

**Weighted
two-mode
projections**

V. Batagelj

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On weighted two-mode network projections

Vladimir Batagelj
UP FAMNIT Koper

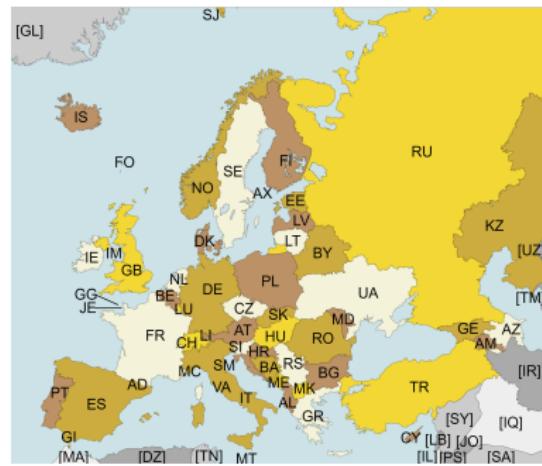
1338. sredin seminar
Ljubljana, August 9, 2023

Outline

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Vladimir Batagelj: vladimir.batagelj@fmf.uni-lj.si
Current version of slides (August 9, 2023 at 02:46): [PDF](#)
<https://github.com/bavla/biblio/>

Motivation

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In the paper [4] we studied the collaboration (co-authorship) between scientists from different post-Soviet countries. We decided to repeat the study on the European countries. It turned out that there are different ways how we can define a network describing the co-authorship collaboration between countries. Some options are discussed in this paper.

From the Centre for Science & Technology Studies (CWTS), Leiden University, NL we obtained data for the years 2000-2022.

Some exploratory analyses

Europe 2019 by levels

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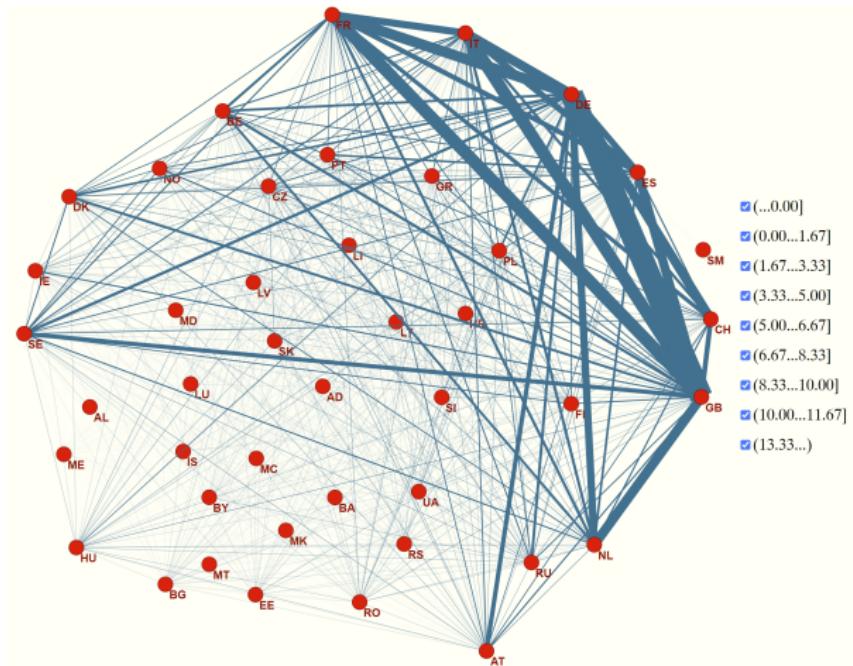
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Europe by levels

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Europe 2019 closest neighbor skeleton

