Analysis of weighted multiway networks

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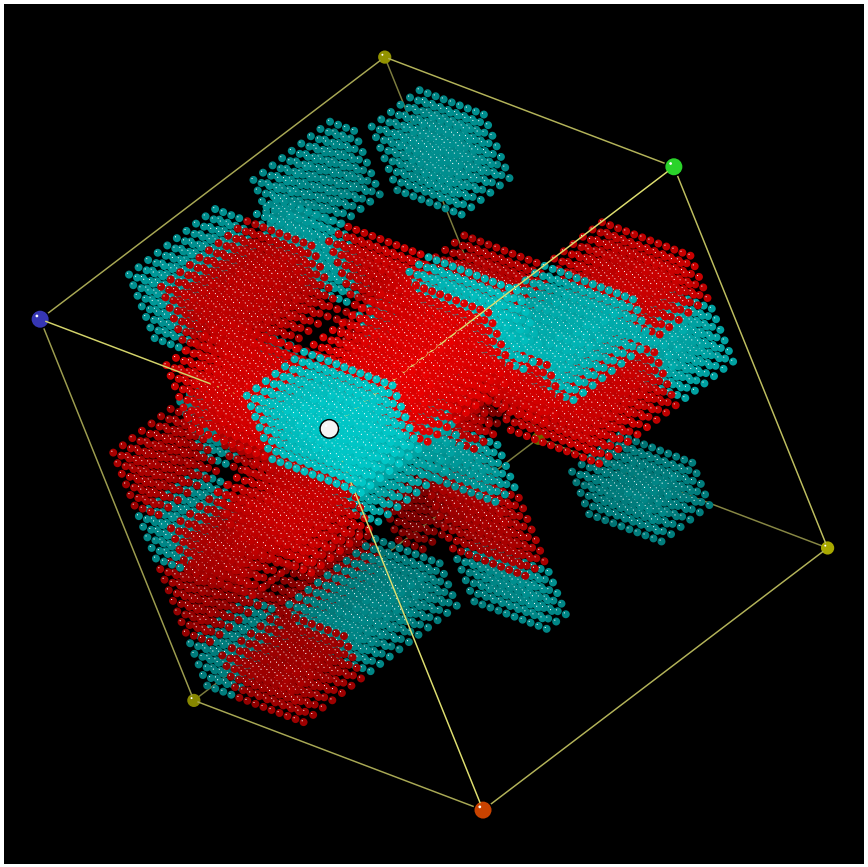
Outline



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Current version of slides (November 9, 2022 at 17 : 24): slides PDF
https://github.com/bavla/ibm3m/

IFCS 2022

This seminar is a result of joint work with colleagues from Napoli and Salerno. At the IFCS 2022 conference in July in Porto, they were presenting their research on Italian students' mobility. They developed their own approach and asked me for comments and alternatives.

Italian students' data

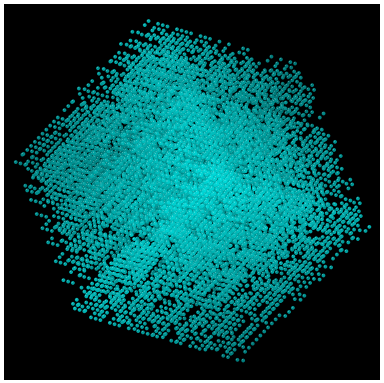
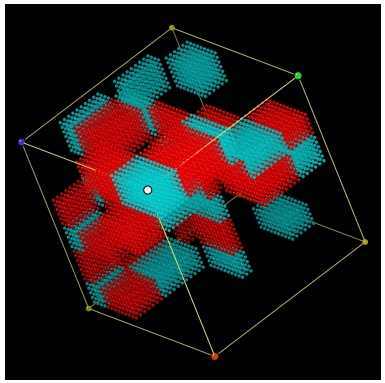
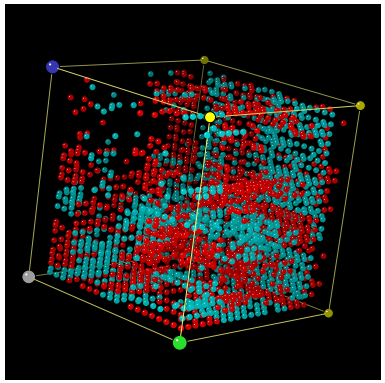
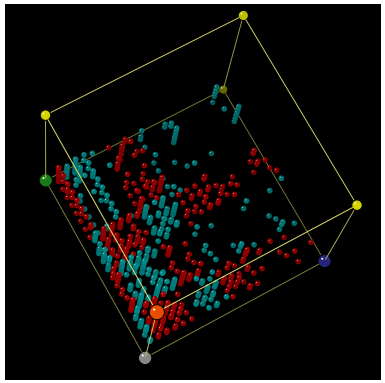
Their data consists of quadruples of *ways* (province, university, study program, year) with the corresponding value (weight) counting how many students from a given province selected in a given year to study a given program at a given university. Such data are called *weighted multiway networks*.

