



## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

# Analysis of bibliographic networks

Vladimir Batagelj

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# Outline

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

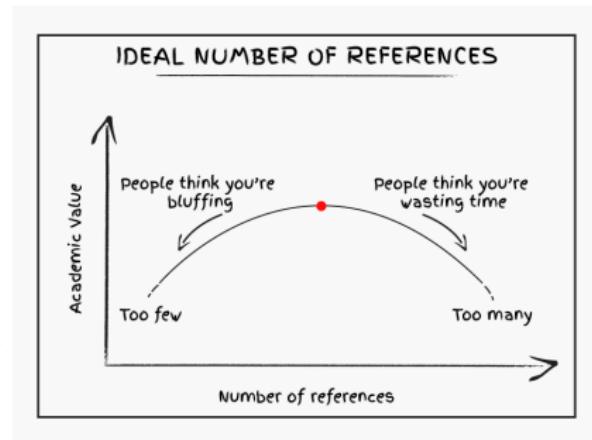
Normalizations

Groups

Bibliographic  
coupling  
and co-citation

Resources

- 1 Introduction
- 2 Statistics
- 3 Citation
- 4 Binary projections
- 5 Fractional approach
- 6 Normalizations
- 7 Groups
- 8 Bibliographic coupling and co-citation
- 9 Resources



J-Gate

**Vladimir Batagelj:** [vladimir.batagelj@fmf.uni-lj.si](mailto:vladimir.batagelj@fmf.uni-lj.si)

**Current version of slides (September 14, 2023 at 04:19):** [slides PDF](#)  
<https://github.com/bavla/biblio/>

## Networks

## Analysis of bibliographic networks

V. Batageli

## Introduction

Statistics

Citation

## Binary projections

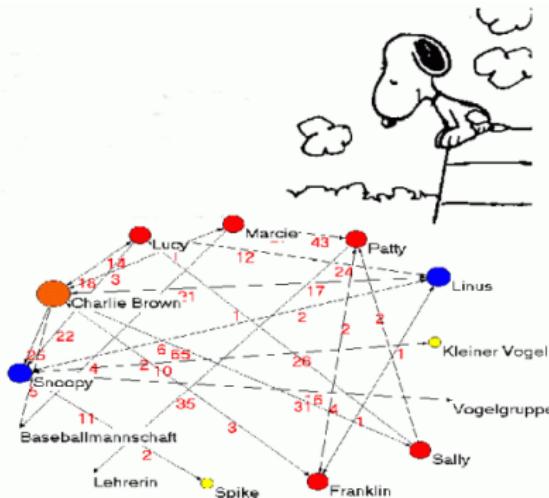
## Fractional approach

## Normalizations

## Groups

## Bibliographic coupling and co-citation

## Resources



## Alexandra Schuler/ Marion Laging-Glaser: Analyse von Snoopy Comics

A *network* is based on two sets – a set of *nodes* (vertices), that represent the selected *units*, and a set of *links* (lines), that represent *ties* between units. They determine a *graph*. A link can be *directed* – an *arc*, or *undirected* – an *edge*. Additional data about nodes or links may be known – their *properties* (attributes). For example: name/label, type, age, value, ...

## Network = Graph + Data



# Network

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

A **network**  $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{P}, \mathcal{W})$  consists of:

- a **graph**  $\mathcal{G} = (\mathcal{V}, \mathcal{L})$ , where  $\mathcal{V}$  is the set of nodes,  $\mathcal{A}$  is the set of arcs,  $\mathcal{E}$  is the set of edges, and  $\mathcal{L} = \mathcal{E} \cup \mathcal{A}$  is the set of links.  
 $n = |\mathcal{V}|$ ,  $m = |\mathcal{L}|$
- $\mathcal{P}$  **node value functions** / properties:  $p: \mathcal{V} \rightarrow A$
- $\mathcal{W}$  **link value functions** / weights:  $w: \mathcal{L} \rightarrow B$

# Graph

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

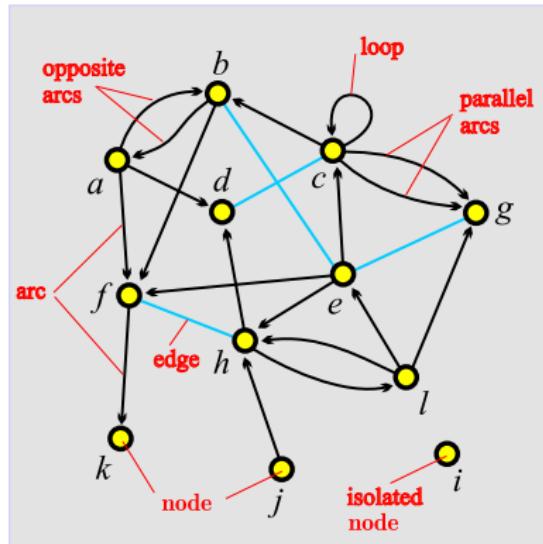
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



unit, actor – node, vertex  
tie, line – link, edge, arc

**arc** = directed link,  $(a, d)$   
a is the *initial* node,  
d is the *terminal* node.

**edge** = undirected link,  
 $(c: d)$   
c and d are *end* nodes.

# Types of networks

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

In a *two-mode* network  $\mathcal{N} = ((\mathcal{V}_1, \mathcal{V}_2), \mathcal{L}, \mathcal{P}, \mathcal{W})$  its set of nodes is split to two subsets. Each link has its end-nodes in both sets.

In a *multi-relational* network  $\mathcal{N} = (\mathcal{V}, (\mathcal{L}_i, i \in I), \mathcal{P}, \mathcal{W})$  the set of its links is partitioned into several mutually disjoint subsets – relations. (Subject Verb Object).

In a *temporal* network  $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{T}, \mathcal{P}, \mathcal{W})$  the time  $\mathcal{T}$  is added. To each node and to each link its *activity* set is assigned. Also properties and weights can change through time – temporal quantities.

In a *linked* or *multimodal* network  $\mathcal{N} = ((\mathcal{V}_1, \dots, \mathcal{V}_j), (\mathcal{L}_1, \dots, \mathcal{L}_k), \mathcal{P}, \mathcal{W})$  the set of nodes  $\mathcal{V}$  is partitioned into subsets (*modes*)  $\mathcal{V}_i$ ,  $\mathcal{L}_s \subseteq \mathcal{V}_p \times \mathcal{V}_q$ , and properties and weights are usually partial functions.

A *collection* of networks consists of some networks with common subsets of nodes.

Types of networks can be combined – for example: a temporal two-mode multi-relational network.



# Record from Web of Science

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

PT J  
AU Dipple, H  
Evans, B  
TI The Leicestershire Huntington's disease support group: a social network analysis  
SO HEALTH & SOCIAL CARE IN THE COMMUNITY  
LA English  
DT Article  
C1 Rehabil Serv, Troon Way Business Ctr, Leicester LE4 9HA, Leics, England.  
RP Dipple, H, Rehabil Serv, Troon Way Business Ctr, Sandringham Suite,Humberstone Lane, Leicester LE4 9HA, Leics, England.  
CR BORGATTI SP, 1992, UCINET 4 VERSION 1 0  
FOLSTEIN S, 1989, HUNTINGTONS DIS DISO  
SCOTT J, 1991, SOCIAL NETWORK ANAL  
NR 3  
TC 3  
PU BLACKWELL SCIENCE LTD  
PI OXFORD  
PA P O BOX 88, OSNEY MEAD, OXFORD OX2 ONE, OXON, ENGLAND  
SN 0966-0410  
J9 HEALTH SOC CARE COMMUNITY  
JI Health Soc. Care Community  
PD JUL  
PY 1998  
VL 6  
IS 4  
BP 286  
EP 289  
PG 4  
SC Public, Environmental & Occupational Health; Social Work  
GA 105UP  
UT ISI:000075092200008  
ER

WoS2Pajek





# Networks from bibliographies

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

For data from the **Web of Science** (Knowledge) we can obtain the corresponding networks using the program **WoS2Pajek**:

- citation network **Ci**: works × works;
- authorship network **WA**: works × authors, for works without complete description only the first author is known;
- keywords network **WK**: works × keywords, only for works with complete description;
- journals network **WJ**: works × journals;
- partition *year* of works by the publication year;
- partition *CD* of works – complete description (1) / ISI name only (0);

Similar programs exist also for other bibliographic sources/formats: Scopus, BibTeX, Zentralblatt Math, Google Scholar, DBLP, IMDB, etc.



# Statistics

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

Global/overall properties.

Extreme (minimal/maximal values) units.

Distributions of values.

*degree* of node  $v$ ,  $\deg(v)$  = number of links with  $v$  as an endnode;

*indegree* of node  $v$ ,  $\text{indeg}(v)$  = number of links with  $v$  as a terminal node (endnode is both initial and terminal);

*outdegree* of node  $v$ ,  $\text{outdeg}(v)$  = number of links with  $v$  as an initial node.

*weighted degree* of node  $v$ ,  $w\deg(v)$  = sum of the weights of links with  $v$  as an endnode;

*source* node  $v \Leftrightarrow \text{indeg}(v) = 0$

*sink* node  $v \Leftrightarrow \text{outdeg}(v) = 0$

## PEERE – most cited works: indegree in Ci

| n  | freq | first author    | title   |
|----|------|-----------------|---|
| 1  | 173  | Cohen, J        | Statistical Power Analysis for the Behavioral Sciences. Routledge, 1988                           |
| 2  | 164  | Peters, DP      | Peer-review practices of psychological journals - the fate of ... Behav Brain Sci, 1982           |
| 3  | 151  | Egger, M        | Bias in meta-analysis detected by a simple, graphical test. Brit Med J, 1997                      |
| 4  | 150  | Stroup, DF      | Meta-analysis of observational studies in epidemiology - A proposal for reporting. JAMA, 2000     |
| 5  | 135  | Dersimonian, R  | Metaanalysis in clinical-trials. Control Clin Trials, 1986  |
| 6  | 130  | Zuckerman, H    | Patterns of evaluation in science - institutionalisation, structure ... Minerva, 1971             |
| 7  | 130  | Higgins, JPT    | Cochrane Handbook for Systematic Reviews of Interventions. Cochrane, 2011                         |
| 8  | 126  | Moher, D        | Preferred Reporting Items for Systematic Reviews and Meta-Analyses. Plos Med, 2009                |
| 9  | 125  | Higgins, JPT    | Measuring inconsistency in meta-analyses. Brit Med J, 2003  |
| 10 | 121  | Cicchetti, DV   | The reliability of peer-review for manuscript and grant submissions - ... Behav Brain Sci, 1991   |
| 11 | 119  | Hirsch, JE      | An index to quantify an individual's scientific research output. P Natl Acad Sci Usa, 2005        |
| 12 | 114  | Mahoney, M      | Publication prejudices: An experimental study of confirmatory bias ... Cognitive T & R, 1977      |
| 13 | 114  | van Rooyen, S   | Effect of open peer review on quality of reviews and on reviewers' ... Brit Med J, 1999           |
| 14 | 114  | Easterbrook, PJ | Publication bias in clinical research. Lancet, 1991   |
| 15 | 110  | Landis, JR      | Measurement Of Observer Agreement For Categorical Data. Biometrics, 1977                          |
| 16 | 109  | Godlee, F       | Effect on the quality of peer review of blinding reviewers and asking them to sign ... JAMA, 1998 |
| 17 | 108  | Horrobin, DF    | The philosophical basis of peer-review and the suppression of innovation. JAMA, 1990              |
| 18 | 107  | Moher, D        | Preferred Reporting Items for Systematic Reviews and Meta-Analyses. Ann Intern Med, 2009          |
| 19 | 107  | Jadad, AR       | Assessing the quality of reports of randomized clinical trials. Control Clin Trials, 1996         |
| 20 | 105  | Mcnutt, RA      | The effects of blinding on the quality of peer-review - a randomized trial. JAMA, 1990            |
| 21 | 104  | Cole, S         | Chance and consensus in peer-review. Science, 1981  |
| 22 | 103  | Moher, D        | Improving the quality of reports of meta-analyses of randomised controlled trials. Lancet, 1999   |
| 23 | 98   | Justice, AC     | Does masking author identity improve peer review quality? JAMA, 1998                              |
| 24 | 97   | Lock, S         | A Difficult Balance: Editorial Peer Review in Medicine. Nuffield Trust, 1985                      |
| 25 | 95   | van Rooyen, S   | Effect of blinding and unmasking on the quality of peer review - A randomized trial. JAMA, 1998   |
| 26 | 92   | Black, N        | What makes a good reviewer and a good review for a general medical journal? JAMA, 1998            |
| 27 | 91   | Scherer, RW     | Full publication of results initially presented in abstracts - a metaanalysis. JAMA, 1994         |
| 28 | 90   | Higgins, JPT    | Quantifying heterogeneity in a meta-analysis. Stat Med, 2002                                      |
| 29 | 90   | Smith, R        | Peer review: a flawed process at the heart of science and journals. J Roy Soc Med, 2006           |
| 30 | 87   | Goodman, SN     | Manuscript quality before and after peer-review and editing at .... Ann Intern Med, 1994          |
| 31 | 87   | Chubin, D       | Peerless Science: Peer Review and U.S. Science Policy. SUNY Press, 1990                           |

# Distributions

indegree and outdegree in citation network

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

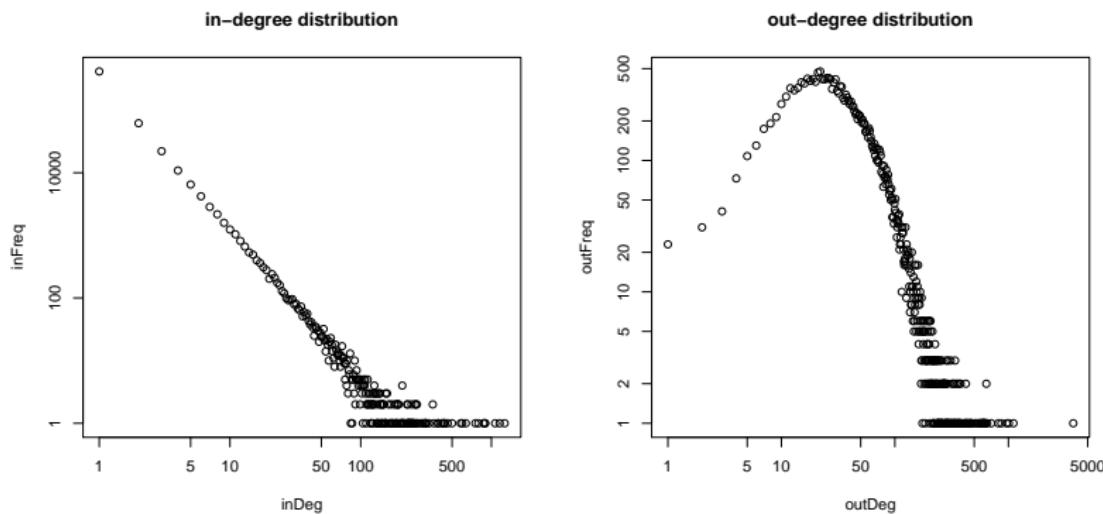
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



The indegree distribution is "scale-free"-like. The parameters can be determined using the package of [Clauset, Shalizi and Newman](#). See also [Stumpf, et al.: Critical Truths About Power Laws](#).

