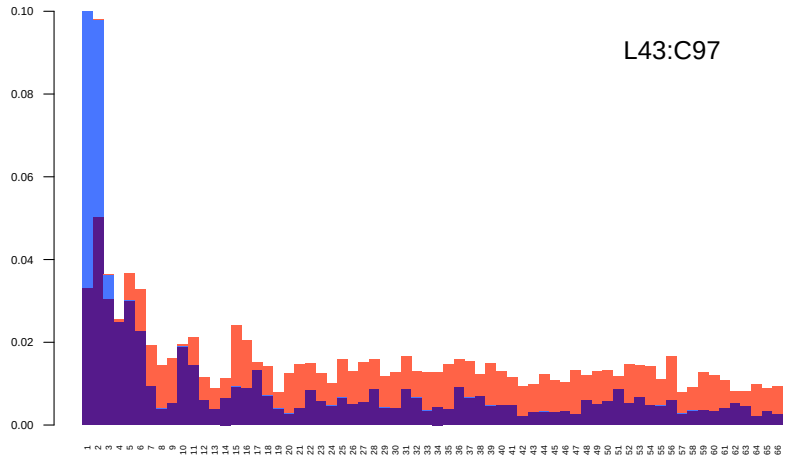
$|C94| = 6018\ , \quad C58, C81 \subset C94$



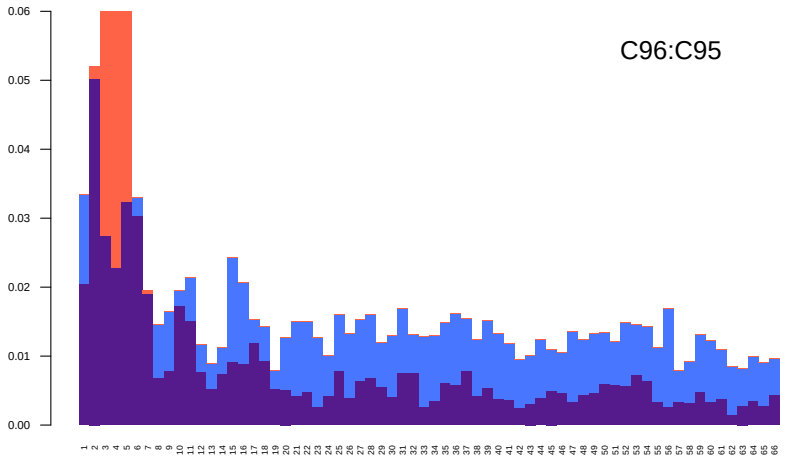
# Comparisons of leaders and cluster representatives L74:C98, L43:C97, C96:C95, C88:C94