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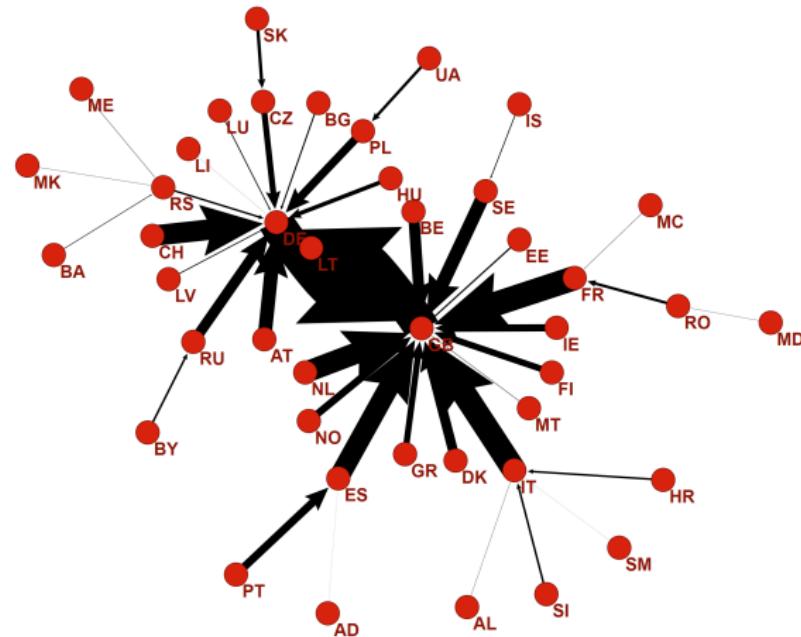
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Europe closest neighbor

Europe 2019 PathFinder skeleton

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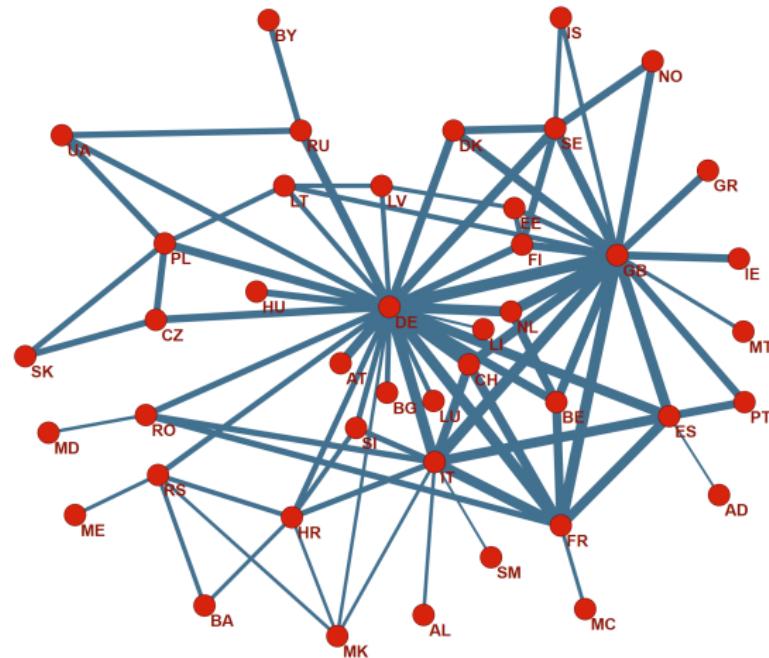
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Europe PathFinder skeleton

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In a *two-mode* (affiliation or bipartite) network $\mathcal{N} = ((U, V), L, w)$ the set of nodes is split into two disjoint sets (*modes*) U and V . Each link $e \in L$ has one end-node $u \in U$ and the other end-node $v \in V$, $\text{ext}(e) = (u, v)$. The function $w : L \rightarrow \mathbb{R}$ assigns to each link its weight.