# Distributions

## SN5 citation network input degrees – scale-free fit

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

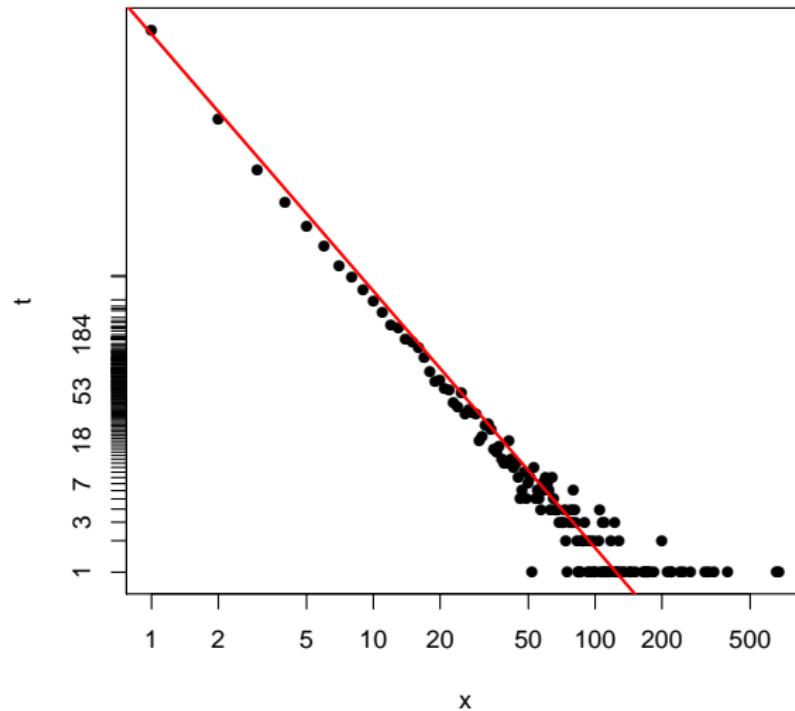
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



# Distributions

year partition lognormal distribution fit

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

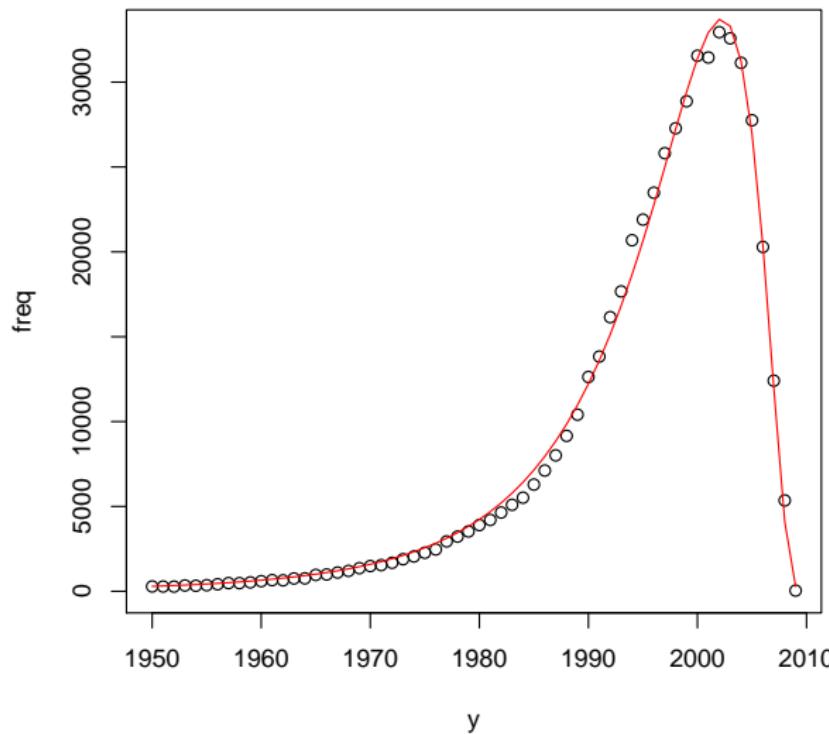
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



# Distributions

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

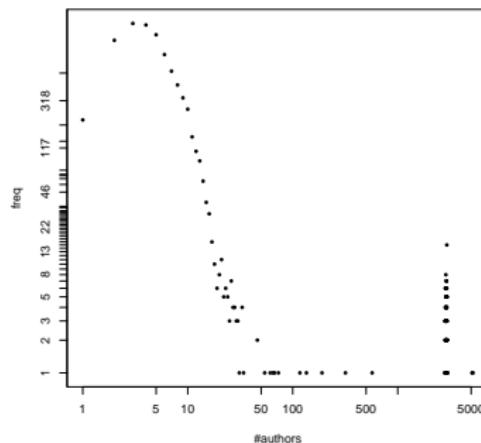
Normalizations

Groups

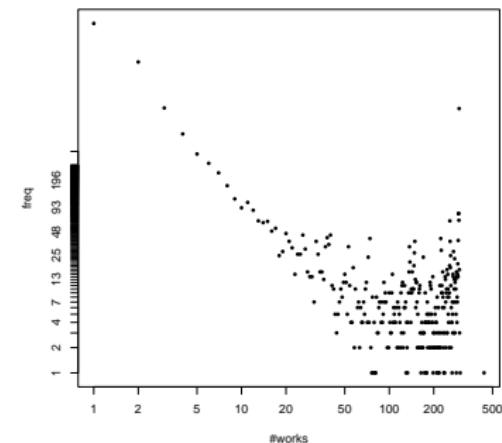
Bibliographic  
coupling and  
co-citation

Resources

HKUST1 – #authors per paper distribution



HKUST1 – #works per author distribution



The Hong Kong University of Science and Technology (HKUST) 2017–2019

5215 co-authors: The ATLAS collaboration, & The CMS collaboration (2019). Combinations of single-top-quark production cross-section measurements and  $|f_{LV} V_{tb}|$  determinations at  $\sqrt{s} = 7$  and 8 TeV with the ATLAS and CMS experiments. Journal of High Energy Physics, 2019(5), Article 88.  
[https://doi.org/10.1007/JHEP05\(2019\)088](https://doi.org/10.1007/JHEP05(2019)088) / Springer

# Distributions

number of keywords in **ZBMath**

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

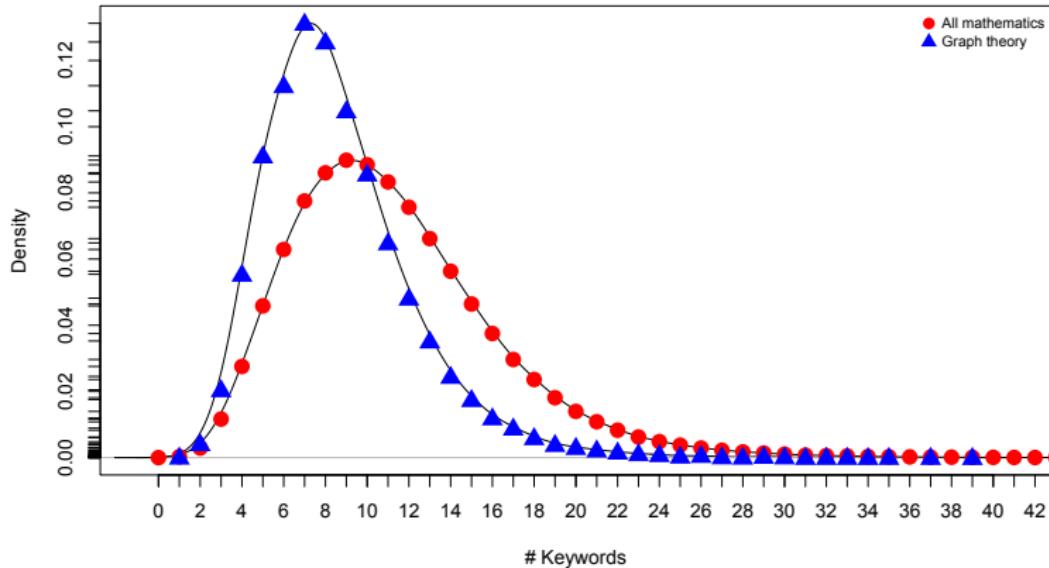
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



# Distributions

keywords by the number of works using a keyword in their description

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

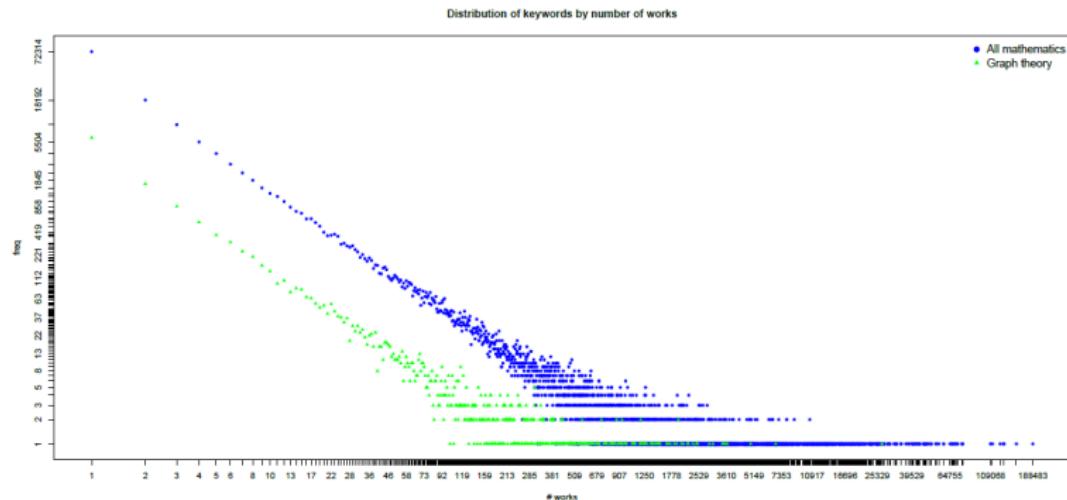
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



# Temporal distribution

Number of authors SNA 2018

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

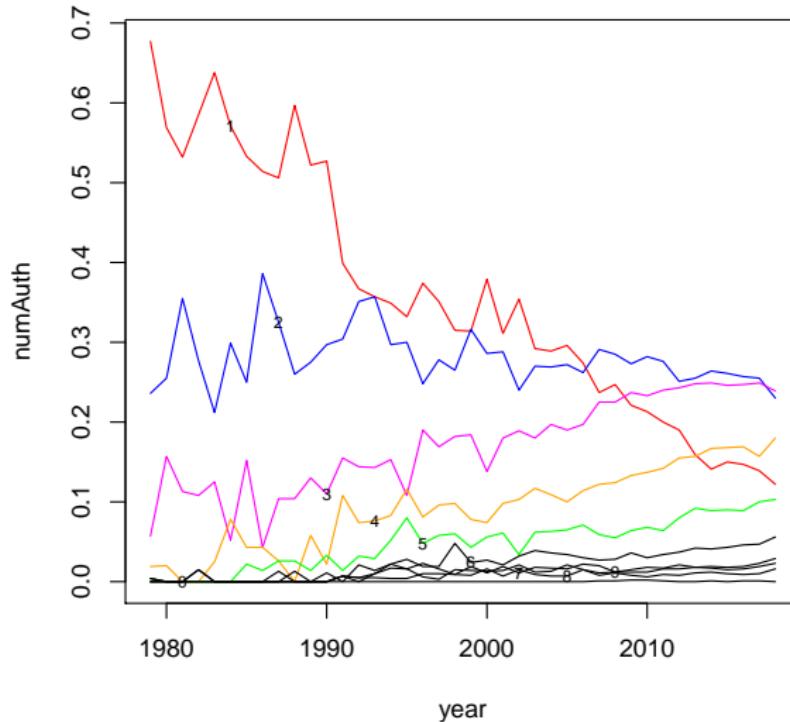
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



SocNet

# Citations between years

SNA 2018

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

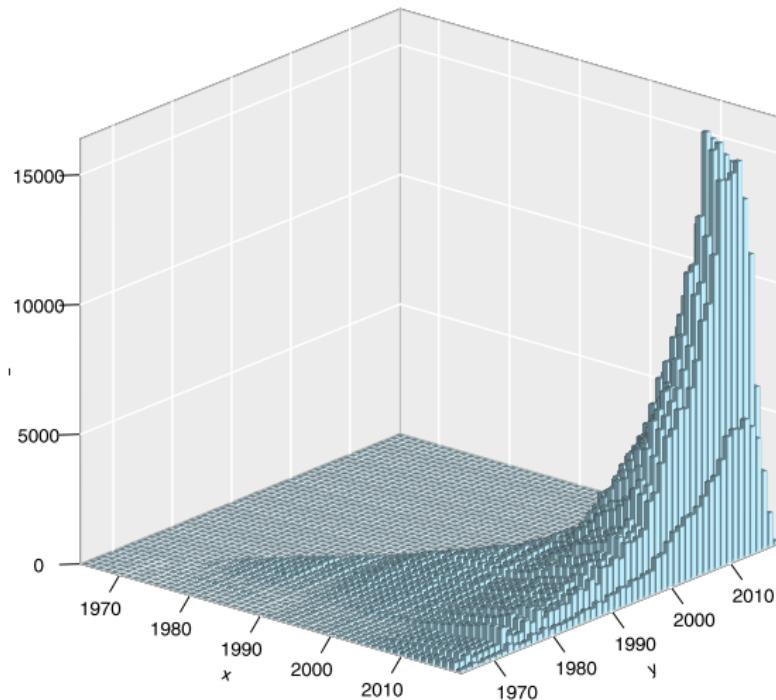
Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

SocNet



# Citations per year

SNA 2018; normalized curves

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

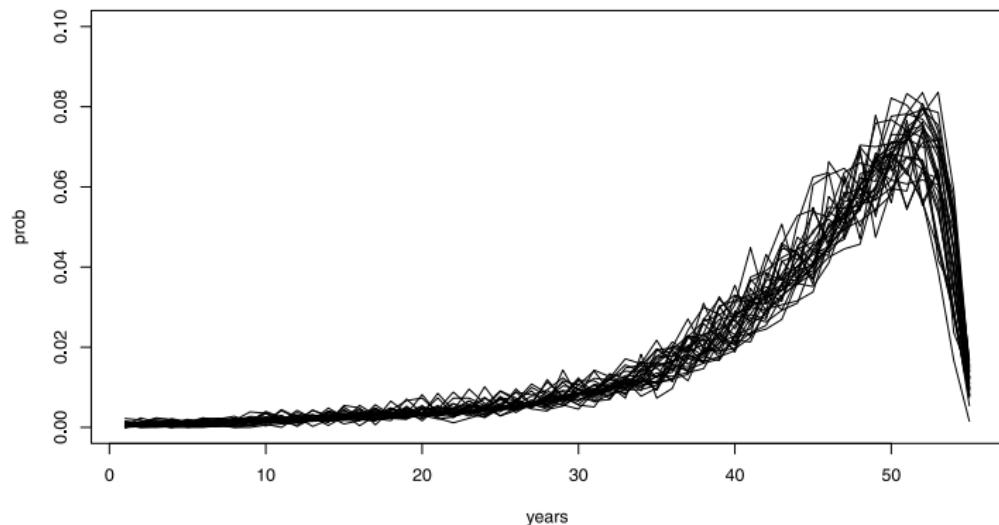
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



SocNet



# Citation networks

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

In a given set of works/nodes  $W$  (articles, books, reports, etc.) we introduce a *citing relation*  $\mathbf{Ci} \subseteq W \times W$

$$u \mathbf{Ci} v \equiv u \text{ cites } v$$

which determines a *citation network*  $\mathcal{N} = (W, \mathbf{Ci})$ .

A citing relation is usually *irreflexive* (no loops) and (almost) *acyclic*. We shall assume that it has these two properties.

Since in real-life citation networks, the strong components are small (usually 2 or 3 nodes) we can transform such a network into an acyclic network by shrinking strong components and deleting loops.

A better way is the preprint transformation.

# Citation network

## Preprint transformation

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

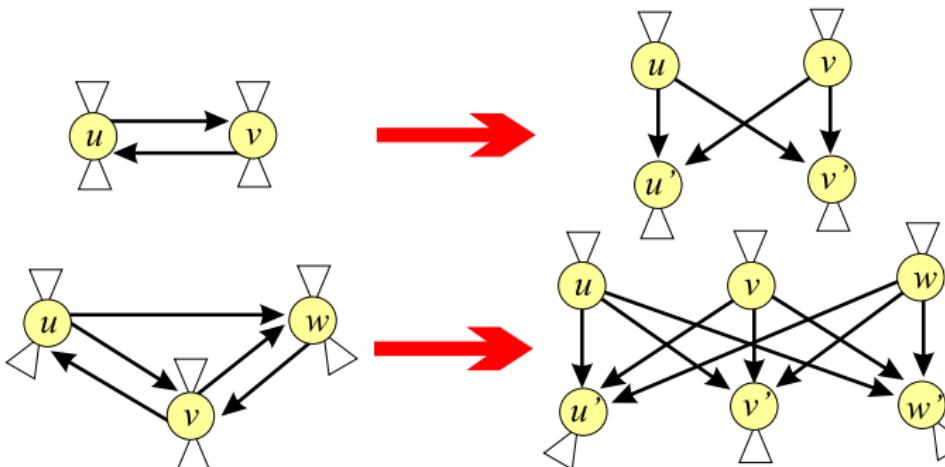
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



Transforming a citation network into an acyclic network using the preprint transformation.

# Standardized citation network

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

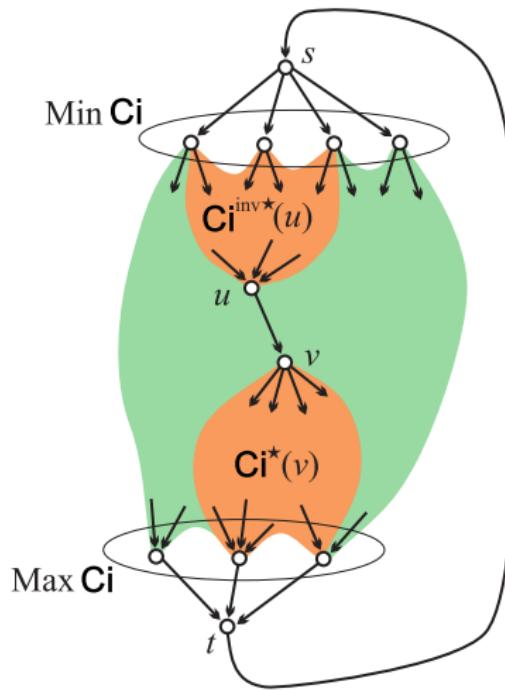
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



We assume that the citation relation  $\mathbf{Ci}$  is acyclic. It is useful to transform a citation network to its *standardized* form by adding a common *source* node  $s \notin W$  and a common *sink* node  $t \notin W$ . The source  $s$  is linked by an arc to all minimal elements of  $\mathbf{Ci}$ ; and all maximal elements of  $\mathbf{Ci}$  are linked to the sink  $t$ . We add also the ‘feedback’ arc  $(t, s)$ .