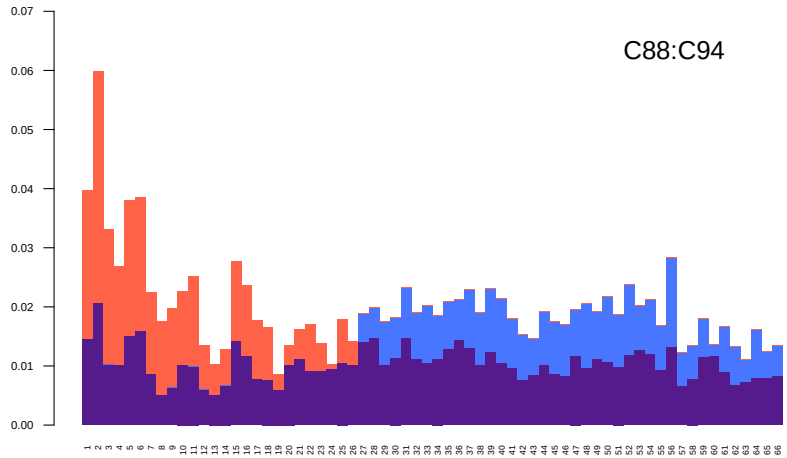





$L74[2] = 0.5894, \quad L43[1] = 0.4393, \quad L43[2] = 0.0981$

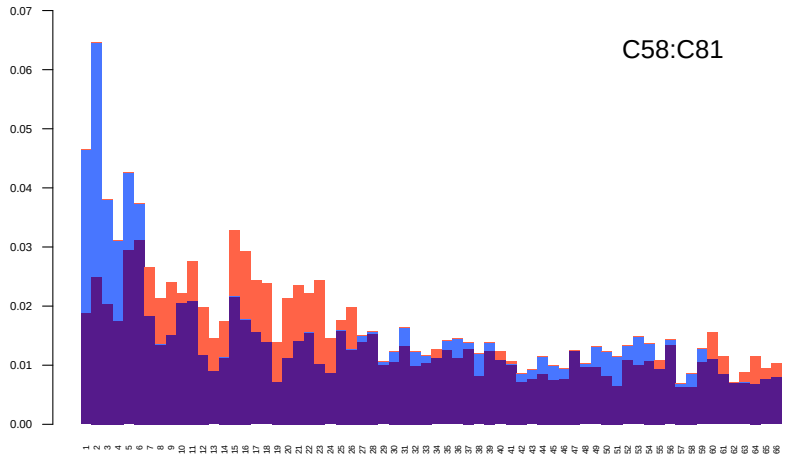


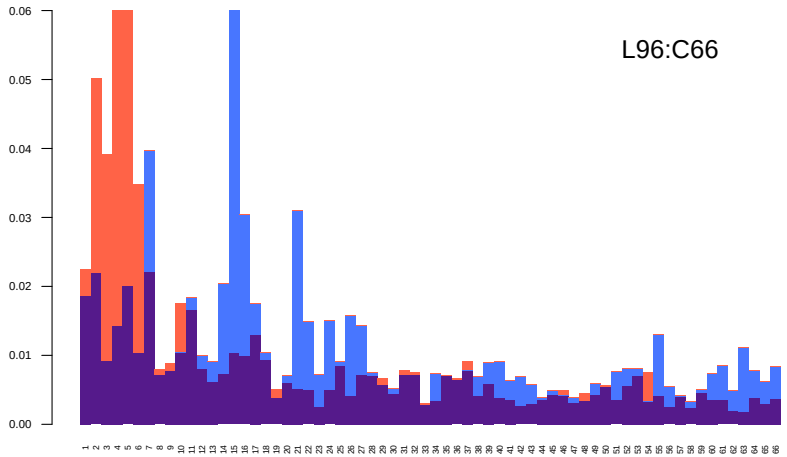




$C95[3] = 0.1665, \quad C95[4] = 0.1570, \quad C95[5] = 0.2250$

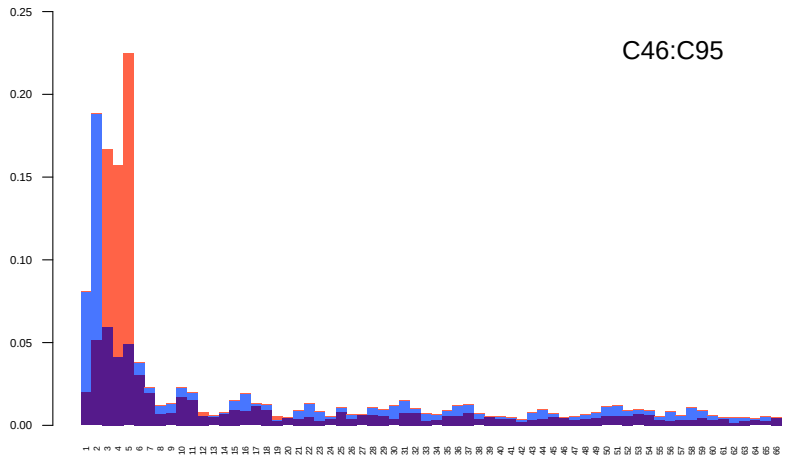
# Comparisons of leaders and cluster representatives C58:C81, L96:C66, C46:C955, C46

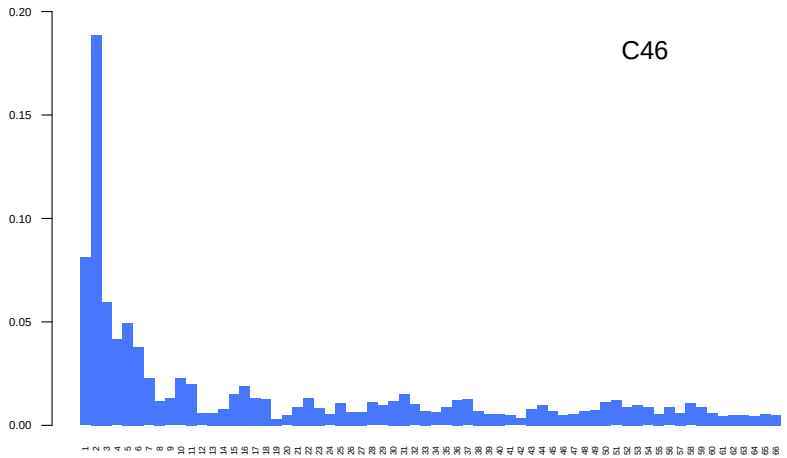





$L96[15] = 0.3524, \quad C66[4] = 0.1961, \quad C66[5] = 0.2917$



# Block model

To produce a block model we have to specify the values on its links. Because the original network is undirected, so is the BM. There are different options.

