Derived bibliographic networks

The impact of multi-person units and truncated networks

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Outline

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https://github.com/bavla/biblio/

The multipersonality's effect on the results of bibliographic analyses

We would like to study the effect of multipersons in derived networks [5]. Let $\mathbf{M} = [m[u, v]]$ is a matrix on $U \times V$ and $\mathbf{C}_U = \{C_1, C_2, \ldots, C_k\}$ a partition of the set U, $\emptyset \subset C_i \subseteq U$, $C_i \cap C_j = \emptyset$ for $i \neq j$, and $\bigcup_i C_i = U$. The set U is the (ground truth) set of real units (persons). The partition $\mathbf{C}_U$ corresponds to units (for example authors) identified by the network construction process. A cluster $C \in \mathbf{C}_U$ with $|C| > 1$ represents a multi-unit; and for $|C| = 1$ a correctly identified unit.

We introduce the *shrinking* transformation S_r of matrix $\mathbf{M}$ by the **r**ows partition $\mathbf{C}_U$ into $S_r(\mathbf{M}, \mathbf{C}_U) = \mathbf{S} = [s[C, v]]$ on $\mathbf{C}_U \times V$ determined by the rule

$$s[C, v] = \sum_{u \in C} m[u, v]$$

The shrinking transformation can be extended to a **c**olumns partition $\mathbf{C}_V$ of the set V by

$$s[u, C] = \sum_{v \in C} m[u, v]$$

Multiunits' effect

or

$$S_c(\mathbf{M}, \mathbf{C}_V) = S_r(\mathbf{M}^T, \mathbf{C}_V)^T$$

and to partitions $\mathbf{C}_U$ and $\mathbf{C}_V$ of both sets by

$$S(\mathbf{M}, (\mathbf{C}_U, \mathbf{C}_V)) = S_c(S_r(\mathbf{M}, \mathbf{C}_U), \mathbf{C}_V)$$

Consider now the case of two compatible matrices $\mathbf{M} = [m[u, t]]$ on $U \times T$ and $\mathbf{N} = [n[t, v]]$ on $T \times V$. For a partition $\mathbf{C}_U$ of the set U it holds

$$S_r(\mathbf{M} \cdot \mathbf{N}, \mathbf{C}_U) = S_r(\mathbf{M}, \mathbf{C}_U) \cdot \mathbf{N}$$

To check this let's denote with $\mathbf{L}$ and $\mathbf{R}$ the left and right sides of this expression. We have

$$l[C, v] = \sum_{u \in C} \mathbf{M} \cdot \mathbf{N}[u, v] = \sum_{u \in C} \sum_{t \in t} m[u, t] \cdot n[t, v]$$

and

$$r[C, v] = \sum_{t \in T} S_r(\mathbf{M}, \mathbf{C}_U)[C, t] \cdot n[t, v] = \sum_{t \in T} (\sum_{u \in C} m[u, t]) \cdot n[t, v] = l[C, v]$$

Multiunits' effect

For the partition $\mathbf{C}_V$ of the set V we get

$$S_c(\mathbf{M}\cdot\mathbf{N},\mathbf{C}_V) = S_r((\mathbf{M}\cdot\mathbf{N})^T,\mathbf{C}_V)^T = S_r(\mathbf{N}^T\cdot\mathbf{M}^T,\mathbf{C}_V)^T =$$

$$= (S_r(\mathbf{N}^T,\mathbf{C}_V)\cdot\mathbf{M}^T)^T = \mathbf{M}\cdot S_r(\mathbf{N}^T,\mathbf{C}_V)^T = \mathbf{M}\cdot S_c(\mathbf{N},\mathbf{C}_V)$$

Therefore

$$S_c(\mathbf{M}\cdot\mathbf{N},\mathbf{C}_V) = \mathbf{M}\cdot S_c(\mathbf{N},\mathbf{C}_V)$$

For partitions of both sets U and V we have

$$S(\mathbf{M}\cdot\mathbf{N},(\mathbf{C}_U,\mathbf{C}_V)) = S_c(S_r(\mathbf{M}\cdot\mathbf{N},\mathbf{C}_U),\mathbf{C}_V) =$$

$$= S_c(S_r(\mathbf{M},\mathbf{C}_U)\cdot\mathbf{N},\mathbf{C}_V) = S_r(\mathbf{M},\mathbf{C}_U)\cdot S_c(\mathbf{N},\mathbf{C}_V)$$

and finally

$$S(\mathbf{M}\cdot\mathbf{N},(\mathbf{C}_U,\mathbf{C}_V)) = S_r(\mathbf{M},\mathbf{C}_U)\cdot S_c(\mathbf{N},\mathbf{C}_V)$$