# Search path count method

Hummon and Doreian

The *search path count* (SPC) method is based on counters  $n(u, v)$  that count the number of different paths from  $s$  to  $t$  through the arc  $(u, v)$ .

The values of counters  $n(u, v)$  form a flow in the citation network – the *Kirchoff's vertex law* holds: For every node  $u$  in a standardized citation network *incoming flow = outgoing flow*:

$$\sum_{v: v \text{ Ci } u} n(v, u) = \sum_{v: u \text{ Ci } v} n(u, v)$$

The weight  $n(t, s)$  equals to the total flow through network and provides a natural normalization of weights

$$w(u, v) = \frac{n(u, v)}{n(t, s)} \Rightarrow 0 \leq w(u, v) \leq 1$$

and if  $C$  is a minimal arc-cut-set  $\sum_{(u,v) \in C} w(u, v) = 1$ .

The value  $w(u, v)$  is equal to the probability that a random  $s-t$  path passes through the arc  $(u, v)$ .



# Cuts

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

The standard approach to find interesting groups inside a network is based on properties/weights – they can be *measured* or *computed* from network structure (for example Kleinberg's **hubs and authorities**).

The *node cut* of a network  $\mathcal{N} = (\mathcal{V}, \mathcal{L}, p)$ ,  $p : \mathcal{V} \rightarrow \mathbb{R}$ , at selected level  $t$  is a subnetwork  $\mathcal{N}(t) = (\mathcal{V}', \mathcal{L}(\mathcal{V}'), p)$ , determined by the set

$$\mathcal{V}' = \{v \in \mathcal{V} : p(v) \geq t\}$$

and  $\mathcal{L}(\mathcal{V}')$  is the set of links from  $\mathcal{L}$  that have both endnodes in  $\mathcal{V}'$ .

The *link cut* of a network  $\mathcal{N} = (\mathcal{V}, \mathcal{L}, w)$ ,  $w : \mathcal{L} \rightarrow \mathbb{R}$ , at selected level  $t$  is a subnetwork  $\mathcal{N}(t) = (\mathcal{V}(\mathcal{L}'), \mathcal{L}', w)$ , determined by the set

$$\mathcal{L}' = \{e \in \mathcal{L} : w(e) \geq t\}$$

and  $\mathcal{V}(\mathcal{L}')$  is the set of all endnodes of the links from  $\mathcal{L}'$ .

## Citation weights

## Analysis of bibliographic networks

V. Batageli

Introduction

Statistics

## Citation

## Binary projections

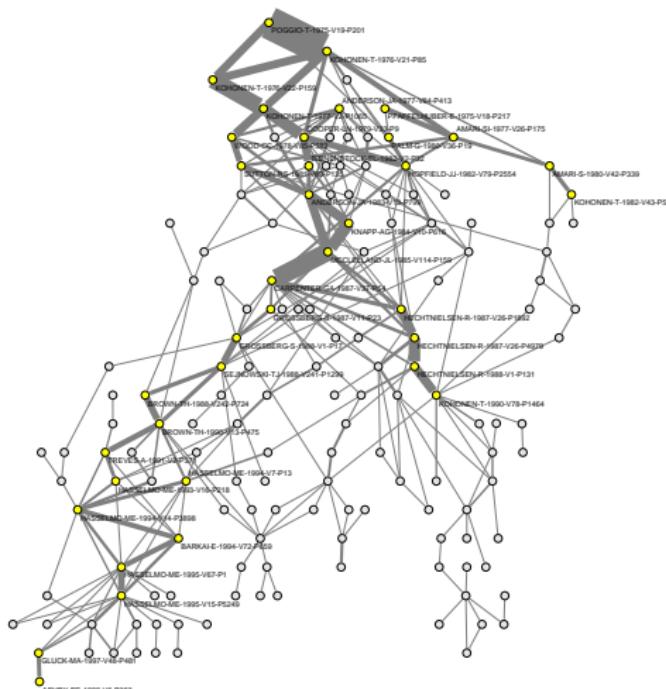
Fractional  
approach

## Normalizations

## Groups

## Bibliographic coupling and co-citation

## Resources



Main subnetwork (arc cut at level 0.007) of the SOM (selforganizing maps) citation network (4470 nodes, 12731 arcs).

For visualization of acyclic networks in Pajek use the macro layers.

See paper.

# Islands

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

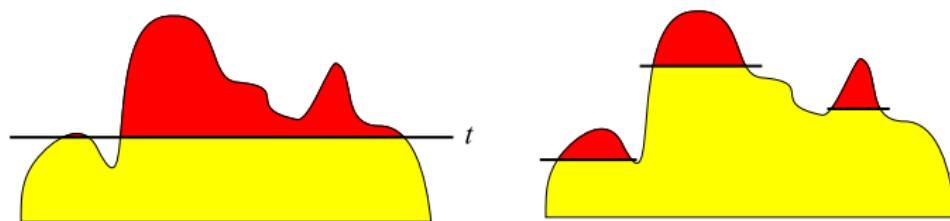
Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

If we represent a given or computed value of nodes / links as a height of nodes / links and we immerse the network into a water up to selected level we get *islands*. Varying the level we get different islands.



We developed very efficient algorithms to determine the islands hierarchy and to list all the islands of selected sizes.  
See [details](#).



# ... Islands

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

Islands are very general and efficient approach to determine the 'important' subnetworks in a given network.

We have to express the goals of our analysis with a related property of the nodes or weight of the links. Using this property we determine the islands of an appropriate size (in the interval  $k$  to  $K$ ).

In large networks we can get many islands which we have to inspect individually and interpret their content.

An important property of the islands is that they identify locally important subnetworks on different levels. Therefore they detect also emerging groups.

The set of nodes  $\mathcal{C} \subseteq \mathcal{V}$  is a *local node peak*, if it is a regular node island and all of its nodes have the same value. Node island with a single local node peak is called a *simple node island*. In similar way we define simple link island.



# ... Islands

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

A set of nodes  $C \subseteq \mathcal{V}$  is a *regular node island* in a network

$\mathcal{N} = (\mathcal{V}, \mathcal{L}, p)$ ,  $p : \mathcal{V} \rightarrow \mathbb{R}$  iff it induces a connected subgraph and the nodes from the island are 'higher' than the neighboring nodes

$$\max_{u \in N(C)} p(u) < \min_{v \in C} p(v)$$

A set of nodes  $C \subseteq \mathcal{V}$  is a *regular link island* in a network

$\mathcal{N} = (\mathcal{V}, \mathcal{L}, w)$ ,  $w : \mathcal{L} \rightarrow \mathbb{R}$  iff it induces a connected subgraph and the links inside the island are 'stronger related' among them than with the neighboring nodes – in  $\mathcal{N}$  there exists a spanning tree  $\mathcal{T}$  over  $C$  such that

$$\max_{(u,v) \in \mathcal{L}, u \notin C, v \in C} w(u,v) < \min_{(u,v) \in \mathcal{T}} w(u,v)$$

A *simple island* is an island with only one peak.

# US patents

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

US patents network ([Nber, US Patents](#)) has 3774768 nodes and 16522438 arcs (1 loop). Without the loop it is acyclic. The weight of an arc is the proportion of paths through the arc from some initial node to some terminal node. We determined all (2,90)-link islands. The corresponding subnetwork has 470137 nodes, 307472 arcs and different  $k$ :  $C_2 = 187610$ ,  $C_5 = 8859$ ,  $C_{30} = 101$ ,  $C_{50} = 30$ , ... islands. [Rolex](#)

|      |     |        |       |      |      |      |     |     |     |     |
|------|-----|--------|-------|------|------|------|-----|-----|-----|-----|
| [1]  | 0   | 139793 | 29670 | 9288 | 3966 | 1827 | 997 | 578 | 362 | 250 |
| [11] | 190 | 125    | 104   | 71   | 47   | 37   | 36  | 33  | 21  | 23  |
| [21] | 17  | 16     | 8     | 7    | 13   | 10   | 10  | 5   | 5   | 5   |
| [31] | 12  | 3      | 7     | 3    | 3    | 3    | 2   | 6   | 6   | 2   |
| [41] | 1   | 3      | 4     | 1    | 5    | 2    | 1   | 1   | 1   | 1   |
| [51] | 2   | 3      | 3     | 2    | 0    | 0    | 0   | 0   | 0   | 1   |
| [61] | 0   | 0      | 0     | 0    | 1    | 0    | 0   | 2   | 0   | 0   |
| [71] | 0   | 0      | 1     | 1    | 0    | 0    | 0   | 1   | 0   | 0   |
| [81] | 2   | 0      | 0     | 0    | 0    | 1    | 2   | 0   | 0   | 7   |

The **Main path** starts in a link with the largest SPC weight and expands in both directions following the adjacent link with the largest SPC weight.

The **CPM path** is determined using the Critical Path Method from Operations Research (the sum of SPC weights along a path is maximal).

# Distribution of island size

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

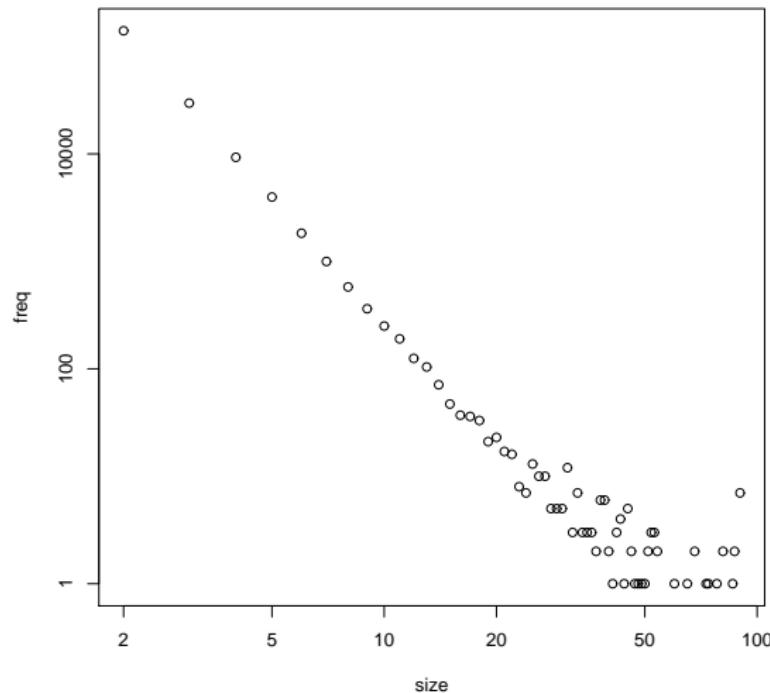
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



Main path and main island in US Patents  
Nber, US Patents;  $n = 3774768$ ,  $m = 16522438$

## Analysis of bibliographic networks

V. Batagelj

## Introduction

Statistics

## Citation

## Binary projections

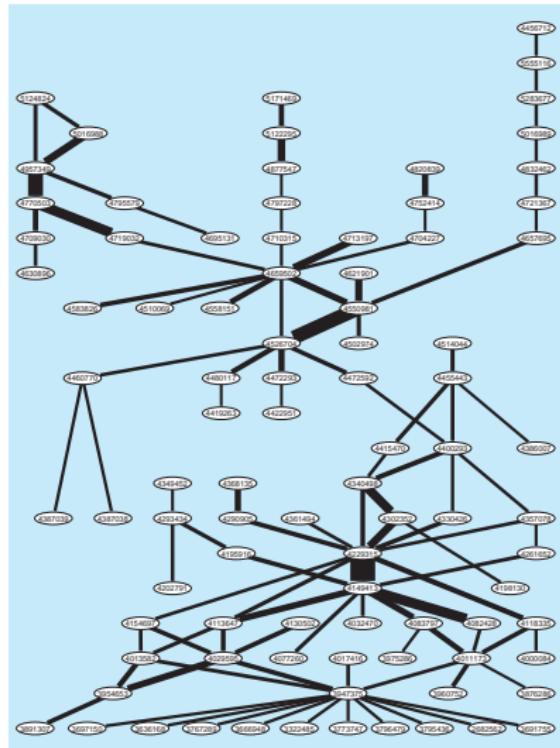
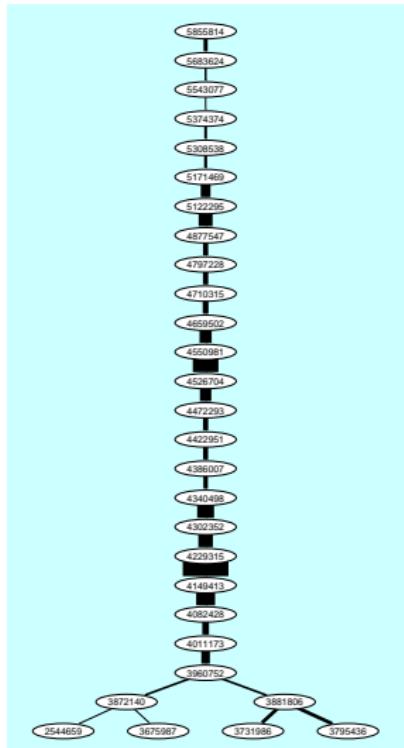
## Fractional approach

## Normalizations

## Groups

## Bibliographic coupling and co-citation

## Resources



# Main island – Liquid crystal display

## Analysis of bibliographic networks

V. Batagelj

## Introduction

## Statistics

## Citation

## Binary projections

## Fractional approach

## Normalizations

## Groups

## Bibliographic coupling and co-citation

## Resources

Table 1: Patents on the liquid-crystal display

| patent  | date         | author(s) and title  |
|---------|--------------|--|
| 2049629 | Mar 13, 1951 | Arivazhagan, et al. Electro-optical device containing sheet and the like and the formation and use thereof   |
| 2082562 | Jul 29, 1954 | Wender, et al. Reduction of aromatic carbonyl compounds having heteroatoms or other elements utilizing an organic nematic compound                         |
| 3322485 | May 30, 1967 | Josephson. Preparation of polyacrylic aromatic compounds   |
| 3636168 | Jun 18, 1972 | Goto, et al. Liquid crystal compositions and devices   |
| 3691755 | Sep 19, 1972 | Tokutomi, et al. Novel liquid crystal compositions   |
| 3697150 | Oct 19, 1972 | Wysoki. Electro-optic systems in which an electrophoretic particle having an undisturbed image on a disturbed background                                   |
| 3697397 | Jul 11, 1973 | Goto, et al. Liquid crystal compositions and devices   |
| 3705426 | Aug 1, 1973  | Strelzow. Substituted nancy benzene which exhibits the Kerr effect at isotropic temperatures   |
| 3731966 | May 8, 1973  | Fengus. Display devices utilizing liquid crystal light   |
| 3767289 | Oct 23, 1973 | Arivazhagan, et al. Class of stearo-ethoxy compounds, some displaying nematic, metallocatic at or near room temperature and others, a range of 0°C to 60°C |
| 3773477 | Nov 20, 1973 | Steinmesser. Substituted nancy benzene which exhibits the Kerr effect at isotropic temperatures  |
| 3795436 | Mar 3, 1974  | Hofrichter, et al. Electro-optical liquid-crystal materials which exhibit the Kerr effect at isotropic temperatures  |
| 3797479 | Mar 12, 1974 | Hofrichter, et al. Electro-optical liquid-crystal materials which exhibit the Kerr effect at isotropic temperatures  |
| 3872140 | Mar 18, 1975 | Klaesnerman, et al. Liquid crystalline compositions and methods of preparing same  |
| 3876286 | Aug 8, 1975  | Dentzschek, et al. Use of nematic liquid crystalline substances in electro-optical devices   |
| 3880337 | Mar 28, 1976 | Yamada, et al. Liquid crystal materials and devices  |
| 3891307 | Jun 24, 1976 | Tanashima, et al. Phase control of the voltages applied to opposite electrodes for a cholesteric to nematic phase transition                               |
| 3947375 | Mar 30, 1976 | Goto, et al. Liquid crystal materials and devices  |
| 3959633 | May 4, 1976  | Yamada, et al. Liquid crystal materials and devices having high dielectric constants   |
| 3960752 | Jun 1, 1976  | Klaesnerman, et al. Liquid crystal compositions  |
| 3972326 | Aug 17, 1976 | Goto, et al. Liquid crystal compositions and method of synthesis   |
| 4000894 | Dec 28, 1976 | Held, et al. Liquid crystal mixtures for electro-optical devices   |
| 4011173 | Mar 8, 1977  | Steinmesser. Modified nematic mixtures with increased viscosity  |
| 4013582 | Mar 22, 1977 | Gavrilev. Liquid crystal compounds and electro-optic devices incorporating them  |
| 4017416 | Apr 12, 1977 | Method for preparing some liquid crystal compositions using same   |
| 4029095 | Jun 14, 1977 | Watanabe, et al. Novel liquid crystal compounds and electro-optic devices incorporating them   |
| 4032470 | Jun 28, 1977 | Goto, et al. Liquid crystal compositions   |
| 4077260 | Mar 7, 1978  | Goto, et al. Optically active cyano-biphenyl compounds and liquid crystal materials containing them  |
| 4082428 | Aug 4, 1978  | Goto, et al. Liquid crystal composition and method   |