The approach proposed by my Italian colleagues was to analyze the data for each year separately by joining the *ways* university and study program into a single way [university, study program] thus obtaining a weighted two-mode network that can be analyzed using available methods.

I first transformed their data into Pajek format and applied some additional methods on them. Genova

Afterward, I transformed the data back into a multiway network. I saved the data in JSON format.

Blockmodeling

I already worked (with Anuška and Pat, in 2006) on the indirect approach to blockmodeling of binary 3-way networks (Github bavla/ibm3m) [3]. Lazega, Krackhardt [4]

We were dealing with 3-way networks also at the INSNA 2009 Viszards session analyzing the Bibsonomy data.

Drilling into the data of my Italian friends I got some ideas for possible approaches.

Examples

- **shoping**: customers, sale items, shops, dates, value
- **books** (Amazon): authors, publishers, dates, price, # pages
- **flights**: departure APs, arrival APs, companies, dates, # of passengers
- **international trade** (WTO): exporters, importers, categories, years, value

Multiway networks

A *weighted multiway network* $N = (V, L, w)$ is based on *nodes* from k pairwise disjoint sets (ways or dimensions)

$$V = \{V_1, V_2, \ldots, V_k\}, \quad V_i \cap V_j = \emptyset \text{ for } i \neq j$$

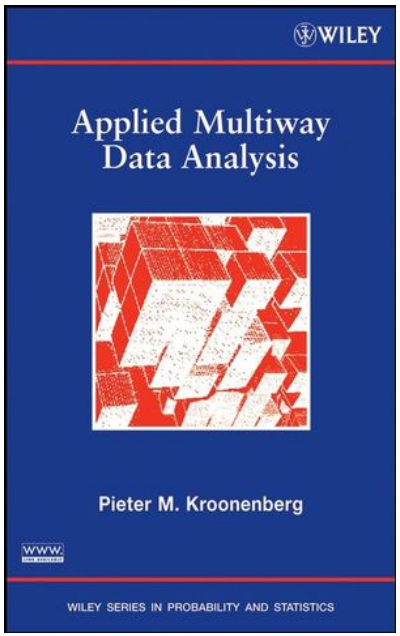
The set of *links* $L \subseteq V_1 \times V_2 \times \cdots \times V_k$. The weight $w : L \to \mathbb{R}$. It can be represented by a k-dimensional array W

$$W[v_1, v_2, \ldots, v_k] = w(v_1, v_2, \ldots, v_k) \text{ for } (v_1, v_2, \ldots, v_k) \in L$$

otherwise $W[v_1, v_2, \ldots, v_k] = 0$.

In a general multiway network, different additional data can be known for nodes and/or links.

Multiway analysis



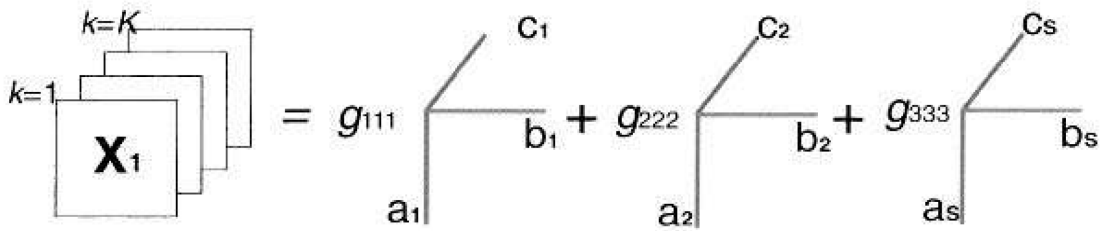


Figure 4.9 The Parafac model.

In data analysis already exists an approach called *multiway analysis* which deals with generalizations of ideas from the multivariate analysis to multiway arrays. They are known as Tucker, Parafac, SimPCA, . . . models [6, 7].

We will continue in another direction – a reduction of multiway networks analysis to weighted networks analysis.