**Temporal quantities:**

$$\mathbf{a}(([i]:[j])) = \text{activity}(C_i, C_j) = \sum_{u \in C_i, v \in C_j} \mathbf{a}(u:v), \quad \text{for } i \neq j$$

and $\mathbf{a}(([i]:[i])) = \frac{1}{2}\text{activity}(C_i, C_i)$ .

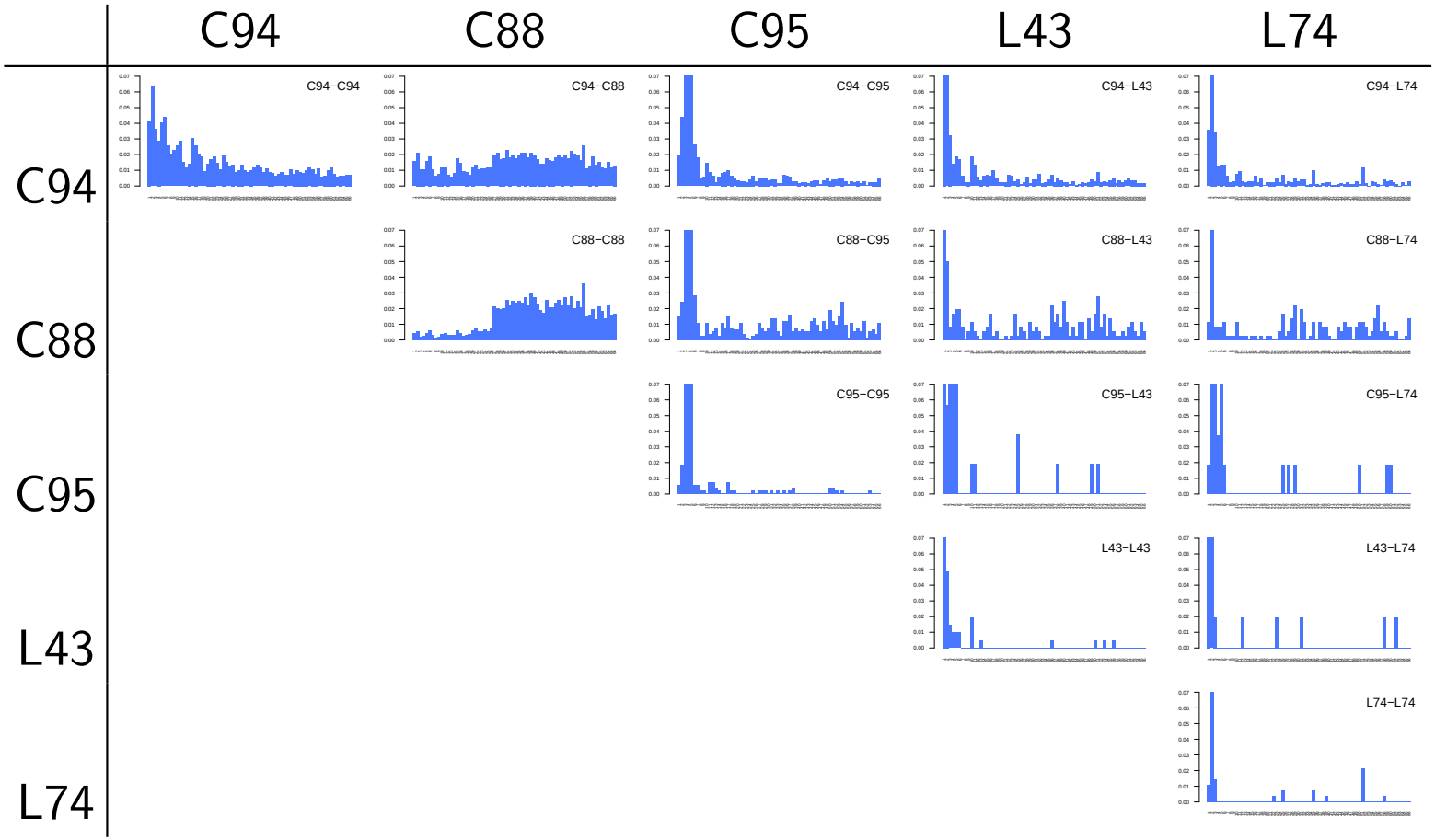
**Total intensities:** $a(([i]:[j])) = \text{tot}(\mathbf{a}(([i]:[j])))$ .