Multiunits' effect

For $C_u \in \mathbf{C}_U$ and $C_v \in \mathbf{C}_V$ we have

$$S(\mathbf{M}\cdot\mathbf{N},(\mathbf{C}_U,\mathbf{C}_V))[C_u,C_v] = S_c(S_r(\mathbf{M}\cdot\mathbf{N},\mathbf{C}_U),\mathbf{C}_V)[C_u,C_v] =$$

$$\sum_{z\in C_v} S_r(\mathbf{M}\cdot\mathbf{N},\mathbf{C}_U)[C_u,z] = \sum_{z\in C_v}\sum_{w\in C_u}\mathbf{M}\cdot\mathbf{N}[w,z] = \sum_{w\in C_u}\sum_{z\in C_v}\mathbf{M}\cdot\mathbf{N}[w,z]$$

In a special case of singelton clusters $C_u = \{u\}$ and $C_v = \{v\}$ we get

$$S(\mathbf{M}\cdot\mathbf{N},(\mathbf{C}_U,\mathbf{C}_V))[\{u\},\{v\}] = \sum_{w\in\{u\}}\sum_{z\in\{v\}}\mathbf{M}\cdot\mathbf{N}[w,z] = \mathbf{M}\cdot\mathbf{N}[u,v]$$

We see that **the multi-units don't affect the values of relations between singletons in the derived networks**.

Fractional approach

Assume that we have the authorship network represented by a matrix $\mathbf{WA} = [wa[w,a]]$ where $wa[w,a] = 1$ iff the author a is (co)author of the work w, and $wa[w,a] = 0$ otherwise.

We will discuss two normalizations. The *standard* normalization $n(\mathbf{WA}) = [nwa[w,a]]$ where

$$nwa[w,a] = \frac{wa[w,a]}{\max(1, \deg(w))}$$

and *strict* or *Newman*'s normalization $n'(\mathbf{WA}) = [nwa'[w,a]]$ where

$$nwa'[w,a] = \frac{wa[w,a]}{\max(1, \deg(w) - 1)}$$

We have $\sum_{a \in A} nwa[w,a] = \operatorname{sign}(\deg(w))$ and

$$\sum_{a \in A} nwa'[w,a] = \frac{\deg(w)}{\max(1, \deg(w) - 1)}.$$

. . . Fractional approach

The meaning of the weighted degree of node a in the network $n(\mathbf{WA})$, $\text{wideg}_{n(WA)}(a) = \sum_{w \in W} nwa[w, a]$, is the *fractional contribution of the author* a to all works.

Note that if $\deg(w) = 0$ then both $nwa[w, a] = 0$ and $nwa'[w, a] = 0$, and if $\deg(w) = 1$ then both $nwa[w, a] = 1$ and $nwa'[w, a] = 1$.

The standard co-appearance network matrix $\mathbf{Cn} = [cn[a, b]]$ is obtained as

$$\mathbf{Cn} = n(\mathbf{WA})^T \cdot n(\mathbf{WA})$$

and the *strict* co-appearance network matrix $\mathbf{Ct} = [ct[a, b]]$ is obtained as

$$\mathbf{Ct} = D_0(n(\mathbf{WA})^T \cdot n'(\mathbf{WA}))$$

where $D_0(\mathbf{M})$ sets the diagonal of matrix $\mathbf{M}$ to 0.

Let's look at an entry of $\mathbf{Cn}$

$$cn[a, b] = \sum_{w \in W} nwa^T[a, w] \cdot nwa[w, b] = \sum_{w \in W} nwa[w, a] \cdot nwa[w, b] = cn[b, a]$$

... Fractional approach

and for **Ct**

$$ct[a,b] = \sum_{w\in W} nwa^T[a,w]\cdot nwa'[w,b] = \sum_{w\in W} \frac{wa[w,a]}{\max(1,\deg(w))}\cdot\frac{wa[w,b]}{\max(1,\deg(w)-1)}$$

$$= \sum_{w\in W} \frac{wa[w,b]}{\max(1,\deg(w))}\cdot\frac{wa[w,a]}{\max(1,\deg(w)-1)} = ct[b,a]$$

The fractional co-appearance matrices **Cn** and **Ct** are symmetric.

From

$$\mathrm{wdeg}_{Cn}(a) = \sum_{b\in A} cn[a,b] = \sum_{w\in W} nwa[w,a]\cdot\sum_{b\in A} nwa[w,b] =$$

$$= \sum_{w\in W} nwa[w,a]\cdot \mathrm{sign}(\deg(w)) = \sum_{w\in W} nwa[w,a] = \mathrm{wideg}_{n(WA)}(a)$$

we see that the authors have the same weighted degree in networks $n(\mathbf{WA})$ and **Cn**.

