# Analysis of bibliographic networks

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# Outline



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**Current version of slides (July 19, 2019 at 07 : 16)**: slides PDF
https://github.com/bavla/biblio/blob/master/doc/SS/bibnet19b.pdf

# Networks

A *network* is based on two sets – a set of *nodes* (vertices), that represent the selected *units*, and a set of *links* (lines), that represent *ties* between units. They determine a *graph*. A link can be *directed* – an *arc*, or *undirected* – an *edge*.
Additional data about nodes or links may be known – their *properties* (attributes). For example: name/label, type, age, value, ...

**Network = Graph + Data**

A *network* $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{P}, \mathcal{W})$ consists of:

- a *graph* $\mathcal{G} = (\mathcal{V}, \mathcal{L})$, where $\mathcal{V}$ is the set of nodes, $\mathcal{A}$ is the set of arcs, $\mathcal{E}$ is the set of edges, and $\mathcal{L} = \mathcal{E} \cup \mathcal{A}$ is the set of links.
  $n = |\mathcal{V}|$, $m = |\mathcal{L}|$
- $\mathcal{P}$ *node value functions* / properties: $p : \mathcal{V} \to A$
- $\mathcal{W}$ *link value functions* / weights: $w : \mathcal{L} \to B$

# Types of networks



In a *two-mode* network $\mathcal{N} = ((\mathcal{V}_1, \mathcal{V}_2), \mathcal{L}, \mathcal{P}, \mathcal{W})$ its set of nodes is split to two subsets. Each link has its end-nodes in both sets.

In a *multi-relational* network $\mathcal{N} = (\mathcal{V}, (\mathcal{L}_i, i \in I), \mathcal{P}, \mathcal{W})$ the set of its links is partitioned into several mutally disjoint subsets – relations. (Subject Verb Object).

In a *temporal* network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{T}, \mathcal{P}, \mathcal{W})$ the time $\mathcal{T}$ is added. To each node and to each link its *activity* set is assigned. Also properties and weights can change through time – temporal quantities.

A *collection* of networks consists of some networks with common subsets of nodes.

Types of networks can be combined – for example: a temporal two-mode multi-relational network.

# Analyses



The saved records from a data base can still contain some inconsistencies. Some of them are detected as results of the analyses. The simplest way to deal with them is to correct them in the saved data base file and rerun the creation of `Pajek`'s files and analyses.

To improve the quality of the data some tools for detecting (possible) inconsistencies could be developed.

Check (in `Pajek`) the obtained networks for multiple links and remove them, if they exist. Remove also the loops from the citation network.

# Cleaning networks



Using on a file $N$`.net` the commands

```
Info/Network/General
Net/Transform/Remove/Loops
Net/Transform/Remove lines/Single line
```

we get the information about the number of loops and multiple links. Remove loops, and replace multiple links with single links. The obtained network we save (`Options - Save coordinates [OFF]`) to file $Nclean$`.net`.

# Network boundary problem

If the obtained collection of networks contains also the citation network it can be used to address the *network boundary problem*. Otherwise we assume that we included in our bibliography (all/most of) the relevant works.

The first option is to limit the network to the works with complete descriptions – records from the WoS file, $DC > 0$. Since for cited-only works only the first author (no keywords, ...) is known this option is used for most analyses.

# Network boundary problem

We can get a richer network if we decide to include also some cited-only works that are cited often – at least $k$ times; we delete nodes for which it holds $(0 < \text{indeg}(v) < k) \wedge (\text{outdeg}(v) = 0)$.

```
Net/Partition/Degree/Input
Partition/Binarize  [1-(k-1)]
Net/Partition/Degree/Output
Partition/Binarize  [0]
[select partition 1]
[select partition 2]
Partitions/Min(V1,V2)
Operations/Extract from Network/Partition [0]
```

For some (most frequent) of these additional works we can augment the WoS file with their descriptions (without CR data).

# PEERE network



To the *Web of Science* (WoS) we put the query `"peer review*"`. In May and June 2015 we got (from Web of Science Core Collection) 17053 hits, and additional 2867 hits for the query `refereeing`.
In March 2016 we updated the data by adding hits for the years 2015 and 2016 and manually prepared short descriptions for the most cited works (fields: AU, PU, TI, PY, PG, KW; but without CR data).
The analysis in 2015 revealed many papers without WoS descriptions having large indegrees in the citation network. We manually searched in WoS for each of them (with indegree larger or equal to 20) and, if found, we added them into the data set. Important earlier papers often did not use the now established terminology and were therefore overlooked by our queries.
The final run of the program `WoS2Pajek` produced networks with sets of the following sizes: works $|W| = 721547$, authors $|A| = 295849$, journals $|J| = 39988$, and keywords $|K| = 36279$. In both phases 22981 records were collected. There were 887 duplicates (considered only once).
We removed multiple links and loops (resulting from homonyms) from the networks. The cleaned citation network **CiteAll** has $n = 721547$ nodes and $m = 869821$ arcs. paper

# PEERE – most cited works



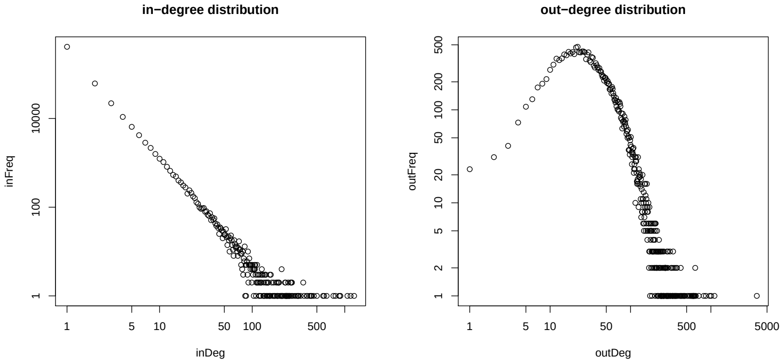
| n | freq | first author | title |
|---|---|---|---|
| 1 | 173 | Cohen, J | Statistical Power Analysis for the Behavioral Sciences. Routledge,1988 |
| 2 | 164 | Peters, DP | Peer-review practices of psychological journals - the fate of . . . Behav Brain S |
| 3 | 151 | Egger, M | Bias in meta-analysis detected by a simple, graphical test. Brit Med J, 1997 |
| 4 | 150 | Stroup, DF | Meta-analysis of observational studies in epidemiology - A proposal for repor |
| 5 | 135 | Dersimonian, R | Metaanalysis in clinical-trials. Control Clin Trials, 1986 |
| 6 | 130 | Zuckerma.H | Patterns of evaluation in science - institutionalisation, structure and function |
| 7 | 130 | Higgins, JPT | Cochrane Handbook for Systematic Reviews of Interventions. Cochrane, 201 |
| 8 | 126 | Moher, D | Preferred Reporting Items for Systematic Reviews and Meta-Analyses: The P |
| 9 | 125 | Higgins, JPT | Measuring inconsistency in meta-analyses. Brit Med J, 2003 |
| 10 | 121 | Cicchetti, DV | The reliability of peer-review for manuscript and grant submissions - . . . Beha |
| 11 | 119 | Hirsch, JE | An index to quantify an individual's scientific research output. P Natl Acad S |
| 12 | 114 | Mahoney, M | Publication prejudices: An experimental study of confirmatory bias . . . Cognit |
| 13 | 114 | van Rooyen, S | Effect of open peer review on quality of reviews and on reviewers' recommen |
| 14 | 114 | Easterbrook, PJ | Publication bias in clinical research. Lancet, 1991 |
| 15 | 110 | Landis, JR | Measurement Of Observer Agreement For Categorical Data. Biometrics, 197 |
| 16 | 109 | Godlee, F | Effect on the quality of peer review of blinding reviewers and asking them to |
| 17 | 108 | Horrobin, DF | The philosophical basis of peer-review and the suppression of innovation. JA |
| 18 | 107 | Moher, D | Preferred Reporting Items for Systematic Reviews and Meta-Analyses: PRISI |
| 19 | 107 | Jadad, AR | Assessing the quality of reports of randomized clinical trials: Is blinding nece |
| 20 | 105 | Mcnutt, RA | The effects of blinding on the quality of peer-review - a randomized trial. JA |
| 21 | 104 | Cole, S | Chance and consensus in peer-review. Science, 1981 |
| 22 | 103 | Moher, D | Improving the quality of reports of meta-analyses of randomised controlled tr |
| 23 | 98 | Justice, AC | Does masking author identity improve peer review quality? - A randomized c |
| 24 | 97 | Lock, S | A Difficult Balance: Editorial Peer Review in Medicine. Nuffield Trust, 1985 |
| 25 | 95 | van Rooyen, S | Effect of blinding and unmasking on the quality of peer review - A randomize |
| 26 | 92 | Black, N | What makes a good reviewer and a good review for a general medical journa |
| 27 | 91 | Scherer, RW | Full publication of results initially presented in abstracts - a metaanalysis. JA |
| 28 | 90 | Higgins, JPT | Quantifying heterogeneity in a meta-analysis. Stat Med, 2002 |
| 29 | 90 | Smith, R | Peer review: a flawed process at the heart of science and journals. J Roy Soc |
| 30 | 87 | Goodman, SN | Manuscript quality before and after peer-review and editing at annals of inte |

# Distributions

We read in `Pajek` the citation network `cite.net` for ‘centrality’ literature. First we remove loops and multiple lines. Then we determine the indegrees and outdegrees. We dispose the normalized degree vectors, transform both partitions into vectors and call R from `Pajek` submitting all vectors.

```
########################################
 R called from Pajek
 The following vectors read:
 v3   :  From partition 1 (548600)
 v4   :  From partition 2 (548600)
---------------------------------------------------------------------
> inTab <- table(v3)
> indeg <- as.integer(names(inTab))
> inDeg <- indeg[indeg>0]
> inFreq <- as.vector(inTab[indeg>0])
> plot(inDeg,inFreq,log='xy',main="in-degree distribution")
> ouTab <- table(v4)
> outdeg <- as.integer(names(ouTab))
> outDeg <- outdeg[outdeg>0]
> outFreq <- as.vector(ouTab[outdeg>0])
> plot(outDeg,outFreq,log='xy',main="out-degree distribution")
```

# Distributions




The in-degree distribution is "scale-free"-like. The parameters can be determined using the package of Clauset, Shalizi and Newman. See also Stumpf, et al.: Critical Truths About Power Laws.

## SN5 citation network input degrees – scale-free fit

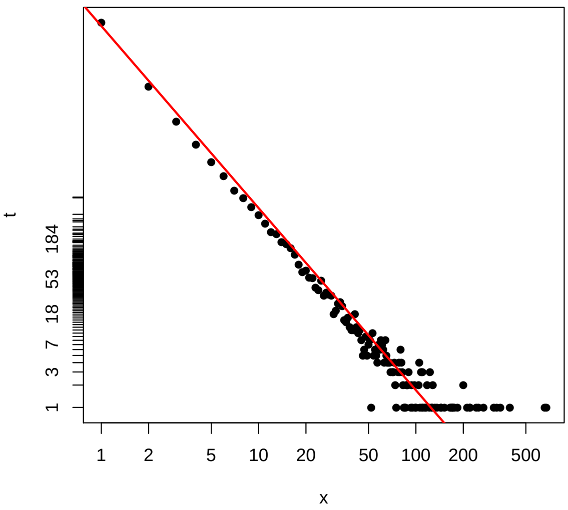


SN5 ("social network*" AND SO=(Social networks)) plus most frequently cited works plus around 100 SNA researchers (collected January 2008). The function `plfit(x)` fits the distribution $p(x) = \frac{\alpha-1}{x_{min}}(\frac{x}{x_{min}})^{-\alpha}$. It returns estimates of *alpha*, $x_{min}$ and $D$ - the Kolmogorov-Smirnov goodness-of-fit statistic. Install library VGAM (zeta function).

```
> source("http://tuvalu.santafe.edu/~aaronc/powerlaws/plfit.r")
> infile <- "https://raw.githubusercontent.com/bavla/biblio/master/dat/indegCite.vec"
> d <- read.table(infile,skip=1,header=FALSE)
> t <- table(d)
> x <- as.numeric(names(t))
> plot(x,t,log="xy",pch=16,main="SN5 cite input degrees / logs")
> len <- length(t)
> z <- t[len:1]; y <- cumsum(z); z <- y[len:1]
> plot(x,z,log="xy",pch=16,cex=0.6,xlab='deg',ylab='num',main="BA cum logs")
> D <- as.vector(d$V1); D <- D[D>0]
> r <- plfit(D)
> names(r)
[1] "xmin"  "alpha" "D"
> r$alpha
[1] 2.45
> r$xmin
[1] 3
> r$D
[1] 0.006133737
> plot(x,t,log="xy",pch=16,main="SN5 cite input degrees / logs")
> b <- (r$alpha-1)*r$xmin**(r$alpha-1)
> abline(b-r$xmin+1,-r$alpha,col="red",lw=2)
```

# Distributions

## SN5 citation network input degrees – scale-free fit




Bibliographic networks

V. Batagelj, D. Maltseva

Networks
Statistics
Citation
Two-mode Ns
Multiplication
Derived Ns
Temporal Ns
References

# Distributions

From the file `Year.clu` we can get the distribution of *citations by years*. For the centrality network we get:

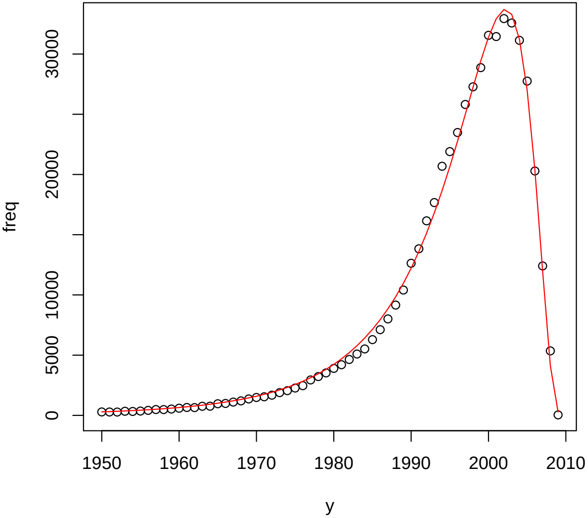
```
> setwd("C:/Users/Batagelj/work/Python/WoS/Central")
> years <- read.table(file="Year.clu",header=FALSE,skip=2)$V1
> t <- table(years)
> year <- as.integer(names(t))
> freq <- as.vector(t[1950<=year & year<=2009])
> y <- 1950:2009
> plot(y,freq)
> model <- nls(freq~c*dlnorm(2010-y,a,b),start=list(c=350000,a=2,b=0.7))
> model
Nonlinear regression model
  model:  freq ~ c * dlnorm(2010 - y, a, b)
   data:  parent.frame()
        c         a         b
5.427e+05 2.491e+00 6.624e-01
 residual sum-of-squares: 20474181

Number of iterations to convergence: 7
Achieved convergence tolerance: 3.978e-06
> lines(y,predict(model,list(x=2010-y)),col='red')
```

It can be well approximated by the *lognormal distribution*, but also by the *generalized reciprocal power exponential curve* $c * (x+d)^{\frac{a}{b+x}}$.

# Distributions





# Zentralblatt MATH networks

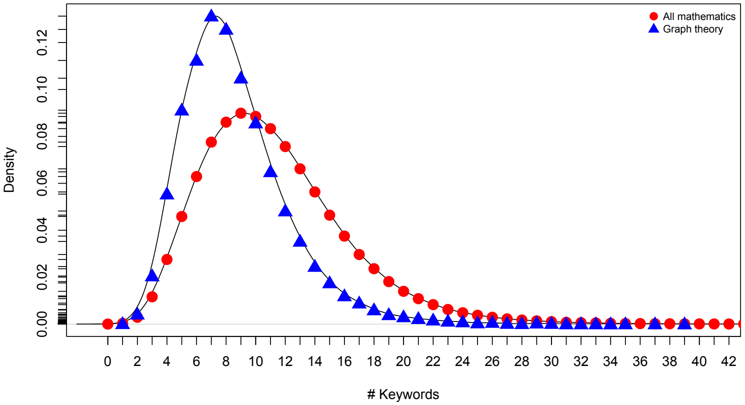
Data from the ZB data base for years 1990–2010.

| Network | **WA** | **WJ** | **WK** | **WM** |
|---|---|---|---|---|
| Size of the first set | 1339201 | 1339201 | 1339201 | 1339201 |
| Size of the second set | 557104 | 3158 | 143513 | 12390 |
| Number of arcs | 2550437 | 1331036 | 15062377 | 3370820 |

See the paper.

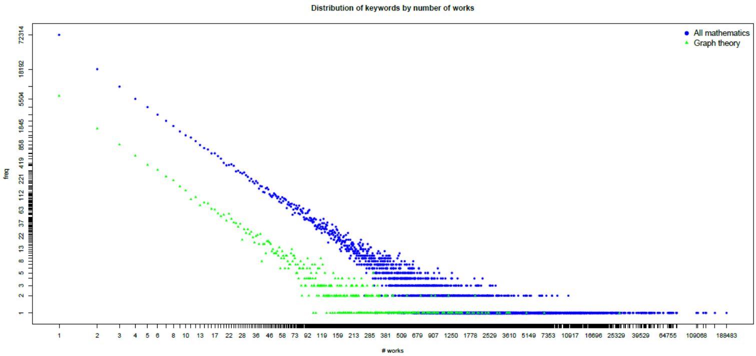
# Distributions – number of keywords





# Distributions – keywords by the number of works using a keyword in their description





# Number of authors: temporal distribution

## SNA 2018

Combining the number of authors with the publication year we get their temporal distribution, describing how the number of authors is changing through years. See next slide.

The results show that since 1980s, the number of single author papers droped from 70% to almost 10%. The number of papers authored by a pair of authors is relatively constant – around 25%. The numbers of papers authored by 3 and more authors are increasing (3: from 6% to 25%, 4: from 2% to 17%, 5: from 0% to 10%, etc.).

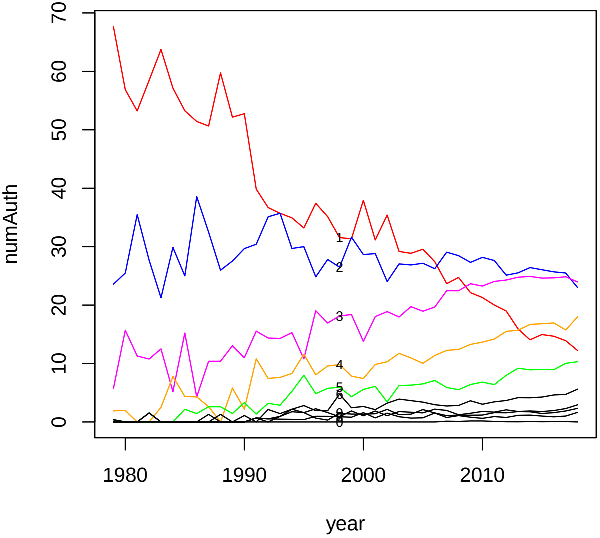
Besides the general trend to higher collaboration the reason could be also the expansion of SNA to other disciplines (physics, computer science, neuro science, biology, chemistry, etc.) with different writing cultures.

SocNet

# Number of authors: temporal distribution

## SNA 2018

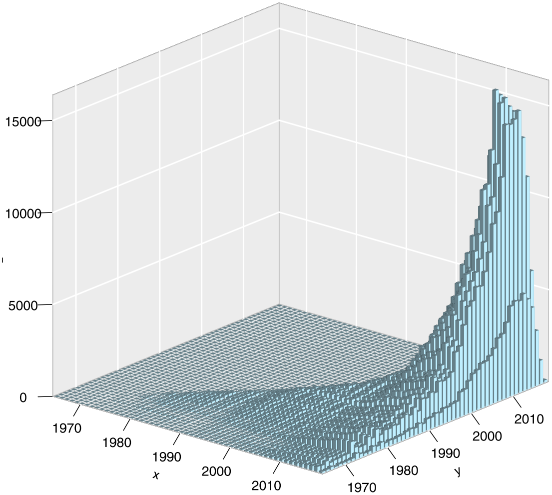




It is interesting to observe how many citations are made per year. We combined CiteB network with the partition on the year of publication and constructed a network of citations between years, where the values are equal to the number of times that all works published in one year were cited in all works published in another year (the network is directed, only later years can cite previous years).

Figure on the next slide presents the distribution of citations between years in a three-dimensional space. The majority of citations in recent works are made to recent works as well. The years having the largest amount of citations from other years are 2007 (80,129), 2008 (77,595), 2009 (82,294), 2010 (88,840), 2011 (79,843). The largest number of citations are from 2015 and 2016 to 2010 (16,384 and 15,755, respectively) and 2011 (16,026 and 15,944).

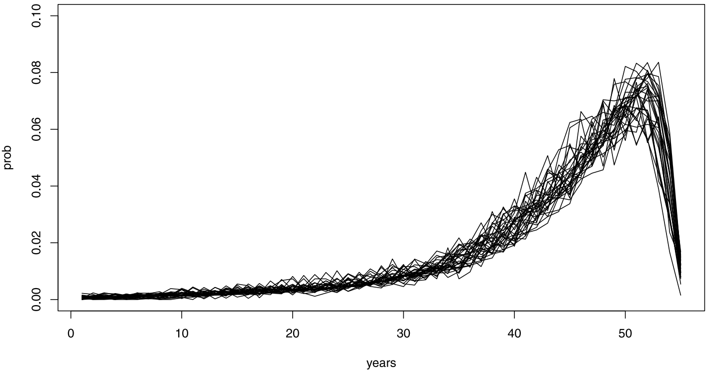
Figure on the next slide presents the normalized curves of values of citations per year in the period 1985-2018 (54 years in total). The result shows that the yearly citation patterns do not vary significantly from year to year – there are always noticeably more citations made of recent works, than of works published previously.