Table 2: Patents on the liquid-crystal display

| patent  | date         | author(s) and title   |
|---------|--------------|---|
| 4113647 | Sep 12, 1979 | Custodis, et al. Liquid crystalline materials   |
| 4118320 | Oct 3, 1979  | Krause, et al. Liquid crystalline materials of reduced viscosity  |
| 4120200 | Oct 17, 1979 | Goto, et al. Optically active liquid crystal mixture and liquid crystal devices containing them   |
| 4149412 | May 15, 1979 | Goto, et al. Liquid crystal derivative having hydroxylphenyl derivatives  |
| 4159012 | Aug 1, 1979  | Custodis, et al. Liquid crystalline materials   |
| 4189130 | Aug 15, 1980 | Goto, et al. Liquid crystal mixtures  |
| 4202794 | May 13, 1980 | Osawa, et al. Nematic liquid crystal materials  |
| 4211262 | Jun 10, 1980 | Goto, et al. Liquid crystal compositions and methods of preparation   |
| 4213632 | Aug 4, 1981  | Goto, et al. Liquid crystal compounds and materials and devices containing them   |
| 4269005 | Sep 22, 1981 | Goto, et al. Liquid crystal compositions and materials and devices containing them  |
| 4293434 | Oct 6, 1981  | Dentzschek, et al. Liquid crystal compounds   |
| 4302325 | Nov 24, 1981 | Eldenbach, et al. Fluorophenylketone compounds, the preparation of their esters and esterates of liquid crystal dioxetanes  |
| 4304042 | Jan 11, 1982 | Eldenbach, et al. Cyclohexylbiphenyl, their preparation and use in electro-optical devices  |
| 4304049 | Jul 20, 1982 | Sugimoto, et al. Halogenated ether derivatives  |
| 4349452 | Sep 14, 1982 | Osumai, et al. Cyclohexylbicyclohexanes   |
| 4352075 | Nov 2, 1982  | Goto, et al. Liquid crystal compositions containing an alicyclic ring and exhibiting a low dielectric anisotropy and liquid crystal materials and devices incorporating such compositions |
| 4361494 | Nov 20, 1982 | Orman, et al. Anisotropic cyclohexylbiphenyl ethers   |
| 4363815 | Jan 11, 1983 | Osawa. Anisotropic compounds with negative or positive dielectric anisotropy  |
| 4366007 | May 31, 1983 | Krause, et al. Liquid crystalline naphthalene derivatives   |
| 4367028 | Jun 7, 1983  | Li, et al. 4-(Trans-4'-alkylcyclohexyl)-benzoic acid  |
| 4367029 | Jun 7, 1983  | Sugimoto, et al. Trans-4-(trans-4'-alkylcyclohexyl)-cyclohexane carboxylic acid and -esters   |
| 4367030 | Jun 7, 1983  | Eldenbach, et al. Liquid crystal fluorine-containing carboxylic acids and esters  |
| 4400253 | Aug 23, 1983 | Goto, et al. Liquid crystal mixtures and electro-optical display elements based thereon   |
| 4413470 | Nov 15, 1983 | Prufrock, et al. Liquid crystalline cyclohexylcarboxylic acid   |
| 4413925 | Dec 6, 1983  | Held, et al. Liquid crystal mixtures for electro-optical devices  |
| 4422953 | Dec 27, 1983 | Sugimoto, et al. Liquid crystal benzene derivatives   |
| 4435441 | Jan 19, 1984 | Takatori, et al. Nematic liquid crystal composition   |
| 4435442 | Jan 19, 1984 | Takatori, et al. Nematic liquid crystal composition   |
| 4460770 | Jul 17, 1984 | Petrzilka, et al. Liquid crystal mixture  |
| 4472250 | Sep 18, 1984 | Goto, et al. Liquid crystal compositions containing the same  |
| 4472302 | Sep 18, 1984 | Takatori, et al. Nematic liquid crystal compositions  |
| 4481370 | Oct 2, 1984  | Sugimoto, et al. High temperature liquid-crystaline ester compounds   |
| 4502974 | Mar 5, 1985  | Goto, et al. Liquid crystal compositions  |
| 4530069 | Aug 9, 1985  | Goto, et al. Cyclohexane derivatives  |

Table 3: Patents on the liquid-crystal display

| patent  | date         | author(s) and title  |
|---------|--------------|--|
| 6114944 | Apr 30, 1987 | Jul 4, 1987. 4-(trans-4'-alkylcyclohexyl)-2-hydroxy-4-(p-substituted phenyl)cyclohexylcarboxylic acid and liquid crystal mixtures                              |
| 6120794 | Jul 2, 1989  | Petrzilka, et al. Melting liquid crystal esters  |
| 6121030 | Oct 10, 1989 | Li, et al. Liquid crystal compositions and mixtures  |
| 6155131 | Dec 10, 1989 | Takatori, et al. Novel liquid crystalline compounds  |
| 6158326 | Dec 22, 1989 | Petrzilka, et al. Phenylhexanes  |
| 6160306 | Jan 10, 1990 | Goto, et al. Liquid crystal compositions and mixtures  |
| 6169096 | Jan 23, 1990 | Petrzilka, et al. Benzene derivatives  |
| 6170095 | Jan 14, 1990 | Saito, et al. Substituted pyridines  |
| 6170100 | Jan 14, 1990 | Li, et al. Electro-optical pyridines   |
| 6095131 | Sep 22, 1990 | Balwali, et al. Dibenzotetralin ethanes and their use in liquid crystal compositions   |
| 6170427 | Nov 3, 1990  | Krasen, et al. Liquid crystal compounds  |
| 6170930 | Nov 24, 1990 | Petrzilka, et al. Novel liquid crystal mixture   |
| 6171030 | Dec 4, 1990  | Goto, et al. Liquid crystal compositions containing liquid crystal mixtures themselves   |
| 6213137 | May 15, 1991 | Eldenbach, et al. Cyclohexane derivatives  |
| 7199362 | Dec 18, 1992 | Yoshimura, et al. Liquid crystal device  |
| 7213367 | Feb 17, 1993 | Yoshimura, et al. Fluorophenylketone heterocyclic compounds  |
| 7213414 | Feb 17, 1993 | Bachelder, et al. Liquid crystalline compounds   |
| 7270000 | Sep 15, 1993 | Bachelder, et al. Liquid crystalline compounds   |
| 7305379 | Jan 3, 1993  | Yoshimura, et al. 2,2-dihydro-4-alkoxy-4-hydroxypheophenyl and 2,2-dihydro-4-alkoxy-4-hydroxylphenyl compounds and their use in liquid crystal display devices |
| 7377228 | Jan 10, 1993 | Goto, et al. Cyclohexane derivative and liquid crystal   |
| 8028039 | Aug 11, 1993 | Krasen, et al. Nitro-phenyl substituted heterocyclic ethers  |
| 8028040 | Aug 11, 1993 | Li, et al. Liquid crystal compositions   |
| 8077547 | Oct 21, 1993 | Weber, et al. Liquid crystal display element   |
| 9357349 | Sep 18, 1993 | Chen, et al. Active matrix screen for the color display of liquid crystal displays, control system and process for producing said screen                       |
| 5010098 | May 21, 1991 | Imamura, et al. Liquid crystal display device with a liquid crystal layer  |
| 5010099 | May 21, 1991 | Ohsaki, et al. Liquid crystal elements with improved contrast and liquid crystal display devices   |
| 5122295 | Jan 16, 1992 | Weber, et al. Matrix liquid crystal display  |
| 5124924 | Jan 23, 1992 | Kozaki, et al. Liquid crystal display device comprising a polymer film having a different refractive index in the thickness direction                          |
| 5128387 | Feb 1, 1994  | Takatori, et al. Liquid crystal materials  |
| 5130168 | Feb 1, 1994  | Takatori, et al. Liquid crystal compositions   |
| 5171409 | Dec 15, 1992 | Yoshimura, et al. Liquid crystal compositions  |
| 5283877 | Feb 2, 1994  | Yoshimura, et al. Liquid crystal compositions  |
| 5300539 | May 3, 1994  | Weber, et al. Separated liquid-crystal display   |
| 5374374 | Dec 20, 1994 | Weber, et al. Separated liquid-crystal display   |
| 5434977 | Aug 6, 1996  | Hoyer, et al. Nematic liquid-crystal composition   |
| 5501168 | Oct 13, 1998 | Yoshimura, et al. Liquid crystal compositions  |
| 5503000 | Jan 5, 1999  | Yoshimura, et al. Liquid crystal compositions  |
| 5523494 | Jan 5, 1999  | Mitsui, et al. Liquid crystal compositions and liquid crystal display elements   |
| 5525414 | Jan 5, 1999  | Mitsui, et al. Liquid crystal compositions and liquid crystal display elements   |

V. Batagelj

Analysis of bibliographic networks

## Word clouds for LCD island and foam island

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

## Citation

## Binary projections

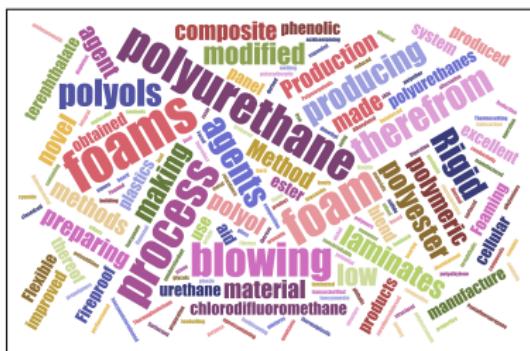
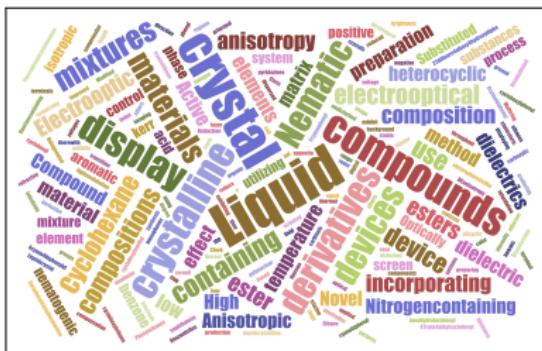
## Fractional approach

### Normalizations

Groups

## Bibliographic coupling and co-citation

## Resources



# Main SPC island in SN5

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

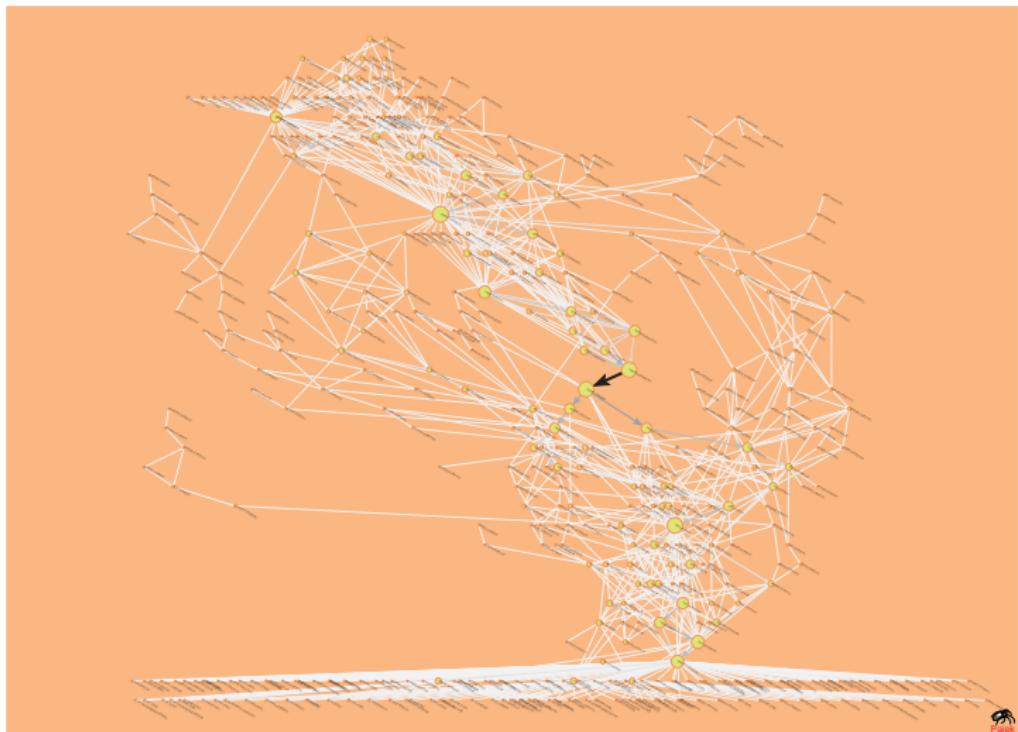
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



# PEERE – Main path

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

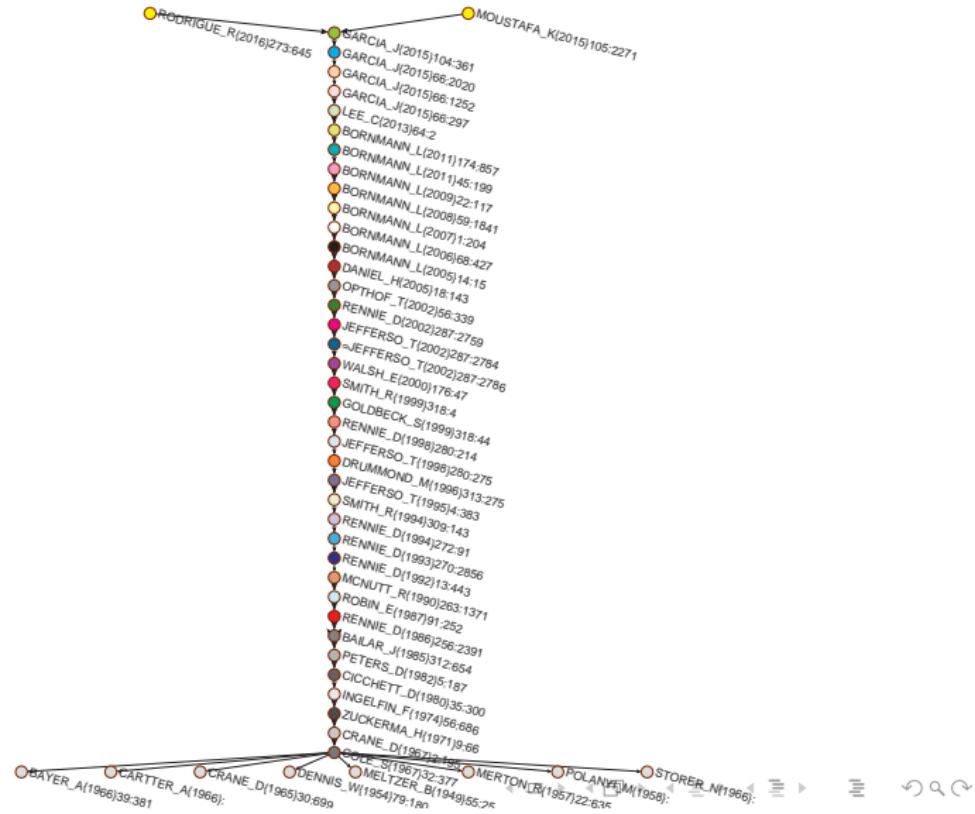
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