. . . Fractional approach

Similarly for the network **Ct**. Because by definition, $ct[a,a] = 0$, we have

$$\mathrm{wdeg}_{Ct}(a) = \sum_{b \in A \setminus \{a\}} ct[a,b] = \sum_{w \in W} \frac{nwa[w,a]}{\max(1, \deg(w) - 1)} \sum_{b \in A \setminus \{a\}} wa[w,b] =$$

If $wa[w,a] = 0$ the term in the $\sum_{w \in W}$ has value 0. So we can assume $wa[w,a] = 1$. This means that $a \in N(w)$ and $wa[w,b] = 1$ means that also $b \in N(w)$. Therefore

$$\sum_{b \in A \setminus \{a\}} wa[w,b] = |N(w) \setminus \{a\}| = \deg(w) - 1$$

Now, we can continue ($W_2 = \{w \in W : deg(w) \geq 2\}$)

$$= \sum_{w \in W_2} \frac{nwa[w,a]}{\max(1, \deg(w) - 1)} (\deg(w) - 1) = \sum_{w \in W_2} nwa[w,a] =$$

$$= \mathrm{wideg}_{n(WA)}(a) - |S|$$

where $S = \{w \in W : \deg(w) = 1\}$ – single author works.

Package bibmat

```
normalize <- function(M) t(apply(M,1,function(x) x/max(1,sum(x))))
newman <- function(M) t(apply(M,1,function(x) x/max(1,sum(x)-1)))
D0 <- function(M) {diag(M) <- 0; return(M)}
binary <- function(M) {B <- t(apply(M,1,function(x) as.integer(x!=0)))
  colnames(B) <- colnames(M); return(B)}
wodeg <- function(M) apply(M,1,sum)
wideg <- function(M) apply(M,2,sum)
odeg <- function(M) wodeg(binary(M))
ideg <- function(M) wideg(binary(M))
wdeg <- wodeg
Co <- function(M) t(M)%*%M
Cn <- function(M) Co(normalize(M))
Ct <- function(M) D0(t(normalize(M))%*%newman(M))
through <- function(M,S) t(M)%*%S%*%M
arit <- function(a,b) mean(c(a,b))
amin <- function(a,b) min(c(a,b))
amax <- function(a,b) max(c(a,b))
geom <- function(a,b) sqrt(a*b)
harm <- function(a,b) ifelse(a*b==0,0,2/(1/a+1/b))
jacc <- function(a,b) ifelse(a*b==0,0,1/(1/a+1/b - 1))
symm <- function(A,M) {n <- nrow(M); S <- M
  for(i in 1:(n-1)) for(j in (i+1):n)
    S[i,j] <- S[j,i] <- A(M[i,j],M[j,i])
  return(S)}
```

Multiunits and truncated networks

V. Batagelj

Multiple units

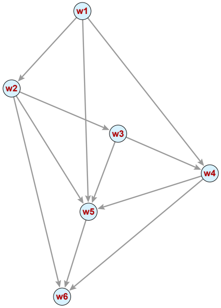
Fractional approach

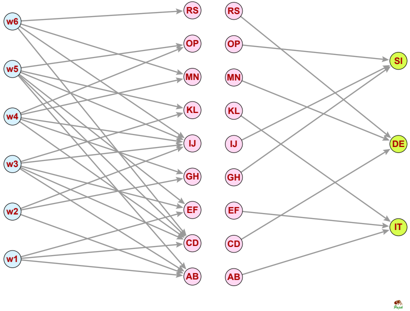
Truncated networks

Linking through a network

Conclusions

References






```
> Ci
   w1 w2 w3 w4 w5 w6
w1  0  1  0  1  1  0
w2  0  0  1  0  1  1
w3  0  0  0  1  1  0
w4  0  0  0  0  1  1
w5  0  0  0  0  0  1
w6  0  0  0  0  0  0
```

```
> WA
   AB CD EF GH IJ KL MN OP RS
w1  1  1  1  0  0  0  0  0  0
w2  1  0  0  1  1  0  0  0  0
w3  1  1  1  0  1  1  0  0  0
w4  0  1  0  1  1  0  1  1  0
w5  1  1  1  0  1  1  0  1  0
w6  0  1  0  0  1  0  1  0  1
```

```
> AC
   IT DE SI
AB  1  0  0
CD  0  1  0
EF  1  0  0
GH  0  0  1
IJ  0  0  1
KL  1  0  0
MN  0  1  0
OP  0  0  1
RS  0  1  0
```

Github/bavla/biblio

. . . Fractional approach

```
> wdir <- "C:/Users/vlado/test/biblio"
> setwd(wdir)
> source(
  "https://raw.githubusercontent.com/bavla/biblio/master/code/bibmat.R")
> urlEx <-
  "https://github.com/bavla/biblio/raw/master/Eu/Data/ExNets.RDS"
> download.file(url=urlEx,destfile=paste0(wdir,"/ExNets.RDS",sep=""))
> Ex <- readRDS("ExNets.RDS")
> Ci <- Ex$Ci; WA <- Ex$WA; AC <- Ex$AC

> WAn <- normalize(WA)
> Cn <- t(WAn)%*%WAn
> wideg(WAn)
    AB     CD     EF     GH     IJ     KL     MN     OP     RS
1.0333 1.1500 0.7000 0.5333 1.1500 0.3667 0.4500 0.3667 0.2500
> wideg(Cn)
    AB     CD     EF     GH     IJ     KL     MN     OP     RS
1.0333 1.1500 0.7000 0.5333 1.1500 0.3667 0.4500 0.3667 0.2500
> WAt <- newman(WA)
> Ct <- DO(t(WAn)%*%WAt)
> wideg(Ct)
    AB     CD     EF     GH     IJ     KL     MN     OP     RS
1.0333 1.1500 0.7000 0.5333 1.1500 0.3667 0.4500 0.3667 0.2500
> sum(wideg(WAn))
[1] 6
```