NAME OF PARTICIPANTS OF GROUP I	CODE NUMBERS AND DATES OF SOCIAL EVENTS REPORTED IN <i>Old City Herald</i>													
	(1) 6/27	(2) 3/2	(3) 4/12	(4) 9/26	(5) 2/25	(6) 5/19	(7) 3/15	(8) 9/16	(9) 4/8	(10) 6/10	(11) 2/23	(12) 4/7	(13) 11/21	(14) 8/3
1. Mrs. Evelyn Jefferson.....	X	X	X	X	X	X	X		X					
2. Miss Laura Mandeville.....	X	X	X		X	X	X	X						
3. Miss Theresa Anderson.....		X	X	X	X	X	X	X						
4. Miss Brenda Rogers.....	X		X	X	X	X	X	X						
5. Miss Charlotte McDowd.....			X	X	X		X							
6. Miss Frances Anderson.....			X		X		X		X					
7. Miss Eleanor Nye.....				X	X	X	X	X						
8. Miss Pearl Oglethorpe.....					X			X		X				
9. Miss Ruth DeSand.....						X	X	X	X					
10. Miss Verne Sanderson.....							X	X	X	X			X	
11. Miss Mym Liddell.....								X	X	X	X		X	
12. Miss Katherine Rogers.....									X	X	X		X	X
13. Mrs. Sylvia Avondale.....									X	X	X		X	X
14. Mrs. Nora Fayette.....									X	X	X	X	X	X
15. Mrs. Helen Lloyd.....									X	X		X	X	
16. Mrs. Dorothy Murchison.....									X	X				
17. Mrs. Olivia Carleton.....									X		X			
18. Mrs. Flora Price.....									X		X			

Davis: Southern women, 1941

Two-mode network matrix and some notions

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In general, the weight can be measured on different measurement scales (counts, ratio, interval, ordinal, nominal, binary, TQ, etc.).

We extend the semiring $(\mathbb{R} \cup \{\square\}, +, \cdot)$ with a new zero \square with rules $\square + a = a$ and $\square \cdot a = a \cdot \square = \square$.

The network **matrix** $\mathbf{M} = [m[u, v]]$ of a two-mode network \mathcal{N} is defined as

$$m[u, v] = \begin{cases} \sum_{e \in L : \text{ext}(e) = (u, v)} w(u, v) & \exists e \in L : \text{ext}(e) = (u, v) \\ \square & \text{otherwise} \end{cases}$$

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The *product* $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ of two compatible matrices $\mathbf{A}_{I \times K}$ and $\mathbf{B}_{K \times J}$ is defined in the standard way

$$C[i, j] = \sum_{k \in K} A[i, k] \cdot B[k, j]$$

(it can be extended to semirings !!!)

The product of two compatible networks $\mathcal{N}_A = ((I, K), L_A, a)$ and $\mathcal{N}_B = ((K, J), L_B, b)$ is the network $\mathcal{N}_C = ((I, J), L_C, c)$ where $L_C = \{(i, j) : c[i, j] \neq \square\}$ and the weight c is determined by the matrix \mathbf{C} , $c(i, j) = C[i, j]$.

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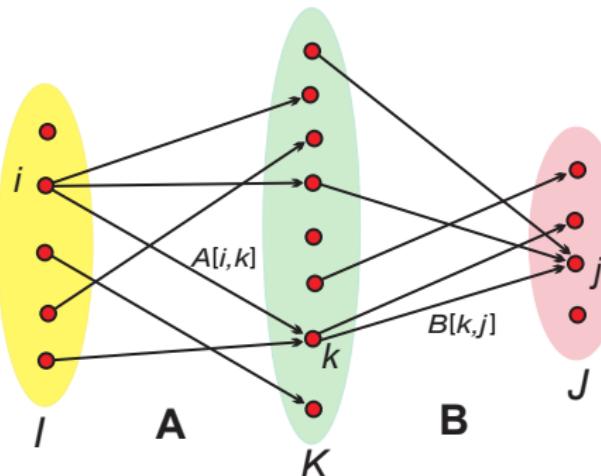
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In binary networks \mathcal{N}_A and \mathcal{N}_B , the value of $C[i,j]$ of $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ counts the number of ways we can go from the node $i \in I$ to the node $j \in J$ passing through K , $C[i,j] = |\mathcal{N}_A(i) \cap \mathcal{N}_B(j)|$.

$$C[i,j] = \sum_{k \in \mathcal{N}_A(i) \cap \mathcal{N}_B(j)} A[i,k] \cdot B[k,j]$$

