



Salton index

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Salton cosine index in network analysis

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Current version of slides (June 28, 2025 at 03:44): [slides PDF](#)

<https://github.com/bavla/ibm3m>

The *inner product* of vectors $x, y \in \mathbb{R}^n$ is defined as

$$\langle x, y \rangle = \sum_{i=1}^n x_i \cdot y_i$$

The following four properties hold for all $x, y, z \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$:

- 1 $\langle x, x \rangle \geq 0$ and $\langle x, x \rangle = 0$ if and only if $x = 0$,
- 2 $\langle x, y + z \rangle = \langle x, y \rangle + \langle x, z \rangle$,
- 3 $\langle x, \alpha y \rangle = \alpha \langle x, y \rangle$,
- 4 $\langle x, y \rangle = \langle y, x \rangle$.

An inner product $\langle \cdot, \cdot \rangle$ induces the *norm* of x

$$\|x\| = \sqrt{\langle x, x \rangle}$$

Cauchy-Schwarz inequality

$$|\langle x, y \rangle| \leq \|x\| \cdot \|y\|$$

Salton or cosine index

$$S(x, y) = \frac{\langle x, y \rangle}{\|x\| \cdot \|y\|} \in [-1, 1]$$

The Salton index has the following properties

- 1 $S(x, y) \in [-1, 1]$
- 2 $S(x, y) = S(y, x)$
- 3 $S(x, x) = 1$
- 4 $x, y \in (\mathbb{R}_0^+)^n \Rightarrow S(x, y) \in [0, 1]$
- 5 $S(\alpha x, \beta y) = S(x, y), \quad \alpha, \beta > 0$
- 6 $S(\alpha x, x) = 1, \quad \alpha > 0$

Property 5 says that the value of the Salton index doesn't depend on the size of vectors but on their "shape". It is very useful when analyzing data with units of different "sizes" – for example, trade between world countries.

The Salton index is a measure of similarity.

It is usually transformed into a dissimilarity d using the transformation

$$d(x, y) = \frac{1 - S(x, y)}{2}$$

or the angular distance θ **wp**

$$\theta(x, y) = \frac{\arccos(S(x, y))}{\pi}$$



Matrix based (dis)similarities

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Let $\mathbf{W} = [w[u, v]]$ be a matrix describing (one/two-mode) (weighted) network \mathcal{N} . For clustering units (nodes) we need a dissimilarity matrix \mathbf{D} . For two-mode networks, we can get it from the Salton matrix $\mathbf{S} = [s[u, v]]$ where

$$s[u, v] = S(w[u, \cdot], w[v, \cdot])$$

If the matrix \mathbf{W} is one-mode we have a problem

$$w[u, \cdot] = [w[u, 1], \dots, w[u, i], \dots, w[u, u], \dots, w[u, v], \dots, w[u, k]]$$

$$w[v, \cdot] = [w[v, 1], \dots, w[v, i], \dots, w[v, u], \dots, w[v, v], \dots, w[v, k]]$$

In traditional (dis)similarities, comparing $w[u, i]$ and $w[v, i]$ we are comparing how u relates to i with how v relates to i . The problem arises for $i = u$ and $i = v$. We would need to compare $w[u, u]$ with $w[v, v]$ and $w[u, v]$ with $w[v, u]$. This leads to *corrected* (dis)similarities.

Corrected Salton index of the link $(u, v) \in \mathcal{L}$

$$S'(u, v) = \frac{\langle w[u, \cdot], w[v, \cdot] \rangle + (w[u, u] - w[u, v]) \cdot (w[v, v] - w[v, u])}{\|w[u, \cdot]\| \cdot \|w[v, \cdot]\|}$$

It also has the usual Salton index properties 1-6.

Projections of two-mode networks

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Let $\mathcal{N} = ((\mathcal{U}, \mathcal{V}), \mathcal{L}, w)$ be a weighted two-mode network with a matrix $\mathbf{W} = [w[u, v]]$. A usual approach to its analysis is to *project* it to the set \mathcal{U} or to the set \mathcal{V} and analyze the so obtained ordinary (one-mode) weighted network.

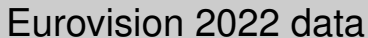
The projection to the set \mathcal{U} is determined by the matrix $\mathbf{C} = [c[u, t]] = \mathbf{W}\mathbf{W}^T$; and to the set \mathcal{V} by the matrix $\mathbf{W}^T\mathbf{W}$.