Totals

For a matrix $\mathbf{M}$ on $U \times V$ we define its *total* $T(\mathbf{M})$ as the sum of all its entries, $T(\mathbf{M}) = \sum_{u \in U} \sum_{v \in V} m[u, v]$. Let's compute

$$T(\mathbf{Cn}) = \sum_{a \in A} \sum_{b \in A} cn[a, b] = \sum_{w \in W} \sum_{a \in A} \sum_{b \in A} nwa[w, a] \cdot nwa[w, b] = \sum_{w \in W} T(w)$$

where

$$T(w) = \sum_{a \in A} \sum_{b \in A} nwa[w, a] \cdot nwa[w, b] = \sum_{a \in A} nwa[w, a] \cdot \sum_{b \in A} nwa[w, b] =$$

$$= \mathrm{sign}(\deg(w))^2 = \mathrm{sign}(\deg(w))$$

We see that the contribution of each work $w \in W$ with $\deg(w) > 0$ is 1. Therefore

$$T(\mathbf{Cn}) = \sum_{w \in W} \mathrm{sign}(\deg(w)) = |W_1|$$

where $W_1 = \{w \in W : \deg(w) \geq 1\}$.

Totals

For matrices $n(\mathbf{WA})$ and $\mathbf{Cn}$ we have

$$T(n(\mathbf{WA})) = \sum_{a\in A}\sum_{w\in W} nwa[w,a] = \sum_{a\in A} \mathrm{wideg}_{n(WA)}(a) =$$

$$= \sum_{a\in A} \mathrm{wdeg}_{Cn}(a) = \sum_{a\in A}\sum_{b\in A} cn[a,b] = T(\mathbf{Cn})$$

And

$$T(\mathbf{Ct}) = \sum_{a\in A}\sum_{b\in A\setminus\{a\}} ct[a,b] =$$

$$\sum_{w\in W}\sum_{a\in A}\sum_{b\in A\setminus\{a\}} nwa[w,a]\cdot nwa'[w,b] = \sum_{w\in W} T'(w)$$

where

$$T'(w) = \sum_{a\in A}\sum_{b\in A\setminus\{a\}} nwa[w,a]\cdot nwa'[w,b] =$$

$$\sum_{a\in A}\frac{nwa[w,a]}{\max(1,\deg(w)-1)}\cdot\sum_{b\in A\setminus\{a\}} wa[w,b]$$

Totals

If $wa[w,a] = 0$ or $wa[w,b] \leq 1$ the term in $\sum_{a \in A}$ has value 0. For $wa[w,a] = 1$ and $wa[w,b] \geq 2$ (or $deg(w) \geq 2$) we have $\sum_{b \in A \setminus \{a\}} wa[w,b] = \deg(w) - 1$. Therefore

$$T'(w) = \begin{cases} 1 & deg(w) \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

and finally

$$T(\mathbf{Ct}) = \sum_{w \in W} T'(w) = |W_2|$$

where $W_2 = \{w \in W : deg(w) \geq 2\}$.