Projections of binary two-mode networks

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Most of the bibliometric networks are obtained by a projection of a non-weighted network represented by a binary matrix. For example from the authorship network WA describing the authorship relation of the set of works (papers, books, reports, etc.) W by the authors from the sets A . It is represented by a matrix $WA = [wa[w, a]]$ where $wa[w, a] = 1$ iff a is an author of the work w and 0 otherwise. We get the co-authorship (counting) network Co_A determined by the projection

$$Co_A = WA^T \cdot WA$$

... Projections of binary two-mode networks

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As we know [2]

- For $a \neq b$, $co_A[a, b] =$ number of works co-authored by authors a and b .
- $co_A[a, a] =$ number of works from W written by the author a .
- The works with a large number of coauthors are "overrepresented" in the network Co_A – for example, the co-authorship of authors of a paper with 2 authors counts the same as the co-authorship between any pair of authors of the paper with 1000 co-authors; a paper with 1000 co-authors adds 1000000 links to projection network; while a single author paper only a loop. For this reason, the number $co_A[a, b]$ is not the best measure for measuring the collaboration intensity.

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The case of collaboration between countries is slightly different because the two-mode network WC is weighted. Actually, we could get it as $WC = WA \cdot AC$ where AC is the author-to-country affiliation network. This view opens a possibility to deal with authors affiliated to different countries provided that $\sum_c ac[a, c] = 1$. If the affiliations are changing through time the temporal quantities can be used [3].

To obtain a collaboration network between a set of countries C based on a set of works W , we start with a two-mode network WC described by a matrix $WC = [wc[w, c]]$ where

$wc[w, c] = \text{number of authors of the work } w \text{ from the country } c$

Authors counting collaboration

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In the network WC we can consider all authors of selected works W by adding to the set of countries C also the "country" Others. Instead of countries, other partitions of the set of authors can be used, for example, institutions.

We will use $T(N) = \sum_{e \in L} w(e)$ to denote the total sum of weights of all links of the network $N = (V, L, w)$.

The *authors counting collaboration* network Co_C described by the matrix \mathbf{Co}_C is obtained by projection

$$\mathbf{Co}_C = \mathbf{WC}^T \cdot \text{bin}(\mathbf{WC})$$

where $\text{bin}(\mathbf{WC}) = [\widehat{wc}[w, c]]$, and $\widehat{wc}[w, c] = 1$ iff $wc[w, c] \neq 0$ and 0 otherwise.

... Authors counting collaboration

What are the meaning of the entries $co_C[a, b]$ and their properties?

- a For $a \neq b$, $co_C[a, b] = \sum_w wc[w, a] \cdot \widehat{wc}[w, b]$ – number of appearances of **authors** from the country a in works co-authored also by some author from the country b . We will denote this number $wdeg_{WC}(a/b)$.
- b $co_C[a, a] = \sum_w wc[w, a] \cdot \widehat{wc}[w, a] = wdeg_{WC}(a)$ – number of appearances of authors from the country a in works from W ; a column sum for country a in the matrix **WC**.
- c From a simple example

$$WC = \begin{matrix} & \begin{matrix} c_1 & c_2 & c_3 \end{matrix} \\ \begin{matrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{matrix} & \left[\begin{matrix} 0 & 2 & 1 \\ 2 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 0 & 2 \\ 2 & 3 & 1 \\ 1 & 0 & 3 \end{matrix} \right] \end{matrix}$$

$$Co_C = \begin{matrix} & \begin{matrix} c_1 & c_2 & c_3 \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} & \left[\begin{matrix} 9 & 5 & 7 \\ 7 & 9 & 8 \\ 7 & 3 & 8 \end{matrix} \right] \end{matrix}$$

we see that the matrix **CoC** is in general not symmetric – there can exist pairs a, b such that $co_C[a, b] \neq co_C[b, a]$.