15000
10000
5000
0
1970
1980
1990
2000
2010
x
1970
1980
1990
2000
2010
y

SocNet

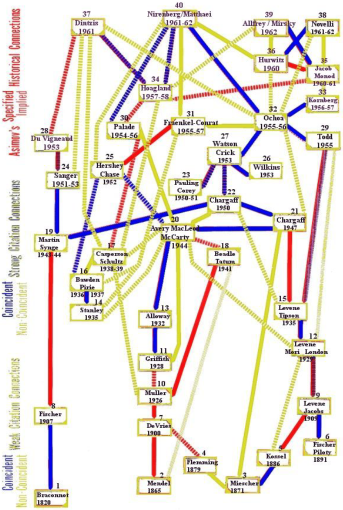
# Citations per year; normalized curves

## SNA 2018




SocNet

# Citation networks




The citation network analysis started in 1964 with the paper of Garfield et al. In 1989 Hummon and Doreian proposed three indices – weights of arcs that provide us with automatic way to identify the (most) important part of the citation network. We developed an algorithm to efficiently compute SPC weights, provided the citation network is acyclic.

# . . . Citation networks

In a given set of works/nodes $W$ (articles, books, works, etc.) we introduce a *citing relation*/set of arcs $\mathbf{Ci} \subseteq W \times W$

$$u \, \mathbf{Ci} \, v \equiv u \text{ cites } v$$

which determines a *citation network* $\mathcal{N} = (W, \mathbf{Ci})$.

A citing relation is usually *irreflexive* (no loops) and (almost) *acyclic*. We shall assume that it has these two properties. Since in real-life citation networks the strong components are small (usually 2 or 3 nodes) we can transform such network into an acyclic network by shrinking strong components and deleting loops.

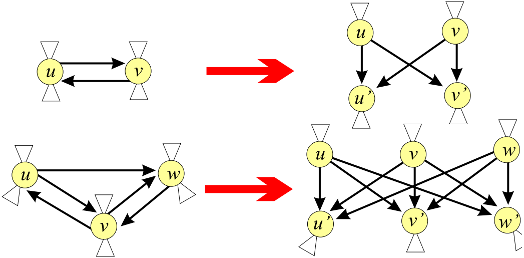
# Preparing the citation network

For further analysis the citation network has to be acyclic – has no nontrivial strong component. To identify nontrivial strong components and extract them use the commands:

```
Net/Components/Strong [2]
Operations/Extract from Network/Partition [1-*]
Operations/Transform/Remove Lines/Between Clusters
```

Save the obtained network to a file *N*`strong.net`.

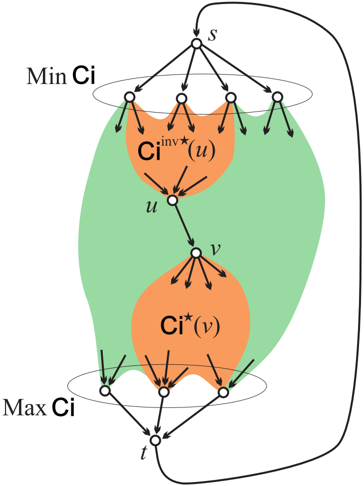
# . . . Preparing the citation network




To transform the network *PR*`citeR.net` into an acyclic network using the preprint transformation use the `Pajek`'s command

`Network/Acyclic Network/Transform/Preprint Transformation`

# Standardized citation network




We assume that the citation relation **Ci** is acyclic. It is useful to transform a citation network to its *standardized* form by adding a common *source* node $s \notin W$ and a common *sink* node $t \notin W$. The source $s$ is linked by an arc to all minimal elements of **Ci**; and all maximal elements of **Ci** are linked to the sink $t$. We add also the 'feedback' arc $(t, s)$.

The *search path count* (SPC) method is based on counters $n(u, v)$ that count the number of different paths from $s$ to $t$ through the arc $(u, v)$. To compute $n(u, v)$ we introduce two auxiliary quantities: $n^-(v)$ counts the number of different paths from $s$ to $v$, and $n^+(v)$ counts the number of different paths from $v$ to $t$. Then

$$n(u, v) = n^-(u) \cdot n^+(v)$$

There exists a very efficient algorithm to compute counters $n^-(v)$ and $n^+(v)$.

The quantities used to compute the arc weights $n$ can be used also to define the corresponding *node weight* $t_c$

$$t_c(u) \quad = \quad n^-(u) \cdot n^+(u)$$

It is counting the number of paths through the node $u$.

# Properties of SPC weights

The values of counters $n(u,v)$ form a flow in the citation network – the *Kirchoff's vertex law* holds: For every node $u$ in a standardized citation network *incoming flow* = *outgoing flow*:

$$\sum_{v:v\,\mathbf{Ci}\,u} n(v,u) = \sum_{v:u\,\mathbf{Ci}\,v} n(u,v) = n^-(u) \cdot n^+(u) = t_c(u)$$

The weight $n(t,s)$ equals to the total flow through network and provides a natural normalization of weights

$$w(u,v) = \frac{n(u,v)}{n(t,s)} \quad \Rightarrow \quad 0 \leq w(u,v) \leq 1$$

and if $C$ is a minimal arc-cut-set $\sum_{(u,v)\in C} w(u,v) = 1$.
In large networks the values of weights can grow very large. This should be considered in the implementation of the algorithms.

# Cuts



The standard approach to find interesting groups inside a network was based on properties/weights – they can be *measured* or *computed* from network structure (for example Kleinberg's hubs and authorities).

The *node-cut* of a network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, p)$, $p : \mathcal{V} \to \mathbb{R}$, at selected level $t$ is a subnetwork $\mathcal{N}(t) = (\mathcal{V}', \mathcal{L}(\mathcal{V}'), p)$, determined by the set

$$\mathcal{V}' = \{v \in \mathcal{V} : p(v) \geq t\}$$

and $\mathcal{L}(\mathcal{V}')$ is the set of links from $\mathcal{L}$ that have both endnodes in $\mathcal{V}'$.

The *link-cut* of a network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, w)$, $w : \mathcal{L} \to \mathbb{R}$, at selected level $t$ is a subnetwork $\mathcal{N}(t) = (\mathcal{V}(\mathcal{L}'), \mathcal{L}', w)$, determined by the set

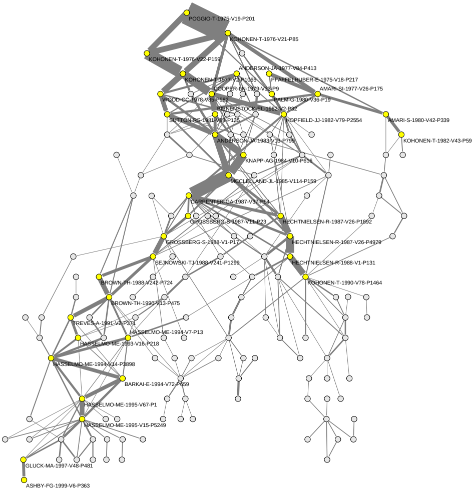
$$\mathcal{L}' = \{e \in \mathcal{L} : w(e) \geq t\}$$

and $\mathcal{V}(\mathcal{L}')$ is the set of all endnodes of the links from $\mathcal{L}'$.

```
File/Network/read  eatRS.net
Info/Network/Line values         ... >= 70
Net/Transform/Remove/Lines with Value/lower than   70
Net/Partitions/Degree/All
Operations/Extract from Network/Partition   1-*
Net/Components/Weak
Draw/Draw-Partition
```

# Citation weights



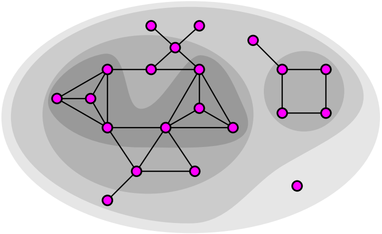



Main subnetwork (arc cut at level 0.007) of the SOM (selforganizing maps) citation network (4470 nodes, 12731 arcs).

For visualization of acyclic networks in Pajek use the macro `layers`.

See paper.

# Cores and generalized cores



The notion of core was introduced by Seidman in 1983. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a graph. A subgraph $\mathcal{H} = (C, \mathcal{E}|C)$ induced by the set $C$ is a $k$-*core* or a *core of order* $k$ iff $\forall v \in C : \deg_{\mathcal{H}}(v) \geq k$, and $\mathcal{H}$ is a maximal subgraph with this property. The core of maximum order is also called the *main* core.

The *core number* of a node $v$ is the highest order of a core that contains this node. The degree $\deg(v)$ can be: in-degree, out-degree, in-degree + out-degree, etc., determining different types of cores.

# Properties of cores

From the figure, representing 0, 1, 2 and 3 core, we can see the following properties of cores:

- The cores are nested: $i < j \implies \mathcal{H}_j \subseteq \mathcal{H}_i$
- Cores are not necessarily connected subgraphs.

An efficient algorithm for determining the cores hierarchy is based on the following property:

> *If from a given graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ we recursively delete all nodes, and edges incident with them, of degree less than $k$, the remaining graph is the $k$-core.*

# ... Properties of cores

The cores, because they can be determined very efficiently, are one among few concepts that provide us with meaningful decompositions of large networks. We expect that different approaches to the analysis of large networks can be built on this basis. For example: we get the following bound on the chromatic number of a given graph $\mathcal{G}$

$$\chi(\mathcal{G}) \leq 1 + \mathrm{core}(\mathcal{G})$$

Cores can also be used to localize the search for interesting subnetworks in large networks since: if it exists, a $k$-component is contained in a $k$-core; and a $k$-clique is contained in a $k$-core.
For details see the paper.

# Generalized cores

The notion of core can be generalized to networks. Let $\mathcal{N} = (\mathcal{V}, \mathcal{E}, w)$ be a network, where $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is a graph and $w : \mathcal{E} \to \mathbb{R}$ is a function assigning values to edges. A *node property function* on **N**, or a *p-function* for short, is a function $p(v, U)$, $v \in \mathcal{V}$, $U \subseteq \mathcal{V}$ with real values. Let $N_U(v) = N(v) \cap U$. Besides degrees and (corrected) clustering coefficient, here are some examples of $p$-functions:

$$p_S(v, U) = \sum_{u \in N_U(v)} w(v, u), \text{ where } w : \mathcal{E} \to \mathbb{R}_0^+$$

$$p_M(v, U) = \max_{u \in N_U(v)} w(v, u), \text{ where } w : \mathcal{E} \to \mathbb{R}$$

$$p_k(v, U) = \text{number of cycles of length } k \text{ through the node } v \text{ in } (U, $$

The subgraph $\mathcal{H} = (C, \mathcal{E}|C)$ induced by the set $C \subseteq \mathcal{V}$ is a *p-core at level* $t \in \mathbb{R}$ iff $\forall v \in C : t \leq p(v, C)$ and $C$ is a maximal such set.

# Additional $p$-functions



relative density

$$p_\gamma(v,\mathcal{C}) = \frac{\deg(v,\mathcal{C})}{\max_{u\in N(v)} \deg(u)}, \text{ if } \deg(v) > 0;\ 0, \text{ otherwise}$$

diversity

$$p_\delta(v,\mathcal{C}) = \max_{u\in N^+(v,\mathcal{C})} \deg(u) - \min_{u\in N^+(v,\mathcal{C})} \deg(u)$$

average weight

$$p_a(v,\mathcal{C}) = \frac{1}{|N(v,\mathcal{C})|} \sum_{u\in N(v,\mathcal{C})} w(v,u), \text{ if } N(v,\mathcal{C}) \neq \emptyset;\ 0, \text{ otherwise}$$

# Generalized cores algorithm



The function $p$ is *monotone* iff it has the property

$$C_1 \subset C_2 \Rightarrow \forall v \in \mathcal{V} : (p(v, C_1) \leq p(v, C_2))$$

The degrees and the functions $p_S$, $p_M$ and $p_k$ are monotone. For a monotone function the $p$-core at level $t$ can be determined, as in the ordinary case, by successively deleting nodes with value of $p$ lower than $t$; and the cores on different levels are nested

$$t_1 < t_2 \Rightarrow \mathcal{H}_{t_2} \subseteq \mathcal{H}_{t_1}$$

The $p$-function is *local* iff $p(v, U) = p(v, N_U(v))$ .
The degrees, $p_S$ and $p_M$ are local; but $p_k$ is **not** local for $k \geq 4$. For a local $p$-function an $O(m \max(\Delta, \log n))$ algorithm for determining the $p$-core levels exists, assuming that $p(v, N_C(v))$ can be computed in $O(\deg_C(v))$.
For details see the paper.

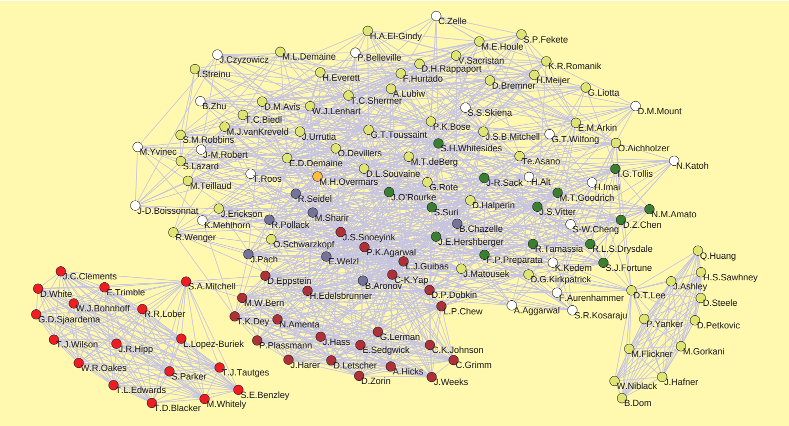
# Cores and generalized cores / Pajek commands



```
File/Network/Read  [Geom.net]
Net/Partitions/Core/All
Info/Partition
Operations/Extract from Network/Partition [13-*]
Draw/Draw-Partition
Layout/Energy/Kamada-Kawai
Options/Values of lines/Similarities
Layout/Energy/Kamada-Kawai
Operations/Extract from Network/Partition [21]
Draw
Layout/Energy/Kamada-Kawai
Options/Values of lines/Forget
Layout/Energy/Kamada-Kawai
[select Geom.net]
Net/Vector/PCore/Sum/All
Info/Vector
Vector/Make Partition/by Intervals/Selected Thresholds [45]
Info/Partition
Operations/Extract from Network/Partition [2]
Draw
Options/Values of lines/Similarities
Layout/Energy/Fruchterman-Reingold
```

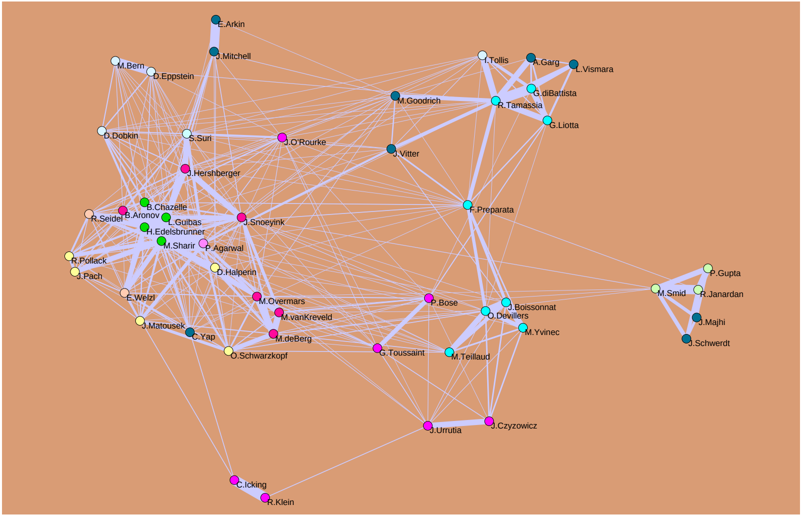
# Cores of orders 10–21 in Computational Geometry





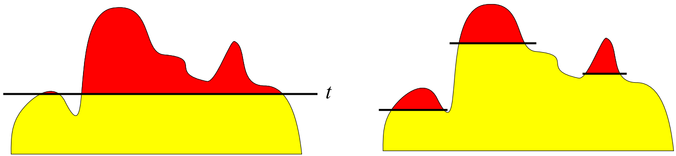
# $p_S$-core at level 46 in Computational Geometry network





# Islands

If we represent a given or computed value of nodes / links as a height of nodes / links and we immerse the network into a water up to selected level we get *islands*. Varying the level we get different islands.




We developed very efficient algorithms to determine the islands hierarchy and to list all the islands of selected sizes.
See details.

# . . . Islands



Islands are very general and efficient approach to determine the 'important' subnetworks in a given network.
We have to express the goals of our analysis with a related property of the nodes or weight of the links. Using this property we determine the islands of an appropriate size (in the interval $k$ to $K$).

In large networks we can get many islands which we have to inspect individually and interpret their content.
An important property of the islands is that they identify locally important subnetworks on different levels. Therefore they detect also emerging groups.

The set of nodes $\mathcal{C} \subseteq \mathcal{V}$ is a *local node peak*, if it is a regular node island and all of its nodes have the same value. Node island with a single local node peak is called a *simple node island*. In similar way we define simple link island.

# . . . Islands

A set of nodes $C \subseteq \mathcal{V}$ is a *regular node island* in a network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, p)$, $p : \mathcal{V} \to \mathbb{R}$ iff it induces a connected subgraph and the nodes from the island are 'higher' than the neighboring nodes

$$\max_{u \in N(C)} p(u) < \min_{v \in c} p(v)$$

A set of nodes $C \subseteq \mathcal{V}$ is a *regular link island* in a network $\mathcal{N} = (\mathcal{V}, \mathcal{L}, w)$, $w : \mathcal{L} \to \mathbb{R}$ iff it induces a connected subgraph and the links inside the island are 'stronger related' among them than with the neighboring nodes – in $\mathcal{N}$ there exists a spanning tree $\mathcal{T}$ over $C$ such that

$$\max_{(u,v) \in \mathcal{L}, u \notin C, v \in C} w(u, v) < \min_{(u,v) \in \mathcal{T}} w(u, v)$$

`Network/Create Partition/Islands/Line Weights`
`Operations/Network+Vector/Islands/Vertex Property`

# Some properties of islands

- The sets of nodes of connected components of node/link-cut at selected level $t$ are regular node/link islands.
- The set $\mathcal{H}_p(\mathcal{N})$ of all regular node islands of a network $\mathcal{N}$ is a complete hierarchy:
  - two islands are disjoint or one of them is a subset of the other
  - each node belongs to at least one island
- The set $\mathcal{H}_w(\mathcal{N})$ of all nondegenerated regular link islands of a network $\mathcal{N}$ is a hierarchy (not necessarily complete):
  - two islands are disjoint or one of them is a subset of the other
- Node/link islands are invariant for the strictly increasing transformations of the property $p$ / weight $w$.
- Two linked nodes cannot/may belong to two disjoint regular node/link islands.

A *simple island* is an island with only one peak.

# US patents



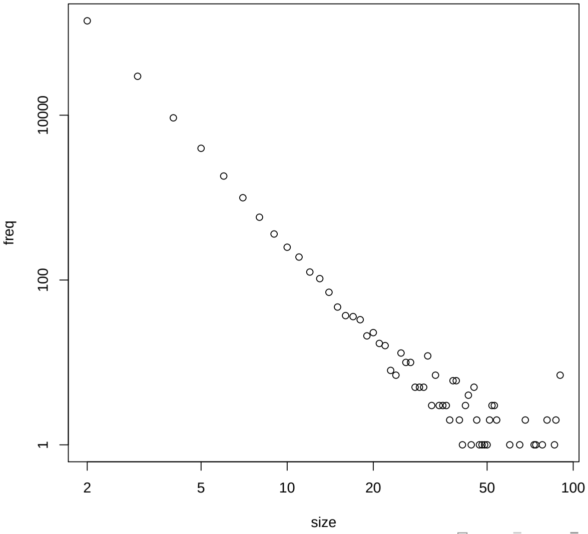
US patents network (Nber, US Patents) has 3774768 nodes and16522438 arcs (1 loop). Without the loop it is acyclic. The weight of an arc is the proportion of paths through the arc from some initial node to some terminal node. We determined al (2,90)-link islands. The corresponding subnetwork has 470137 nodes, 307472 arcs and different $k$: $C_2$ =187610, $C_5$ =8859, $C_{30}$ =101, $C_{50}$ =30, ... islands. Rolex

```
 [1]    0 139793  29670 9288 3966 1827 997 578 362 250
[11]  190    125    104   71   47   37  36  33  21  23
[21]   17     16      8    7   13   10  10   5   5   5
[31]   12      3      7    3    3    3   2   6   6   2
[41]    1      3      4    1    5    2   1   1   1   1
[51]    2      3      3    2    0    0   0   0   0   1
[61]    0      0      0    0    1    0   0   2   0   0
[71]    0      0      1    1    0    0   0   1   0   0
[81]    2      0      0    0    0    1   2   0   0   7
```

The *Main path* starts in a link with the largest SPC weight and expands in both directions following the adjacent link with the largest SPC weight.

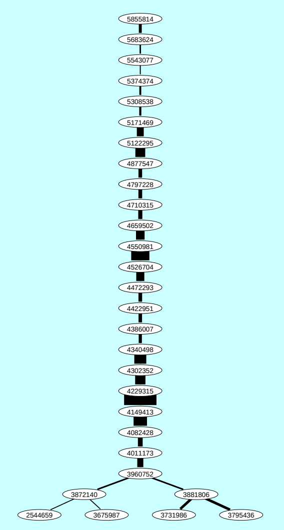
The *CPM path* is determined using the Critical Path Method from Operations Research (the sum of SPC weights along a path is maximal).