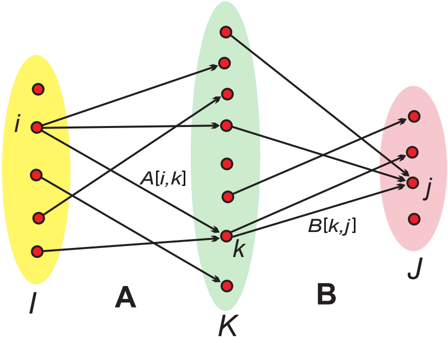
Note also that

$$\sum_{a \in A} \mathrm{wideg}_{n(WA)}(a) = T(n(\mathbf{WA}))$$

... Totals

```
> sum(Cn)
[1] 6
> sum(Ct)
[1] 6
> empty <- rep(0,length(A)); WA1 <- rbind(WA,empty,empty)
> rownames(WA1)[7:8] <- c("w7","w8"); WA1["w8","CD"] <- 1
> WA1
   AB CD EF GH IJ KL MN OP RS
w1  1  1  1  0  0  0  0  0  0
w2  1  0  0  1  1  0  0  0  0
w3  1  1  1  0  1  1  0  0  0
w4  0  1  0  1  1  0  1  1  0
w5  1  1  1  0  1  1  0  1  0
w6  0  1  0  0  1  0  1  0  1
w7  0  0  0  0  0  0  0  0  0
w8  0  1  0  0  0  0  0  0  0
> WAn1 <- normalize(WA1); Cn1 <- t(WAn1)%*%WAn1
> sum(Cn1)
[1] 7
> WAt1 <- newman(WA1); Ct1 <- DO(t(WAn1)%*%WAt1)
> sum(Ct1)
[1] 6
> wideg(WAn1)
    AB     CD     EF     GH     IJ     KL     MN     OP     RS
1.0333 2.1500 0.7000 0.5333 1.1500 0.3667 0.4500 0.3667 0.2500
> wdeg(Cn1)
    AB     CD     EF     GH     IJ     KL     MN     OP     RS
1.0333 2.1500 0.7000 0.5333 1.1500 0.3667 0.4500 0.3667 0.2500
> wdeg(Ct1)
    AB     CD     EF     GH     IJ     KL     MN     OP     RS
1.0333 1.1500 0.7000 0.5333 1.1500 0.3667 0.4500 0.3667 0.2500
```

Multiplication of networks



In binary networks $\mathcal{N}_A$ and $\mathcal{N}_B$, the value of $C[i,j]$ of $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ counts the number of ways we can go from the node $i \in I$ to the node $j \in J$ passing through K, $C[i,j] = |N_A(i) \cap N_B(j)|$.

$$C[i,j] = \sum_{k \in N_A(i) \cap N_B(j)} A[i,k] \cdot B[k,j]$$

Outer product decomposition

For vectors $x = [x_1, x_2, \ldots, x_n]$ and $y = [y_1, y_2, \ldots, y_m]$ their *outer product* $x \circ y$ is defined as a matrix

$$x \circ y = [x_i \cdot y_j]_{n \times m}$$

then we can express the product $\mathbf{C}$ of two compatible matrices $\mathbf{A}$ and $\mathbf{B}$ as the *outer product decomposition*

$$\mathbf{C} = \mathbf{A} \cdot \mathbf{B} = \sum_k \mathbf{H}_k \quad \text{where} \quad \mathbf{H}_k = \mathbf{A}[\cdot, k] \circ \mathbf{B}[k, \cdot],$$

$\mathbf{A}[\cdot, k]$ is the k-th column of matrix $\mathbf{A}$, and $\mathbf{B}[k, \cdot]$ is the k-th row of matrix $\mathbf{B}$.
On the basis of outer product decomposition, we have

$$T(\mathbf{C}) = T(\sum_k \mathbf{H}_k) = \sum_k T(\mathbf{H}_k) \quad \text{and} \quad T(\mathbf{H}_k) = \mathrm{wid}_A(k) \cdot \mathrm{wod}_B(k)$$

...Outer product decomposition

```
> Cin <- normalize(Ci); WAn <- normalize(WA)
> WAn["w4",]
 AB  CD  EF  GH  IJ  KL  MN  OP  RS
0.0 0.2 0.0 0.2 0.2 0.0 0.2 0.2 0.0
> Cin["w4",]
 w1  w2  w3  w4  w5  w6
0.0 0.0 0.0 0.0 0.5 0.5
> WAn["w4",] %o% Cin["w4",]
   w1 w2 w3 w4  w5  w6
AB  0  0  0  0 0.0 0.0
CD  0  0  0  0 0.1 0.1
EF  0  0  0  0 0.0 0.0
GH  0  0  0  0 0.1 0.1
IJ  0  0  0  0 0.1 0.1
KL  0  0  0  0 0.0 0.0
MN  0  0  0  0 0.1 0.1
OP  0  0  0  0 0.1 0.1
RS  0  0  0  0 0.0 0.0
> WAn["w5",] %o% WAn["w5",]
> sum(WAn["w5",] %o% WAn["w5",])
> sum(WAn["w3",] %o% WAn["w3",])
> sum(WAn["w4",] %o% Cin["w4",])
```

Truncated fractional co-appearance networks

A usual approach to the analysis of a two-mode network is its projection to its selected mode. The obtained weighted one-mode network is afterward analyzed using standard methods. Most bibliographic two-mode networks are sparse – they have a small average degree. If the other mode has some nodes of very large degree the projection can "explode" – it is not a sparse network (increased time and space complexity) [3]. In fractional projections, the contribution of the nodes of large degree is very small and mostly doesn't affect the resulting important subnetworks – the important part of the result can be obtained by projection of a two-mode subnetwork on a subset of important nodes – a truncated projection. This idea is elaborated in the following.