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- d Consider a row sum $R(a)$ for the country a in the matrix \mathbf{CoC} .
We get

$$R(a) = \sum_b co_C[a, b] = \sum_w wc[w, a] \cdot \sum_b \widehat{wc}[w, b] = \sum_w wc[w, a] \cdot \deg_{WC}(w)$$

Since in the network WC only works W with co-authors from at least 2 countries are considered, we have $\deg_{WC}(w) \geq 2$ and we can continue $R(a) \geq 2 \sum_w wc[w, a] = 2 \text{wdeg}_{WC}(a)$. Now, combined with \mathbf{b} , we finally get

$$\sum_{b: b \neq a} co_C[a, b] \geq \text{wdeg}_{WC}(a) = co_C[a, a]$$

The sum of the out-diagonal entries in the a row of the matrix \mathbf{CoC} is larger or equal to its diagonal entry.

From the example in c we see that this property does not hold for columns – see the column c_2 .

... Authors and works counting collaboration

- e For the diagonal values of the network \mathbf{Co}_C it holds
 $coc[c, c] = \text{wdeg}_{WC}(c)$

$$coc[c, c] = \sum_w wc[w, c] \cdot \widehat{wc}[w, c] = \sum_w wc[w, c] = \text{wdeg}_{WC}(c)$$

Therefore $\sum_c coc[c, c] = T(WC)$.

- f In the case when also the matrix WC is binary,
 $\text{bin}(WC) = WC$, we deal with the standard projection
 mentioned in the introduction $\mathbf{Co}_C = WC^T \cdot WC$. In the **works
counting collaboration** network $\mathbf{Co}_b = \text{bin}(WC)^T \cdot \text{bin}(WC)$ its
 weight $co_b[a, b]$ counts **works**: $co_b[a, b] =$ number of works
 from W co-authored by authors from countries a and b , and
 $co_b[a, a] =$ number of works from W co-authored by authors
 from the country a . Note that the inequality from d still holds
 (and also for columns).

Normalizations

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For binary networks, we define their normalized versions: *standard*
 $n(\mathbf{WA}) = [wan[w, a]]$

$$wan[w, a] = \frac{wa[w, a]}{\max(1, \deg_{\mathbf{WA}}(w))}$$

and *strict* (or Newman's) $N(\mathbf{WA}) = [waN[w, a]]$

$$waN[w, a] = \frac{wa[w, a]}{\max(1, \deg_{\mathbf{WA}}(w) - 1)}$$

Using the normalized networks we define the *standard fractional projection*

$$\mathbf{Co}_n = n(\mathbf{WA})^T \cdot n(\mathbf{WA})$$

and the *strict fractional projection*

$$\mathbf{Co}_N = D_0(n(\mathbf{WA})^T \cdot N(\mathbf{WA}))$$

where the function $D_0(\mathbf{M})$ sets the diagonal of a square matrix \mathbf{M} to 0.

We know [1] that if $\deg_{\mathbf{WA}}(w) > 0$, each work $w \in W$ contributes equally, a unit 1, to the total weight of links in \mathbf{Co}_n . The same holds for \mathbf{Co}_N if $\deg_{\mathbf{WA}}(w) > 1$.

Standard weighted projection

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To extend the fractional projections to weighted two-mode networks we define for the *standard* case $n(\mathbf{WC}) = [wcn[w, c]]$

$$wcn[w, c] = \frac{wc[w, c]}{\max(1, \text{wdeg}_{\mathbf{WC}}(w))}$$

Again we have $T(\mathbf{Co}_n) = |W|$ for $\mathbf{Co}_n = n(\mathbf{WC})^T \cdot n(\mathbf{WC})$.

$$\mathbf{Co}_b = \begin{matrix} & c_1 & c_2 & c_3 \\ c_1 & \left[\begin{array}{ccc} 5 & 3 & 4 \\ 3 & 4 & 3 \\ 4 & 3 & 5 \end{array} \right] \\ c_2 & & & \\ c_3 & & & \end{matrix} \quad \mathbf{Co}_n = \begin{matrix} & c_1 & c_2 & c_3 \\ c_1 & \left[\begin{array}{ccc} 1.0180556 & 0.5088889 & 0.5230556 \\ 0.5088889 & 1.1655556 & 0.4255556 \\ 0.5230556 & 0.4255556 & 0.9013889 \end{array} \right] \\ c_2 & & & \\ c_3 & & & \end{matrix}$$

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There is no obvious way how to define the strict normalization for weighted networks.