$\mathcal{N}_C = (\mathcal{U}, \mathcal{L}_C, c)$, where $\mathcal{L}_C = \{(u, t) : c[u, t] \neq 0\}$ and for $(u, t) \in \mathcal{L}_C : c(u, t) = c[u, t]$.

$$c[u, t] = \sum_{v \in \mathcal{V}} w[u, v] \cdot w^T[v, t] = \sum_{v \in \mathcal{V}} w[u, v] \cdot w[t, v] = \langle w[u, \cdot], w[t, \cdot] \rangle$$

from which we can get the corresponding Salton matrix

$$s[u, v] = \frac{c[u, v]}{\sqrt{c[u, u] \cdot c[v, v]}}$$



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

















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Total

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1		Ukraine	631	17	16	19	12	18	18	12	18	19	16	17	12	12	22	18	22	10	22	15	19	12	24	24	8	24	16	12	8	15	24	20	22	14	7	18	12	12	13	12		
2		United Kingdom	466	14	1	7	20	20	15	13		9	14	12	9	14	14	16	18	5	14	14	20	12	11	14	20	13	5	12	8	12	11	16	11	14	1	2	11	14	11	19		
3		Spain	459	12	20	18	3	15	16	16	13	14	13	5	1	3	12	11	10	17	6	17	11		4	11	19	11	16	12	22	8	7	22	9	22	14	5		14	14	1	15	
4		Sweden	438	8	12	15	13	12	12			9	5	15	17	22	20		8	12	4	20	13	17	5	16	13	15	13	8	3	10	16	20	7	16	8	6	5	13		10	4	16
5		Serbia	312	3	6	16	10	6	6	10	24	11	10				14	8	7	7	8	1	6	7	11	2	1	6	24	3	22	3	5	8	7	8		18	8	12	12	2		
6		Italy	268	20	10	4	6	15	12	3	10	6					10	2	3	10	3	8	6	6	11		7	17	16	13		4	3	8	8	4	18	15		10				
7		Moldova	253	1	6	4	7		8	7	8		8	5	6	5	10	8	10	9	6	7	6	15	7	5			7	12	7	4	7	8	12	4	12	7	7		4	6	8	
8		Greece	215	12	5		4	6		18	3	22	3	12	6	6	3	4							10	7		1		12	19	2	4	2	17		12	2		12	7	4		
9		Portugal	207	1	12	6	10	7	6	4	10		5	8	7		7	7		3	5	4		1	6	15		2	9	8		10	8		7	7	3	4		12	10	3		
10		Norway	182		10	10	7		3	5		5	8	4	10			4	6	18	4	4	10	2	4	6	2	8	2	4	2	2	2	8	3	3	1	3	13	1	2	6		
11		Netherlands	171	12	4		8	4	10			2		10	4	5	4	4	8	4	9		3	14	4	7			3		5	3	11	3		8		1	10	11				
12		Poland	151			3	4	3	7			3	7	4			12	2	8	10	4	10	7	3			6		4	6	2	5			4		1	5	1	12	18			
13		Estonia	141	5	15	1						2	12	3	10					8	8	5	12	3	1	3	2	8		5	16					10	5		5	7				
14		Lithuania	128		2	2						10	2		10	9	2		5		3	8			11		4	2		1	12	2	6			5	12	3		10	7			
15		Australia	125					2	1			8	1		8	8			5	8		2	5	2	2		6	6		7	5		6		6	6			8	10	4	3	6	
16		Azerbaijan	106	3		2			3	3	10		3	3	3	6	12									2	1	1		4					7	12		12	7		5	7		
17		Switzerland	78				1			7	1	6	2			2						3	3	7			7	10	1	1	5	2	5	1		5	1		6	2				
18		Romania	65						4	4						1	1	9		2	8					3	10								1	5		14			3			
19		Belgium	64	2		5			6	2						6					4	7					5		10	3					8		5	1						
20		Armenia	61	4		6			10	1				1	12	10								8							4		1						2	2				
21		Finland	38	2								1		13		2								1	1	1										6		7		7	4			
22		Czech Republic	38			4			2							2			2	2	7	2					3				5					3			5		1			
23		Iceland	20									3											1				2						6								8			
24		France	17		7		1		1					1	1	1													2							6			1	2				
25		Germany	6				2						2																													2		

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Corrected Salton 2-neighbors