V. Batagelj

Analysis of bibliographic networks



# PEERE – List of publications on main path

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projectionsFractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

|  | year | first author   | title   | journal           |
|--|------|----------------|---|-------------------|
|  | 1949 | Meltzer BN     | The productivity of social scientists                                     | AmJSociol         |
|  | 1954 | Dennis W       | Bibliographies of eminent scientists                                      | ScientificM       |
|  | 1957 | Merton RK      | Priorities in scientific discovery - a chapter in the sociology of sci... | AmSociolRev       |
| V. Batagelj                            | 1958 | Polanyi M      | Personal Knowledge: Towards a Post-Critical Philosophy                    | UPChicago         |
| Introduction                           | 1965 | Crane D        | Scientists at major and minor universities                                | AmSociolRev       |
| Statistics                             | 1966 | Bayer AE       | Some correlates of citation measure of productivity in science            | SociolEduc        |
| Citation                               | 1966 | Storer NW      | The Social System of Science  | HRW               |
| Binary projections                     | 1966 | Carter A       | An Assessment of Quality in Graduate Education                            | ACE               |
| Fractional approach                    | 1967 | Crane D        | Gatekeepers of science - some factors affecting selection...              | AmSociol          |
| Normalizations                         | 1967 | Cole S         | Scientific output and recognition - study in operation of reward...       | AmSociolRev       |
| Groups                                 | 1971 | Zuckerman H    | Patterns of evaluation in science - institutionalisation, struct...       | Minerva           |
| Bibliographic coupling and co-citation | 1974 | Ingelfinger FJ | Peer review in biomedical publication                                     | AmJMed            |
| Resources                              | 1980 | Cicchetti DV   | Reliability of reviews for the american-psychologist                      | AmPsychol         |
|  | 1982 | Peters DP      | Peer-review practices of psychological journals - the fate...             | BehavBrainSci     |
|  | 1985 | Bailar JC      | Journal peer-review - the need for a research agenda                      | NewEnglJMed       |
|  | 1986 | Rennie D       | Guarding the guardians - a conference on editorial peer-review            | Jama              |
|  | 1987 | Robin ED       | Peer-review in medical journals   | Chest             |
|  | 1990 | Mcnutt RA      | The effects of blinding on the quality of peer-review                     | Jama              |
|  | 1992 | Rennie D       | Suspended judgment - editorial peer-review - let us put it on trial       | ControlClinTrials |
|  | 1993 | Rennie D       | More peering into editorial peer-review                                   | Jama              |
|  | 1994 | Rennie D       | The 2nd international-congress on peer-review in biomedical...            | Jama              |
|  | 1994 | Smith R        | Promoting research into peer-review                                       | BritMedJ          |
|  | 1995 | Jefferson T    | Are guidelines for peer-reviewing economic evaluations necessary          | HealthEcon        |
|  | 1996 | Drummond M     | Guidelines for authors and peer reviewers of economic submis...           | BritMedJ          |
|  | 1998 | Jefferson T    | Evaluating the BMJ guidelines for economic submissions                    | Jama              |
|  | 1998 | Rennie D       | Peer review in Prague   | Jama              |



# PEERE – . . . List of works on main path

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projectionsFractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

|  | year | first author   | title  | journal         |
|--|------|----------------|--|-----------------|
|  | 1999 | Smith R        | Opening up BMJ peer review - A beginning that should lead to...      | BritMedJ        |
|  | 1999 | Goldbeck-W. S  | Evidence on peer review - scientific quality control or smokescreen? | BritMedJ        |
|  | 2000 | Walsh E        | Open peer review: a randomised controlled trial                      | BritJPsiychiat  |
| Introduction                           | 2002 | Jefferson T    | Effects of editorial peer review - A systematic review               | Jama            |
| Statistics                             | 2002 | Rennie D       | Fourth International Congress on Peer Review in Biomedical Pub...    | Jama            |
| Citation                               | 2002 | Ophof T        | The significance of the peer review process against ... bias         | CardiovascRes   |
| Binary projections                     | 2002 | Jefferson T    | Measuring the quality of editorial peer review                       | Jama            |
| Fractional approach                    | 2005 | Bornmann L     | Committee peer review at an international research foundation        | ResEvaluat      |
| Normalizations                         | 2005 | Daniel HD      | Publications as a measure of scientific advancement and of...        | LearnPubl       |
| Groups                                 | 2006 | Bornmann L     | Selecting scientific excellence through committee peer review        | Scientometrics  |
| Bibliographic coupling and co-citation | 2007 | Bornmann L     | Convergent validation of peer review decisions using the h index     | JInformatr      |
| Resources                              | 2008 | Bornmann L     | Selecting manuscripts for a high-impact journal through peer review  | JAmSocInfSciTe  |
|  | 2009 | Bornmann L     | The luck of the referee draw: the effect of exchanging reviews       | LearnPubl       |
|  | 2011 | Bornmann L     | Scientific Peer Review   | AnnuRevInform   |
|  | 2011 | Bornmann L     | A multilevel modelling approach to investigating the predictive...   | JRStatSocAStat  |
|  | 2013 | Lee CJ         | Bias in peer review  | JAmSocInfSciTe  |
|  | 2015 | Garcia JA      | The Principal-Agent Problem in Peer Review                           | JAssocInfSciTec |
|  | 2015 | Garcia JA      | Adverse selection of reviewers                                       | JAssocInfSciTec |
|  | 2015 | Garcia JA      | Bias and effort in peer review                                       | JAssocInfSciTec |
|  | 2015 | Garcia JA      | The author-editor game   | Scientometrics  |
|  | 2015 | Moustafa K     | Don't infer anything from unavailable data                           | Scientometrics  |
|  | 2016 | Rodriguez-S. R | Evolutionary games between authors and their editors                 | ApplMathComp    |

# PEERE – Main paths for 100 largest weights

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

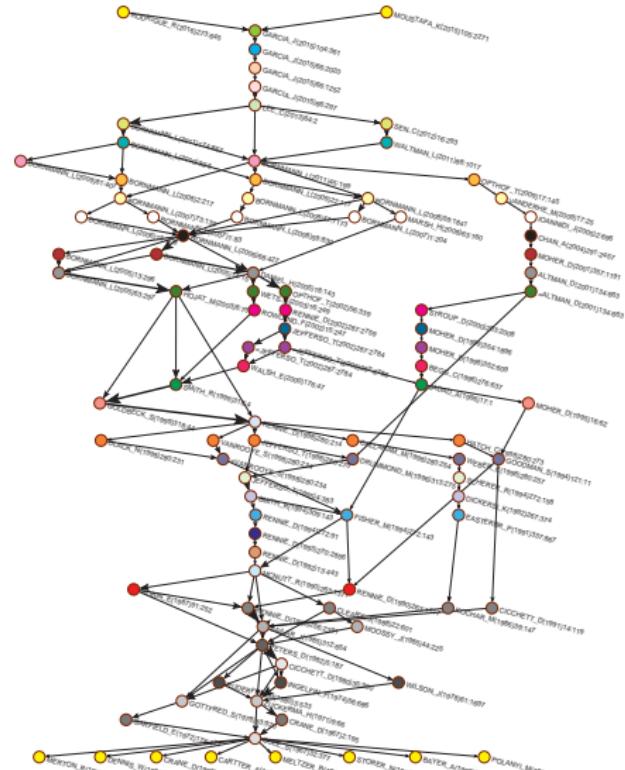
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



PEERE – SPC – Link island1

$$w_{max} = 0.297$$

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

## Citation

## Binary projections

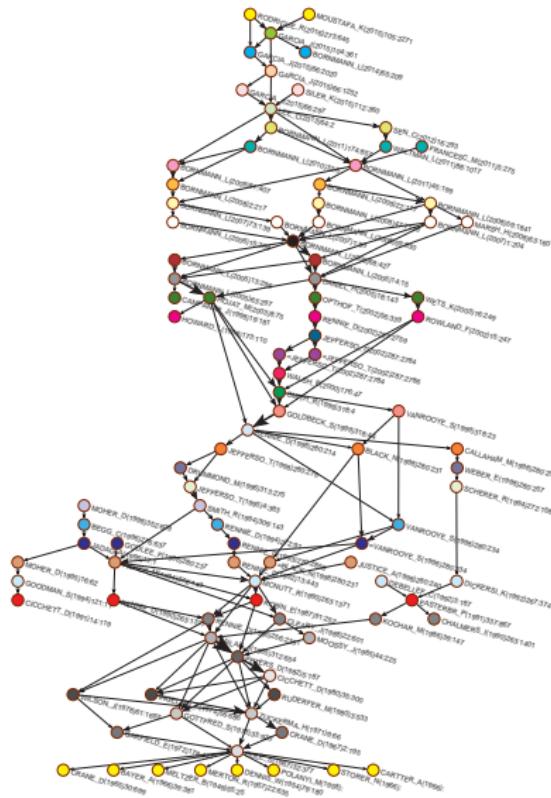
## Fractional approach

## Normalizations

## Groups

## Bibliographic coupling and co-citation

## Resources



This island is very similar to the main paths for 100 largest weights and includes main path.



# Matrix representation of a network

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

A simple (no parallel links) two-mode network

$\mathcal{N}_A = ((I, K), L_A, a)$  can be represented by a matrix

$\mathbf{A} = [A[i, k]]_{I \times K}$  where  $A[i, k] = a(i, k)$  iff  $(i, k) \in L_A$  and  $\square$  otherwise.

We extend the codomain  $\mathbb{R}$  of the weight  $a : L_A \rightarrow \mathbb{R}$  with a *structural zero*  $\square$  such that

$$\square + x = x \quad \text{and} \quad \square \cdot x = \square$$

In the case  $a : L_A \rightarrow \mathbb{R}^+$  we can set  $\square = 0$ .



# Multiplication of networks

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

The product  $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$  of two compatible matrices  $\mathbf{A}_{I \times K}$  and  $\mathbf{B}_{K \times J}$  is defined in the standard way

$$C[i, j] = \sum_{k \in K} A[i, k] \cdot B[k, j]$$

(semirings !!!)

The product of two compatible networks  $\mathcal{N}_A = ((I, K), L_A, a)$  and  $\mathcal{N}_B = ((K, J), L_B, b)$  is the network  $\mathcal{N}_C = ((I, J), L_C, c)$  where  $L_C = \{(i, j) : c[i, j] \neq \square\}$  and the weight  $c$  is determined by the matrix  $\mathbf{C}$ ,  $c(i, j) = C[i, j]$ .

# Multiplication of networks

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

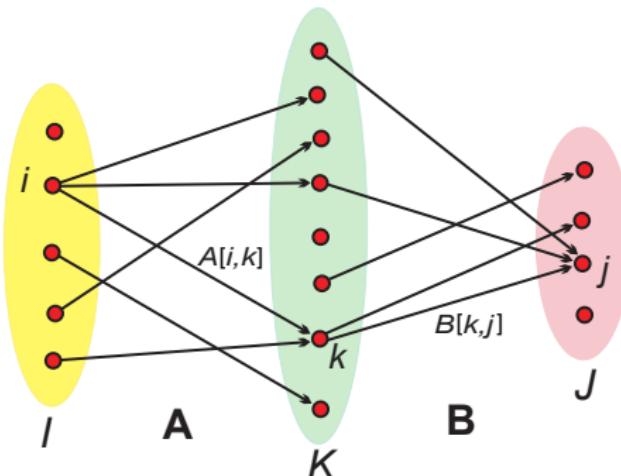
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



If all weights in networks  $\mathcal{N}_A$  and  $\mathcal{N}_B$  are equal to 1, the value of  $C[i, j]$  of  $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$  counts the number of ways we can go from the node  $i \in I$  to the node  $j \in J$  passing through  $K$ ,  $C[i, j] = |N_A(i) \cap N_B^-(j)|$ .

$$C[i, j] = \sum_{k \in N_A(i) \cap N_B^-(j)} A[i, k] \cdot B[k, j]$$



# Multiplication of networks

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

The standard matrix multiplication has the complexity  $O(|\mathcal{I}| \cdot |\mathcal{K}| \cdot |\mathcal{J}|)$  – it is too slow to be used for large networks. For sparse large networks, we can multiply much faster considering only nonzero elements.

```
for k in K do
    for (i,j) in NA-(k) × NB(k) do
        if ∃ci,j then ci,j := ci,j + ai,k · bk,j
        else new ci,j := ai,k · bk,j
```

In general, the multiplication of large sparse networks is a 'dangerous' operation since the result can 'explode' – it is not sparse.

# Multiplication of networks

From the network multiplication algorithm we see that each intermediate node  $k \in \mathcal{K}$  adds to a product network a complete two-mode subgraph  $K_{N_A^-(k), N_B(k)}$  (or, in the case  $\mathcal{I} = \mathcal{J}$ , a complete subgraph  $K_{N(k)}$ ). If both degrees  $\deg_A(k) = |N_A^-(k)|$  and  $\deg_B(k) = |N_B(k)|$  are large then already the computation of this complete subgraph has a quadratic (time and space) complexity – the result 'explodes'.

If at least one of the sparse networks  $\mathcal{N}_A$  and  $\mathcal{N}_B$  has small maximal degree on  $\mathcal{K}$  then also the resulting product network  $\mathcal{N}_C$  is sparse.

If for the sparse networks  $\mathcal{N}_A$  and  $\mathcal{N}_B$  there are in  $\mathcal{K}$  only few nodes with large degree and no one among them with large degree in both networks then also the resulting product network  $\mathcal{N}_C$  is sparse.



# Outer product decomposition

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

For vectors  $x = [x_1, x_2, \dots, x_n]$  and  $y = [y_1, y_2, \dots, y_m]$  their *outer product*  $x \circ y$  is defined as a matrix

$$x \circ y = [x_i \cdot y_j]_{n \times m}$$

then we can express the product **C** of two compatible matrices **A** and **B** as the *outer product decomposition*

$$\mathbf{C} = \mathbf{A} \cdot \mathbf{B} = \sum_k \mathbf{H}_k \quad \text{where} \quad \mathbf{H}_k = \mathbf{A}[:, k] \circ \mathbf{B}[k, :],$$

**A**[:,  $k$ ] is the  $k$ -th column of matrix **A**, and **B**[ $k$ , :] is the  $k$ -th row of matrix **B**.

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

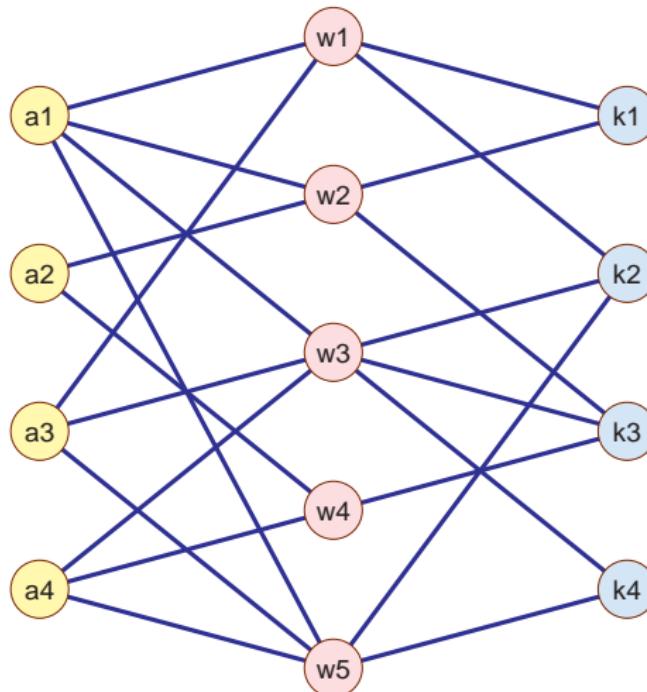
Binary  
projectionsFractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



# Example

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

$$\mathbf{H} = \mathbf{A}\mathbf{W} \cdot \mathbf{W}\mathbf{K}, \quad \mathbf{WA} = \begin{matrix} w_1 & a_1 & a_2 & a_3 & a_4 \\ w_2 & 1 & 0 & 1 & 0 \\ w_3 & 1 & 1 & 0 & 0 \\ w_4 & 0 & 1 & 0 & 1 \\ w_5 & 1 & 0 & 1 & 1 \end{matrix}, \quad \mathbf{WK} = \begin{matrix} w_1 & k_1 & k_2 & k_3 & k_4 \\ w_2 & 1 & 1 & 0 & 0 \\ w_3 & 1 & 0 & 1 & 0 \\ w_4 & 0 & 1 & 1 & 1 \\ w_5 & 0 & 1 & 0 & 1 \end{matrix}$$

$$\mathbf{H} = \begin{matrix} k_1 & k_2 & k_3 & k_4 \\ a_1 & 2 & 3 & 2 & 2 \\ a_2 & 1 & 0 & 2 & 0 \\ a_3 & 1 & 3 & 1 & 2 \\ a_4 & 0 & 2 & 2 & 2 \end{matrix} = \begin{matrix} \mathbf{H}_1 & k_1 & k_2 & k_3 & k_4 \\ a_1 & 1 & 1 & 0 & 0 \\ a_2 & 0 & 0 & 0 & 0 \\ a_3 & 1 & 1 & 0 & 0 \\ a_4 & 0 & 0 & 0 & 0 \end{matrix} + \begin{matrix} \mathbf{H}_2 & k_1 & k_2 & k_3 & k_4 \\ a_1 & 1 & 0 & 1 & 0 \\ a_2 & 1 & 0 & 1 & 0 \\ a_3 & 0 & 0 & 0 & 0 \\ a_4 & 0 & 0 & 0 & 0 \end{matrix} + \dots$$

$$\begin{matrix} \mathbf{H}_3 & k_1 & k_2 & k_3 & k_4 \\ a_1 & 0 & 1 & 1 & 1 \\ a_2 & 0 & 0 & 0 & 0 \\ a_3 & 0 & 1 & 1 & 1 \\ a_4 & 0 & 1 & 1 & 1 \end{matrix} + \begin{matrix} \mathbf{H}_4 & k_1 & k_2 & k_3 & k_4 \\ a_1 & 0 & 0 & 0 & 0 \\ a_2 & 0 & 0 & 1 & 0 \\ a_3 & 0 & 0 & 0 & 0 \\ a_4 & 0 & 0 & 1 & 0 \end{matrix} + \begin{matrix} \mathbf{H}_5 & k_1 & k_2 & k_3 & k_4 \\ a_1 & 0 & 1 & 0 & 1 \\ a_2 & 0 & 0 & 0 & 0 \\ a_3 & 0 & 1 & 0 & 1 \\ a_4 & 0 & 1 & 0 & 1 \end{matrix}$$



# Derived networks

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

**WA** – works × authors – authorship network

**WK** – works × keywords

**Ci** – works × works – citation network

**JD** – journal × discipline

**AK** =  $\mathbf{WA}^T * \mathbf{WK}$  – authors × keywords

**Co** =  $\mathbf{WA}^T * \mathbf{WA}$  – coauthorship

**ACi** =  $\mathbf{WA}^T * \mathbf{Ci} * \mathbf{WA}$  – citations between authors

**DCi** =  $\mathbf{JD}^T * \mathbf{WJ}^T * \mathbf{Ci} * \mathbf{WJ} * \mathbf{JD}$  – citations between disciplines

Interpretation!!!