# Distribution of island size



# Main path and main island in US Patents

## Nber, US Patents; $n = 3774768$, $m = 16522438$



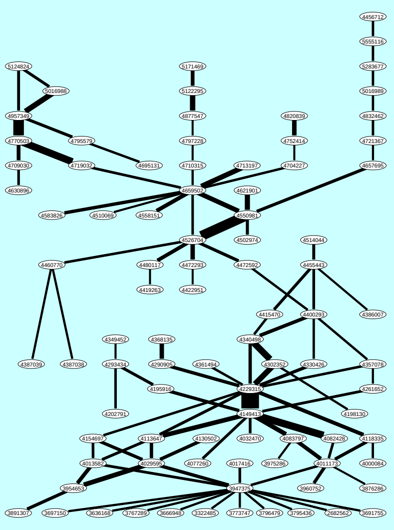



Table 1: Patents on the liquid-crystal display

| patent | date | author(s) and title |
|---|---|---|
| 2544659 | Mar 13, 1951 | Dreyer. Dichroic light-polarizing sheet and the like and the formation and use thereof |
| 2682562 | Jun 29, 1954 | Wender, et al. Reduction of aromatic carbinols |
| 3322485 | May 30, 1967 | Williams. Electro-optical elements utilazing an organic nematic compound |
| 3636168 | Jan 18, 1972 | Josephson. Preparation of polynuclear aromatic compounds |
| 3666948 | May 30, 1972 | Mechlowitz, et al. Liquid crystal termal imaging system having an undisturbed image on a disturbed background |
| 3675987 | Jul 11, 1972 | Rafuse. Liquid crystal compositions and devices |
| 3691755 | Sep 19, 1972 | Girard. Clock with digital display |
| 3697150 | Oct 10, 1972 | Wysochi. Electro-optic systems in which an electrophoretic-like or dipolar material is dispersed throughout a liquid crystal to reduce the turn-off time |
| 3731986 | May 8, 1973 | Fergason. Display devices utilizing liquid crystal light modulation |
| 3767289 | Oct 23, 1973 | Aviram, et al. Class of stable trans-stilbene compounds, some displaying nematic mesophases at or near room temperature and others in a range up to 100°C |
| 3773747 | Nov 20, 1973 | Steinstrasser. Substituted azoxy benzene compounds |
| 3795436 | Mar 5, 1974 | Boller, et al. Nematogenic material which exhibit the Kerr effect at isotropic temperatures |
| 3796479 | Mar 12, 1974 | Helfrich, et al. Electro-optical light-modulation cell utilizing a nematogenic material which exhibits the Kerr effect at isotropic temperatures |
| 3872140 | Mar 18, 1975 | Klanderman, et al. Liquid crystalline compositions and method |
| 3876286 | Apr 8, 1975 | Deutscher, et al. Use of nematic liquid crystalline substances |
| 3881806 | May 6, 1975 | Suzuki. Electro-optical display device |
| 3891307 | Jun 24, 1975 | Tsukamoto, et al. Phase control of the voltages applied to opposite electrodes for a cholesteric to nematic phase transition display |
| 3947375 | Mar 30, 1976 | Gray, et al. Liquid crystal materials and devices |
| 3954653 | May 4, 1976 | Yamazaki. Liquid crystal composition having high dielectric anisotropy and display device incorporating same |
| 3960752 | Jun 1, 1976 | Klanderman, et al. Liquid crystal compositions |
| 3975286 | Aug 17, 1976 | Oh. Low voltage actuated field effect liquid crystals compositions and method of synthesis |
| 4000084 | Dec 28, 1976 | Hsieh, et al. Liquid crystal mixtures for electro-optical display devices |
| 4011173 | Mar 8, 1977 | Steinstrasser. Modified nematic mixtures with positive dielectric anisotropy |
| 4013582 | Mar 22, 1977 | Gavrilovic. Liquid crystal compounds and electro-optic devices incorporating them |
| 4017416 | Apr 12, 1977 | Inukai, et al. P-cyanophenyl 4-alkyl-4'-biphenylcarboxylate, method for preparing same and liquid crystal compositions using same |
| 4029595 | Jun 14, 1977 | Ross, et al. Novel liquid crystal compounds and electro-optic devices incorporating them |
| 4032470 | Jun 28, 1977 | Bloom, et al. Electro-optic device |
| 4077260 | Mar 7, 1978 | Gray, et al. Optically active cyano-biphenyl compounds and liquid crystal materials containing them |
| 4082428 | Apr 4, 1978 | Hsu. Liquid crystal composition and method |

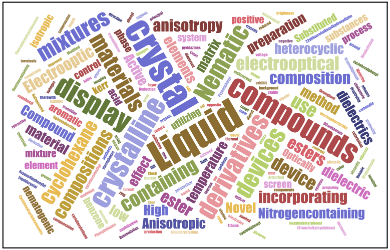
Table 2: Patents on the liquid-crystal display

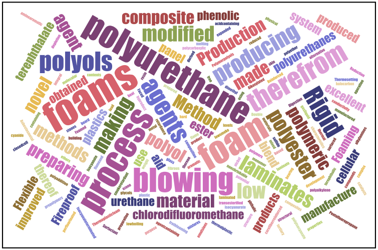
| patent | date | author(s) and title |
|---|---|---|
| 4083797 | Apr 11, 1978 | Oh. Nematic liquid crystal compositions |
| 4113647 | Sep 12, 1978 | Coates, et al. Liquid crystalline materials |
| 4118335 | Oct 3, 1978 | Krause, et al. Liquid crystalline materials of reduced viscosity |
| 4130502 | Dec 19, 1978 | Eidenschink, et al. Liquid crystalline cyclohexane derivatives |
| 4149413 | Apr 17, 1979 | Gray, et al. Optically active liquid crystal mixtures and liquid crystal devices containing them |
| 4154697 | May 15, 1979 | Eidenschink, et al. Liquid crystalline hexahydroterphenyl derivatives |
| 4195916 | Apr 1, 1980 | Coates, et al. Liquid crystal compounds |
| 4198130 | Apr 15, 1980 | Boller, et al. Liquid crystal mixtures |
| 4202791 | May 13, 1980 | Sato, et al. Nematic liquid crystalline materials |
| 4229315 | Oct 21, 1980 | Krause, et al. Liquid crystalline cyclohexane derivatives |
| 4261652 | Apr 14, 1981 | Gray, et al. Liquid crystal compounds and materials and devices containing them |
| 4290905 | Sep 22, 1981 | Kanbe. Ester compound |
| 4293434 | Oct 6, 1981 | Deutscher, et al. Liquid crystal compounds |
| 4302352 | Nov 24, 1981 | Eidenschink, et al. Fluorophenylcyclohexanes, the preparation thereof and their use as components of liquid crystal dielectrics |
| 4330426 | May 18, 1982 | Eidenschink, et al. Cyclohexylbiphenyls, their preparation and use in dielectrics and electrooptical display elements |
| 4340498 | Jul 20, 1982 | Sugimori. Halogenated ester derivatives |
| 4349452 | Sep 14, 1982 | Osman, et al. Cyclohexylcyclohexanoates |
| 4357078 | Nov 2, 1982 | Carr, et al. Liquid crystal compounds containing an alicyclic ring and exhibiting a low dielectric anisotropy and liquid crystal materials and devices incorporating such compounds |
| 4361494 | Nov 30, 1982 | Osman, et al. Anisotropic cyclohexyl cyclohexylmethyl ethers |
| 4368135 | Jan 11, 1983 | Osman. Anisotropic compounds with negative or positive DC-anisotropy and low optical anisotropy |
| 4386007 | May 31, 1983 | Krause, et al. Liquid crystalline naphthalene derivatives |
| 4387038 | Jun 7, 1983 | Fukui, et al. 4-(Trans-4'-alkylcyclohexyl) benzoic acid 4'''-cyano-4''-biphenylyl esters |
| 4387039 | Jun 7, 1983 | Sugimori, et al. Trans-4-(trans-4'-alkylcyclohexyl)-cyclohexane carboxylic acid 4'''-cyanobiphenyl ester |
| 4400293 | Aug 23, 1983 | Romer, et al. Liquid crystalline cyclohexylphenyl derivatives |
| 4415470 | Nov 15, 1983 | Eidenschink, et al. Liquid crystalline fluorine-containing cyclohexylbiphenyls and dielectrics and electro-optical display elements based thereon |
| 4419263 | Dec 6, 1983 | Praefcke, et al. Liquid crystalline cyclohexylcarbonitrile derivatives |
| 4422951 | Dec 27, 1983 | Sugimori, et al. Liquid crystal benzene derivatives |
| 4455443 | Jun 19, 1984 | Takatsu, et al. Nematic halogen Compound |
| 4456712 | Jun 26, 1984 | Christie, et al. Bismaleimide triazine composition |
| 4460770 | Jul 17, 1984 | Petrzilka, et al. Liquid crystal mixture |
| 4472293 | Sep 18, 1984 | Sugimori, et al. High temperature liquid crystal substances of four rings and liquid crystal compositions containing the same |
| 4472592 | Sep 18, 1984 | Takatsu, et al. Nematic liquid crystalline compounds |
| 4480117 | Oct 30, 1984 | Takatsu, et al. Nematic liquid crystalline compounds |
| 4502974 | Mar 5, 1985 | Sugimori, et al. High temperature liquid-crystalline ester compounds |
| 4510069 | Apr 9, 1985 | Eidenschink, et al. Cyclohexane derivatives |

Table 3: Patents on the liquid-crystal display

| patent | date | author(s) and title |
|---|---|---|
| 4514044 | Apr 30, 1985 | Gunjima, et al. 1-(Trans-4-alkylcyclohexyl)-2-(trans-4'-(p-sub stituted phenyl) cyclohexyl)ethane and liquid crystal mixture |
| 4526704 | Jul 2, 1985 | Petrzilka, et al. Multiring liquid crystal esters |
| 4550981 | Nov 5, 1985 | Petrzilka, et al. Liquid crystalline esters and mixtures |
| 4558151 | Dec 10, 1985 | Takatsu, et al. Nematic liquid crystalline compounds |
| 4583826 | Apr 22, 1986 | Petrzilka, et al. Phenylethanes |
| 4621901 | Nov 11, 1986 | Petrzilka, et al. Novel liquid crystal mixtures |
| 4630896 | Dec 23, 1986 | Petrzilka, et al. Benzonitriles |
| 4657695 | Apr 14, 1987 | Saito, et al. Substituted pyridazines |
| 4659502 | Apr 21, 1987 | Fearon, et al. Ethane derivatives |
| 4695131 | Sep 22, 1987 | Balkwill, et al. Disubstituted ethanes and their use in liquid crystal materials and devices |
| 4704227 | Nov 3, 1987 | Krause, et al. Liquid crystal compounds |
| 4709030 | Nov 24, 1987 | Petrzilka, et al. Novel liquid crystal mixtures |
| 4710315 | Dec 1, 1987 | Schad, et al. Anisotropic compounds and liquid crystal mixtures therewith |
| 4713197 | Dec 15, 1987 | Eidenschink, et al. Nitrogen-containing heterocyclic compounds |
| 4719032 | Jan 12, 1988 | Wachtler, et al. Cyclohexane derivatives |
| 4721367 | Jan 26, 1988 | Yoshinaga, et al. Liquid crystal device |
| 4752414 | Jun 21, 1988 | Eidenschink, et al. Nitrogen-containing heterocyclic compounds |
| 4770503 | Sep 13, 1988 | Buchecker, et al. Liquid crystalline compounds |
| 4795579 | Jan 3, 1989 | Vauchier, et al. 2,2'-difluoro-4-alkoxy-4'-hydroxydiphenyls and their derivatives, their production process and their use in liquid crystal display devices |
| 4797228 | Jan 10, 1989 | Goto, et al. Cyclohexane derivative and liquid crystal composition containing same |
| 4820839 | Apr 11, 1989 | Krause, et al. Nitrogen-containing heterocyclic esters |
| 4832462 | May 23, 1989 | Clark, et al. Liquid crystal devices |
| 4877547 | Oct 31, 1989 | Weber, et al. Liquid crystal display element |
| 4957349 | Sep 18, 1990 | Clerc, et al. Active matrix screen for the color display of television pictures, control system and process for producing said screen |
| 5016988 | May 21, 1991 | Iimura. Liquid crystal display device with a birefringent compensator |
| 5016989 | May 21, 1991 | Okada. Liquid crystal element with improved contrast and brightness |
| 5122295 | Jun 16, 1992 | Weber, et al. Matrix liquid crystal display |
| 5124824 | Jun 23, 1992 | Kozaki, et al. Liquid crystal display device comprising a retardation compensation layer having a maximum principal refractive index in the thickness direction |
| 5171469 | Dec 15, 1992 | Hittich, et al. Liquid-crystal matrix display |
| 5283677 | Feb 1, 1994 | Sagawa, et al. Liquid crystal display with ground regions between terminal groups |
| 5308538 | May 3, 1994 | Weber, et al. Supertwist liquid-crystal display |
| 5374374 | Dec 20, 1994 | Weber, et al. Supertwist liquid-crystal display |
| 5543077 | Aug 6, 1996 | Rieger, et al. Nematic liquid-crystal composition |
| 5555116 | Sep 10, 1996 | Ishikawa, et al. Liquid crystal display having adjacent electrode terminals set equal in length |
| 5683624 | Nov 4, 1997 | Sekiguchi, et al. Liquid crystal composition |
| 5855814 | Jan 5, 1999 | Matsui, et al. Liquid crystal compositions and liquid crystal display elements |

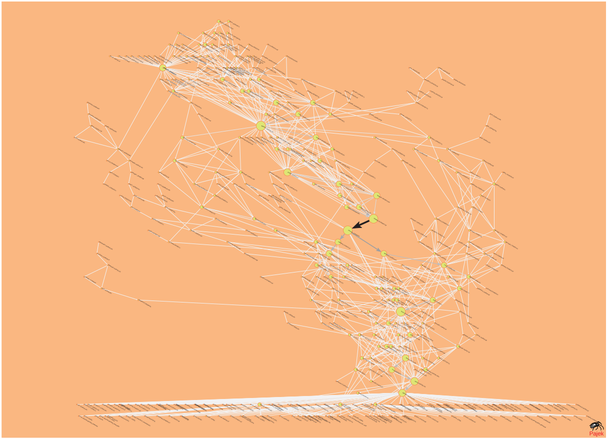
# Word clouds for LCD island and foam island



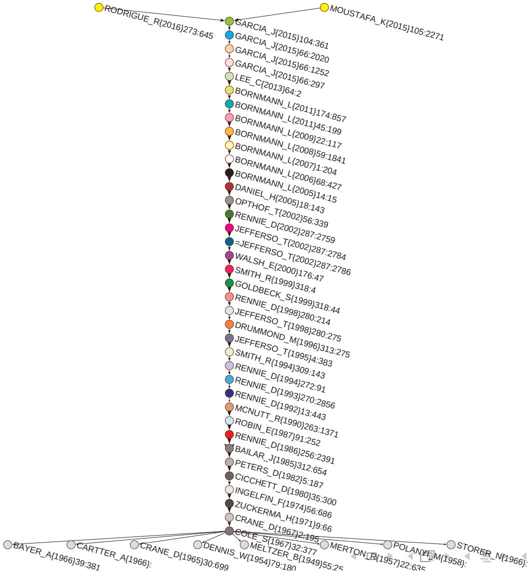
# Main SPC island in SN5

**Network** SN5 (2008): for "social network*" + most frequent references + around 100 social networkers; $|W| = 193376$, $|C| = 7950$, $|A| = 75930$, $|J| = 14651$, $|K| = 29267$. Citation networks are acyclic. Acyclic networks can be displayed by levels – run macro Layers1.mcr from the map Geneo.



# PEERE – Main path





# List of publications on main path



| year | first author | title | journal |
|---|---|---|---|
| 1949 | Meltzer BN | The productivity of social scientists | AmJSociol |
| 1954 | Dennis W | Bibliographies of eminent scientists | ScientificM |
| 1957 | Merton RK | Priorities in scientific discovery - a chapter in the sociology of sci... | AmSociolRev |
| 1958 | Polanyi M | Personal Knowledge: Towards a Post-Critical Philosophy | UPChicago |
| 1965 | Crane D | Scientists at major and minor universities | AmSociolRev |
| 1966 | Bayer AE | Some correlates of citation measure of productivity in science | SociolEduc |
| 1966 | Storer NW | The Social System of Science | HRW |
| 1966 | Cartter A | An Assessment of Quality in Graduate Education | ACE |
| 1967 | Crane D | Gatekeepers of science - some factors affecting selection... | AmSociol |
| 1967 | Cole S | Scientific output and recognition - study in operation of reward... | AmSociolRev |
| 1971 | Zuckerman H | Patterns of evaluation in science - institutionalisation, struct... | Minerva |
| 1974 | Ingelfinger FJ | Peer review in biomedical publication | AmJMed |
| 1980 | Cicchetti DV | Reliability of reviews for the american-psychologist | AmPsychol |
| 1982 | Peters DP | Peer-review practices of psychological journals - the fate... | BehavBrainS |
| 1985 | Bailar JC | Journal peer-review - the need for a research agenda | NewEnglJMe |
| 1986 | Rennie D | Guarding the guardians - a conference on editorial peer-review | Jama |
| 1987 | Robin ED | Peer-review in medical journals | Chest |
| 1990 | Mcnutt RA | The effects of blinding on the quality of peer-review | Jama |
| 1992 | Rennie D | Suspended judgment - editorial peer-review - let us put it on trial | ControlClinT |
| 1993 | Rennie D | More peering into editorial peer-review | Jama |
| 1994 | Rennie D | The 2nd international-congress on peer-review in biomedical... | Jama |
| 1994 | Smith R | Promoting research into peer-review | BritMedJ |
| 1995 | Jefferson T | Are guidelines for peer-reviewing economic evaluations necessary | HealthEcon |
| 1996 | Drummond M | Guidelines for authors and peer reviewers of economic submis... | BritMedJ |
| 1998 | Jefferson T | Evaluating the BMJ guidelines for economic submissions | Jama |
| 1998 | Rennie D | Peer review in Prague | Jama |

# List of works on main path cont.



| year | first author | title | journal |
|---|---|---|---|
| 1999 | Smith R | Opening up BMJ peer review - A beginning that should lead to... | BritMedJ |
| 1999 | Goldbeck-W. S | Evidence on peer review - scientific quality control or smokescreen? | BritMedJ |
| 2000 | Walsh E | Open peer review: a randomised controlled trial | BritJPsych |
| 2002 | Jefferson T | Effects of editorial peer review - A systematic review | Jama |
| 2002 | Rennie D | Fourth International Congress on Peer Review in Biomedical Pub... | Jama |
| 2002 | Opthof T | The significance of the peer review process against . . . bias | Cardiovasc |
| 2002 | Jefferson T | Measuring the quality of editorial peer review | Jama |
| 2005 | Bornmann L | Committee peer review at an international research foundation | ResEvaluat |
| 2005 | Daniel HD | Publications as a measure of scientific advancement and of... | LearnPubl |
| 2006 | Bornmann L | Selecting scientific excellence through committee peer review | Scientomet |
| 2007 | Bornmann L | Convergent validation of peer review decisions using the h index | JInformetr |
| 2008 | Bornmann L | Selecting manuscripts for a high-impact journal through peer review | JAmSocInf |
| 2009 | Bornmann L | The luck of the referee draw: the effect of exchanging reviews | LearnPubl |
| 2011 | Bornmann L | Scientific Peer Review | AnnuRevIn |
| 2011 | Bornmann L | A multilevel modelling approach to investigating the predictive... | JRStatSoc |
| 2013 | Lee CJ | Bias in peer review | JAmSocInf |
| 2015 | Garcia JA | The Principal-Agent Problem in Peer Review | JAssocInfS |
| 2015 | Garcia JA | Adverse selection of reviewers | JAssocInfS |
| 2015 | Garcia JA | Bias and effort in peer review | JAssocInfS |
| 2015 | Garcia JA | The author-editor game | Scientomet |
| 2015 | Moustafa K | Don't infer anything from unavailable data | Scientomet |
| 2016 | Rodriguez-S. R | Evolutionary games between authors and their editors | ApplMathC |

# The main path publications

## Phases

- before 1982: social science journals;
- from 1982 to 2002: biomedical journals;
- after 2002: specialized journals on science studies.

# The main path publications

## till 1982

**Journals:** social science journals (sociological, psychological, educational,...) and three books.

The most **influential authors:** Meltzer (1949), Dennis (1954), Merton (1957), Polany (1958), Crane (1965, 1967), Bayer and Folger (1966), Storer (1966), Cartter (1966), Cole and Cole (1967), Zuckerman and Merton (1971), Ingelfinger (1974), Cicchetti (1980) and Peters and Ceci (1982).

**Topics:** scientific productivity, bibliographies, knowledge, citation measures as measures of scientific accomplishement, scientific output and recognition, evaluation in science, referee system, journal evaluation, peer-evaluation system, review process, peer review practices.

# The main path publications from 1983 to 2002

**Journals:** biomedical journals, mainly JAMA. From 1986 the International Congress on Peer Review and Biomedical Publication is organized every four years.

The most **influential authors:** Rennie (1986, 1992, 1993, 1994, 1998, 2002), Smith (1994, 1999), Jefferson (1995, 1998, 2002), and their collaborators.

**Topics:** the effects of blinding on review quality, research into peer review, guidelines for peer reviewing, monitoring the peer review performance, open peer review, bias in peer review system, measuring the quality of editorial peer review. Development of meta-analysis and systematic reviews studies of peer-review.

# The main path publications from 2003 on

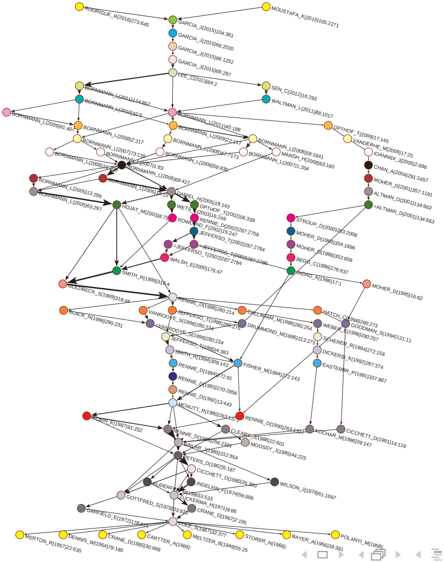


**Journals:** specialized journals on science studies: Scientometrics, Research Evaluation, Journal of Informetrics, JASIST.

The most **influential authors:** Bornmann and Daniel (2005, 2006, 2007, 2008, 2009, 2011) and Garcia, Rodriguez-Sanchez and Fdez-Valdivia (4 papers in 2015, 2016). Others are Lee et al. (2013) and Moustafa (2015).