**Geometric average intensities:** $g(([i]:[j])) = \dfrac{a(([i]:[j]))}{\sqrt{|C_i| \cdot |C_j|}}$ .

# Block model



## Temporal quantities:



# Block model

**Total intensities:**

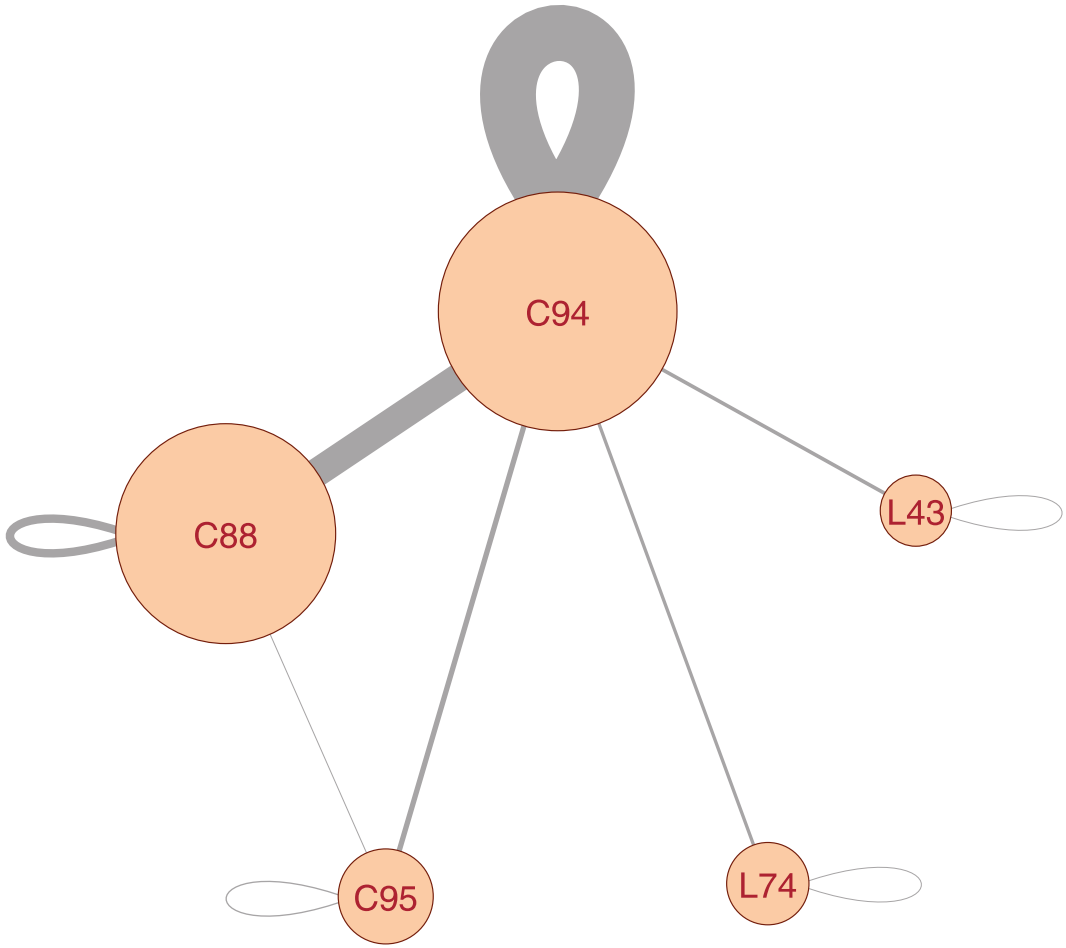
| *i* | *cluster* | *size* | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|
| 1 | *C*94 | 6018 | 143549 | 67801 | 5422 | 2816 | 2939 |
| 2 | *C*88 | 5109 | 0 | 18288 | 739 | 357 | 357 |
| 3 | *C*95 | 954 | 0 | 0 | 535 | 53 | 54 |
| 4 | *L*43 | 535 | 0 | 0 | 0 | 205 | 51 |
| 5 | *L*74 | 716 | 0 | 0 | 0 | 0 | 281 |