Truncated standard fractional network

Let's split the set of authors A into two sets A_1 (selected authors) and A_0 (remaining authors), $A_1 \cup A_0 = A$ and $A_1 \cap A_0 = \emptyset$. We call a *truncated (standard) fractional network* the network

$$\mathbf{Cn}_{11} = \mathbf{Cn}[A_1, A_1] = n(\mathbf{WA})[W, A_1]^T \cdot n(\mathbf{WA})[W, A_1]$$

For a selected author $a \in A_1$, we denote with $\alpha(a) = \mathrm{wdeg}_{Cn_{11}}(a)$ her/his *internal* fractional contribution, and with $\beta(a) = \mathrm{wdeg}_{n(WA)}(a) - \alpha(a)$ her/his *external* fractional contribution.

We reorder the nodes of the network **Cn** according to the A_1, A_0 split (see next slide). The network matrix is split into four submatrices $\mathbf{Cn}_{ij}, i, j \in \{0, 1\}$. We denote their totals $T_{ij} = T(\mathbf{Cn}_{ij})$. Because the matrix **Cn** is symmetric we have $T_{01} = T_{10}$. T_{11} is the fractional contribution of collaboration among selected authors, $T_{10} + T_{01} = 2T_{10}$ is the fractional contribution of collaboration of selected authors with remaining authors, and T_{00} is the fractional contribution of collaboration among remaining authors.

. . . Truncated standard fractional network

$$\begin{array}{c|c|c|}
 & A_1 & A_0 \\ \hline
A_1 & \mathbf{Cn}_{11} & \mathbf{Cn}_{10} \\ \hline
A_0 & \mathbf{Cn}_{01} & \mathbf{Cn}_{00} \\ \hline
\end{array}$$

... Truncated standard fractional network

We can compute $T_{11} = T(\mathbf{Cn}_{11})$ and $T_{10} = \sum_{a \in A_1} \beta(a)$, and finally $T_{00} = T(n(\mathbf{WA})) - T_{11} - 2T_{10}$. Note that we used only information from $n(\mathbf{WA})$ and $\mathbf{Cn}_{11}$.

A primary application of the truncated standard fractional network scheme is for the set of the most active authors

$$A_1 = \{a \in A : \mathrm{wideg}_{n(WA)}(a) \geq t\}$$

where t is a selected threshold value.

Note that the computation of the vector $\mathbf{wideg}_{n(WA)}$ is cheap.

Truncated strict fractional network

Again we have $\text{wdeg}_{Ct}(a) = \text{wideg}_{n(WA)}(a)$. Therefore the scheme used for the truncated standard fractional network can be applied also for *truncated strict fractional network*

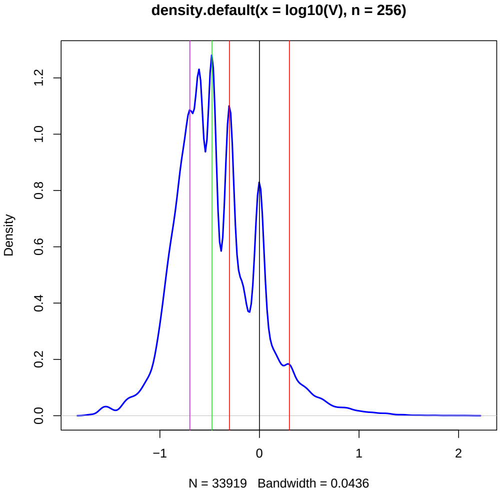
$$\mathbf{Ct}_{11} = \mathbf{Ct}[A_1, A_1] = D_0(n(\mathbf{WA})[W, A_1]^T \cdot n'(\mathbf{WA})[W, A_1])$$

Sizes of truncated networks

iMetrics network [2] and Nataliya's HKUST1 network.

	iMetrics			*HKUST1*		
interval	n	$m = \|A\|$	*avdeg*	n	$m = \|A\|$	*avdeg*
≥ 0	33919	225931	13.32	28108	45365272	3227.92
$\geq 1/10$	32418	191888	11.84	17656	3216796	364.39
$\geq 1/5$	26247	134049	10.21	10213	86529	16.94
$\geq 1/3$	14381	71967	10.01	5171	45845	17.73
$\geq 1/2$	12781	60587	9.48	4032	32806	16.27
≥ 1	6211	32395	10.43	1799	13723	15.26
≥ 2	1832	14900	16.27	689	4195	12.18
≥ 3	964	9306	19.31	369	1743	9.45
≥ 5	446	4646	20.83	172	646	7.51
≥ 10	162	1450	17.90	55	125	4.55

Density of $\log_{10}(\mathrm{wdeg}_{WAn}(a))$



Linking through a network

Assume that another network **S** on $W \times W$ is given. The network

$$\mathbf{Q} = n(\mathbf{WA})^T \cdot \mathbf{S} \cdot n(\mathbf{WA})$$

links authors to authors *through* the network **S**.
From [1] we know that

- If **S** is symmetric, $\mathbf{S}^T = \mathbf{S}$, then also **Q** is symmetric, $\mathbf{Q}^T = \mathbf{Q}$.
- $T(\mathbf{Q}) = T(\mathbf{S})$.