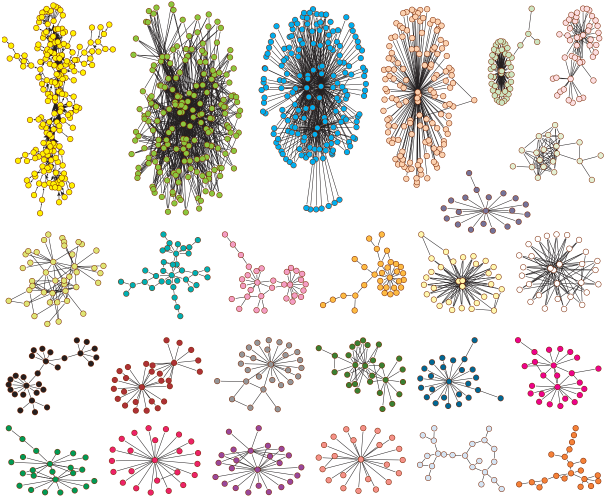
**Topics:** Bornmann and Daniel studied the validity of commitee peer review process for awarding long-term fellowship to post-graduate researchers, the use of h-index and pre-screening of applications at Boehringer Ingelheim Fonds. They also analysed citations of accepted and rejected papers at a prime chemistry journal, the effect of exchanging reviews, the peer review process in this journal, the validity of its editorial decisions. The other papers studied bias in peer review, selection of reviewers, and the author-editor game.

# PEERE – Main paths for 100 largest weights



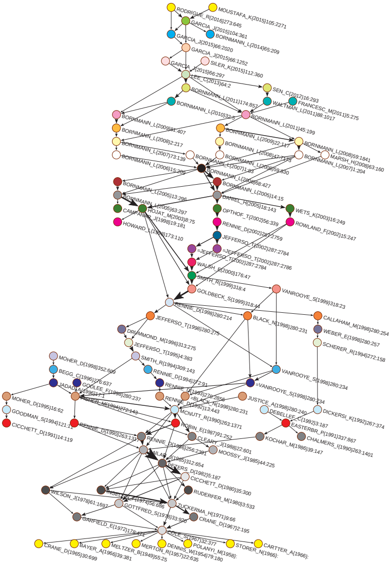
# PEERE – SPC link islands [20 200]





# PEERE – SPC – Link island1

$w_{max} = 0.297$




This island is very similar to the main paths for 100 largest weights and includes main path.

# Islands

## positioning

A recent extension of the main paths approach

```
Network/Acyclic network/Create (sub)network/Main paths/
   Local search/Through vertices in Cluster
Network/Acyclic network/Create (sub)network/Main paths/
   Global search/Through vertices in Cluster
```

enables a researcher to determine main paths through a given set of nodes (works) in a citation network. This can be used to position a given set of nodes in a citation network – it can, either attach to the principal main paths, or form a separate stream.

In the literature on network clustering we considered works on valued networks, { ZIBERNA_A(2007)29:105, NORDLUND_C(2007)29:59, ZIBERNA_A (2008)32:57, ZIBERNA_A(2009)6:99, ZIBERNA_A(2013):, ZIBERNA_A (2013)10:99, ZIBERNA_A(2014)39:46, ZIBERNA_A(2016)12:137, NORDLUND_C(2016)44:160 }, as shown in figure in the following slide.

# Islands

## positioning

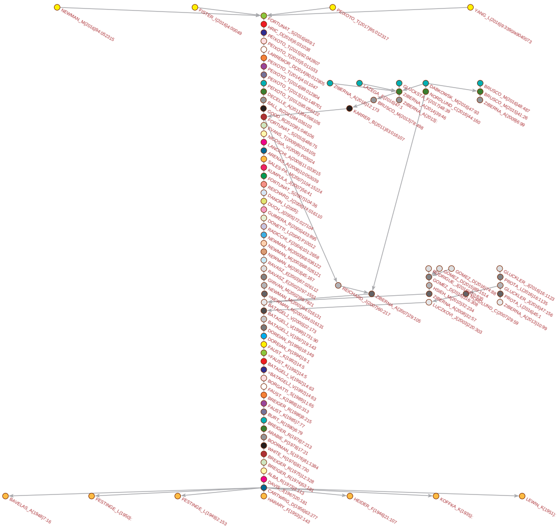
The corresponding cluster of nodes is produced manually using the Pajek's command

```
Network / Info / Vertex Label -> Vertex Number
```

# Islands

## positioning – valued networks in clustering





# Two-mode networks



In a *two-mode* network $\mathcal{N} = ((\mathcal{U}, \mathcal{V}), \mathcal{L}, \mathcal{P}, \mathcal{W})$ the set of nodes consists of two disjoint sets of nodes $\mathcal{U}$ and $\mathcal{V}$, and all the links from $\mathcal{L}$ have one end-node in $\mathcal{U}$ and the other in $\mathcal{V}$. Often also a *weight* $w : \mathcal{L} \to \mathbb{R} \in \mathcal{W}$ is given; if not, we assume $w(u, v) = 1$ for all $(u, v) \in \mathcal{L}$.

A two-mode network can also be described by a rectangular matrix $\mathbf{A} = [a_{uv}]_{\mathcal{U} \times \mathcal{V}}$.

$$a_{uv} = \begin{cases} w_{uv} & (u, v) \in \mathcal{L} \\ 0 & \text{otherwise} \end{cases}$$

Examples: (persons, societies, years of membership), (buyers/consumers, goods, quantity), (parlamentarians, problems, positive vote), (persons, journals, reading), (papers, keywords, is described by), etc.

# Deep South

Classical example of two-mode network are the Southern women (Davis 1941).

Davis.paj. Freeman's overview.

| Names of Participants of Group I | Code Numbers and Dates of Social Events Reported in *Old City Herald* | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) 6/27 | (2) 3/2 | (3) 4/12 | (4) 9/26 | (5) 2/25 | (6) 5/19 | (7) 3/15 | (8) 9/16 | (9) 4/8 | (10) 6/10 | (11) 2/23 | (12) 4/7 | (13) 11/21 | (14) 8/3 |
| 1. Mrs. Evelyn Jefferson | × | × | × | × | × | × | .... | × | × | .... | .... | .... | .... | .... |
| 2. Miss Laura Mandeville | × | × | × | .... | × | × | × | × | .... | .... | .... | .... | .... | .... |
| 3. Miss Theresa Anderson | .... | × | × | × | × | × | × | × | × | .... | .... | .... | .... | .... |
| 4. Miss Brenda Rogers | × | .... | × | × | × | × | × | × | .... | .... | .... | .... | .... | .... |
| 5. Miss Charlotte McDowd | .... | .... | × | × | × | .... | × | .... | .... | .... | .... | .... | .... | .... |
| 6. Miss Frances Anderson | .... | .... | × | .... | × | × | .... | × | .... | .... | .... | .... | .... | .... |
| 7. Miss Eleanor Nye | .... | .... | .... | .... | × | × | × | × | .... | .... | .... | .... | .... | .... |
| 8. Miss Pearl Oglethorpe | .... | .... | .... | .... | .... | × | .... | × | × | .... | .... | .... | .... | .... |
| 9. Miss Ruth DeSand | .... | .... | .... | .... | × | .... | × | × | × | .... | .... | .... | .... | .... |
| 10. Miss Verne Sanderson | .... | .... | .... | .... | .... | .... | × | × | × | .... | .... | × | .... | .... |
| 11. Miss Myra Liddell | .... | .... | .... | .... | .... | .... | .... | × | × | × | .... | × | .... | .... |
| 12. Miss Katherine Rogers | .... | .... | .... | .... | .... | .... | .... | × | × | × | .... | × | × | × |
| 13. Mrs. Sylvia Avondale | .... | .... | .... | .... | .... | .... | × | × | × | × | .... | × | × | × |
| 14. Mrs. Nora Fayette | .... | .... | .... | .... | .... | × | × | .... | × | × | × | × | × | × |
| 15. Mrs. Helen Lloyd | .... | .... | .... | .... | .... | .... | × | × | .... | × | × | × | .... | .... |
| 16. Mrs. Dorothy Murchison | .... | .... | .... | .... | .... | .... | .... | × | × | .... | .... | .... | .... | .... |
| 17. Mrs. Olivia Carleton | .... | .... | .... | .... | .... | .... | .... | .... | × | .... | × | .... | .... | .... |
| 18. Mrs. Flora Price | .... | .... | .... | .... | .... | .... | .... | .... | × | .... | × | .... | .... | .... |

# Approaches to two-mode network analysis

The usual approach to analyze a two-mode network is to transform it to a one-mode network and use standard methods on it.

For direct analysis of two-mode networks we can use the *eigen-vector approach* – a two-mode variant of Kleinberg's hubs and authorities. The weight vector $(\mathbf{x}, \mathbf{y})$ on $\mathcal{U} \cup \mathcal{V}$ is determined by relations $\mathbf{y} = \mathbf{A}\mathbf{x}$ and $\mathbf{x} = \mathbf{A}^T\mathbf{y}$.

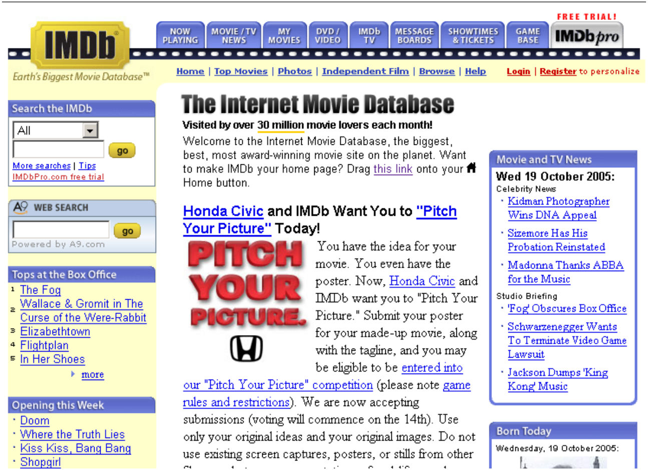
`Network/2-Mode Network/Important Vertices`

There are also special methods for *clustering* and *blockmodeling* in two-mode networks.

In this lecture we will present two additional direct methods: *two-mode cores* and *4-rings*.

# Internet Movie Database http://www.imdb.com/






12th Annual Graph Drawing Contest, 2005. The IMDb network is two-mode and has $1324748 = 428440 + 896308$ nodes and 3792390 arcs.

# Two-mode cores



The subset of nodes $C \subseteq \mathcal{V}$ is a *$(p, q)$-core* in a two-mode network $\mathcal{N} = (\mathcal{V}_1, \mathcal{V}_2; \mathcal{L})$, $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$ iff

**a**. in the induced subnetwork $\mathcal{K} = (C_1, C_2; \mathcal{L}(C))$, $C_1 = C \cap \mathcal{V}_1$, $C_2 = C \cap \mathcal{V}_2$ it holds $\forall v \in C_1 : \deg_{\mathcal{K}}(v) \geq p$ and $\forall v \in C_2 : \deg_{\mathcal{K}}(v) \geq q$ ;

**b**. $C$ is the maximal subset of $\mathcal{V}$ satisfying condition **a**.

Properties of two-mode cores:

- $C(0,0) = \mathcal{V}$
- $\mathcal{K}(p, q)$ is not always connected
- $(p_1 \leq p_2) \wedge (q_1 \leq q_2) \Rightarrow C(p_1, q_1) \subseteq C(p_2, q_2)$
- $\mathcal{C} = \{C(p, q) : p, q \in \mathbb{N}\}$. If all nonempty elements of $\mathcal{C}$ are different it is a lattice.

# Algorithm for two-mode cores

To determine a $(p, q)$-core the procedure similar to the ordinary core procedure can be used:

**repeat**
  remove from the first set all nodes of degree less than $p$,
    and from the second set all nodes of degree less than $q$
**until** no node was deleted

It can be implemented to run in $O(m)$ time.
Interesting $(p, q)$-cores? Table of cores' characteristics $n_1 = |C_1(p, q)|$, $n_2 = |C_2(p, q)|$ and $k$ – number of components in $\mathcal{K}(p, q)$:

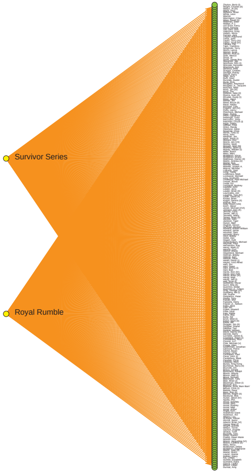
- $n_1 + n_2 \leq$ selected threshold
- 'border line' in the $(p, q)$-table.

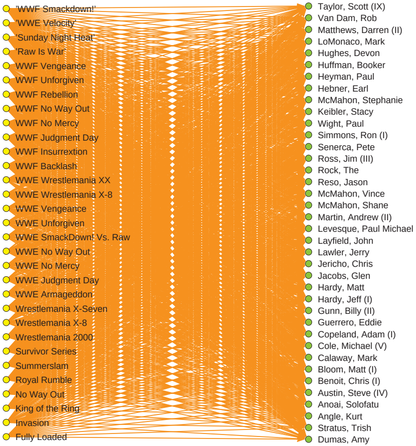
# Table $(p, q : n_1, n_2)$ for Internet Movie Database

```
Network/2-Mode Network/Core/2-Mode Border
```

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1590: | 1590 | 1 | 16 | 39: | 2173 | 678 | 44 | 14: | 29 | 83 |
| 2 | 516: | 788 | 3 | 17 | 35: | 2791 | 995 | 46 | 13: | 29 | 94 |
| 3 | 212: | 1705 | 18 | 18 | 32: | 2684 | 1080 | 49 | 12: | 26 | 95 |
| 4 | 151: | 4330 | 154 | 19 | 30: | 2395 | 1063 | 52 | 11: | 16 | 79 |
| 5 | 131: | 4282 | 209 | 20 | 28: | 2216 | 1087 | 56 | 10: | 34 | 162 |
| 6 | 115: | 3635 | 223 | 21 | 26: | 1988 | 1087 | 62 | 9: | 31 | 177 |
| 7 | 101: | 3224 | 244 | 22 | 24: | 1854 | 1153 | 66 | 8: | 29 | 198 |
| 8 | 88: | 2860 | 263 | 24 | 23: | 34 | 39 | 72 | 7: | 22 | 203 |
| 9 | 77: | 3467 | 393 | 27 | 22: | 31 | 38 | 96 | 6: | 7 | 114 |
| 10 | 69: | 3150 | 428 | 29 | 20: | 35 | 52 | 119 | 5: | 6 | 137 |
| 11 | 63: | 2442 | 382 | 32 | 19: | 34 | 57 | 141 | 4: | 8 | 258 |
| 12 | 56: | 2479 | 454 | 35 | 18: | 33 | 61 | 186 | 3: | 3 | 186 |
| 13 | 50: | 3330 | 716 | 36 | 17: | 33 | 65 | 247 | 2: | 2 | 247 |
| 14 | 46: | 2460 | 596 | 39 | 16: | 29 | 70 | 1334 | 1: | 1 | 1334 |
| 15 | 42: | 2663 | 739 | 42 | 15: | 28 | 76 | | | | |

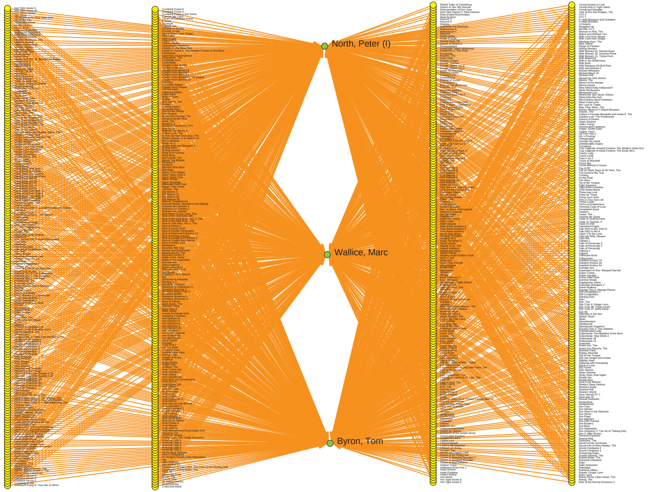
# (247,2)-core and (27,22)-core




IMDb: $n_1 = 428440,\ n_2 = 896308,\ m = 3792390.$

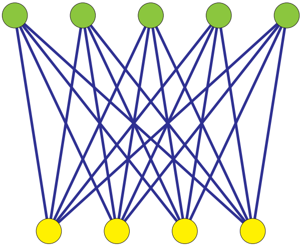
# (2,516)-Hard core



# 4-rings and analysis of two-mode networks

A *k-ring* is a simple closed chain of length $k$. Using $k$-rings we can define a weight of edges as
$w_k(e) = \#$ of different $k$-rings containing the edge $e \in \mathcal{E}$

In two-mode network there are no 3-rings. The densest substructures are complete bipartite subgraphs $K_{p,q}$. They contain many 4-rings.



There are

$$\binom{p}{2}\binom{q}{2} = \frac{1}{4}p(p-1)q(q-1)$$

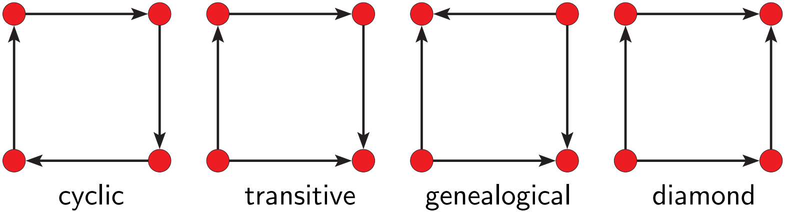
4-rings in $K_{p,q}$; and each of its edges $e$ has weight

$$w_4(e) = (p-1)(q-1)$$

`Network/Create New Network/with Ring Counts.../4-Rings/Undirected`

# Directed 4-rings

There are 4 types of directed 4-rings:




In the case of transitive rings `Pajek` provides a special weight counting on how many transitive rings the arc is a *shortcut*.

`Network/Create New Network/with Ring Counts/4-Rings/Directed`

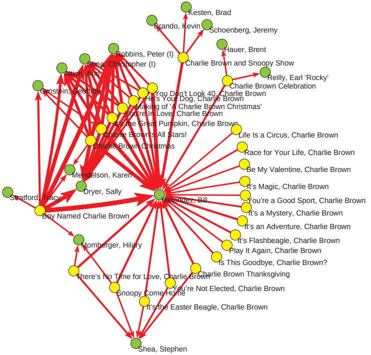
# Simple link islands in IMDb for $w_4$

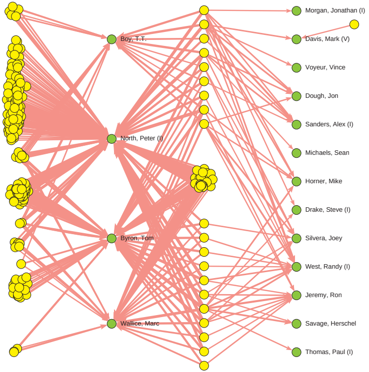
We obtained 12465 simple link islands on 56086 nodes. Here is their size distribution.

| Size | Freq | Size | Freq | Size | Freq | Size | Freq |
|---|---|---|---|---|---|---|---|
| 2 | 5512 | 20 | 19 | 38 | 4 | 59 | 2 |
| 3 | 1978 | 21 | 18 | 39 | 3 | 61 | 1 |
| 4 | 1639 | 22 | 15 | 40 | 2 | 64 | 1 |
| 5 | 968 | 23 | 9 | 42 | 2 | 67 | 1 |
| 6 | 666 | 24 | 13 | 43 | 3 | 70 | 1 |
| 7 | 394 | 25 | 12 | 45 | 3 | 73 | 1 |
| 8 | 257 | 26 | 6 | 46 | 4 | 76 | 1 |
| 9 | 209 | 27 | 6 | 47 | 5 | 82 | 1 |
| 10 | 148 | 28 | 5 | 48 | 1 | 86 | 1 |
| 11 | 118 | 29 | 6 | 49 | 2 | 106 | 1 |
| 12 | 87 | 30 | 3 | 50 | 2 | 122 | 1 |
| 13 | 55 | 31 | 6 | 51 | 1 | 135 | 1 |
| 14 | 62 | 32 | 5 | 52 | 2 | 144 | 1 |
| 15 | 46 | 33 | 3 | 53 | 1 | 163 | 1 |
| 16 | 39 | 34 | 1 | 54 | 2 | 269 | 1 |
| 17 | 27 | 35 | 5 | 55 | 1 | 301 | 1 |
| 18 | 28 | 36 | 4 | 57 | 1 | 332 | 2 |
| 19 | 29 | 37 | 7 | 58 | 1 | 673 | 1 |

# Example: Islands for $w_4$ Charlie Brown and Adult

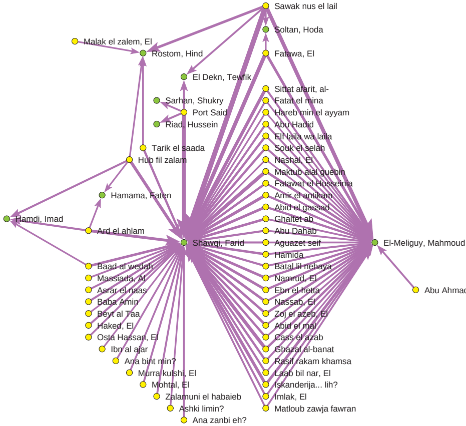




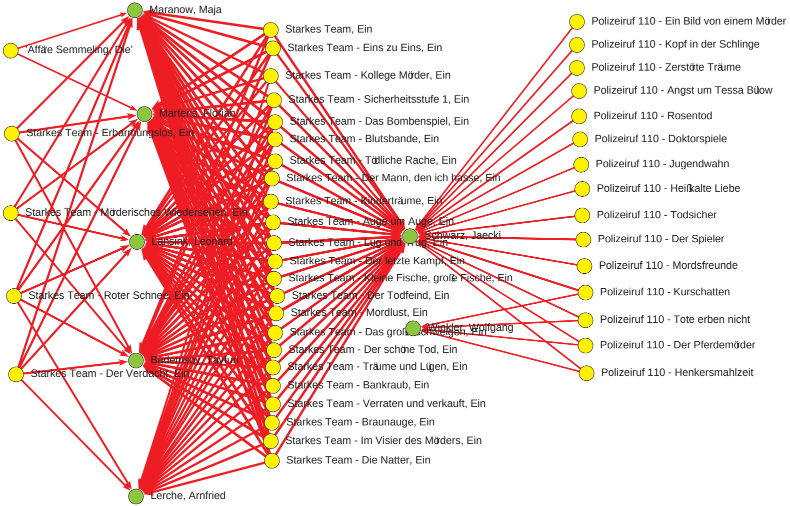



# Example: Islands for $w_4$ Mark Twain and Abid



# Example: Island for $w_4$ Polizeiruf 110 and Starkes Team



# Multiplication of networks

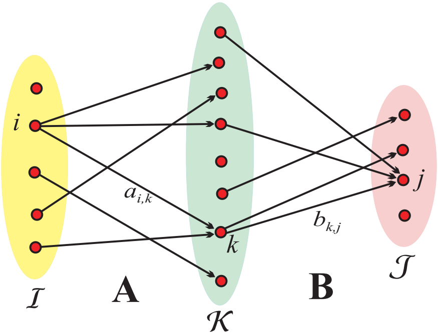
To a simple (no parallel arcs) two-mode *network* $\mathcal{N} = (\mathcal{I}, \mathcal{J}, \mathcal{A}, w)$; where $\mathcal{I}$ and $\mathcal{J}$ are sets of *nodes*, $\mathcal{A}$ is a set of *arcs* linking $\mathcal{I}$ and $\mathcal{J}$, and $w : \mathcal{A} \to \mathbb{R}$ (or some other semiring) is a *weight*; we can assign a *network matrix* $\mathbf{W} = [w_{i,j}]$ with elements: $w_{i,j} = w(i,j)$ for $(i,j) \in \mathcal{A}$ and $w_{i,j} = 0$ otherwise.