# Projections

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

A standard approach to the analysis of a two-mode network  $\mathcal{N}$  is to transform it into the corresponding one-mode networks determined by:

*row projection* to  $U$ :  $\text{row}(\mathbf{A}) = \mathbf{A} \cdot \mathbf{A}^T$ , or  
*column projection* to  $V$ :  $\text{col}(\mathbf{A}) = \mathbf{A}^T \cdot \mathbf{A}$

and analyze the obtained weighted network.

$$\text{col}(\mathbf{A}) = \mathbf{A}^T \cdot \mathbf{A} = \mathbf{A}^T \cdot (\mathbf{A}^T)^T = \text{row}(\mathbf{A}^T), \quad \text{row}(\mathbf{A}) = \text{col}(\mathbf{A}^T)$$



# Binary projections

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

In the following, we will consider a two-mode network  $\mathcal{N} = ((W, A), L, wa)$  described with the corresponding binary matrix  $\mathbf{WA}$ . To support our intuition, we can interpret it as an authorship network linking a work  $w \in W$  to its authors from  $A$ .

Its column projection  $\mathbf{Co} = \text{col}(\mathbf{WA}) = \mathbf{WA}^T \cdot \mathbf{WA}$  has entries

$Co[a, b] = |N(a) \cap N(b)| = Co[b, a] = \# \text{ of works that authors } a \text{ and } b \text{ coauthored}$

$\mathbf{Co}$  is a *co-appearance* / coauthorship matrix.

$Co[a, a] = |N(a)| = \text{indeg}_{WA}(a) = \# \text{ of works (co)authored by the author } a$ .



# ... nodes

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

A real-life network  $\mathcal{N}$  can contain nodes  $w \in W$  of degree 0 (works with no author) and 1 (single author works). Works with no author do not contribute to the matrix **Co**. Single author works contribute only to the author's diagonal entry in the matrix **Co**.

We denote  $W_{[d]} = \{w \in W : \deg(w) \geq d\}$  and  
 $\mathcal{N}_{[d]} = ((W_{[d]}, A), L(W_{[d]}), wa/W_{[d]})$ .

Using the entries of the matrix **Co** we can express the *Salton's cosine similarity* between nodes  $a_i$  and  $a_j$  as

$$S(i, j) = \cos[a_i, a_j] = \frac{\text{Co}[a_i, a_j]}{\sqrt{\text{Co}[a_i, a_i] \cdot \text{Co}[a_j, a_j]}}$$



# Properties of Salton's cosine similarity

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

The Salton's cosine has the following properties

- ①  $S(i, j) \in [-1, 1]$
- ②  $S(i, j) = S(j, i)$
- ③  $S(i, i) = 1$
- ④  $wa_{ki} \in \mathbb{R}_0^+ \Rightarrow S(i, j) \in [0, 1]$
- ⑤  $S(\alpha i, \beta j) = S(i, j), \quad \alpha, \beta > 0$
- ⑥  $S(\alpha i, i) = 1, \quad \alpha > 0$



# Total weight

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

The **total weight** of links in the network  $\mathcal{N} = (V, L, w)$ ,  $w: L \rightarrow \mathbb{R}$

$$T(\mathcal{N}) = \sum_{(u,v) \in L} w(u,v)$$

On the basis of outer product decomposition, we have

$$T(\mathbf{C}) = T\left(\sum_k \mathbf{H}_k\right) = \sum_k T(\mathbf{H}_k)$$

$$T(\mathbf{H}_k) = \left(\sum_i A[i, k]\right) \cdot \left(\sum_j B[k, j]\right) = \text{windeg}_A(k) \cdot \text{woutdeg}_B(k)$$

Therefore for **Co** we have  $\mathbf{H}_w = \mathbf{K}_{N(w)}$  and

$$T(\mathbf{Co}) = \sum_{w \in W} \text{outdeg}_{WA}(w)^2$$

# $p_S$ -core at level 46 in Computational Geometry co-authorship network

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

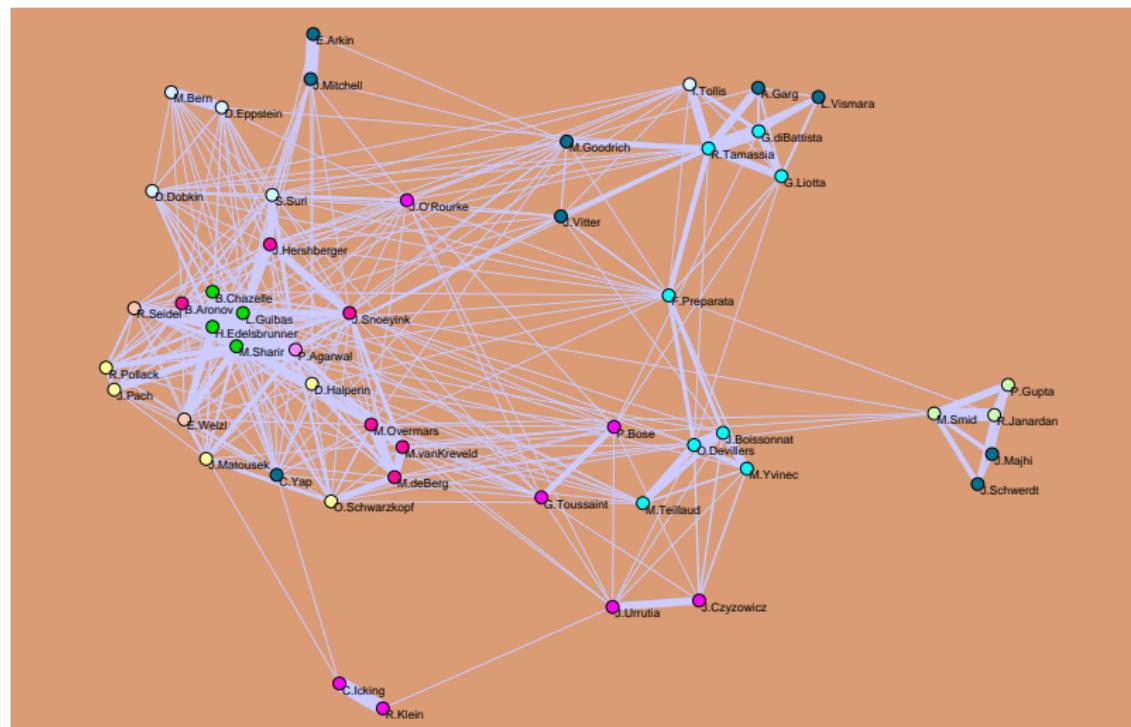
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources





# Structure of projection

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

## In words

- a projection network is a sum of complete subgraphs;
- the contribution of a node  $w$  from the other set  $W$  to the total is  $\text{outdeg}(w)^2$ .

This means that the nodes with a large degree in  $W$  are over-represented in the projection.

To make a contribution of each active node  $k \in W_{[1]}$  equal to 1 we normalize matrices **A** and **B** such that for the normalized matrices  $n(\mathbf{A})$  and  $n(\mathbf{B})$  we have

$$\text{windeg}_{n(A)}(k) = 1 \quad \text{and} \quad \text{woutdeg}_{n(B)}(k) = 1$$

– *fractional approach*.

# ... fractional approach

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

We define the *normalized* matrices

$$n(\mathbf{WA}) = \text{diag}\left(\frac{1}{\deg_{WA}}\right)\mathbf{WA}, \quad n(\mathbf{WK}) = \text{diag}\left(\frac{1}{\deg_{WK}}\right)\mathbf{WK}$$

Then the *normalized product* matrix is

$$\mathbf{AKn} = n(\mathbf{WA})^T \cdot n(\mathbf{WK})$$

Denoting  $\mathbf{F}_w = \frac{1}{\deg_{WA}(w) \deg_{WK}(w)} \mathbf{H}_w$  the outer product decomposition gets form

$$\mathbf{AKn} = \sum_w \mathbf{F}_w$$

# Example

## normalized matrices

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

$$\text{diag}\left(\frac{1}{\deg_{WA}}\right) = \begin{bmatrix} w_1 & w_1 & w_2 & w_3 & w_4 & w_5 \\ w_2 & 0 & 1/2 & 0 & 0 & 0 \\ w_3 & 0 & 0 & 1/3 & 0 & 0 \\ w_4 & 0 & 0 & 0 & 1/2 & 0 \\ w_5 & 0 & 0 & 0 & 0 & 1/3 \end{bmatrix}$$

$$\text{diag}\left(\frac{1}{\deg_{WK}}\right) = \begin{bmatrix} w_1 & w_1 & w_2 & w_3 & w_4 & w_5 \\ w_2 & 1/2 & 0 & 0 & 0 & 0 \\ w_3 & 0 & 1/2 & 0 & 0 & 0 \\ w_4 & 0 & 0 & 1/3 & 0 & 0 \\ w_5 & 0 & 0 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 0 & 1/2 \end{bmatrix}$$

$$n(\mathbf{WA}) = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ w_1 & 1/2 & 0 & 1/2 \\ w_2 & 1/2 & 1/2 & 0 \\ w_3 & 1/3 & 0 & 1/3 \\ w_4 & 0 & 1/2 & 0 \\ w_5 & 1/3 & 0 & 1/3 \end{bmatrix}, \quad n(\mathbf{WK}) = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \\ w_1 & 1/2 & 1/2 & 0 \\ w_2 & 1/2 & 0 & 1/2 \\ w_3 & 0 & 1/3 & 1/3 \\ w_4 & 0 & 0 & 1 \\ w_5 & 0 & 1/2 & 0 \end{bmatrix}$$

# ... example

## normalized matrices

Introduction

Statistics

Citation

Binary  
projectionsFractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

$$\mathbf{F}_1 = \begin{matrix} & k_1 & k_2 & k_3 & k_4 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} 1/4 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/4 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$
$$\mathbf{F}_5 = \begin{matrix} & k_1 & k_2 & k_3 & k_4 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} 0 & 1/6 & 0 & 1/6 \\ 0 & 0 & 0 & 0 \\ 0 & 1/6 & 0 & 1/6 \\ 0 & 1/6 & 0 & 1/6 \end{bmatrix} \end{matrix}$$

$$\mathbf{AKn} = \mathbf{F} = \begin{matrix} & k_1 & k_2 & k_3 & k_4 \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{matrix} & \begin{bmatrix} 0.50000 & 0.52778 & 0.36111 & 0.27778 \\ 0.25000 & 0.00000 & 0.75000 & 0.00000 \\ 0.25000 & 0.52778 & 0.11111 & 0.27778 \\ 0.00000 & 0.27778 & 0.61111 & 0.27778 \end{bmatrix} \end{matrix}$$



# Fractional approach

The normalized 2-mode network  $n(\mathbf{WA})$  has weights

$$n(\mathbf{WA})[w, a] = \frac{\mathbf{WA}[w, a]}{\max(1, \text{outdeg}_{\mathbf{WA}}(w))}$$

$$\sum_{a \in A} n(\mathbf{WA})[w, a] = 1, \quad w \in W_{[1]}$$

*Normalized projection*     $\mathbf{Cn} = n(\mathbf{WA})^T \cdot n(\mathbf{WA})$

Network: opposite arcs replaced by an edge with double weight.

$$T(\mathbf{Cn}) = |W_{[1]}|$$

Each work (with at least one author) has a value 1 that is distributed over the links of the projection.

# ... fractional approach

## Collaborativeness

Note: instead of the matrix  $n(\mathbf{WA})$  we could use a matrix of authors' contributions  $\mathbf{N} = [N[w, a]]$  such that

$\sum_{a \in A} N[w, a] = 1$ . Unfortunately, it is very difficult to get such data.

In the network  $n(\mathbf{WA})$ ,

- $\text{indeg}_{n(\mathbf{WA})}(a) = \# \text{ of works (co-)authored by the author } a$
- $\text{windeg}_{n(\mathbf{WA})}(a) = \text{fractional contribution of the author } a \text{ to his/her works}$

$$\text{Self-sufficiency: } S(a) = \frac{\text{windeg}_{n(\mathbf{WA})}(a)}{\text{indeg}_{\mathbf{WA}}(a)}$$

$$\text{Collaborativeness: } K(a) = 1 - S(a)$$

# ... fractional approach

## SNA18 collaborativness

### Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

| #  | Author     | Total # works | Total contribution | Collaborativeness, % | #  | Author      | Total # works | Total contribution | Collaborativeness, % |
|----|------------|---------------|--------------------|----------------------|----|-------------|---------------|--------------------|----------------------|
| 1  | LATKIN_C   | 130           | 32.99              | <b>75</b>            | 31 | EVERETT_M   | 44            | 22.58              | 49                   |
| 2  | VALENTE_T  | 97            | 34.96              | <b>64</b>            | 32 | MORRIS_M    | 43            | 17.22              | <b>60</b>            |
| 3  | DUNBAR_R   | 91            | 40.02              | <b>56</b>            | 33 | CONTRACT_N  | 43            | 14.15              | <b>67</b>            |
| 4  | NEWMAN_M   | 81            | <b>50.02</b>       | 38                   | 34 | WHITE_H     | 42            | 27.28              | 35                   |
| 5  | CHRISTAK_N | 74            | 22.89              | <b>69</b>            | 35 | SKVORETZ_J  | 42            | 20.07              | <b>52</b>            |
| 6  | DOREIAN_P  | 72            | <b>46.19</b>       | 36                   | 36 | PENTLAND_A  | 41            | 14.12              | <b>66</b>            |
| 7  | CARLEY_K   | 72            | 28.11              | <b>61</b>            | 37 | MOODY_J     | 40            | 17.7               | <b>56</b>            |
| 8  | BURT_R     | 71            | <b>55.73</b>       | 22                   | 38 | SMITH_A     | 40            | 14.2               | <b>65</b>            |
| 9  | BORGATTI_S | 71            | 29.72              | <b>58</b>            | 39 | MARSDEN_P   | 39            | 30.17              | 23                   |
| 10 | SNIJDERS_T | 67            | 29.63              | <b>56</b>            | 40 | BERKMAN_L   | 39            | 14.3               | <b>63</b>            |
| 11 | BARABASI_A | 67            | 27.61              | <b>59</b>            | 41 | SMITH_M     | 39            | 13.29              | <b>66</b>            |
| 12 | FOWLER_J   | 65            | 20.14              | <b>69</b>            | 42 | KRACKHAR_D  | 38            | 18.24              | <b>52</b>            |
| 13 | KAZENKO_P  | 64            | 21.97              | <b>66</b>            | 43 | JACKSON_M   | 38            | 17.78              | <b>53</b>            |
| 14 | ROBINS_G   | 64            | 19.67              | <b>69</b>            | 44 | THELWALL_M  | 37            | 18.41              | <b>50</b>            |
| 15 | WELLMAN_B  | 63            | 36.43              | 42                   | 45 | FRIEDKIN_N  | 36            | 28.17              | 22                   |
| 16 | FALOUTSO_C | 60            | 17.86              | <b>70</b>            | 46 | SINGH_A     | 36            | 14.5               | <b>60</b>            |
| 17 | RAHMAN_M   | 59            | 19.18              | <b>67</b>            | 47 | WASSERMA_S  | 35            | 15.64              | <b>55</b>            |
| 18 | PATTISON_P | 58            | 18.94              | <b>67</b>            | 48 | BRANDES_U   | 35            | 14.39              | <b>59</b>            |
| 19 | JOHNSON_J  | 54            | 21.19              | <b>61</b>            | 49 | GONZALEZ_A  | 35            | 14.13              | <b>60</b>            |
| 20 | MARTINEZ_M | 53            | 21.9               | <b>59</b>            | 50 | KLEINBERG_J | 34            | 15.05              | <b>56</b>            |
| 21 | GONZALEZ_M | 52            | 17.76              | <b>66</b>            | 51 | FARINE_D    | 34            | 14.04              | <b>59</b>            |
| 22 | RODRIGUE_J | 52            | 15.9               | <b>69</b>            | 52 | BATAGELJ_V  | 33            | 14.64              | <b>56</b>            |
| 23 | SCHNEIDE_J | 52            | 13.89              | <b>73</b>            | 53 | BREIGER_R   | 31            | 19.73              | 36                   |
| 24 | LEYDESDO_L | 51            | 33.28              | 35                   | 54 | WILLIAMS_A  | 31            | 14.5               | <b>53</b>            |
| 25 | LITWIN_H   | 50            | 32.42              | 35                   | 55 | SCOTT_J     | 28            | 17.54              | 37                   |
| 26 | RICE_E     | 48            | 13.09              | <b>73</b>            | 56 | MASUDA_N    | 28            | 14.26              | 49                   |
| 27 | BONACICH_P | 46            | 34                 | 26                   | 57 | FREEMAN_L   | 27            | 20.03              | 26                   |
| 28 | RODRIGUE_M | 46            | 13.21              | <b>71</b>            | 58 | WATTS_D     | 27            | 13.67              | 49                   |
| 29 | NGUYEN_H   | 46            | 12.76              | 72                   | 59 | LAZEGA_E    | 26            | 14.17              | 46                   |
| 30 | CROFT_D    | 46            | 11.6               | <b>75</b>            | 60 | FAUST_K     | 25            | 13.5               | 46                   |