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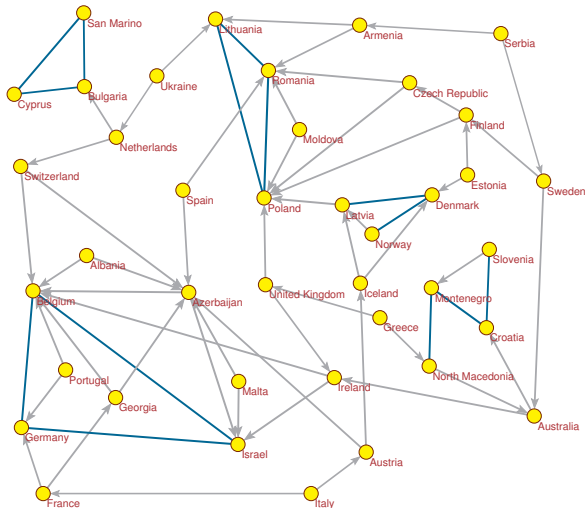
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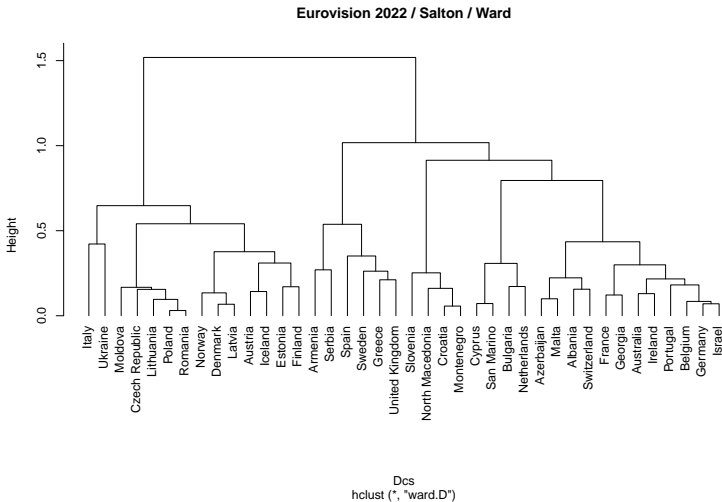
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Matrix network display

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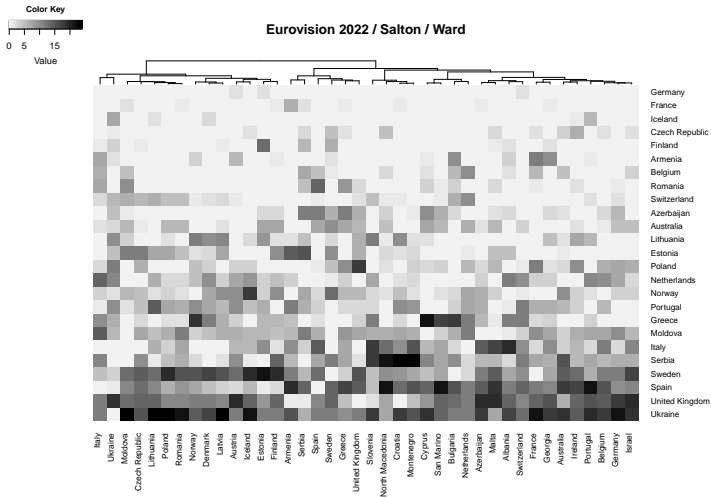
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A **weighted multiway network** $\mathcal{N} = (\mathcal{V}, \mathcal{L}, w)$ is based on **nodes** from k finite sets (ways or dimensions) $\mathcal{V} = (\mathcal{V}_1, \mathcal{V}_2, \dots, \mathcal{V}_k)$, the set of **links** \mathcal{L} , and the **weight** $w : \mathcal{L} \rightarrow \mathbb{R}$. The incidence function $l : \mathcal{L} \rightarrow \mathcal{V}_1 \times \mathcal{V}_2 \times \dots \times \mathcal{V}_k$ assigns to each link $e \in \mathcal{L}$ a k -tuple of its nodes $l(e) = (e(1), e(2), \dots, e(i), \dots, e(k))$, $e(i) \in \mathcal{V}_i$. If for $i \neq j$, $\mathcal{V}_i = \mathcal{V}_j$, we say that \mathcal{V}_i and \mathcal{V}_j are of the same **mode**.

In a general multiway network, different additional data can be known for nodes and/or links $\mathcal{N} = (\mathcal{V}, \mathcal{L}, \mathcal{P}, \mathcal{W})$, where \mathcal{P} is a set of node properties $p : \mathcal{V}_i \rightarrow S_p$, and \mathcal{W} is a set of link weights $w : \mathcal{L} \rightarrow S_w$.



MWnets

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To support the analysis of multi-way networks, I started developing an R package **MWnets**. It is available at GitHub/Bavla

<https://github.com/bavla/ibm3m>

```
> source("https://raw.githubusercontent.com/bavla/ibm3m/master/multiway/MWnets.R")
```

Multiway network data sets

<https://github.com/bavla/ibm3m/tree/master/data>

For user-friendly inspection of a JSON file, you can use the Firefox browser or application JSON in Chrome.