Given a pair of compatible networks $\mathcal{N}_A = (\mathcal{I}, \mathcal{K}, \mathcal{A}_A, w_A)$ and $\mathcal{N}_B = (\mathcal{K}, \mathcal{J}, \mathcal{A}_B, w_B)$ with corresponding matrices $\mathbf{A}_{\mathcal{I}\times\mathcal{K}}$ and $\mathbf{B}_{\mathcal{K}\times\mathcal{J}}$ we call a *product of networks* $\mathcal{N}_A$ and $\mathcal{N}_B$ a network $\mathcal{N}_C = (\mathcal{I}, \mathcal{J}, \mathcal{A}_C, w_C)$, where $\mathcal{A}_C = \{(i,j) : i \in \mathcal{I}, j \in \mathcal{J}, c_{i,j} \neq 0\}$ and $w_C(i,j) = c_{i,j}$ for $(i,j) \in \mathcal{A}_C$. The product matrix $\mathbf{C} = [c_{i,j}]_{\mathcal{I}\times\mathcal{J}} = \mathbf{A} * \mathbf{B}$ is defined in the standard way

$$c_{i,j} = \sum_{k\in\mathcal{K}} a_{i,k} \cdot b_{k,j}$$

In the case when $\mathcal{I} = \mathcal{K} = \mathcal{J}$ we are dealing with ordinary one-mode networks (with square matrices).

# Multiplication of networks




$$c_{i,j} = \sum_{k \in N_A(i) \cap N_B^-(j)} a_{i,k} \cdot b_{k,j}$$

If all weights in networks $\mathcal{N}_A$ and $\mathcal{N}_B$ are equal to 1 the value of $c_{i,j}$ counts the number of ways we can go from $i \in \mathcal{I}$ to $j \in \mathcal{J}$ passing through $\mathcal{K}$.

# Multiplication of networks



The standard matrix multiplication has the complexity $O(|\mathcal{I}| \cdot |\mathcal{K}| \cdot |\mathcal{J}|)$ – it is too slow to be used for large networks. For sparse large networks we can multiply much faster considering only nonzero elements. In general the multiplication of large sparse networks is a 'dangerous' operation since the result can 'explode' – it is not sparse.

If at least one of the sparse networks $\mathcal{N}_A$ and $\mathcal{N}_B$ has small maximal degree on $\mathcal{K}$ then also the resulting product network $\mathcal{N}_C$ is sparse.
If for the sparse networks $\mathcal{N}_A$ and $\mathcal{N}_B$ there are in $\mathcal{K}$ only few nodes with large degree and no one among them with large degree in both networks then also the resulting product network $\mathcal{N}_C$ is sparse.

# Multiplication of networks – details

From the network multiplication algorithm we see that each intermediate node $k \in \mathcal{K}$ adds to a product network a complete two-mode subgraph $K_{N_A^-(k),N_B(k)}$ (or, in the case $\mathcal{I} = \mathcal{J}$, a complete subgraph $K_{N(k)}$). If both degrees $\deg_A(k) = |N_A^-(k)|$ and $\deg_B(k) = |N_B(k)|$ are large then already the computation of this complete subgraph has a quadratic (time and space) complexity – the result 'explodes'.

For more details see the paper.

# Two-mode network analysis by conversion to one-mode network – projections

Often we transform a two-mode network $\mathcal{N} = (\mathcal{U}, \mathcal{V}, \mathcal{E}, w)$ into an ordinary (one-mode) network $\mathcal{N}_1 = (\mathcal{U}, \mathcal{E}_1, w_1)$ or/and $\mathcal{N}_2 = (\mathcal{V}, \mathcal{E}_2, w_2)$, where $\mathcal{E}_1$ and $w_1$ are determined by the matrix $\mathbf{W}^{(1)} = \mathbf{W}\mathbf{W}^T$, $w^{(1)}_{uv} = \sum_{z \in \mathcal{V}} w_{uz} \cdot w^T_{zv}$. Evidently $w^{(1)}_{uv} = w^{(1)}_{vu}$. There is an edge $(u : v) \in \mathcal{E}_1$ in $\mathcal{N}_1$ iff $N(u) \cap N(v) \neq \emptyset$. Its weight is $w_1(u, v) = w^{(1)}_{uv}$.

The network $\mathcal{N}_2$ is determined in a similar way by the matrix $\mathbf{W}^{(2)} = \mathbf{W}^T\mathbf{W}$.
The networks $\mathcal{N}_1$ and $\mathcal{N}_2$ are analyzed using standard methods.

`Network/2-Mode Network/2-Mode to 1-Mode/Rows`

# Authorship networks



Let **WA** be the works $\times$ authors two mode authorship network; $wa_{pi} \in \{0, 1\}$ is describing the authorship of author $i$ of work $p$.

$$\forall p \in W : \sum_{i \in A} wa_{pi} = \text{outdeg}_{WA}(p) = \# \text{ authors of work } p$$

Let **N** be its normalized version

$$\forall p \in W : \sum_{i \in A} n_{pi} \in \{0, 1\}$$

obtained from **WA** by $n_{pi} = wa_{pi} / \max(1, \text{outdeg}_{WA}(p))$, or by some other rule determining the author's contribution – the *fractional* approach.

# Some transformations of networks

*Binarization* $b(\mathcal{N})$ is a network obtained from the $\mathcal{N}$ in which all weights are set to 1.

*Transposition* $\mathcal{N}^T$ or $t(\mathcal{N})$ is a network obtained from $\mathcal{N}$ in which to all arcs their direction is reversed. $\mathbf{AW} = \mathbf{WA}^T$, $\mathbf{KW} = \mathbf{WK}^T$, ...

*(Out) normalization* $n(\mathcal{N})$ is a network obtained from $\mathcal{N}$ in which the weight of each arc $a$ is divided by the sum of weights of all arcs having the same initial node as the arc $a$. For binary networks

$$n(\mathbf{A}) = \mathrm{diag}(\frac{1}{\max(1, \mathrm{outdeg}_{WA}(i))})_{i \in \mathcal{I}} * \mathbf{A}$$

$\mathbf{N} = n(\mathbf{WA})$, $\mathbf{WA} = b(\mathbf{N})$

# First co-authorship network

$\mathbf{Co} = \mathbf{AW} * \mathbf{WA}$

$$co_{ij} = \sum_{p \in W} wa_{pi} wa_{pj} = \sum_{p \in N^-(i) \cap N^-(j)} 1$$

$co_{ij}$ = the number of works that authors $i$ and $j$ wrote together
$co_{ii}$ = the total number of works that author $i$ wrote
It holds: $co_{ij} = co_{ji}$.
Using the weights $co_{ij}$ we can determine the Salton's cosine similarity or Ochiai coefficient between authors $i$ and $j$ as

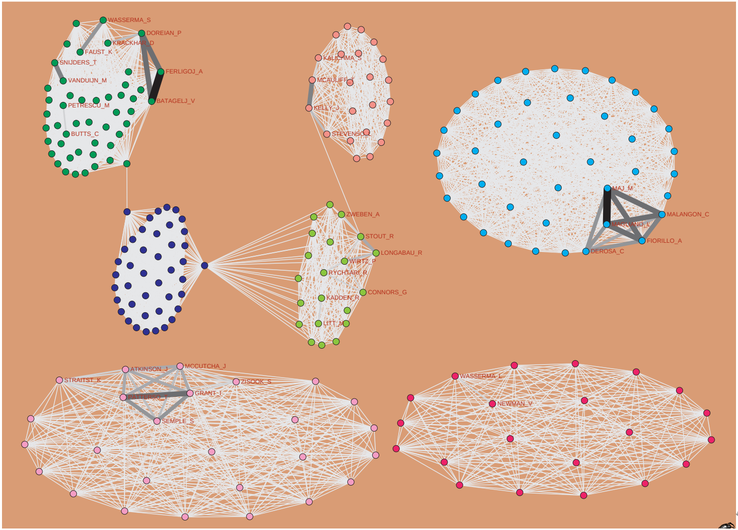
$$cos(i,j) = \frac{co_{ij}}{\sqrt{co_{ii} co_{jj}}}, \qquad \text{for } co_{ij} > 0$$

# Cores of orders 20–47 in **Co**(SN5)





**Network** SN5 (2008): for "social network*" + most frequent references + around 100 social networkers; $|W| = 193376$, $|C| = 7950$, $|A| = 75930$, $|J| = 14651$, $|K| = 29267$



# Papers by number of authors



**Problem:** The **Co** network is composed of complete graphs on the set of work's authors. Works with many authors produce large complete subgraphs and are over-represented, thus bluring the collaboration structure.

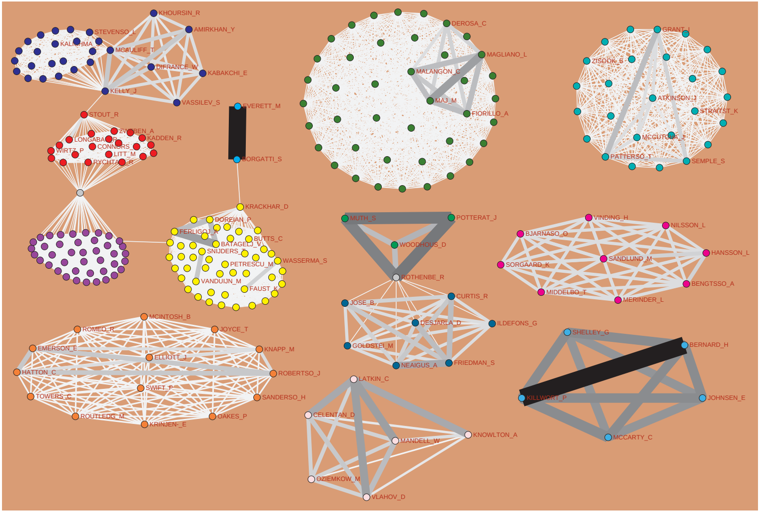
| outdeg | frequency | outdeg | frequency | paper |
|---|---|---|---|---|
| 1 | 2637 | 12 | 8 | |
| 2 | 2143 | 13 | 4 | |
| 3 | 1333 | 14 | 3 | |
| 4 | 713 | 15 | 2 | |
| 5 | 396 | 21 | 1 | Pierce et al. (2007) |
| 6 | 206 | 22 | 1 | Allen et al. (1998) |
| 7 | 114 | 23 | 1 | Kelly et al. (1997) |
| 8 | 65 | 26 | 1 | Semple et al. (1993) |
| 9 | 43 | 41 | 1 | Magliano et al. (2006) |
| 10 | 24 | 42 | 1 | Doll et al. (1992) |
| 11 | 10 | 48 | 1 | Snijders et al. (2007) |

# Snijders et al. (2007)

**Snijders et al.(2007):** Snijders, T.A.B., Robinson, T., Atkinson, A.C., Riani, M., Gormley, I.C., Murphy, T.B., Sweeting, T., Leslie, D.S., Longford, N.T., Kent, J.T., Lawrance, T., Airoldi, E.M., Besag, J., Blei, D., Fienberg, S.E., Breiger, R., Butts, C.T., Doreian, P., Batagelj, V., Ferligoj, A., Draper, D., van Duijn, M.A.J., Faust, K., Petrescu-Prahova, M., Forster, J.J., Gelman, A., Goodreau, S. M., Greenwood, P.E., Gruenberg, K., Francis, B., Hennig, C., Hoff, P.D., Hunter, D.R., Husmeier, D., Glasbey, C., Krackhardt, D., Kuha, J., Skrondal, A., Lawson, A., Liao, T. F., Mendes, B., Reinert, G., Richardson, S., Lewin, A., Titterington, D.M., Wasserman, S., Werhli, A.V. and Ghazal, P.. *Discussion on the paper by Handcock, Raftery and Tantrum*. Journal of the Royal Statistical Society: Series A - Statistics in Society, 170 (2007), pp. 322-354.

# $p_S$-core at level 20 of **Co**(SN5)





# Second co-authorship network

**Cn = AW ∗ N**

$$cn_{ij} = \sum_{p\in W} wa_{pi} n_{pj} = \sum_{p\in N^-(i)\cap N^-(j)} n_{pj}$$

$cn_{ij}$ = contribution of author $j$ to works, that (s)he wrote together with the author $i$.

It holds $\sum_{j\in A}\sum_{j\in A} wa_{pi} n_{pj} = \mathrm{outdeg}_{WA}(p)$ and $\sum_{j\in A} cn_{ij} = \mathrm{indeg}_{WA}(i)$

$cn_{ii} = \sum_{p\in N(i)} n_{pi}$ is the contribution of author $i$ to his/her works.

*Self-sufficiency*: $S_i = \dfrac{cn_{ii}}{\mathrm{indeg}_{WA}(i)}$

*Collaborativness*: $K_i = 1 - S_i$

$$\sum_{i\in A}\sum_{j\in A} cn_{ij} = \sum_{i\in A} \mathrm{indeg}_{WA}(i) = m_{WA}$$

To compute the table we prepared a macro in `Pajek`.

# The "best" authors in Social Networks

| i | author | $cn_{ii}$ | total | $K_i$ | i | author | $cn_{ii}$ | total | $K_i$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Burt,R | 43.83 | 53 | 0.173 | 26 | Latkin,C | 10.14 | 37 | 0.726 |
| 2 | Newman,M | 36.77 | 60 | 0.387 | 27 | Morris,M | 9.98 | 20 | 0.501 |
| 3 | Doreian,P | 34.44 | 47 | 0.267 | 28 | Rothenberg,R | 9.82 | 28 | 0.649 |
| 4 | Bonacich,P | 30.17 | 41 | 0.264 | 29 | Kadushin,C | 9.75 | 11 | 0.114 |
| 5 | Marsden,P | 29.42 | 37 | 0.205 | 30 | Faust,K | 9.72 | 18 | 0.460 |
| 6 | Wellman,B | 26.87 | 41 | 0.345 | 31 | Batagelj,V | 9.69 | 20 | 0.516 |
| 7 | Leydesdorf,L | 24.37 | 35 | 0.304 | 32 | Mizruchi,M | 9.67 | 15 | 0.356 |
| 8 | White,H | 23.50 | 33 | 0.288 | 33 | [Anon] | 9.00 | 9 | 0.000 |
| 9 | Friedkin,N | 20.00 | 23 | 0.130 | 34 | Johnson,J | 8.89 | 21 | 0.577 |
| 10 | Borgatti,S | 19.20 | 41 | 0.532 | 35 | Fararo,T | 8.83 | 16 | 0.448 |
| 11 | Everett,M | 16.92 | 31 | 0.454 | 36 | Lazega,E | 8.50 | 12 | 0.292 |
| 12 | Litwin,H | 16.00 | 21 | 0.238 | 37 | Knoke,D | 8.33 | 11 | 0.242 |
| 13 | Freeman,L | 15.53 | 20 | 0.223 | 38 | Ferligoj,A | 8.19 | 19 | 0.569 |
| 14 | Barabasi,A | 14.99 | 35 | 0.572 | 39 | Brewer,D | 8.03 | 11 | 0.270 |
| 15 | Snijders,T | 14.99 | 30 | 0.500 | 40 | Klovdahl,A | 7.96 | 17 | 0.532 |
| 16 | Valente,T | 14.80 | 34 | 0.565 | 41 | Hammer,M | 7.92 | 10 | 0.208 |
| 17 | Breiger,R | 14.44 | 20 | 0.278 | 42 | White,D | 7.83 | 15 | 0.478 |
| 18 | Skvoretz,J | 14.43 | 27 | 0.466 | 43 | Holme,P | 7.42 | 14 | 0.470 |
| 19 | Krackhardt,D | 13.65 | 25 | 0.454 | 44 | Boyd,J | 7.37 | 13 | 0.433 |
| 20 | Carley,K | 12.93 | 28 | 0.538 | 45 | Kilduff,M | 7.25 | 16 | 0.547 |
| 21 | Pattison,P | 12.10 | 27 | 0.552 | 46 | Small,H | 7.00 | 7 | 0.000 |
| 22 | Wasserman,S | 11.72 | 26 | 0.549 | 47 | Iacobucci,D | 7.00 | 12 | 0.417 |
| 23 | Berkman,L | 11.21 | 30 | 0.626 | 48 | Pappi,F | 6.83 | 10 | 0.317 |
| 24 | Moody,J | 10.83 | 15 | 0.278 | 49 | Chen,C | 6.78 | 12 | 0.435 |
| 25 | Scott,J | 10.47 | 15 | 0.302 | 50 | Seidman,S | 6.75 | 9 | 0.250 |

# Outer product decomposition

For vectors $x = [x_1, x_2, \ldots, x_n]$ and $y = [y_1, y_2, \ldots, y_m]$ their *outer product* $x \circ y$ is defined as

$$x \circ y = [x_i \cdot y_j]_{n \times m}$$

Let $S_x = \sum_i x_i$ and $S_y = \sum_j y_j$ then

$$S = \sum_{i,j} (x \circ y)_{ij} = \sum_i \sum_j x_i \cdot y_j = \sum_i x_i \cdot \sum_j y_j = S_x \cdot S_y$$

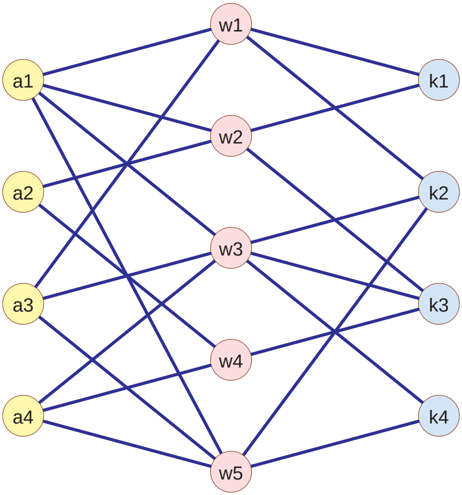
Therefore, if $S_x = S_y = 1$ then also $S = 1$.
It is easy to verify that the *outer product decomposition* holds

$$\mathbf{H}_w = \mathbf{WA}[w, \cdot] \circ \mathbf{WK}[w, \cdot] \quad \text{and} \quad \mathbf{AK} = \mathbf{AW} \cdot \mathbf{WK} = \sum_w \mathbf{H}_w$$

For binary (weights) networks we have

$$\mathbf{H}_w = K_{N_{WA}(w), N_{WK}(w)}$$

# $WA^T \cdot WK$



# Example



$$\mathbf{H} = \mathbf{AW}\cdot\mathbf{WK}, \quad \mathbf{WA} = \begin{array}{c|cccc} & a_1 & a_2 & a_3 & a_4 \\ \hline w_1 & 1 & 0 & 1 & 0 \\ w_2 & 1 & 1 & 0 & 0 \\ w_3 & 1 & 0 & 1 & 1 \\ w_4 & 0 & 1 & 0 & 1 \\ w_5 & 1 & 0 & 1 & 1 \end{array}, \quad \mathbf{WK} = \begin{array}{c|cccc} & k_1 & k_2 & k_3 & k_4 \\ \hline w_1 & 1 & 1 & 0 & 0 \\ w_2 & 1 & 0 & 1 & 0 \\ w_3 & 0 & 1 & 1 & 1 \\ w_4 & 0 & 0 & 1 & 0 \\ w_5 & 0 & 1 & 0 & 1 \end{array}$$

$$\begin{array}{c|cccc} \mathbf{H} & k_1 & k_2 & k_3 & k_4 \\ \hline a_1 & 2 & 3 & 2 & 2 \\ a_2 & 1 & 0 & 2 & 0 \\ a_3 & 1 & 3 & 1 & 2 \\ a_4 & 0 & 2 & 2 & 2 \end{array} = \begin{array}{c|cccc} \mathbf{H}_1 & k_1 & k_2 & k_3 & k_4 \\ \hline a_1 & 1 & 1 & 0 & 0 \\ a_2 & 0 & 0 & 0 & 0 \\ a_3 & 1 & 1 & 0 & 0 \\ a_4 & 0 & 0 & 0 & 0 \end{array} + \begin{array}{c|cccc} \mathbf{H}_2 & k_1 & k_2 & k_3 & k_4 \\ \hline a_1 & 1 & 0 & 1 & 0 \\ a_2 & 1 & 0 & 1 & 0 \\ a_3 & 0 & 0 & 0 & 0 \\ a_4 & 0 & 0 & 0 & 0 \end{array} +$$

$$\begin{array}{c|cccc} \mathbf{H}_3 & k_1 & k_2 & k_3 & k_4 \\ \hline a_1 & 0 & 1 & 1 & 1 \\ a_2 & 0 & 0 & 0 & 0 \\ a_3 & 0 & 1 & 1 & 1 \\ a_4 & 0 & 1 & 1 & 1 \end{array} + \begin{array}{c|cccc} \mathbf{H}_4 & k_1 & k_2 & k_3 & k_4 \\ \hline a_1 & 0 & 0 & 0 & 0 \\ a_2 & 0 & 0 & 1 & 0 \\ a_3 & 0 & 0 & 0 & 0 \\ a_4 & 0 & 0 & 1 & 0 \end{array} + \begin{array}{c|cccc} \mathbf{H}_5 & k_1 & k_2 & k_3 & k_4 \\ \hline a_1 & 0 & 1 & 0 & 1 \\ a_2 & 0 & 0 & 0 & 0 \\ a_3 & 0 & 1 & 0 & 1 \\ a_4 & 0 & 1 & 0 & 1 \end{array}$$

# Fractional approach



In general each term $\mathbf{H}_w$ in the outer product decomposition has different value $S(\mathbf{H}_w) = \sum_{a,k} (\mathbf{H}_w)_{ak}$ leading to over-representation of works with large values.