# $p_S$ -core at level 0.75 in **Cn(SN5)**

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

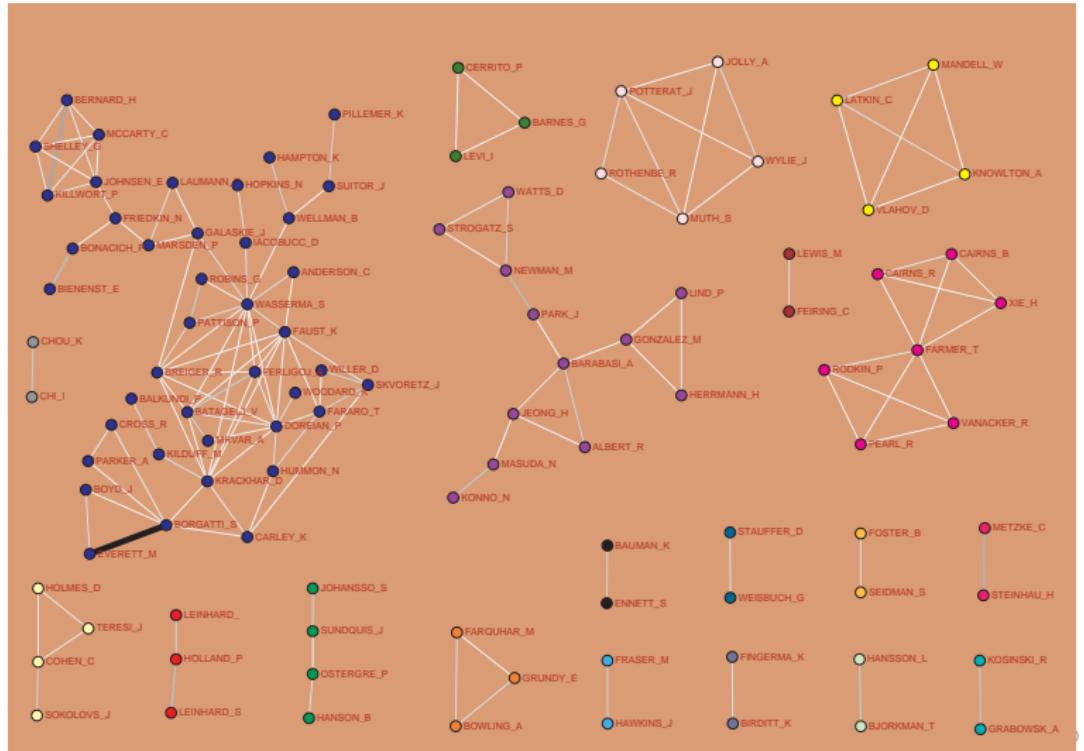
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources





# Newman's normalization – strict coauthorship

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

Mark Newman proposed an alternative normalization that considers only coauthorship between different authors – single-author works and self-coauthorship are excluded.

The *Newman's normalized* 2-mode network  $n'(\mathbf{WA})$  has weights

$$n'(\mathbf{WA})[w, a] = \begin{cases} \frac{\mathbf{WA}[w, a]}{\deg(w) - 1} & w \in W_{[2]} \\ 0 & \text{otherwise} \end{cases}$$

*Strict projection*  $\mathbf{Ct} = D_0(n(\mathbf{WA})^T \cdot n'(\mathbf{WA}))$ .

$D_0(\mathbf{M})$  sets the diagonal of the square matrix  $\mathbf{M}$  to 0.

Network: opposite arcs replaced by an edge with double weight.

$$T(\mathbf{Ct}) = |W_{[2]}|$$

Example: SNA18[17] /selected strict islands 10-30

$$|W| = 70792, |A| = 93011$$

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

## Citation

## Binary projections

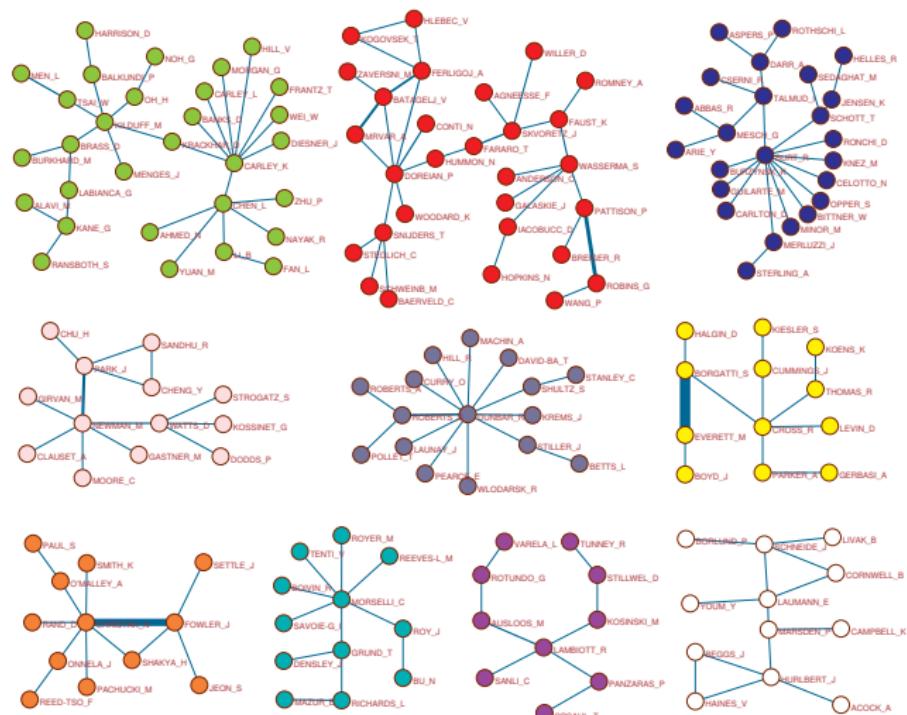
Fractional  
approach

### Normalizations

Groups

Bibliographic  
coupling and  
co-citation

## Resources





# Normalizations

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

In networks obtained from large two-mode networks, there are often huge differences in weights. Therefore it is not interesting to compare the nodes according to the raw data – the nodes with large weights will prevail. First, we have to *normalize* the network to make the weights comparable.

There exist several ways how to do this. Assume that the network is described with a square matrix  $\mathbf{C}_{V \times V}$ .

*Balassa* or activity normalization:

$$A[u, v] = \frac{C[u, v] \cdot T(C)}{\text{woutdeg}(u) \cdot \text{windeg}(v)}$$

If  $A[u, v] > 1$  the measured weight is larger than expected.

$$c_B[u, v] = \log_2 A[u, v] \quad \text{for } A[u, v] > 0$$



# ... Normalizations

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

$$c_{\min}[u, v] = \frac{C[u, v]}{\min(C[u, u], C[v, v])} \quad c_{\max}[u, v] = \frac{C[u, v]}{\max(C[u, u], C[v, v])}$$

$$c_{\min\text{Dir}}[u, v] = \begin{cases} \frac{C[u, v]}{C[u, u]} & C[u, u] \leq C[v, v] \\ 0 & \text{otherwise} \end{cases}$$

$$c_{\max\text{Dir}}[u, v] = \begin{cases} \frac{C[u, v]}{C[v, v]} & C[u, u] \leq C[v, v] \\ 0 & \text{otherwise} \end{cases}$$

After a selected normalization the important parts of the network are obtained by link-cuts or islands approaches.

# minDir of Slovenian journals 2000

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

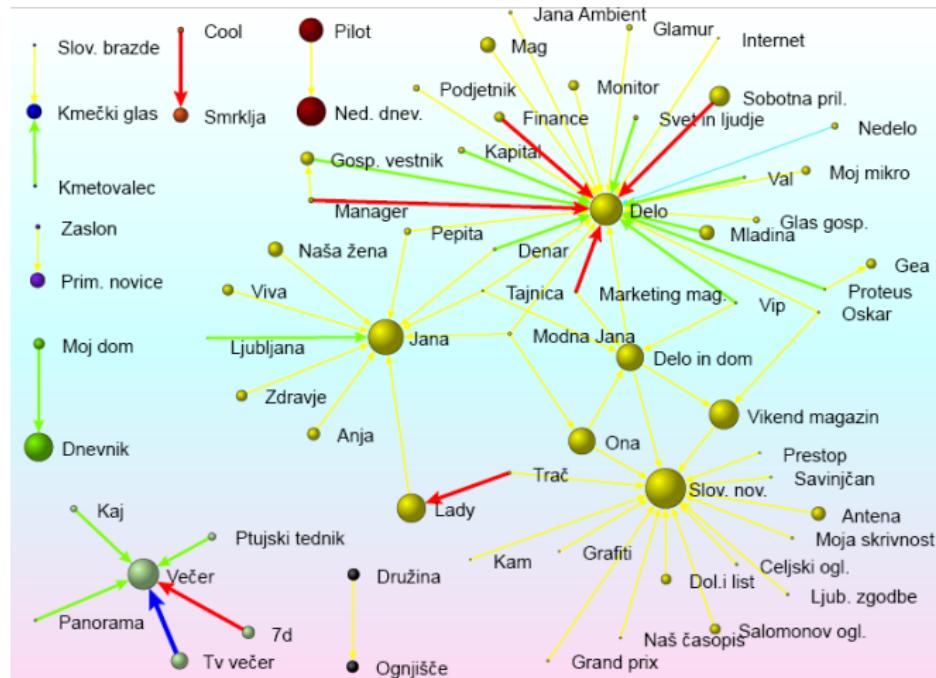
Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



Over 100000 people were asked in the years 1999 and 2000 about the journals they read.  
They mentioned 124 different journals. (source Cati)

# Keywords and journals for selected groups of authors

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

To uncover the research topic and disciplinary identities of a selected group of authors we constructed networks of authors and keywords

**AKn** =  $n(\mathbf{WA})^T * n(\mathbf{WK})$  and authors and journals

**AJn** =  $n(\mathbf{WA})^T * n(\mathbf{WJ})$ . Both networks are normalized. In the network **AKn**, the weight  $AKn[a, k]$  is equal to the fractional use (in publishing) of author  $a$  of keyword  $k$ . In the network **AJn**, the weight  $AJn[a, j]$  is equal to the fractional use of author  $a$  of journal  $j$ . For a given keyword  $k$  and journal  $j$ , it can be extended to a group of authors  $C \subseteq A$

$$\mathbf{AKn}[C, k] = \sum_{a \in C} AKn[a, k] \quad \text{and} \quad \mathbf{AJn}[C, j] = \sum_{a \in C} AJn[a, j]$$

# SNA18[17] / keywords for 3 selected islands

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

| Wasserman et al. |       | Dunbar et al.        |       | Newman et al.        |       |
|------------------|-------|----------------------|-------|----------------------|-------|
| Rank             | Value | Keyword              | Value | Keyword              | Value |
| 1                | 30.02 | <i>network</i>       | 3.80  | <i>social</i>        | 11.27 |
| 2                | 20.11 | <i>social</i>        | 3.60  | <i>network</i>       | 5.05  |
| 3                | 8.62  | <i>model</i>         | 2.97  | <i>size</i>          | 4.13  |
| 4                | 7.36  | <i>analysis</i>      | 1.41  | <i>human</i>         | 3.80  |
| 5                | 6.01  | <i>graph</i>         | 1.31  | <b>evolution</b>     | 3.54  |
| 6                | 5.50  | <i>structure</i>     | 1.17  | <i>brain</i>         | 2.68  |
| 7                | 3.19  | <i>datum</i>         | 1.13  | <i>group</i>         | 2.51  |
| 8                | 3.03  | <i>structural</i>    | 0.90  | <b>primate</b>       | 2.43  |
| 9                | 3.00  | <b>correction</b>    | 0.84  | <b>support</b>       | 2.31  |
| 10               | 2.96  | <i>exchange</i>      | 0.79  | <i>perspective</i>   | 2.24  |
| 11               | 2.80  | <b>equivalence</b>   | 0.75  | <b>constraint</b>    | 1.96  |
| 12               | 2.68  | <i>random</i>        | 0.69  | <i>neocortex</i>     | 1.82  |
| 13               | 2.54  | <i>theory</i>        | 0.69  | <i>relationship</i>  | 1.46  |
| 14               | 2.53  | <i>power</i>         | 0.61  | <b>communication</b> | 1.39  |
| 15               | 2.51  | <i>markov</i>        | 0.60  | <b>hypothesis</b>    | 1.28  |
| 16               | 2.41  | <i>evolution</i>     | 0.57  | <i>evolutionary</i>  | 1.16  |
| 17               | 2.28  | <i>group</i>         | 0.55  | <i>behavior</i>      | 1.10  |
| 18               | 2.25  | <i>statistical</i>   | 0.52  | <i>cooperation</i>   | 1.03  |
| 19               | 2.19  | <i>method</i>        | 0.51  | <i>cognitive</i>     | 0.93  |
| 20               | 2.18  | <b>dynamics</b>      | 0.49  | <i>difference</i>    | 0.92  |
| 21               | 1.88  | <i>generalized</i>   | 0.47  | <i>kinship</i>       | 0.90  |
| 22               | 1.82  | <i>journal</i>       | 0.44  | <i>altruism</i>      | 0.74  |
| 23               | 1.80  | <b>regression</b>    | 0.43  | <i>bond</i>          | 0.70  |
| 24               | 1.78  | <i>exponential</i>   | 0.40  | <i>organization</i>  | 0.69  |
| 25               | 1.78  | <i>blockmodel</i>    | 0.40  | <b>emotional</b>     | 0.67  |
| 26               | 1.76  | <i>logit</i>         | 0.38  | <i>female</i>        | 0.67  |
| 27               | 1.73  | <i>balance</i>       | 0.38  | <i>closeness</i>     | 0.66  |
| 28               | 1.73  | <i>p</i>             | 0.36  | <i>analysis</i>      | 0.65  |
| 29               | 1.68  | <i>measure</i>       | 0.36  | <i>individual</i>    | 0.64  |
| 30               | 1.66  | <i>algorithm</i>     | 0.35  | <b>baboon</b>        | 0.64  |
| 31               | 1.66  | <b>cluster</b>       | 0.35  | <i>internet</i>      | 0.62  |
| 32               | 1.64  | <i>approach</i>      | 0.35  | <b>dynamics</b>      | 0.60  |
| 33               | 1.62  | <i>actor</i>         | 0.34  | <i>structure</i>     | 0.60  |
| 34               | 1.59  | <b>logistic</b>      | 0.34  | <i>personal</i>      | 0.59  |
| 35               | 1.55  | <i>relation</i>      | 0.33  | <i>volume</i>        | 0.56  |
| 36               | 1.54  | <i>introduction</i>  | 0.33  | <i>world</i>         | 0.56  |
| 37               | 1.54  | <i>bias</i>          | 0.32  | <b>community</b>     | 0.56  |
| 38               | 1.51  | <i>dynamic</i>       | 0.32  | <b>friendship</b>    | 0.55  |
| 39               | 1.45  | <i>blockmodeling</i> | 0.31  | <i>theory</i>        | 0.53  |
| 40               | 1.44  | <i>friendship</i>    | 0.30  | <i>society</i>       | 0.52  |