There is another possible view on fractional projections. The definition of matrix $n(\mathbf{WA})$ can be written as $n(\mathbf{WA}) = \mathbf{d}_n \cdot \mathbf{WA}$ and similarly $N(\mathbf{WA}) = \mathbf{d}_N \cdot \mathbf{WA}$ where \mathbf{d}_n is a diagonal $W \times W$ matrix with $d_n[w, w] = 1 / \max(1, \deg_{\mathbf{WA}}(w))$ and \mathbf{d}_N with $d_N[w, w] = 1 / \max(1, \deg_{\mathbf{WA}}(w) - 1)$. In both cases we get ($\mathbf{d}^T = \mathbf{d}$)

$$\mathbf{Co}_n = n(\mathbf{WA})^T \cdot n(\mathbf{WA}) = \mathbf{WA}^T \cdot \mathbf{d}_n \cdot \mathbf{d}_n \cdot \mathbf{WA}$$

$$\mathbf{Co}_N = n(\mathbf{WA})^T \cdot N(\mathbf{WA}) = \mathbf{WA}^T \cdot \mathbf{d}_n \cdot \mathbf{d}_N \cdot \mathbf{WA}$$

Because a product of diagonal matrices is a diagonal matrix, $\text{diag}(a_w) \cdot \text{diag}(b_w) = \text{diag}(a_w \cdot b_w)$, both cases have a common form $\mathbf{WA}^T \cdot \mathbf{d} \cdot \mathbf{WA}$. It can be related to the weighted scalar product. Maybe this form can lead also to the extension of strict projection for weighted two-mode networks.

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Let us look at a simple example. Assume, that a work w has authors from three countries a , b , and c . Then, since the co-authors inside the same country do not count, its contribution $T(w)$ to the total weight, see the contribution matrix

$$\text{Co}_C(w) = \begin{bmatrix} & a & & b & & c \\ a & \mathbf{0} & & wc[w, a] \cdot wc[w, b] & & wc[w, a] \cdot wc[w, c] \\ b & & wc[w, b] \cdot wc[w, a] & \mathbf{0} & & wc[w, b] \cdot wc[w, c] \\ c & & wc[w, c] \cdot wc[w, a] & & wc[w, c] \cdot wc[w, b] & \mathbf{0} \end{bmatrix},$$

is $T(w) = \sum_{e,f \in \{a,b,c\} \wedge e \neq f} wc[w, e] \cdot wc[w, f]$. By the rule of product and the rule of sum from basic combinatorics [5], $T(W)$ is equal to twice the number of all co-authorships of authors from different countries – pairs (a, b) and (b, a) are representing co-authorship of authors a and b .

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To make $T_N(w) = 1$ we must set the entry $d_N[w, w]$ of the diagonal matrix \mathbf{d}_N for the weighted network \mathbf{WC} to
 $d_N[w, w] = 1/T(w) = 1/(\text{wdeg}_{WC}(w)^2 - \sum_c wc[w, c]^2)$.

Note that $\sum_c wc[w, c] = \text{wdeg}_{WC}(w)$ and

$$\text{wdeg}_{WC}(w)^2 - \sum_c wc[w, c]^2 = \sum_{e, f: e \neq f} wc[w, e] \cdot wc[w, f]$$

The left side of this equality is computationally more convenient.