To make the contributions of all works equal we can apply the *fractional* approach by normalizing the weights: setting $x' = x/S_x$ we get $S_{x'} = 1$ and therefore $S(\mathbf{H}'_w) = 1$ for all works $w$.

In the case of two-mode networks **WA** and **WK** we denote

$$S_w^{\mathbf{WA}} = \begin{cases} \sum_a \mathbf{WA}[w,a] & \mathrm{outdeg}_{\mathbf{WA}}(w) > 0 \\ 1 & \mathrm{outdeg}_{\mathbf{WA}}(w) = 0 \end{cases}$$

(and similarly $S_w^{\mathbf{WK}}$)

# . . . fractional approach

and define the *normalized* matrices

$$\mathbf{WAn} = \mathrm{diag}(\frac{1}{S^{WA}})\mathbf{WA}, \quad \mathbf{WKn} = \mathrm{diag}(\frac{1}{S^{WK}})\mathbf{WK}$$

Then the *normalized product* matrix is

$$\mathbf{AKt} = \mathbf{WAn}^T \cdot \mathbf{WKn}$$

Denoting $\mathbf{F}_w = \frac{1}{S^{WA}_w S^{WK}_w}\mathbf{H}_w$ the outer product decomposition gets form

$$\mathbf{AKt} = \sum_w \mathbf{F}_w$$

# . . . fractional approach

Since

$$T(\mathbf{F}_w) = \begin{cases} 1 & (\mathrm{outdeg}_{\mathbf{WA}}(w) > 0) \wedge (\mathrm{outdeg}_{\mathbf{WK}}(w) > 0) \\ 0 & \text{otherwise} \end{cases}$$

we have further

$$\sum_{a,k} \mathbf{F}[a,k] = \sum_{a,k} \sum_{w} \mathbf{F}_w[a,k] = \sum_{w} T(\mathbf{F}_w) = |W^+|$$

where $W^+ = \{w \in W : (\mathrm{outdeg}_{\mathbf{WA}}(w) > 0) \wedge (\mathrm{outdeg}_{\mathbf{WK}}(w) > 0)\}$.

In the network **AKt** the contribution of each work to the bibliography is 1. These contributions are redistributed to arcs from authors to keywords.

# Example

## normalized matrices



$$\mathrm{diag}\left(\frac{1}{S^{WA}}\right) = \begin{array}{c c} & \begin{array}{ccccc} w_1 & w_2 & w_3 & w_4 & w_5 \end{array} \\ \begin{array}{c} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{array} & \begin{bmatrix} 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/3 \end{bmatrix} \end{array}$$

$$\mathrm{diag}\left(\frac{1}{S^{WK}}\right) = \begin{array}{c c} & \begin{array}{ccccc} w_1 & w_2 & w_3 & w_4 & w_5 \end{array} \\ \begin{array}{c} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{array} & \begin{bmatrix} 1/2 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1/2 \end{bmatrix} \end{array}$$

$$\mathbf{WAn} = \begin{array}{c c} & \begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \end{array} \\ \begin{array}{c} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{array} & \begin{bmatrix} 1/2 & 0 & 1/2 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1/3 & 0 & 1/3 & 1/3 \\ 0 & 1/2 & 0 & 1/2 \\ 1/3 & 0 & 1/3 & 1/3 \end{bmatrix} \end{array}, \quad \mathbf{WKn} = \begin{array}{c c} & \begin{array}{cccc} k_1 & k_2 & k_3 & k_4 \end{array} \\ \begin{array}{c} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{array} & \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 1 & 0 \\ 0 & 1/2 & 0 & 1/2 \end{bmatrix} \end{array}$$

# . . . example

## normalized matrices

$$\mathbf{F}_1 = \begin{array}{c c} & \begin{array}{cccc} k_1 & k_2 & k_3 & k_4 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \end{array} & \left[ \begin{array}{cccc} 1/4 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/4 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array} \qquad \mathbf{F}_5 = \begin{array}{c c} & \begin{array}{cccc} k_1 & k_2 & k_3 & k_4 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \end{array} & \left[ \begin{array}{cccc} 0 & 1/6 & 0 & 1/6 \\ 0 & 0 & 0 & 0 \\ 0 & 1/6 & 0 & 1/6 \\ 0 & 1/6 & 0 & 1/6 \end{array} \right] \end{array}$$

$$\mathbf{AKt} = \mathbf{F} = \begin{array}{c c} & \begin{array}{cccc} k_1 & k_2 & k_3 & k_4 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \end{array} & \left[ \begin{array}{cccc} 0.50000 & 0.52778 & 0.36111 & 0.27778 \\ 0.25000 & 0.00000 & 0.75000 & 0.00000 \\ 0.25000 & 0.52778 & 0.11111 & 0.27778 \\ 0.00000 & 0.27778 & 0.61111 & 0.27778 \end{array} \right] \end{array}$$

# Third co-authorship network

$\mathbf{Ct} = \mathbf{N}^T * \mathbf{N}$

$ct_{ij}$ = the total contribution of 'collaboration' of authors $i$ and $j$ to works.

It holds $ct_{ij} = ct_{ji}$ and

$$\sum_{i \in A} \sum_{j \in A} n_{pi} n_{pj} = 1$$

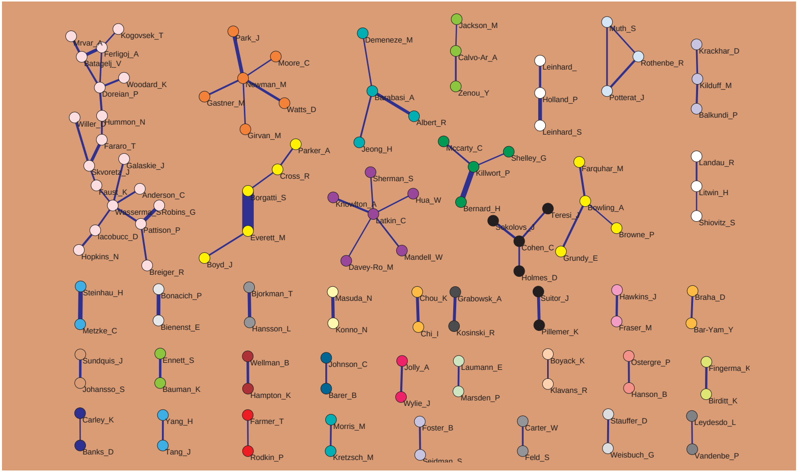
The total contribution of a complete subgraph corresponding to the authors of a work $p$ is 1.

$\sum_{j \in A} ct_{ij} = \sum_{p \in W} n_{pi}$ = the total contribution of author $i$ to works from $W$.

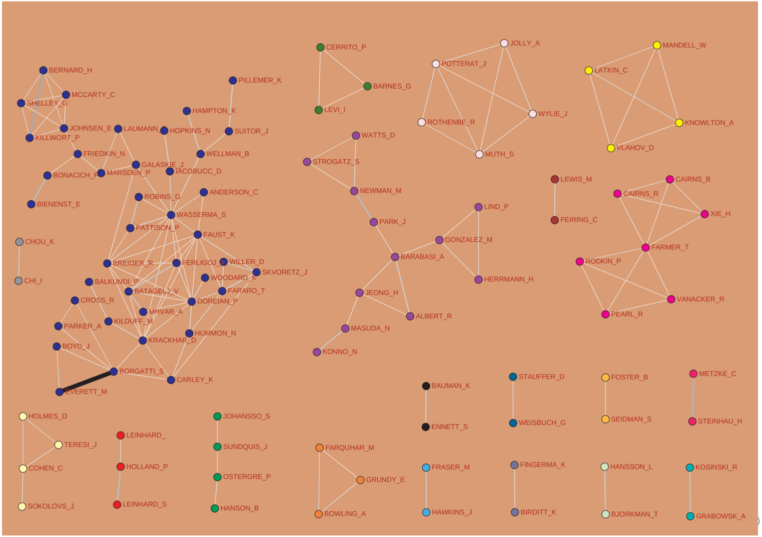
$$\sum_{i \in A} \sum_{j \in A} ct_{ij} = |W|$$

# Components in **Ct**(SN5) cut at level 0.5

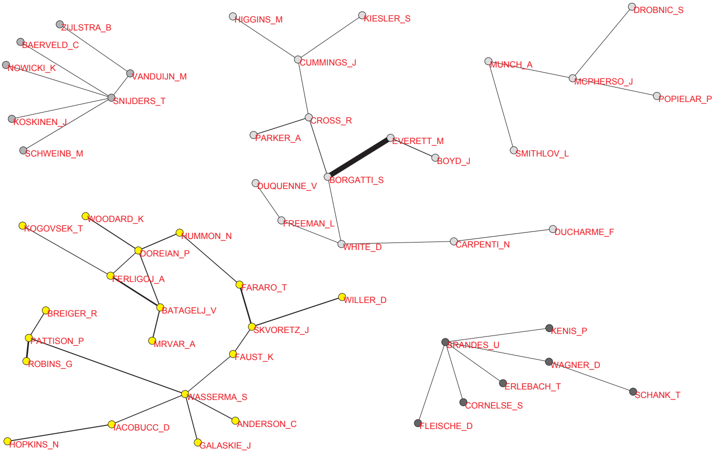
**Network** SN5 (2008): for "social network*" + most frequent references + around 100 social networkers; $|W| = 193376$, $|C| = 7950$, $|A| = 75930$, $|J| = 14651$, $|K| = 29267$



# $p_S$-core at level 0.75 in **Ct**(SN5)



# Some link islands [5,20] in **Ct**(SN5)



# Fourth co-authorship network



$\mathbf{Ct}' = \mathbf{N}^T * \mathbf{N}'$, where $n'_{pi} = wa_{pi} / \max(1, \text{outdeg}_{WA}(p) - 1)$ is the Newman's normalization.

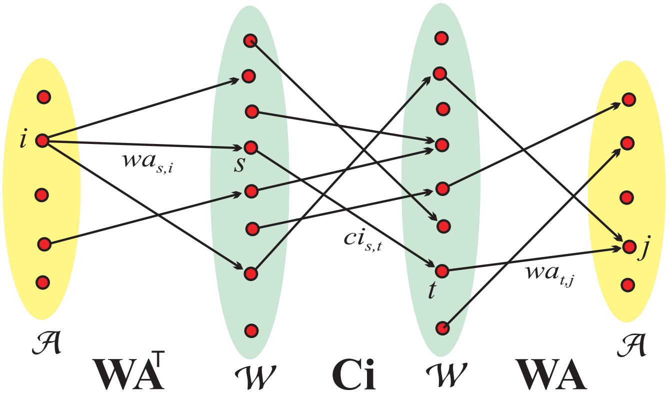
$ct'_{ij}$ = the total contribution of 'strict collaboration' of authors $i$ and $j$ to works.

In `Pajek` we can use macros to save sequences of commands to produce different co-authorship networks.

The final result is returned as an undirected simple network with weights (for $i \neq j$)

$$ct'_{ij} = \sum_p \frac{2 \cdot wa_{pi} \cdot wa_{pj}}{\max(1, \text{outdeg}_{WA}(p)) \cdot \max(1, \text{outdeg}_{WA}(p) - 1)}$$

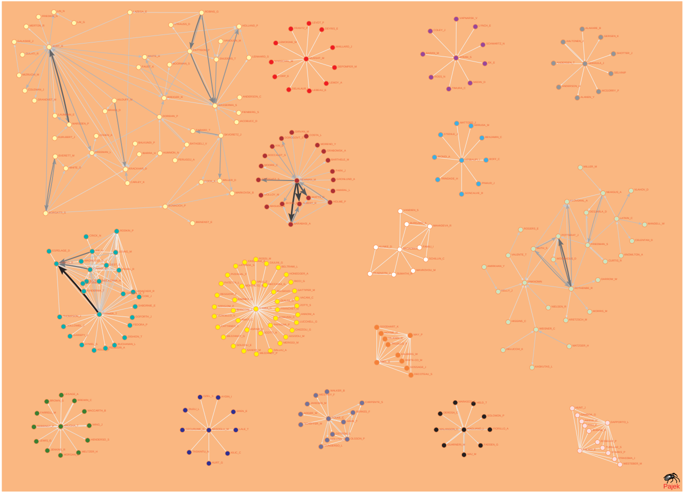
# Authors' citations network




$\mathbf{Ca} = \mathbf{AW} * \mathbf{Ci} * \mathbf{WA}$ is a network of citations between authors. The weight $w(i, j)$ counts the number of times a work authored by $i$ is citing a work authored by $j$.

# Islands in SN5 authors citation network

**Network** `SN5` (2008): for `"social network*"` + most frequent references + around 100 social networkers; $|W| = 193376$, $|C| = 7950$, $|A| = 75930$, $|J| = 14651$, $|K| = 29267$



# Linking through a network



Let a network $\mathbf{S}$ links works to works. The derived network $\mathbf{WA}^T \cdot \mathbf{S} \cdot \mathbf{WA}$ links authors to authors *through* $\mathbf{S}$. Again, the normalization question has to be addressed. Among different options let us consider the derived networks defined as:

$$\mathbf{C} = \mathbf{WAn}^T \cdot \mathbf{S} \cdot \mathbf{WAn}$$

It is easy to verify that:

- if $\mathbf{S}$ is symmetric, $\mathbf{S}^T = \mathbf{S}$, then also $\mathbf{C}$ is symmetric, $\mathbf{C}^T = \mathbf{C}$;

$$\mathbf{C}^T = (\mathbf{WAn}^T \cdot \mathbf{S} \cdot \mathbf{WAn})^T = \mathbf{WAn}^T \cdot \mathbf{S}^T \cdot (\mathbf{WAn}^T)^T = \mathbf{C}$$

- if $W^+ = \{w \in W : \text{outdeg}_{\mathbf{WA}}(w) > 0\} = W$, the total of weights of $\mathbf{S}$ is redistributed in $\mathbf{C}$:

$$T(\mathbf{C}) = \sum_{e \in L(\mathbf{C})} c(e) = \sum_{e \in L(\mathbf{S})} s(e) = T(\mathbf{S})$$

# ... Linking through a network



Since $\sum_{a \in A} wa[p,a] = \text{outdeg}_{\mathbf{WA}}(p)$ and

$$wan[p,a] = \begin{cases} \frac{wa[p,a]}{\text{outdeg}_{\mathbf{WA}}(p)} & \text{outdeg}_{\mathbf{WA}}(p) > 0 \\ 0 & \text{otherwise} \end{cases}$$ we get

$$T(\mathbf{C}) = \sum_{e \in L(\mathbf{C})} c(e) = \sum_{a \in A} \sum_{b \in A} c[a,b] =$$

$$\sum_{a \in A} \sum_{b \in A} \sum_{p \in W} \sum_{q \in W} wan[p,a] \cdot s[p,q] \cdot wan[q,b] =$$

$$= \sum_{p \in W^+} \sum_{q \in W^+} \frac{s[p,q]}{\text{outdeg}_{\mathbf{WA}}(p)\text{outdeg}_{\mathbf{WA}}(q)} \sum_{a \in A} wa[p,a] \sum_{b \in A} wa[q,b] =$$

$$\sum_{p \in W^+} \sum_{q \in W^+} s[p,q]$$

and finally, if $W^+ = W$

$$\sum_{p \in W^+} \sum_{q \in W^+} s[p,q] = \sum_{e \in L(\mathbf{S})} s(e) = T(\mathbf{S})$$

# ... Linking through a network



As special cases we get for normalized author's citation networks with $W^+ = W$: for $\mathbf{S} = \mathbf{Ci}$

$$\sum_{a\in A}\sum_{b\in A} c[a,b] = \sum_{p\in W}\sum_{q\in W} ci[p,q] = |\mathbf{Ci}|$$

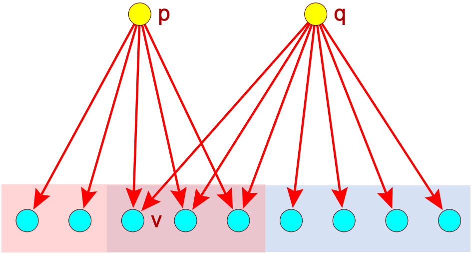
and for $\mathbf{S} = \mathbf{Cin}$

$$\sum_{a\in A}\sum_{b\in A} c[a,b] = \sum_{p\in W}\sum_{q\in W:\ \mathbf{outdeg}_{\mathbf{Ci}}(q)>0} \frac{ci[p,q]}{\text{outdeg}_{\mathbf{Ci}}(p)} =$$

$$\sum_{q\in W:\ \mathbf{outdeg}_{\mathbf{Ci}}(q)>0} 1 = |W^+_{\mathbf{Ci}}|$$

# Bibliographic Coupling

Bibliographic coupling occurs when two works each cite a third work in their bibliographies, see figure. The idea was introduced by Kessler (1963) and has been used extensively since then. See figure where two citing works, $p$ and $q$, are shown. Work $p$ cites five works and $q$ cites seven works. The key idea is that there are three works cited by both $p$ and $q$. This suggests some content communality for the three works cited by both $p$ and $q$. Having more works citing pairs of prior works increases the likelihood of them sharing content.



# Bibliographic Coupling



We assume that the citation relation means
$p \ \mathbf{Ci} \ q \equiv$ work $p$ cites work $q$. Then the *bibliographic coupling* network **biCo** can be determined as

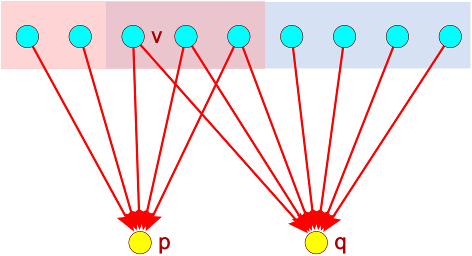
$$\mathbf{biCo} = \mathbf{Ci} * \mathbf{Ci}^T$$

The weight $bico_{pq}$ is equal to the number of works cited by both works $p$ and $q$; $bico_{pq} = |\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|$. Bibliographic coupling weights are symmetric: $bico_{pq} = bico_{qp}$:

$$\mathbf{biCo}^T = (\mathbf{Ci} \cdot \mathbf{Ci}^T)^T = \mathbf{Ci} \cdot \mathbf{Ci}^T = \mathbf{biCo}$$

# Co-citation

Co-citation is a concept with strong parallels with bibliographic coupling (Small and Marshakova 1973), see Figure ??, right. The focus is on the extent to which works are co-cited by later works. The basic intuition is that the more earlier works are cited, the higher the likelihood that they have common content.



# Bibliographic Coupling



The *co-citation* network **coCi** can be determined as

$$\mathbf{coCi} = \mathbf{Ci}^T \cdot \mathbf{Ci}.$$

The weight $coci_{pq}$ is equal to the number of works citing both works $p$ and $q$. The network **coCi** is symmetric $coci_{pq} = coci_{qp}$:

$$\mathbf{coCi}^T = (\mathbf{Ci}^T \cdot \mathbf{Ci})^T = \mathbf{Ci}^T \cdot \mathbf{Ci} = \mathbf{coCi}$$

An important property of co-citation is that $\mathbf{coCi}(\mathbf{Ci}) = \mathbf{biCo}(\mathbf{Ci}^T)$:

$$\mathbf{biCo}(\mathbf{Ci}^T) = \mathbf{Ci}^T \cdot (\mathbf{Ci}^T)^T = \mathbf{Ci}^T \cdot \mathbf{Ci} = \mathbf{coCi}(\mathbf{Ci})$$

Therefore the constructions proposed for bibliographic coupling can be applied also for co-citation.

# Normalization



What about normalizations? Searching for the most coupled works we have again problems with works with many citations, especially with review papers. To neutralize their impact we can introduce normalized measures. The fractional approach works fine for normalized co-citation

$$\mathbf{CoCit} = \mathbf{Cin}^T \cdot \mathbf{Cin}$$

where $\mathbf{Cin} = \mathbf{D} \cdot \mathbf{Ci}$ and $\mathbf{D} = \mathrm{diag}(\frac{1}{\max(1, \mathrm{outdeg}(p))})$. $\mathbf{D}^T = \mathbf{D}$ . In the normalized network every work has value 1 and it is equally distributed to all cited works.

# Normalization of Bibliographic Coupling

The fractional approach can not bi directly applied to bibliographic coupling – to get the outer product decomposition work we would need to normalize **Ci** by columns – a cited work has value 1 which is distributed equally to the citing works – the most cited works give the least. This is against our intuition. To construct a reasonable measure we can proceed as follows. Let us first look at

$$\mathbf{biC} = \mathbf{Cin} \cdot \mathbf{Ci}^T$$

we have

$$\mathbf{biC} = (\mathbf{D} \cdot \mathbf{Ci}) \cdot \mathbf{Ci}^T = \mathbf{D} \cdot \mathbf{biCo}$$

$$\mathbf{biC}^T = (\mathbf{D} \cdot \mathbf{biCo})^T = \mathbf{biCo}^T \cdot \mathbf{D}^T = \mathbf{biCo} \cdot \mathbf{D}$$

# Normalization of Bibliographic Coupling

For $\mathbf{Ci}(p) \neq \emptyset$ and $\mathbf{Ci}(q) \neq \emptyset$ it holds

$$\mathbf{biC}_{pq} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(p)|} \quad \text{and} \quad \mathbf{biC}_{qp} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(q)|} = \mathbf{biC}^T_{pq}$$

and $\mathbf{biC}_{pq} \in [0,1]$. $\mathbf{biC}_{pq}$ is the proportion of its references that the work $p$ shares with the work $q$. The network **biC** is not symmetric.