...Linking through a network

Let's look at

$$\mathrm{wdeg}_Q(a) = \sum_{b \in A} q[a,b] = \sum_{b \in A} \sum_{w \in W} \sum_{z \in W} nwa[w,a] \cdot s[w,z] \cdot nwa[z,b] =$$

$$= \sum_{w \in W} \sum_{z \in W} nwa[w,a] \cdot s[w,z] \cdot \mathrm{sign}(\deg(w)) =$$

$$\sum_{w \in W} nwa[w,a] \cdot \sum_{z \in W} s[w,z] = \sum_{w \in W} nwa[w,a] \cdot \mathrm{wdeg}_S(w)$$

(note $\deg(w) = 0 \Rightarrow nwa[w,a] = 0$) or in a vector form

$$\mathbf{wdeg}_Q = n(\mathbf{WA})^T \cdot \mathbf{wdeg}_S$$

The most active authors are

$$A_1 = \{a \in A : \mathrm{wdeg}_Q(a) \geq t\}$$

Again the truncation scheme can be applied.

Authors co-citation

The *co-citation* network is defined as $\mathbf{coCi} = \mathbf{Ci}^T \cdot \mathbf{Ci}$ and the *fractional co-citation* network as $\mathbf{coCin} = \mathbf{Cin}^T \cdot \mathbf{Cin}$. Both $\mathbf{coCi}$ and $\mathbf{coCin}$ are symmetric. Authors fractional co-citation network is obtained by linking authors through the fractional co-citation network

$$\mathbf{coCan} = n(\mathbf{WA})^T \cdot \mathbf{coCin} \cdot n(\mathbf{WA}) =$$

$$= n(\mathbf{WA})^T \cdot \mathbf{Cin}^T \cdot \mathbf{Cin} \cdot n(\mathbf{WA}) = \mathbf{Can}^T \cdot \mathbf{Can}$$

where $\mathbf{Can} = \mathbf{Cin} \cdot n(\mathbf{WA})$.

As in the case of $\mathbf{WA}$ we have $\mathrm{wdeg}_{Cn}(a) = \mathrm{wdeg}_{n(WA)}(a)$, for $\mathbf{Ci}$ it holds also $\mathrm{wdeg}_{coCin}(w) = \mathrm{wideg}_{n(Ci)}(w)$. Therefore

$$\mathbf{wdeg}_{coCan} = n(\mathbf{WA})^T \cdot \mathbf{wdeg}_{coCin} = n(\mathbf{WA})^T \cdot \mathbf{wideg}_{n(Ci)}$$

and it is easy to see that also

$$\mathbf{wideg}_{Can} = n(\mathbf{WA})^T \cdot \mathbf{wideg}_{n(Ci)} = \mathbf{wdeg}_{coCan}$$

. . . Authors co-citation

```
> Cin <- normalize(Ci)
> coCin <- t(Cin)%*%Cin
> coCan <- t(WAn)%*%coCin%*%WAn
> wdeg_coCan <- (t(WAn)%*%wideg(Cin))[,1]
> wdeg_coCan
    AB     CD     EF     GH     IJ     KL     MN     OP     RS
0.4556 0.9694 0.3444 0.2778 1.0806 0.3444 0.6250 0.4444 0.4583
> wodeg(coCan)
    AB     CD     EF     GH     IJ     KL     MN     OP     RS
0.4556 0.9694 0.3444 0.2778 1.0806 0.3444 0.6250 0.4444 0.4583
> Can <- Cin%*%WAn
> wideg(Can)
    AB     CD     EF     GH     IJ     KL     MN     OP     RS
0.4556 0.9694 0.3444 0.2778 1.0806 0.3444 0.6250 0.4444 0.4583
```

Bibliographic coupling

The *bibliographic coupling* network is defined as $\mathbf{biCo} = \mathbf{Ci} \cdot \mathbf{Ci}^T$. It is symmetric. The fractional approach can not be directly applied to bibliographic coupling – to get the outer product decomposition work we would need to normalize $\mathbf{Ci}$ by columns – a cited work has value 1 which is distributed equally to the citing works – the most cited works give the least. This is against our intuition. To construct a reasonable measure we can proceed as follows.

We consider matrices $\mathbf{biC} = \mathbf{Cin} \cdot \mathbf{Ci}^T$ and $\mathbf{biC}' = \mathbf{Ci} \cdot \mathbf{Cin}^T$. The weight $\mathbf{biC}[p, q] = \sum_w cin[p, w] \cdot ci[q, w]$ measures the fractional citation contribution of work p to work q. Since $\mathbf{M}[p, q] = \mathbf{M}^T[q, p]$, we have

$$\mathbf{biC}'[p, q] = \mathbf{Ci} \cdot \mathbf{Cin}^T[p, q] = (\mathbf{Ci} \cdot \mathbf{Cin}^T)^T[q, p] =$$

$$= \mathbf{Cin} \cdot \mathbf{Ci}^T[q, p] = \mathbf{biC}[q, p] = \mathbf{biC}^T[p, q]$$

Therefore $\mathbf{biC}' = \mathbf{biC}^T$ – we need to compute only the network $\mathbf{biC}$. The network $\mathbf{biC}$ is not symmetric.