**Geometric average intensities:**

| *i* | *cluster* | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| 1 | *C*94 | 23.85 | 12.23 | 2.26 | 1.57 | 1.42 |
| 2 | *C*88 | 0 | 3.58 | 0.33 | 0.22 | 0.19 |
| 3 | *C*95 | 0 | 0 | 0.56 | 0.07 | 0.07 |
| 4 | *L*43 | 0 | 0 | 0 | 0.38 | 0.08 |
| 5 | *L*74 | 0 | 0 | 0 | 0 | 0.39 |

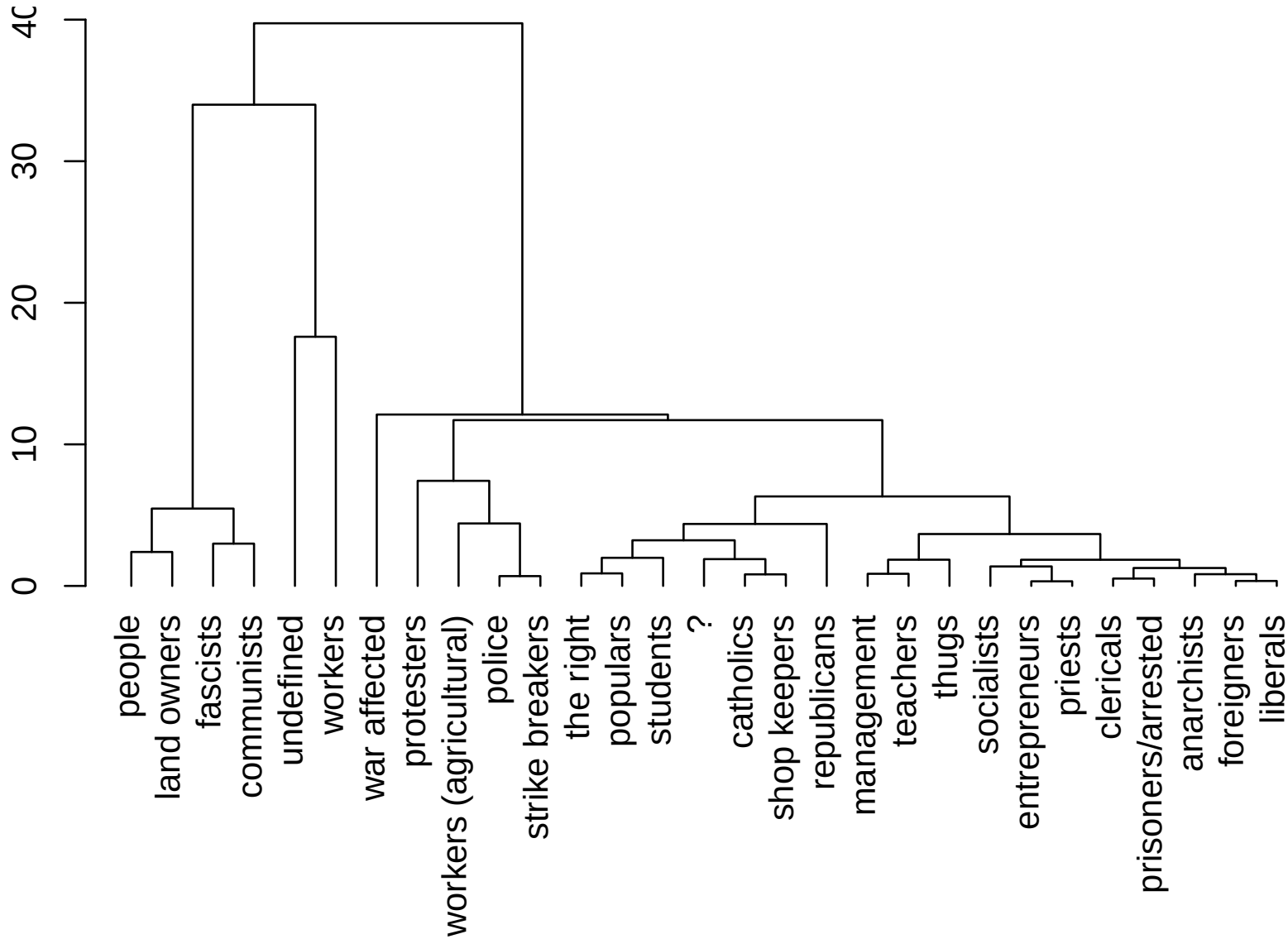
# Block model
## Geometric average intensities, cut at level 0.3





# Hierarchical clustering of Franzosi's violence network





# Conclusions

- only started
- how to present/display the temporal BMs ?
- compatibilty of clustering criterion function $P$ and BM $\mu$

# Acknowledgments



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