We have different options to construct normalized symmetric measures such as

$$\mathbf{biCoa}_{pq} = \frac{1}{2}(\mathbf{biC}_{pq} + \mathbf{biC}_{qp}) \quad \text{Average}$$

$$\mathbf{biCom}_{pq} = \min(\mathbf{biC}_{pq}, \mathbf{biC}_{qp}) \quad \text{Minimum}$$

$$\mathbf{biCoM}_{pq} = \max(\mathbf{biC}_{pq}, \mathbf{biC}_{qp}) \quad \text{Maximum}$$

# Normalization of Bibliographic Coupling



or, may be more interesting

$$\mathbf{biCog}_{pq} = \sqrt{\mathbf{biC}_{pq} \cdot \mathbf{biC}_{qp}} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{\sqrt{|\mathbf{Ci}(p)| \cdot |\mathbf{Ci}(q)|}}$$ Geometric mean Salton cosinus

$$\mathbf{biCoh}_{pq} = 2 \cdot (\mathbf{biC}_{pq}^{-1} + \mathbf{biC}_{qp}^{-1})^{-1} = \frac{2|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(p)| + |\mathbf{Ci}(q)|}$$ Harmonic mean

$$\mathbf{biCoj}_{pq} = (\mathbf{biC}_{pq}^{-1} + \mathbf{biC}_{qp}^{-1} - 1)^{-1} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(p) \cup \mathbf{Ci}(q)|}$$ Jaccard index

All these measures are similarities.

It is easy to verify that $biCoX_{pq} \in [0, 1]$ and: $biCoX_{pq} = 1$ iff the works $p$ and $q$ are referencing the same works, $\mathbf{Ci}(p) = \mathbf{Ci}(q)$.

# Normalization of Bibliographic Coupling

From $m \leq H \leq G \leq A \leq M$ and $J \leq m$, $(\frac{|P \cap Q|}{|P \cup Q|} \leq \min(\frac{|P \cap Q|}{|P|}, \frac{|P \cap Q|}{|Q|}))$ we get

$$\mathbf{biCoj}_{pq} \leq \mathbf{biCom}_{pq} \leq \mathbf{biCoh}_{pq} \leq \mathbf{biCog}_{pq} \leq \mathbf{biCoa}_{pq} \leq \mathbf{biCoM}_{pq}$$

The equalities hold iff $\mathbf{Ci}(p) = \mathbf{Ci}(q)$.

To get a dissimilarity we can use transformations $dis = 1 - sim$ or $dis = \frac{1}{sim} - 1$ or $dis = -\log sim$. For example

$$\mathbf{biCod}_{pq} = 1 - \mathbf{biCoj}_{pq} = \frac{|\mathbf{Ci}(p) \oplus \mathbf{Ci}(q)|}{|\mathbf{Ci}(p) \cup \mathbf{Ci}(q)|} \quad \text{Jaccard distance}$$

where $\oplus$ denotes the symmetric difference of sets.

# Bibliographic Coupling, Co-citation and Linking through a network



Bibliographic coupling and co-citation networks are linking works to works. To get linking between authors, journals or keywords considering citation similarity we can apply the construction from *Linking through a network* to the normalized co-citation or bibliographic coupling network.

# Temporal quantities

We introduce a notion of a *temporal quantity*

$$a(t) = \begin{cases} a'(t) & t \in T_a \\ \maltese & t \in \mathcal{T} \setminus T_a \end{cases}$$

where $T_a$ is the *activity time set* of $a$ and $a'(t)$ is the value of $a$ in an instant $t \in T_a$, and $\maltese$ denotes the value *undefined*.

We assume that the values of temporal quantities belong to a set $A$ which is a *semiring* $(A, +, \cdot, 0, 1)$ for binary operations $+ : A \times A \to A$ and $\cdot : A \times A \to A$.
Let $A_\maltese(\mathcal{T})$ denote the set of all temporal quantities over $A_\maltese$ in time $\mathcal{T}$. To extend the operations to networks and their matrices we first define the *sum* (parallel links) $a + b$ as

$$(a+b)(t) = a(t) + b(t) \quad \text{and} \quad T_{a+b} = T_a \cup T_b.$$

The *product* (sequential links) $a \cdot b$ is defined as

$$(a \cdot b)(t) = a(t) \cdot b(t) \quad \text{and} \quad T_{a \cdot b} = T_a \cap T_b.$$

# Sum and product of temporal quantities



```
a = [(1, 5, 2), (6, 8, 1), (11, 12, 3), (14, 16, 2),
     (17, 18, 5), (19, 20, 1)]
b = [(2, 3, 4), (4, 7, 3), (9, 10, 2), (13, 15, 5), (16, 21, 1)]
```

The following are the sum $s = a + b$ and the product $p = a \cdot b$ of temporal quantities $a$ and $b$ over combinatorial semiring.

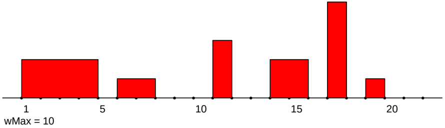
```
s = [(1, 2, 2), (2, 3, 6), (3, 4, 2), (4, 5, 5), (5, 6, 3),
     (6, 7, 4), (7, 8, 1), (9, 10, 2), (11, 12, 3),
     (13, 14, 5), (14, 15, 7), (15, 16, 2), (16, 17, 1),
     (17, 18, 6), (18, 19, 1), (19, 20, 2), (20, 21, 1)]
p = [(2, 3, 8), (4, 5, 6), (6, 7, 3), (14, 15, 10),
     (17, 18, 5), (19, 20, 1)]
```

They are visually displayed at the bottom half of figures on the following slides.

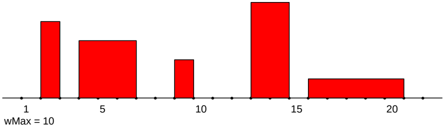
# Addition of temporal quantities.



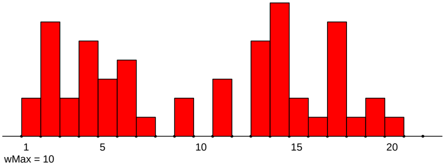
$a:$




$b:$

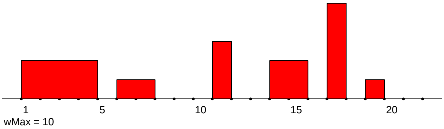



$a+b:$

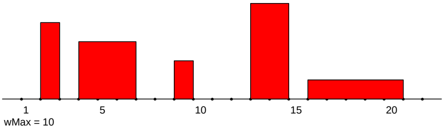


# Multiplication of temporal quantities.

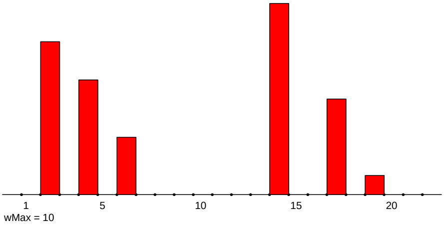
$a$ :




$b$ :




$a \cdot b$ :



# Temporal affiliation networks



Let the binary *affiliation* matrix $\mathbf{A} = [a_{ep}]$ describe a two-mode network on the set of events $E$ and the set of of participants $P$:

$$a_{ep} = \begin{cases} 1 & p \text{ participated in the event } e \\ 0 & \text{otherwise} \end{cases}$$

The function $d : E \to \mathcal{T}$ assigns to each event $e$ the date $d(e)$ when it happened. $\mathcal{T} = [\mathit{first}, \mathit{last}] \subset \mathbb{N}$. Using these data we can construct two temporal affiliation matrices:

- **instantaneous Ai** $= [ai_{ep}]$, where

$$ai_{ep} = \begin{cases} [(d(e), d(e)+1, 1)] & a_{ep} = 1 \\ [\,] & \text{otherwise} \end{cases}$$

- **cumulative Ac** $= [ac_{ep}]$, where

$$ac_{ep} = \begin{cases} [(d(e), \mathit{last}+1, 1)] & a_{ep} = 1 \\ [\,] & \text{otherwise} \end{cases}$$

# Cumulative properties

In general a temporal quantity $a$ is called *cumulative* iff it has for $t, t' \in \mathcal{T}$ the property

$$t \in T_a \ \wedge \ t' > t \quad \Rightarrow \quad t' \in T_a \ \wedge \ a(t') \geq a(t)$$

A sum and product (over combinatorial semiring) of cumulative temporal quantities are cumulative temporal quantities.
For a temporal quantity $a = [(s_i, f_i, v_i)]_{i \in 1..k}$ its *cumulative* $cum(a)$ is defined as

$$cum(a) = [(s_i, s_{i+1}, V_i)]_{i \in 1..k}$$

where $s_{k+1} = last$ and $V_i = \sum_{j=1}^{i} v_j$ .
A temporal network is cumulative for a weight $w$ iff all its values are cumulative.

# Converting Pajek net and clu files into temporal network in netsJSON

The Python code for creating temporal networks from Pajek files for the peer review data is given here.

To set up an environment for computing our examples we have to put in the directory `gdir` Python files (`Nets.py`, `TQ.py`, `search.py`, `coloring.py`, `IndexMinPQ.py`) from the library Nets, and in the subdirectory `cdir` the files `TQchart.html`, `d3.v3.min.js` and `barData.js`. The directory `ndir` contains the network data and the directory `wdir` contains the results.

# ... Converting Pajek net and clu files into temporal network in netsJSON

```
gdir = 'c:/path/Nets'
wdir = 'c:/path/Test/peere'
ndir = 'c:/path/WoS/peere2'
cdir = 'c:/path/Nets/chart'
import sys, os, datetime, json
sys.path = [gdir]+sys.path; os.chdir(wdir)
from TQ import *
from Nets import Network as N
net = ndir+"/WAd.net"
clu = ndir+"/Yeard.clu"
t1 = datetime.datetime.now(); print("started: ",t1.ctime(),"\n")
WAc = N.twoMode2netsJSON(clu,net,'WAcum.json',instant=False)
t2 = datetime.datetime.now()
print("\nconverted to cumulative TN: ",t2.ctime(),"\ntime used: ", t2-t1)
WAi = N.twoMode2netsJSON(clu,net,'WAins.json',instant=True)
t3 = datetime.datetime.now()
print("\nconverted to instantaneous TN: ",t3.ctime(),"\ntime used: ", t3-t2)
cit = ndir+"/CiteD.net"
Citei = N.oneMode2netsJSON(clu,cit,'CiteIns.json',instant=True)
t4 = datetime.datetime.now()
print("\nconverted to instantaneous TN: ",t4.ctime(),"\ntime used: ", t4-t3)
ia = WAi.Index()
ic = Citei.Index()
```

# Temporal properties

Let $\mathbf{N}$ be a temporal network on $E \times P$. On it we can define some interesting temporal quantities such as *in-sum*:

$$iS(\mathbf{N}, p) = \sum_{e \in E} n_{ep}$$

and *out-sum*:

$$oS(\mathbf{N}, e) = \sum_{p \in P} n_{ep}$$

In a special case where $\mathbf{N} \equiv \mathbf{WAi}$ we get the *productivity of an author* $a$

$$pr(a) = iS(\mathbf{WAi}, a) = \text{ number of publications of the author } a \text{ by year}$$

and for $\mathbf{N} \equiv \mathbf{WAc}$ we get the *cumulative productivity of an author* $a$

$$cpr(a) = iS(\mathbf{WAc}, a) = \text{ cumulative number of publications of } a \text{ by year.}$$

It holds $cpr(a) = cum(pr(a))$.

# Group productivity

The productivity of an author can be extended to the *productivity of a group of authors* $C$

$$pr(C) = \sum_{a \in C} pr(a) = \sum_{a \in C} iS(\mathbf{WAi}, a)$$

There is a problem with the productivity of a group. In the case when two authors from a group co-authored the same paper it is counted twice. To account for a "real" contribution of each author the fractional approach is used. It is based on normalized networks (matrices) – in the case of co-authorship on $n(\mathbf{WA}) = \mathbf{WAn} = [wan_{wa}]$

$$wan_{wa} = \frac{wa_{wa}}{\max(1, \text{outdeg}_{\mathbf{WA}}(w))}.$$

This leads to the *fractional productivity of an author* a

$$fpr(a) = iS(\mathbf{WAni}, a) = \text{fractional contribution of publications of } a \text{ by year}$$

# Temporal properties in networks on peer review

## Productivity and cumulative productivity of Lutz Bornmann

In the analysis of the ordinary authorship network **WA** we get that Lutz Bornmann is the author who wrote the largest number, 61, of works on peer review. To see the dynamics of his publishing we compute his productivity

$$
\begin{aligned}
pr \quad = \quad & [(2005, 2006, 4), (2006, 2007, 3), (2007, 2008, 4), (2008, 2009, 9), \\
& (2009, 2010, 4), (2010, 2011, 14), (2011, 2012, 5), (2012, 2013, 7), \\
& (2013, 2014, 2), (2014, 2015, 3), (2015, 2016, 6)]
\end{aligned}
$$

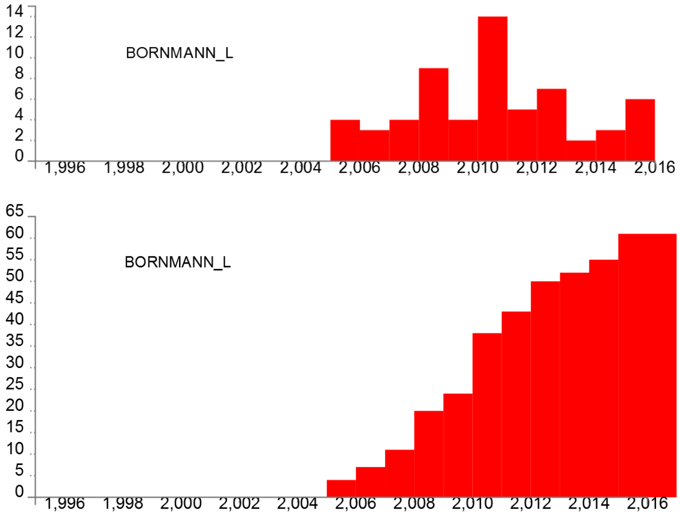
see the top of the figure. The corresponding cumulative productivity is

$$
\begin{aligned}
cpr \quad = \quad & [(2005, 2006, 4), (2006, 2007, 7), (2007, 2008, 11), (2008, 2009, 20), \\
& (2009, 2010, 24), (2010, 2011, 38), (2011, 2012, 43), (2012, 2013, 50), \\
& (2013, 2014, 52), (2014, 2015, 55), (2015, 2017, 61)]
\end{aligned}
$$

see the bottom of the figure. Note that $cpr = cum(pr)$: $7 = 4 + 3$, $11 = 4 + 3 + 4$, ...

# Temporal properties in networks on peer review

## Productivity and cumulative productivity of Lutz Bornmann



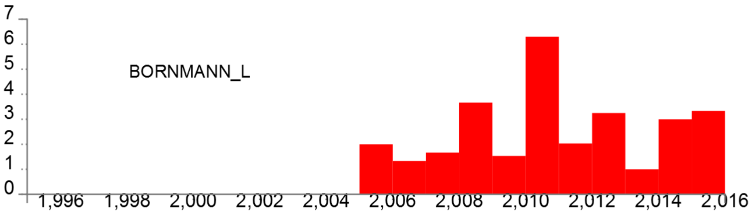
## Fractional productivity of Lutz Bornmann



The fractional productivity of Lutz Bornmann is

$$
\begin{aligned}
fpr \quad = \quad & [(2005, 2006, 2.0), (2006, 2007, 1.333), (2007, 2008, 1.667), \\
& (2008, 2009, 3.667), (2009, 2010, 1.533), (2010, 2011, 6.3), \\
& (2011, 2012, 2.033), (2012, 2013, 3.25), (2013, 2014, 1.0), \\
& (2014, 2015, 3.0), (2015, 2016, 3.333)]
\end{aligned}
$$

see the figure. For the Python code see the next slide.



# Productivities of authors – Python code



```
>>> tit = 'BORNMANN_L';   b = ia[tit]
>>> pr = WAi.TQnetInSum(b)
>>> pr
[(2005, 2006, 4), (2006, 2007, 3), (2007, 2008, 4), (2008, 2009, 9),...
>>> TQ.TqSummary(pr)
(1900, 2017, 0, 14)
>>> TQmax = 15; Tmin = 1995; Tmax = 2016; w = 600; h = 150
>>> N.TQshow(pr,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='red')
>>> cpr = WAc.TQnetInSum(b)
>>> cpr
[(2005, 2006, 4), (2006, 2007, 7), (2007, 2008, 11), (2008, 2009, 20),..
>>> TQmax = 65; Tmin = 1995; Tmax = 2016; w = 600; h = 250
>>> N.TQshow(cpr,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='red')
>>> WAni = WAi.TQnormal()
>>> fpr = WAni.TQnetInSum(b)
>>> fpr
[(2006, 2007, 1.3333333333333333), (2007, 2008, 1.6666666666666665),...
>>> TQmax = 7; Tmin = 1995; Tmax = 2016; w = 600; h = 150
>>> N.TQshow(fpr,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='red')
```

# Citations to a paper through years



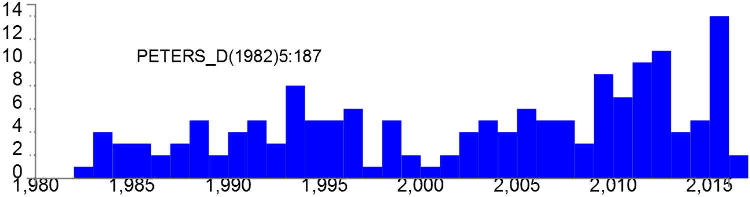
In the citation network **Cite** for the peer review bibliography the most cited, 164, paper is Peters, D. P., Ceci, S. J. (1982). Peer-review practices of psychological journals: The fate of published articles, submitted again. Behavioral and Brain Sciences, 5(2), 187-255. The temporal quantity $citP = iS(\mathbf{Cii}, \mathrm{PETERS\_D(1982)5{:}187})$ describes the number of citations to this paper through years.

$$
\begin{aligned}
citP \quad = \quad & [(1982, 1983, 1), (1983, 1984, 4), (1984, 1986, 3), (1986, 1987, 2), \\
& (1987, 1988, 3), (1988, 1989, 5), (1989, 1990, 2), (1990, 1991, 4), \\
& (1991, 1992, 5), (1992, 1993, 3), (1993, 1994, 8), (1994, 1996, 5), \\
& (1996, 1997, 6), (1997, 1998, 1), (1998, 1999, 5), (1999, 2000, 2), \\
& (2000, 2001, 1), (2001, 2002, 2), (2002, 2003, 4), (2003, 2004, 5), \\
& (2004, 2005, 4), (2005, 2006, 6), (2006, 2008, 5), (2008, 2009, 3), \\
& (2009, 2010, 9), (2010, 2011, 7), (2011, 2012, 10), (2012, 2013, 11), \\
& (2013, 2014, 4), (2014, 2015, 5), (2015, 2016, 14), (2016, 2017, 2)]
\end{aligned}
$$

See the figure at the next slide.

# Peer review

## Citations to Peters




Similarly we could look at the number of works by year $wy = \sum_{j \in J} iS(\mathbf{WJi}, j)$, the popularity of a keyword $k$: $pop(k) = iS(\mathbf{WKi}, k)$, etc.

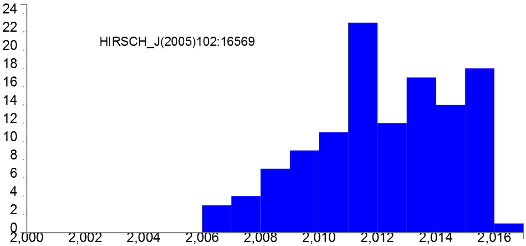
# Peer review

## Citations to Hirsch

Another well known paper is Hirsch, J.E. (2005). An index to quantify an individual's scientific research output. Proc Natl Acad Sci U S A. 2005 Nov 15;102(46):16569-72 with 119 citations and
$citH = iS(\mathbf{Cii}, \text{HIRSCH\_J(2005)102:16569})$

$$citH = [(2006, 2007, 3), (2007, 2008, 4), (2008, 2009, 7), (2009, 2010, 9), (2010, 2011, 11), (2011, 2012, 23), (2012, 2013, 12), (2013, 2014, 17), (2014, 2015, 14), (2015, 2016, 18), (2016, 2017, 1)]$$

For the Python code see the next slide.



# Citations between works – Python code



```
>>> tit = 'PETERS_D(1982)5:187'; c = ic[tit]
>>> ci = Citei.TQnetInSum(c)
>>> ci
[(1982, 1983, 1), (1983, 1984, 4), (1984, 1986, 3), (1986, 1987, 2), ...
>>> TQmax = 15; Tmin = 1980; Tmax = 2016; w = 600; h = 150
>>> N.TQshow(ci,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='blue')
>>> tit = 'HIRSCH_J(2005)102:16569'; c = ic[tit]
>>> ci = Citei.TQnetInSum(c)
>>> ci
[(2005, 2006, 0), (2006, 2007, 3), (2007, 2008, 4), (2008, 2009, 7), ...
>>> TQmax = 25; Tmin = 2000; Tmax = 2017; w = 600; h = 250
>>> N.TQshow(ci,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='blue')
```

# Main journals publishing on peer review

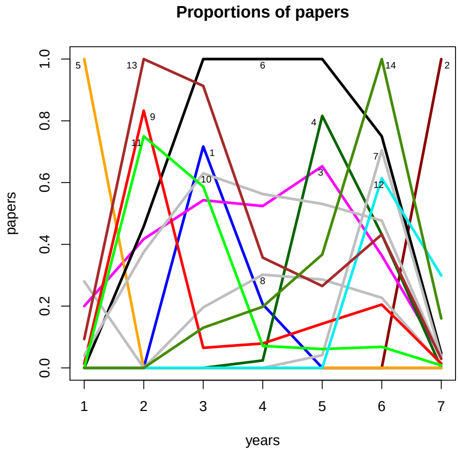
To identify the main journals publishing on peer review we determined first the temporal inSums $Jt$ in the network **JWi** for all journals. An entry $Jt(j) = iS(\mathbf{JWi}, j)$ contains the temporal quantity counting the number of papers on peer review published in the journal $j$ in each year. Because most of frequencies are small (one digit numbers) we decided to change the time scale (granularity) to time intervals: 1: 1900-1970, 2: 1971-1980, 3: 1981-1990, 4: 1991-2000, 5: 2001-2005, 6: 2006-2010, 7:2011-2015. The recoded table $Jt$ is labeled $Jr$. For the table $Jr$ we determined for each time interval three the most frequently used journals – they are listed on the right side of the figure. The corresponding data were exported as `journals.csv` and visualized using R. The picture on the left side presents the trajectories of relative importance (journal's frequency divided with the maximum frequency on the interval) for the selected journals.