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... Strict weighted projection

It is easy to see that we made a good guess – in the corresponding projection $\mathbf{Co}_N = D_0(\mathbf{WC}^T \cdot \mathbf{d}_N \cdot \mathbf{WC})$ each work contributes equally, a unit 1, to the total of link weights.

$$T_N(w) = d_N[w, w] \cdot \sum_{e, f: e \neq f} wc[w, e] \cdot wc[w, f] = 1$$

Therefore

$$T(\mathbf{Co}_N) = \sum_w T_N(w) = |W|$$

For our example from Section 2 c we get

$$\mathbf{Co}_N = \begin{matrix} & c_1 & c_2 & c_3 \\ c_1 & 0.000000 & 0.9870130 & 1.1623377 \\ c_2 & 0.987013 & 0.0000000 & 0.8506494 \\ c_3 & 1.162338 & 0.8506494 & 0.0000000 \end{matrix}$$

with $T(\mathbf{Co}_N) = 6$.

Weighted projections

Weighted two-mode projections

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C is a set of countries of our interest. If it is used, $\text{others} \in C$. \mathcal{W} is a list of metadata about the works from the selected bibliographic data source, co-authored by authors from at least two different countries from $C \setminus \{\text{others}\}$. All four projection matrices \mathbf{Co}_b , \mathbf{Co}_n , \mathbf{Co}_C , and \mathbf{Co}_N can be constructed in a single run through the list using the Algorithm given in the following slides.

Computing weighted projections I

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```
1: function PROJECTIONS( $\mathcal{W}, C$ )
2:    $Cob \leftarrow Con \leftarrow CoC \leftarrow CoN \leftarrow matrix(0, nrow = |C|, ncol = |C|)$ 
3:   for  $w \in \mathcal{W}$  do
4:     from the metadata of work  $w$  determine the list of pairs
        $Lw = ((a, ca)), \quad a - \text{author. } ca - \text{country of } a$ 
5:      $Cw \leftarrow empty$ 
6:     for  $(a, ca) \in Lw$  do
7:        $c \leftarrow ca$  if  $ca \in C$  else others
8:       if  $c \notin Cw$  then
9:          $Cw[c] \leftarrow 0$ 
10:      end if
11:       $Cw[c] \leftarrow Cw[c] + 1$ 
12:    end for
13:    write ID of  $w$  and  $Cw$  to the file WC
14:     $wdegw \leftarrow \sum_{c \in Cw} Cw[c]$ 
15:     $sqw \leftarrow \sum_{c \in Cw} Cw[c]^2$ 
16:     $dnw \leftarrow 1/wdegw^2$ 
17:     $dNw \leftarrow 1/(wdegw^2 - sqw)$ 
```

Computing weighted projections II

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```
18:      for  $e \in Cw$  do
19:          for  $f \in Cw$  do
20:               $Cob[e, f] \leftarrow Cob[e, f] + 1$ 
21:               $CoC[e, f] \leftarrow CoC[e, f] + Cw[e]$ 
22:               $Con[e, f] \leftarrow Con[e, f] + Cw[e] \cdot Cw[f] \cdot dnw$ 
23:              if  $e \neq f$  then
24:                   $CoN[e, f] \leftarrow CoN[e, f] + Cw[e] \cdot Cw[f] \cdot dNw$ 
25:              end if
26:          end for
27:      end for
28:  end for
29:  return  $Cob, Con, CoC, CoN$ 
30: end function
```

Test in R

Notes on the implementation of the algorithm

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- A natural implementation of the counters Cw is as a dictionary.
- Line 13 is optional. It creates the file WC describing the two-mode network WC .
- Networks Co_b , Co_n , and Co_N are symmetric. They can be represented by an undirected network with the weight of an edge $(e : f)$ equal to twice the computed value, except for loops. The computation can be restricted to pairs (e, f) for which $e \leq f$.

Conclusions

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In the paper, we derived the results in terms of the binary authorship network WA and the weighted network WC . The results hold in general for similar weighted two-mode networks such as

- (journals, universities, number of published articles of authors from the university u in the journal j in the selected time interval),
- (territorial units, universities, number of students from the territorial unit t studying this year at the university u),
- (web resources (movies or music tracks), types of resource, number of times the resource r of type t was downloaded in the selected time interval),
- (retail chain customers (chain card owners), (types of) products, the value of the product p bought by the customer c in the selected time interval),

etc.

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Fractional
approach

Strict
fractional
collaboration

Computing

Conclusions

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