# SNA18[17] /journals for 3 selected islands

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

| Wasserman et al. |        |                      | Dunbar et al. |                      |       | Newman et al.        |       |         |
|------------------|--------|----------------------|---------------|----------------------|-------|----------------------|-------|---------|
| Rank             | Value  | Journal              | Value         | Journal              | Value | Journal              | Value | Journal |
| 1                | 122.60 | Soc Networks         | 4.67          | Hum Nature-Int Bios  | 35.02 | Phys Rev E           |       |         |
| 2                | 25.18  | J Math Sociol        | 4.17          | Evol Hum Behav       | 8.62  | P Natl Acad Sci      |       |         |
| 3                | 10.75  | Psychometrika        | 3.33          | Philos T R Soc B     | 5.25  | Nature               |       |         |
| 4                | 8.08   | J Math Psychol       | 2.95          | Soc Networks         | 4.75  | Phys Rev Lett        |       |         |
| 5                | 7.33   | Sociol Method Res    | 2.75          | Anim Behav           | 3.00  | Soc Networks         |       |         |
| 6                | 7.10   | J Classif            | 2.57          | P Roy Soc B-Biol Sci | 2.92  | Science              |       |         |
| 7                | 7.08   | Qual Quant           | 2.33          | Biol Lett-Uk         | 2.83  | Lect Notes Comput Sc |       |         |
| 8                | 5.04   | Sociol Methodol      | 2.25          | J Theor Biol         | 2.75  | Eur Phys J B         |       |         |
| 9                | 4.00   | J Am Stat Assoc      | 2.20          | Roy Soc Open Sci     | 2.50  | Am J Sociol          |       |         |
| 10               | 4.00   | Contemp Sociol       | 2.00          | Pers Indiv Differ    | 2.00  | J Stat Mech-Theory   |       |         |
| 11               | 3.90   | Am Sociol Rev        | 2.00          | Trends Cogn Sci      | 1.97  | Plos One             |       |         |
| 12               | 3.59   | Lect Notes Comput Sc | 2.00          | Behaviour            | 1.67  | Siam Rev             |       |         |
| 13               | 3.58   | Am J Sociol          | 1.70          | Sci Rep-Uk           | 1.33  | Reliab Eng Syst Sa   |       |         |
| 14               | 3.53   | Scientometrics       | 1.67          | Comput Hum Behav     | 1.33  | Secur Commun Netw    |       |         |
| 15               | 3.40   | Soc Forces           | 1.62          | Plos One             | 1.25  | Nat Commun           |       |         |
| 16               | 3.38   | Sociol Method        | 1.58          | Hum Nature           | 1.12  | J Comput Sci-Neth    |       |         |
| 17               | 3.17   | Brit J Math Stat Psy | 1.57          | Adapt Hum Behav Phys | 1.08  | Scientometrics       |       |         |
| 18               | 3.00   | J Am Soc Inform Sci  | 1.53          | P Natl Acad Sci      | 1.03  | J Supercomput        |       |         |
| 19               | 2.98   | Annu Rev Sociol      | 1.50          | Brit J Psychol       | 1.00  | Inform Process Ma    |       |         |
| 20               | 2.75   | Soc Psychol Quart    | 1.25          | Soc Cogn Affect Neur | 1.00  | Comm Com Inf Sc      |       |         |
| 21               | 2.50   | Inform Process Manag | 1.00          | Int J Dev Disabil    | 1.00  | Symmetry-Basel       |       |         |
| 22               | 2.19   | Plos One             | 1.00          | Group Dyn-Theor Res  | 1.00  | Electron Libr        |       |         |
| 23               | 2.17   | Netw Sci             | 1.00          | Curr Dir Psychol Sci | 1.00  | Contemp Phys         |       |         |
| 24               | 2.00   | Poetics              | 1.00          | Brit J Dev Psychol   | 1.00  | Comput Phys Commun   |       |         |
| 25               | 2.00   | Sociol Rev           | 1.00          | Soc Dev              | 1.00  | J Consum Res         |       |         |
| 26               | 2.00   | Soc Sci Res          | 1.00          | J Community Appl Soc | 1.00  | J Stat Phys          |       |         |
| 27               | 2.00   | Comput Stat Data An  | 1.00          | Folia Primatol       | 1.00  | Harvard Bus Rev      |       |         |
| 28               | 1.92   | Methods Ser          | 1.00          | Hist Hum Sci         | 1.00  | Annu Rev Sociol      |       |         |
| 29               | 1.92   | Organ Sci            | 1.00          | Underst Complex Syst | 1.00  | Jpn J Polit Sci      |       |         |
| 30               | 1.75   | Sociol Perspect      | 1.00          | Orig Hum Behav       | 1.00  | Phys Lett A          |       |         |

# Linking through a network

Let a network  $\mathbf{S}$  links works to works. The derived network  $\mathbf{WA}^T \cdot \mathbf{S} \cdot \mathbf{WA}$  links authors to authors *through*  $\mathbf{S}$ . Again, the normalization question has to be addressed. Among different options let us consider the derived networks defined as:

$$\mathbf{C} = \mathbf{WA}^T \cdot \mathbf{S} \cdot \mathbf{WA}$$

It is easy to verify that:

- if  $\mathbf{S}$  is symmetric,  $\mathbf{S}^T = \mathbf{S}$ , then also  $\mathbf{C}$  is symmetric,  $\mathbf{C}^T = \mathbf{C}$ ;
- if  $W_{[1]} = W$ , the total of weights of  $\mathbf{S}$  is redistributed in  $\mathbf{C}$ :

$$T(\mathbf{C}) = \sum_{e \in L(\mathbf{C})} c(e) = \sum_{e \in L(\mathbf{S})} s(e) = T(\mathbf{S})$$

# Bibliographic Coupling

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

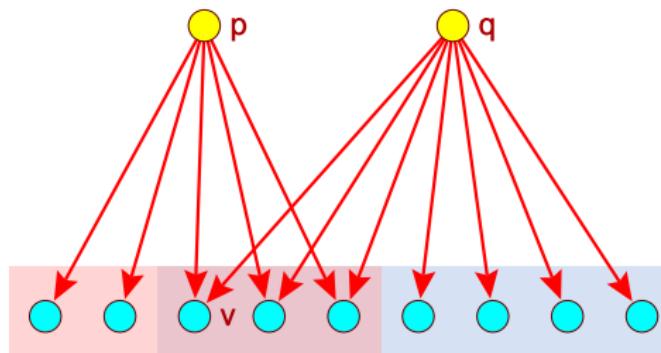
Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

Bibliographic coupling occurs when two works each cite a third work in their bibliographies, see figure. The idea was introduced by Kessler (1963) and has been used extensively since then. See figure where two citing works,  $p$  and  $q$ , are shown. Work  $p$  cites five works and  $q$  cites seven works. The key idea is that there are three works cited by both  $p$  and  $q$ . This suggests some content communality for the three works cited by both  $p$  and  $q$ . Having more works citing pairs of prior works increases the likelihood of them sharing content.





# Bibliographic Coupling

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

We assume that the citation relation means  
 $p \mathbf{Ci} q \equiv$  work  $p$  cites work  $q$ . Then the *bibliographic coupling*  
network **biCo** can be determined as

$$\mathbf{biCo} = \mathbf{Ci} * \mathbf{Ci}^T$$

The weight  $bico_{pq}$  is equal to the number of works cited by both  
works  $p$  and  $q$ ;  $bico_{pq} = |\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|$ . Bibliographic coupling  
weights are symmetric:  $bico_{pq} = bico_{qp}$ :

$$\mathbf{biCo}^T = (\mathbf{Ci} \cdot \mathbf{Ci}^T)^T = \mathbf{Ci} \cdot \mathbf{Ci}^T = \mathbf{biCo}$$

# Co-citation

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

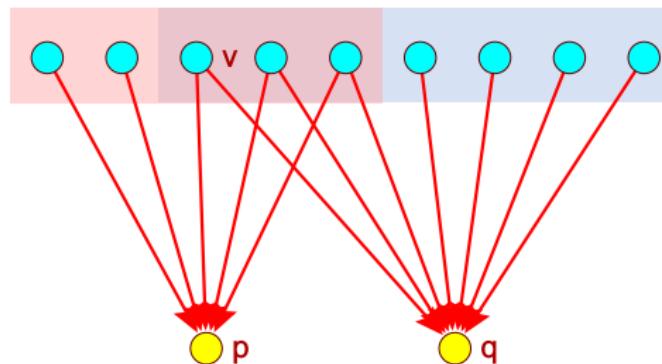
Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

Co-citation is a concept with strong parallels with bibliographic coupling (Small and Marshakova 1973). The focus is on the extent to which works are co-cited by later works. The basic intuition is that the more earlier works are cited, the higher the likelihood that they have common content.





# Bibliographic Coupling

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

The *co-citation* network **coCi** can be determined as

$$\mathbf{coCi} = \mathbf{Ci}^T \cdot \mathbf{Ci}.$$

The weight  $coci_{pq}$  is equal to the number of works citing both works  $p$  and  $q$ . The network **coCi** is symmetric  $coci_{pq} = coci_{qp}$ :

$$\mathbf{coCi}^T = (\mathbf{Ci}^T \cdot \mathbf{Ci})^T = \mathbf{Ci}^T \cdot \mathbf{Ci} = \mathbf{coCi}$$

An important property of co-citation is that  $\mathbf{coCi}(\mathbf{Ci}) = \mathbf{biCo}(\mathbf{Ci}^T)$ :

$$\mathbf{biCo}(\mathbf{Ci}^T) = \mathbf{Ci}^T \cdot (\mathbf{Ci}^T)^T = \mathbf{Ci}^T \cdot \mathbf{Ci} = \mathbf{coCi}(\mathbf{Ci})$$

Therefore the constructions proposed for bibliographic coupling can be applied also for co-citation.



# Normalization

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

What about normalizations? Searching for the most coupled works we have again problems with works with many citations, especially with review papers. To neutralize their impact we can introduce normalized measures. The fractional approach works fine for normalized co-citation

$$\mathbf{CoCit} = \mathbf{Cin}^T \cdot \mathbf{Cin}$$

where  $\mathbf{Cin} = \mathbf{D} \cdot \mathbf{Ci}$  and  $\mathbf{D} = \text{diag}\left(\frac{1}{\max(1, \text{outdeg}(p))}\right)$ .  $\mathbf{D}^T = \mathbf{D}$ . In the normalized network every work has value 1 and it is equally distributed to all cited works.



# Normalization of Bibliographic Coupling

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

The fractional approach can not be directly applied to bibliographic coupling – to get the outer product decomposition work we would need to normalize  $\mathbf{C}_i$  by columns – a cited work has value 1 which is distributed equally to the citing works – the most cited works give the least. This is against our intuition. To construct a reasonable measure we can proceed as follows. Let us first look at

$$\mathbf{biC} = \mathbf{C}_{in} \cdot \mathbf{C}_i^T$$

we have

$$\mathbf{biC} = (\mathbf{D} \cdot \mathbf{C}_i) \cdot \mathbf{C}_i^T = \mathbf{D} \cdot \mathbf{biCo}$$

$$\mathbf{biC}^T = (\mathbf{D} \cdot \mathbf{biCo})^T = \mathbf{biCo}^T \cdot \mathbf{D}^T = \mathbf{biCo} \cdot \mathbf{D}$$



# Normalization of Bibliographic Coupling

For  $\mathbf{Ci}(p) \neq \emptyset$  and  $\mathbf{Ci}(q) \neq \emptyset$  it holds

$$\mathbf{biC}_{pq} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(p)|} \quad \text{and} \quad \mathbf{biC}_{qp} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(q)|} = \mathbf{biC}_{pq}^T$$

and  $\mathbf{biC}_{pq} \in [0, 1]$ .  $\mathbf{biC}_{pq}$  is the proportion of its references that the work  $p$  shares with the work  $q$ . The network  $\mathbf{biC}$  is not symmetric.

We have different options to construct normalized symmetric measures such as

$$\mathbf{biCoa}_{pq} = \frac{1}{2}(\mathbf{biC}_{pq} + \mathbf{biC}_{qp}) \quad \text{Average}$$

$$\mathbf{biCom}_{pq} = \min(\mathbf{biC}_{pq}, \mathbf{biC}_{qp}) \quad \text{Minimum}$$

$$\mathbf{biCoM}_{pq} = \max(\mathbf{biC}_{pq}, \mathbf{biC}_{qp}) \quad \text{Maximum}$$



# Normalization of Bibliographic Coupling

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

or, may be more interesting

$$\mathbf{biCog}_{pq} = \sqrt{\mathbf{biC}_{pq} \cdot \mathbf{biC}_{qp}} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{\sqrt{|\mathbf{Ci}(p)| \cdot |\mathbf{Ci}(q)|}}$$

Geometric mean  
Salton cosinus

$$\mathbf{biCoh}_{pq} = 2 \cdot (\mathbf{biC}_{pq}^{-1} + \mathbf{biC}_{qp}^{-1})^{-1} = \frac{2|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(p)| + |\mathbf{Ci}(q)|}$$

Harmonic mean

$$\mathbf{biCoj}_{pq} = (\mathbf{biC}_{pq}^{-1} + \mathbf{biC}_{qp}^{-1} - 1)^{-1} = \frac{|\mathbf{Ci}(p) \cap \mathbf{Ci}(q)|}{|\mathbf{Ci}(p) \cup \mathbf{Ci}(q)|}$$

Jaccard index

All these measures are similarities.

It is easy to verify that  $\mathbf{biCoX}_{pq} \in [0, 1]$  and:  $\mathbf{biCoX}_{pq} = 1$  iff the works  $p$  and  $q$  are referencing the same works,  $\mathbf{Ci}(p) = \mathbf{Ci}(q)$ .



# Normalization of Bibliographic Coupling

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

From  $m \leq H \leq G \leq A \leq M$  and  $J \leq m$ ,  
 $(\frac{|P \cap Q|}{|P \cup Q|} \leq \min(\frac{|P \cap Q|}{|P|}, \frac{|P \cap Q|}{|Q|}))$  we get

$$\mathbf{biCoj}_{pq} \leq \mathbf{biCom}_{pq} \leq \mathbf{biCoh}_{pq} \leq \mathbf{biCog}_{pq} \leq \mathbf{biCoa}_{pq} \leq \mathbf{biCoM}_{pq}$$

The equalities hold iff  $\mathbf{Ci}(p) = \mathbf{Ci}(q)$ .

To get a dissimilarity we can use transformations  $dis = 1 - sim$  or  
 $dis = \frac{1}{sim} - 1$  or  $dis = -\log sim$ . For example

$$\mathbf{biCod}_{pq} = 1 - \mathbf{biCoj}_{pq} = \frac{|\mathbf{Ci}(p) \oplus \mathbf{Ci}(q)|}{|\mathbf{Ci}(p) \cup \mathbf{Ci}(q)|} \quad \text{Jaccard distance}$$

where  $\oplus$  denotes the symmetric difference of sets.



# Bibliographic Coupling, Co-citation and Linking through a network

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

Bibliographic coupling and co-citation networks are linking works to works. To get linking between authors, journals or keywords considering citation similarity we can apply the construction from *Linking through a network* to the normalized co-citation or bibliographic coupling network.

# Books

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

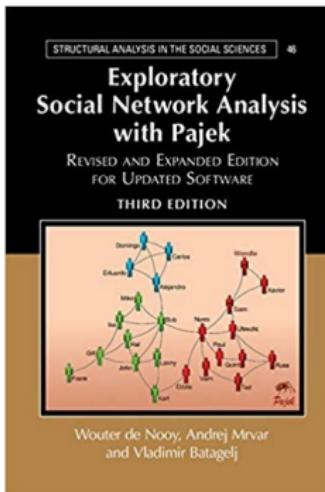
Fractional  
approach

Normalizations

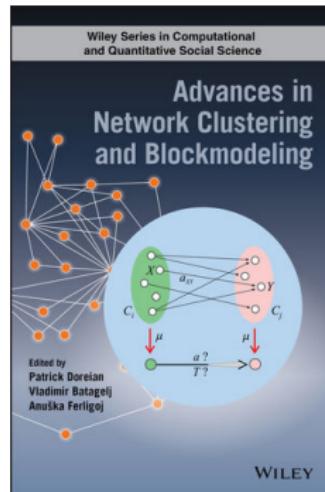
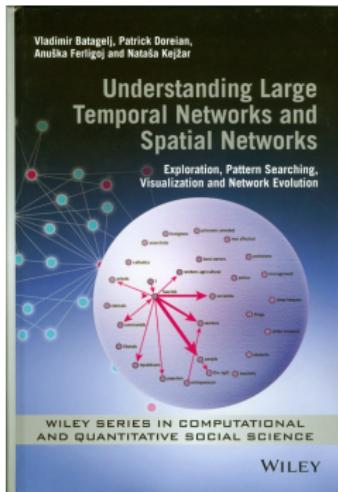
Groups

Bibliographic  
coupling and  
co-citation

Resources



Pajek





# References |

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

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# References II

## Analysis of bibliographic networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

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Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

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# References IV

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources

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# References V

Analysis of  
bibliographic  
networks

V. Batagelj

Introduction

Statistics

Citation

Binary  
projections

Fractional  
approach

Normalizations

Groups

Bibliographic  
coupling and  
co-citation

Resources



Pajek's wiki. <http://pajek.imfm.si>



Vladimir Batagelj, Andrej Mrvar: [Pajek manual](#).