# ... Main journals publishing on peer review



The papers on peer review (refereeing) published till 1970 appeared most often in J ASSOC OFF AGR CHEM. Till 2005 the dominant journals were JAMA, SCIENCE, NATURE, BRIT MED J, and LANCET (general medical and science journals). In the period 2006-2010 the leading role was overtaken by a specialized journal SCIENTOMETRICS. In the last period 2011-2015 the primate is shifted to the mega-journals BMJ OPEN and PLOS ONE. Note that the frequencies for SCIENTOMETRICS are 3: 6, 4: 25, 5:18, 6: 44, 7: 78 and in the period 2011-2015 489 papers on peer review were published in BMJ OPEN.

# Main journals publishing on peer review




| i | journal |
|---|---|
| 1 | BEHAV BRAIN SCI |
| 2 | BMJ OPEN |
| 3 | BRIT MED J |
| 4 | CUTIS |
| 5 | J ASSOC OFF AGR CHEM |
| 6 | JAMA-J AM MED ASSOC |
| 7 | J SEX MED |
| 8 | LANCET |
| 9 | MED J AUSTRALIA |
| 10 | NATURE |
| 11 | NEW ENGL J MED |
| 12 | PLOS ONE |
| 13 | SCIENCE |
| 14 | SCIENTOMETRICS |

# The most important journals – Python code

```
>>> jrn = ndir+"/WJd.net"
>>> WJc = N.twoMode2netJSON(clu,jrn,'WJcum.json',instant=False)
>>> WJi = N.twoMode2netJSON(clu,jrn,'WJins.json',instant=True)
>>> J = list(WJi.nodesMode(2))
>>> Jt = [ (j, WJi._nodes[j][3]['lab'], TQ.cutGT(WJi.TQnetInSum(j),0)) for j in J ]
>>> p = [0,1971,1981,1991,2001,2006,2011,2016,3000]
>>> Jr = [ (j,l,TQ.changeTime(a,p)) for (j,l,a) in Jt ]
>>> I = { Jt[j][1] : j for j in range(len(Jt)) }
>>> JL = [ "BEHAV BRAIN SCI", "BMJ OPEN", "BRIT MED J", "CUTIS",
   "J ASSOC OFF AGR CHEM", "JAMA-J AM MED ASSOC", "J SEX MED",
   "LANCET", "MED J AUSTRALIA", "NATURE", "NEW ENGL J MED",
   "PLOS ONE", "SCIENCE", "SCIENTOMETRICS" ]
>>> IJ = [ I[j] for j in JL ]; Ir = [ Jr[i] for i in IJ]
```

In the library TQ we included a new function `changeTime` that recodes a temporal quantity *a* into new time intervals determined by a sequence *p*.

# Multiplication of temporal networks

Let $\mathbf{A}$ on $A \times P$ and $\mathbf{B}$ on $P \times B$ be (matrices of) co-occurence networks. Then $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ is a temporal network on $A \times B$. What is its meaning? Consider the value of its item in an instant $t$

$$c_{ij}(t) = \sum_{p \in P} a_{ip}(t)^T \cdot b_{pj}(t) = \sum_{p \in P} a_{pi}(t) \cdot b_{pj}(t)$$

For $c_{ij}(t)$ to be defined (different from ⌘) there should be at least one $p \in P$ such that $a_{pi}(t)$ and $b_{pj}(t)$ are both defined, i.e. $t \in T_{a_{pi}} \cap T_{b_{pj}}$. Then there exists $g_{pi}$ such that $(s_{g_{pi}}, f_{g_{pi}}, v_{g_{pi}}) \in a_{pi}$, $t \in [s_{g_{pi}}, f_{g_{pi}})$, and $a_{pi}(t) = v_{g_{pi}}$. Similarly $b_{pj}(t) = v_{h_{pj}}$. Therefore

$$c_{ij}(t) = \sum_{p: t \in T_{a_{pi}} \cap T_{b_{pj}}} v_{g_{pi}} \cdot v_{h_{pj}}$$

# . . . Multiplication of temporal networks

For binary instantaneous two-mode networks $\mathbf{A}$ and $\mathbf{B}$ the value $c_{ij}(t)$ of the product $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ is equal to the number of different members of $P$ with which both $i$ and $j$ have contact in the instant $t$.

The product of cumulative networks is cumulative itself. For binary cumulative two-mode networks $\mathbf{A}$ and $\mathbf{B}$ the value $c_{ij}(t)$ of the product $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$ is equal to the number of different members of $P$ with which both $i$ and $j$ had contact in instants up to including the instant $t$.

# Temporal co-occurrence networks



Using the multiplication of temporal affiliation networks over the combinatorial semiring we get the corresponding instantaneous and cumulative co-occurrence networks

$$\mathbf{Ci} = \mathbf{Ai}^T \cdot \mathbf{Ai} \qquad \text{and} \qquad \mathbf{Cc} = \mathbf{Ac}^T \cdot \mathbf{Ac}$$

The triple $(s, f, v)$ in a temporal quantity $ci_{pq}$ tells that in the time interval $[s, f)$ there were $v$ events in which both $p$ and $q$ took part. The triple $(s, f, v)$ in a temporal quantity $cc_{pq}$ tells that in the time interval $[s, f)$ there were in total $v$ accumulated events in which both $p$ and $q$ took part.

The diagonal (loop) weights $ci_{pp}$ and $cc_{pp}$ contain the temporal quantities counting the number of events in the time intervals in which the participant $p$ took part.
A typical example of such a network is the works authorship network **WA** where $E$ is the set of papers $W$, $P$ is the set of authors $A$, and $d$ is the publication year.

# Example: Temporal coauthorship network



The instantaneous coauthorship network **Coi** is obtained as

$$\mathbf{Coi} = \mathbf{WAi}^T \cdot \mathbf{WAi}$$

Bibliographic networks are usually sparse. Often also the product of sparse networks is sparse itself. Considering in computation only non zero elements it can be computed fast. In our example, the network **WA** has 22104 works, 62106 authors and 80021 arcs. The derived network **Coi** has 633977 edges and was computed on a laptop in 12.7 seconds.

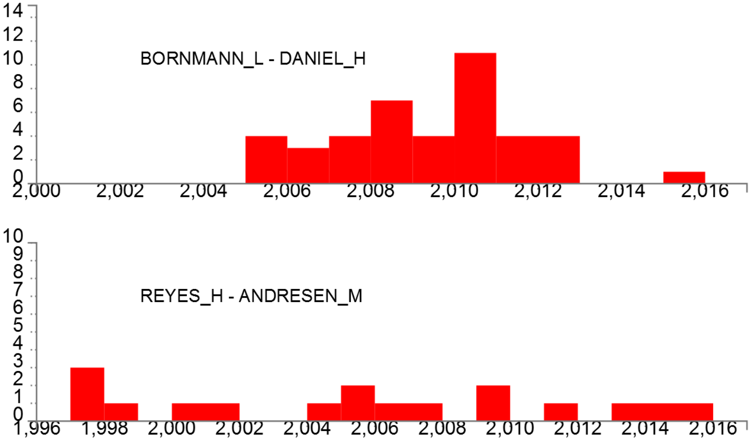
# Example: Temporal coauthorship network

For the peer review data we get the largest values
$co$[BORNMANN_L, DANIEL_H] $= 42$,
$co$[MOHER_D, ALTMAN_D] $= 24$,
$co$[REYES_H, ANDRESEN_M] $= 17$.
The corresponding temporal quantities
$bd = tq$(BORNMANN_L, DANIEL_H) and
$ra = tq$(REYES_H, ANDRESEN_M) are

$$\begin{aligned} bd &= [(2005, 2006, 4), (2006, 2007, 3), (2007, 2008, 4), (2008, 2009, 7), \\ &\quad (2009, 2010, 4), (2010, 2011, 11), (2011, 2013, 4), (2015, 2016, 1)] \\ ra &= [(1997, 1998, 3), (1998, 1999, 1), (2000, 2002, 1), (2004, 2005, 1), \\ &\quad (2005, 2006, 2), (2006, 2008, 1), (2009, 2010, 2), (2011, 2012, 1), \\ &\quad (2013, 2016, 1)] \end{aligned}$$

Both temporal quantities are presented on the next slide. The Python code is given in the following slide.

# Example: Temporal coauthorship network



# Example: Temporal coauthorship network Python code



```
>>> Co = WAi.TQtwo2oneCols()
>>> Co.saveNetsJSON('CoIns.json',indent=2)
>>> Co.delLoops()
>>> C = Co.TQtopLinks(thresh=15)
>>> tit = C[0][2]+' - '+C[0][3]; bd = C[0][5]
>>> TQmax = 15; Tmin = 2000; Tmax = 2017; w = 600; h = 150
>>> N.TQshow(bd,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='red')
>>> tit = C[2][2]+' - '+C[2][3]; ra = C[2][5]
>>> TQmax = 10; Tmin = 1996; Tmax = 2017; w = 600; h = 150
>>> N.TQshow(ra,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='red')
>>> TQ.total(bd), TQ.total(ra)
(42, 17)
```

# Example: Temporal citations between journals



The derived network describing citations between journals is obtained as

$$\mathbf{JCJ} = \mathbf{WJi}^T \cdot \mathbf{CiI} \cdot \mathbf{WJc}$$

Note that the third network in the product is cumulative.
The weight of the element $jcj_{ij}$ is equal to the number of citations per year from works published in journal $i$ to works published in journal $j$.
In a special case when $i = j$ we get a temporal quantity describing selfcitations of journal $i$. In the peer review data the largest number of selfcitations are 320 in JAMA and 148 in Scientometrics.

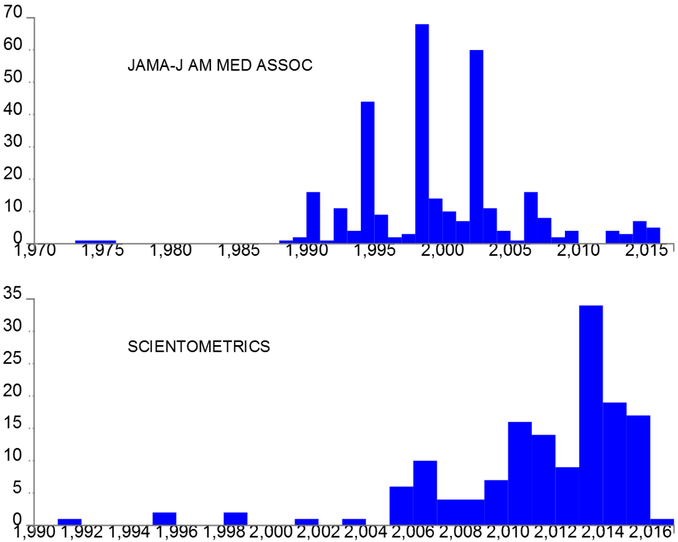
# ... Example: Temporal citations between journals

The corresponding temporal quantities $jm = jcj[\text{JAMA}, \text{JAMA}]$ and $sm = jcj[\text{SCIENTOMETRICS}, \text{SCIENTOMETRICS}]$ are:

$$\begin{aligned}
jm &= [(1973, 1976, 1), (1988, 1989, 1), (1989, 1990, 2), (1990, 1991, 16), (1991, 1992, 1), \\
&\quad (1992, 1993, 11), (1993, 1994, 4), (1994, 1995, 44), (1995, 1996, 9), (1996, 1997, 2), \\
&\quad (1997, 1998, 3), (1998, 1999, 68), (1999, 2000, 14), (2000, 2001, 10), (2001, 2002, 7), \\
&\quad (2002, 2003, 60), (2003, 2004, 11), (2004, 2005, 4), (2005, 2006, 1), (2006, 2007, 16), \\
&\quad (2007, 2008, 8), (2008, 2009, 2), (2009, 2010, 4), (2012, 2013, 4), (2013, 2014, 3), \\
&\quad (2014, 2015, 7), (2015, 2016, 5)] \\
sm &= [(1991, 1992, 1), (1995, 1996, 2), (1998, 1999, 2), (2001, 2002, 1), (2003, 2004, 1), \\
&\quad (2005, 2006, 6), (2006, 2007, 10), (2007, 2009, 4), (2009, 2010, 7), (2010, 2011, 16), \\
&\quad (2011, 2012, 14), (2012, 2013, 9), (2013, 2014, 34), (2014, 2015, 19), (2015, 2016, 17), \\
&\quad (2016, 2017, 1)]
\end{aligned}$$

and are presented on the next slide.

# Example: Journals Selfcitations



# Example: Temporal citations between journals

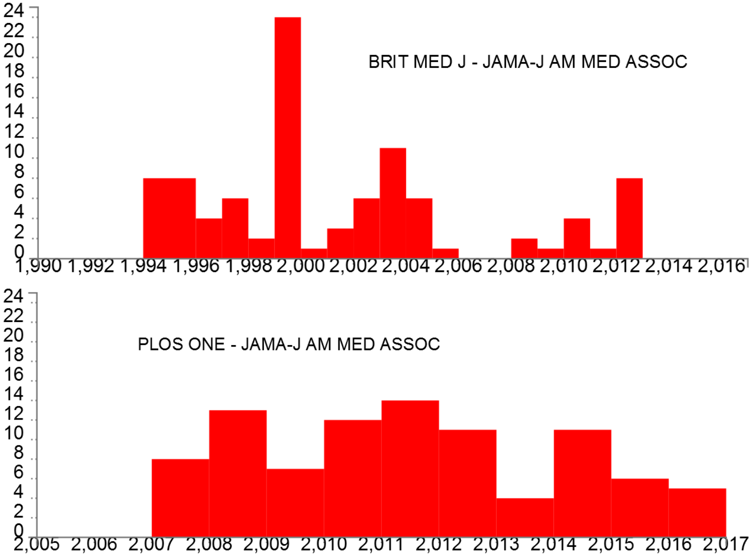


The largest number of citations are from journals BMJ Open (142) and Scientometrics (108) to the unknown journal *****, followed by $bj = jcj$[BRIT MED J, JAMA-J AM MED ASSOC] and $pj = jcj$[PLOS ONE, JAMA-J AM MED ASSOC] with totals 96 and 91.

$$
\begin{aligned}
bj &= [(1994, 1996, 8), (1996, 1997, 4), (1997, 1998, 6), (1998, 1999, 2), (1999, 2000, 24), \\
&\quad (2000, 2001, 1), (2001, 2002, 3), (2002, 2003, 6), (2003, 2004, 11), (2004, 2005, 6), \\
&\quad (2005, 2006, 1), (2008, 2009, 2), (2009, 2010, 1), (2010, 2011, 4), (2011, 2012, 1), \\
&\quad (2012, 2013, 8)] \\
pj &= [(2007, 2008, 8), (2008, 2009, 13), (2009, 2010, 7), (2010, 2011, 12), (2011, 2012, 14), \\
&\quad (2012, 2013, 11), (2013, 2014, 4), (2014, 2015, 11), (2015, 2016, 6), (2016, 2017, 5)]
\end{aligned}
$$

# Example: Temporal citations between journals





# Example: Temporal citations between journals

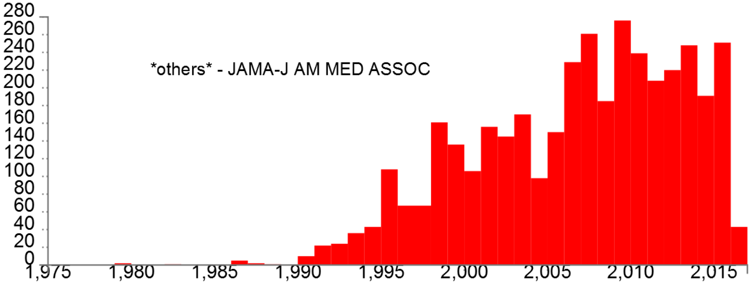


In the peer review data the journal JAMA is the most prominent. To get the temporal quantity describing citations of others to JAMA we compute $jci = iS(\mathbf{JCJ}, \text{JAMA-J AM MED ASSOC})$:

$$\begin{aligned} jci = {} & [(1979, 1980, 2), (1982, 1983, 1), (1986, 1987, 5), (1987, 1988, 2), \\ & (1988, 1989, 1), (1990, 1991, 10), (1991, 1992, 22), (1992, 1993, 24), \\ & (1993, 1994, 36), (1994, 1995, 43), (1995, 1996, 108), (1996, 1998, 67), \\ & (1998, 1999, 161), (1999, 2000, 136), (2000, 2001, 106), (2001, 2002, 156), \\ & (2002, 2003, 145), (2003, 2004, 170), (2004, 2005, 98), (2005, 2006, 150), \\ & (2006, 2007, 229), (2007, 2008, 261), (2008, 2009, 185), (2009, 2010, 276), \\ & (2010, 2011, 239), (2011, 2012, 208), (2012, 2013, 220), (2013, 2014, 248), \\ & (2014, 2015, 191), (2015, 2016, 251), (2016, 2017, 43)] \end{aligned}$$

See the figure on the next slide. The Python code is given in the following slide.

# Example: Temporal citations between journals



# Example: Temporal citations between journals Python code



```
>>> JCJ = N.TQmultiply(N.TQmultiply(WJi.transpose(),
          Citei.one2twoMode()),WJc,True)
>>> L = JCJ.TQtopLoops(thresh=100)
>>> tit = L[0][1]; jm = L[0][3]
>>> TQmax = 70; Tmin = 1970; Tmax = 2017; w = 600; h = 200
>>> N.TQshow(jm,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='blue')
>>> tit = L[1][1]; sm = L[1][3]
>>> TQmax = 35; Tmin = 1990; Tmax = 2017; w = 600; h = 200
>>> N.TQshow(sm,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='blue')
>>> JCJ.delLoops()
>>> T = JCJ.TQtopLinks(thresh=70)
>>> tit = T[2][2]+' - '+T[2][3]; bj = T[2][5]
>>> TQmax = 25; Tmin = 1990; Tmax = 2017; w = 600; h = 200
>>> N.TQshow(bj,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='red')
>>> tit = T[3][2]+' - '+T[3][3]; pj = T[3][5]
>>> TQmax = 25; Tmin = 2005; Tmax = 2017; w = 600; h = 200
>>> N.TQshow(pj,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='red')
>>> jci = TQ.cutGE(JCJ.TQnetInSum(T[2][1]),1e-10)
>>> TQ.TqSummary(jci)
(1979, 2017, 1, 276)
>>> TQ.total(jci)
3861
>>> tit = '*others* - '+T[2][3]
>>> TQmax = 280; Tmin = 1975; Tmax = 2017; w = 600; h = 200
>>> N.TQshow(jci,cdir,TQmax,Tmin,Tmax,w,h,tit,fill='red')
```

# Citations between authors

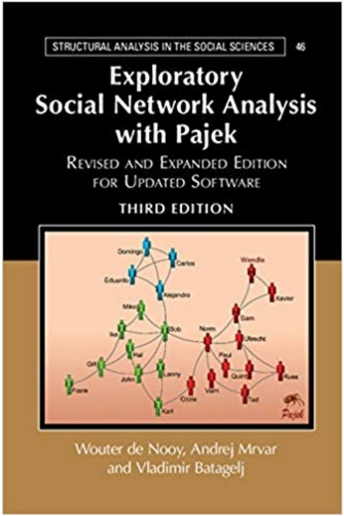


Similarly we get the temporal network describing citations between authors

$$\mathbf{ACA} = \mathbf{WAi}^T \cdot \mathbf{CiI} \cdot \mathbf{WAc}$$

The weight of the element $aca_{ab}$ is equal to the number of citations per year from works coauthored by author $a$ to works coauthored by author $b$.

# ESNA Pajek



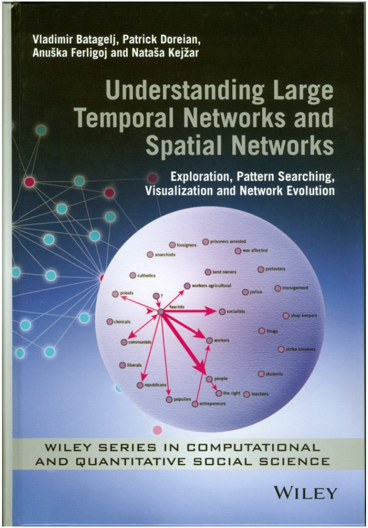

An introduction to social network analysis with `Pajek` is available in the book ESNA 3 (de Nooy, Mrvar, Batagelj, CUP 2005, 2011, 2018).

ESNA in Japanese was published by Tokyo Denki University Press in 2010; and in Chinese by Beijing World Publishing in November 2012.

`Pajek` – program for analysis and visualization of large networks is freely available, for noncommercial use, at its web site.

http://mrvar.fdv.uni-lj.si/pajek/

# Understanding large networks




This lecture is closely related to chapters 2 and 3 in the book:

Vladimir Batagelj, Patrick Doreian, Anuška Ferligoj and Nataša Kejžar: Understanding Large Temporal Networks and Spatial Networks: Exploration, Pattern Searching, Visualization and Network Evolution. Wiley Series in Computational and Quantitative Social Science. Wiley, October 2014.

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# References IV



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