Transformations

In data analysis of multi-way networks, some transformations could prove to be useful:

- slicing
- reordering of ways
- joining the ways
- flattening of a way
- projection to a selected way
- aggregation by a way partition (blockmodeling)
- normalization
- recoding (binarization)

Representation

A natural representation of a weighted multi-way network is by a multi-dimensional array. In real-life networks many (most of) array entries have the value 0. In such cases, this representation is computationally inefficient – takes more space and requires unnecessary computations with 0s.

Because of this, for an experimental implementation of a library for the analysis of weighted multiway networks, I selected the k-tuples representation as a basic data representation. It allows efficient work also with large sparse multiway networks.

Representation

An alternative representation follows the formal definition

```
{
title = "TITLE";

nodes1 = ["v11", ...,"v1n1"];
...
nodesk = ["vk1", ...,"vknk"];
nodes = [ nodes1, ..., nodesk ];
links = [
...
[link: [v1,v2,...,vk], w: W ];
...
]
}
```

lazegaMN.json

Slices

Trivial.

Reordering of ways

Trivial.

Wiki

Joining the ways

Selected ways V_i and V_j, $i < j$, are replaced by a new joint way

$$V_{ij} = \{(u : v) : u \in V_i \wedge v \in V_j \wedge \exists(\ldots, u, \ldots, v, \ldots) \in L\}$$

This transformation reduces the number of ways for 1.

"Commutativity".

Wiki

Flattening of a way

A selected way V_i is removed from the network.

$$V' = \{V_1, V_2, \ldots, V_{i-1}, V_{i+1}, \ldots, V_k\}$$

$$w'(v_1, v_2, \ldots, v_{i-1}, v_{i+1}, \ldots, v_k) = \sum_{v \in V_i} w(v_1, v_2, \ldots, v_{i-1}, v, v_{i+1}, \ldots, v_k)$$

This transformation reduces the number of ways for 1.

"Commutativity", "inequality".

wiki

Projection to a selected way

Because of the reordering option, we can assume that we selected the way V_1. A projection to a selected way is a generalization of the projection of two-mode networks. The projection creates an ordinary weighted network (V_1, A, p), $A \subseteq V_1 \times V_1$ and $p : A \to \mathbb{R}$. Let $u, t \in V_1$ then

$$p(u,t) = \sum_{(v_2,\ldots,v_k) \in V_2 \times \cdots \times V_k} w(u, v_2, \ldots, v_k) \cdot w(t, v_2, \ldots, v_k)$$

This network can be analyzed using traditional methods for the analysis of weighted networks. Sometimes it is more appropriate to apply projection(s) to a normalized version of the original multi-way network.

wiki

The operation can be generalized to the product of compatible multiway networks.

Salton index has the following properties

From the projection p we can get the corresponding measure of similarity – *Salton index* $S(u,t)$ [1]

$$S(u,t) = \frac{p(u,t)}{\sqrt{p(u,u) \cdot p(t,t)}}$$

that can be used for clustering the set V_1. wiki

The Salton index has the following properties

1. $S(u,t) \in [-1,1]$
2. $S(u,t) = S(t,u)$
3. $S(u,u) = 1$
4. $w : L \to \mathbb{R}_0^+ \Rightarrow S(u,t) \in [0,1]$
5. $S(\alpha u, \beta t) = S(u,t), \quad \alpha, \beta > 0$
6. $S(\alpha u, u) = 1, \quad \alpha > 0$

Way partition aggregation

wiki

Normalization

log w

TBA [2]

Recoding

Let $P = \{P_1, P_2, \ldots, P_p\}$ be a partition of $\mathbb{R}$. The recoding transformation transforms the weight function w into a new weight w' determined for a link $(v_1, v_2, \ldots, v_k) \in L$ as

$$w'(v_1, v_2, \ldots, v_k) = i \Leftrightarrow w(v_1, v_2, \ldots, v_k) \in P_i$$

Code 0 corresponds to the case $(v_1, v_2, \ldots, v_k) \notin L$ which is usually equivalent to $w(v_1, v_2, \ldots, v_k) = 0$ in the array representation. If 0 is also a legal weight value we have to introduce another zero, $\square$, that indicates the absence of a link.

A special case is a binarization for which $P_0 = \{0\}$ and $P_1 = \mathbb{R} \setminus P_0$.

Recoding is often used to get more readable matrix visualizations of a given network.

wiki

Conclusions

- relation with operations in databases
- dplyr
- data bases and two-mode networks
- additional examples
- develop libraries in R, Python and Julia

Acknowledgments

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