Fractional bibliographic coupling

We define *fractional bibliographic coupling* as

$$\mathbf{biCon}_A[p,q] = A(\mathbf{biC}[p,q], \mathbf{biC}'[p,q]) = A(\mathbf{biC}[p,q], \mathbf{biC}[q,p])$$

where A is a selected average (arithmetic, geometric, harmonic, min, max, Jaccard, etc.). It is symmetric.

$$\mathbf{biC}[p,q] = \sum_w cin[p,w] \cdot ci[q,w] = \frac{1}{\max(1, \deg_{Ci}(p))} \sum_w ci[p,w] \cdot ci[q,w] =$$

From $\sum_w ci[p,w] \cdot ci[q,w] = |Ci(p) \cap Ci(q)|$ and $\deg_{Ci}(p) = |Ci(p)|$ we get for $Ci(p) \neq \emptyset$

$$\mathbf{biC}[p,q] = \frac{|Ci(p) \cap Ci(q)|}{|Ci(p)|} \in [0,1]$$

and therefore also $\mathbf{biCon}_A[p,q] \in [0,1]$.

Note that the underlying graph of a derived network and its fractional versions is the same. For example, $\mathrm{bin}(\mathbf{biC}) = \mathrm{bin}(\mathbf{biCo})$ $= \mathrm{bin}(\mathbf{biCon}_A)$.

Fractional bibliographic coupling

$$\mathrm{wideg}_{biC}[w] = \sum_{t \in W} ci[w,t] \cdot \sum_{z \in W} cin[z,t] = \sum_{t \in W} ci[w,t]\mathrm{wideg}_{n(Ci)}[t]$$

$$\mathbf{wideg}_{biC} = \mathbf{Ci} \cdot \mathbf{wideg}_{n(Ci)}$$

and

$$\mathrm{wodeg}_{biC}[w] = \sum_{t \in W} cin[w,t] \cdot \sum_{z \in W} ci[z,t] = \sum_{t \in W} cin[w,t]\mathrm{ideg}_{Ci}[t]$$

$$\mathbf{wodeg}_{biC} = n(\mathbf{Ci}) \cdot \mathbf{ideg}_{Ci}$$

Authors fractional bibliographic coupling

$$\mathbf{biCa} = n(\mathbf{WA})^T \cdot \mathbf{biCon}_A \cdot n(\mathbf{WA})$$

Fractional bibliographic coupling

```
> biC <- Cin %*% t(Ci)
> biCo <- Ci %*% t(Ci)
> (wicin <- wideg(Cin))
       w1        w2        w3        w4        w5        w6
0.0000000 0.3333333 0.3333333 0.8333333 1.6666667 1.8333333
> wideg(biC)
      w1       w2       w3       w4       w5       w6
2.833333 3.833333 2.500000 3.500000 1.833333 0.000000
> (Ci %*% wicin)[,1]
      w1       w2       w3       w4       w5       w6
2.833333 3.833333 2.500000 3.500000 1.833333 0.000000
> wodeg(biC)
      w1       w2       w3       w4       w5       w6
2.333333 2.666667 3.000000 3.500000 3.000000 0.000000
> (Cin %*% ideg(Ci))[,1]
      w1       w2       w3       w4       w5       w6
2.333333 2.666667 3.000000 3.500000 3.000000 0.000000
> biConG <- symm(geom,biC)
> (biCa <- through(WAn,biConG))
          AB        CD         EF         GH        IJ         KL
AB 0.5915234 0.5093036 0.38408370 0.33265960 0.5141621 0.18150245 0.1252
CD 0.5093036 0.4693287 0.35854188 0.26154849 0.4273296 0.16578111 0.1107
EF 0.3840837 0.3585419 0.28775510 0.16711538 0.2893263 0.12221088 0.0707
GH 0.3326596 0.2615485 0.16711538 0.25997732 0.3628391 0.10286179 0.0944
IJ 0.5141621 0.4273296 0.28932627 0.36283912 0.5334787 0.17063957 0.1380
KL 0.1815025 0.1657811 0.12221088 0.10286179 0.1706396 0.06777778 0.0435
MN 0.1252199 0.1107868 0.07078678 0.09443311 0.1380033 0.04357023 0.0400
OP 0.1850727 0.1621348 0.09856456 0.15007835 0.2214264 0.07134800 0.0635
RS 0.0000000 0.0000000 0.00000000 0.00000000 0.0000000 0.00000000 0.0000
>
```

Conclusions

- efficient implementation as Pajek macros and in Python Nets package.

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