Projection to a selected way

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Because of the reordering option, we can assume that we selected the way \mathcal{V}_1 . A projection to a selected way is a generalization of the projection of two-mode networks. The projection creates an ordinary weighted network $(\mathcal{V}_1, \mathcal{A}, p)$, $\mathcal{A} \subseteq \mathcal{V}_1 \times \mathcal{V}_1$ and $p : \mathcal{A} \rightarrow \mathbb{R}$. Let $u, t \in \mathcal{V}_1$ then

$$p(u, t) = \sum_{(v_2, \dots, v_k) \in \mathcal{V}_2 \times \dots \times \mathcal{V}_k} w(u, v_2, \dots, v_k) \cdot w(t, v_2, \dots, v_k)$$

Note that the projection network is symmetric $p(u, t) = p(t, u)$ and considering that the right side in the definition of $p(u, t)$ is a inner (scalar) product we get $p(u, t) \leq \sqrt{p(u, u) \cdot p(t, t)}$ – Cauchy-Schwarz inequality. From it it follows $p(u, u) = p(t, t) = 0 \Rightarrow p(u, t) = 0$.

In computing the projection, we consider only existing links

$$p(u, t) = \sum_{(v_2, \dots, v_k) : ((u, v_2, \dots, v_k) \in \mathcal{L} \wedge (t, v_2, \dots, v_k) \in \mathcal{L})} w(u, v_2, \dots, v_k) \cdot w(t, v_2, \dots, v_k)$$

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$$p(u, u) \geq 0.$$

$$(\exists z \in \{z : (u, z) \in \mathcal{L}\} : w(u, z) \neq 0) \rightarrow p(u, u) > 0.$$

This network can be analyzed using traditional methods for analyzing weighted networks. Sometimes it is more appropriate to apply projection(s) to a normalized version of the original multi-way network.

From the projection p we can get the corresponding measure of similarity – **Salton index** $S(u, t)$ **Batagelj and Cerinšek (2013)**

$$S(u, t) = \frac{p(u, t)}{\sqrt{p(u, u) \cdot p(t, t)}}$$

that can be used for clustering the set \mathcal{V}_1 . In the case $p(u, u) = p(t, t) = 0$ we set $S(u, t) = 1$.



European airports and flight companies dataset

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The *European airports and flight companies* dataset was collected by A. Cardillo and his collaborators for the paper

A. Cardillo, J. Gómez-Gardeñes, M. Zanin, M. Romance, D. Papo, F. del Pozo, S. Boccaletti: Emergence of Network Features from Multiplexity, Scientific Reports 3, 1344 (2013).

It is an example of a multiway network. It contains 450 airports making two ways `airA` (departure airports) and `airB` (arrival airports), the third way is `line` (37 flight companies). There are 7176 links $(from, to, line) \in \mathcal{L}$ telling that there is a flight provided by the company *line* from the airport *from* to the airport *to*. Each link has the weight $w = 1$.

The multiway network `AirEu2013` represented in JSON format and an R package `MWnets` for analysis of multiway networks are available at [GitHub/Bavla](https://github.com/Bavla).



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```

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 $ info :List of 7
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 ..$ title : chr "Air Transportation Multiplex"
 ..$ ref : chr "Cardillo A. et al. Emergence of network features from multiplexit
 ..$ href : chr "http://complex.unizar.es/~atnmultiplex/"
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 ..$ airB: chr "second airport"
 ..$ line: chr "airline"
 $ nodes :List of 3
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 .. ..$ ID : chr [1:450] "LCLK" "EDDF" "EDDK" "EGNX" ...
 .. ..$ lon : num [1:450] 33.63 8.57 7.14 -1.33 -3.41 ...
 .. ..$ lat : num [1:450] 34.9 50 50.9 52.8 50.7 ...
 .. ..$ iata : chr [1:450] "ICA" "FRA" "CGN" "EMA" ...
 .. ..$ long : chr [1:450] "Larnaca International Airport" "Frankfurt Airport" "Co
 .. ..$ region : chr [1:450] "Larnaka" "Hessen" "Nordrhein-Westfalen" "England" ...
 .. ..$ country: chr [1:450] "CY" "DE" "DE" "GB" ...
 ..$ airB:'data.frame': 450 obs. of 2 variables:
 .. ..$ ID : chr [1:450] "LCLK" "EDDF" "EDDK" "EGNX" ...
 .. ..$ long: chr [1:450] "Larnaca International Airport" "Frankfurt Airport" "Colog
 ..$ line:'data.frame': 37 obs. of 1 variable:
 .. ..$ ID: chr [1:37] "Lufthansa" "Ryanair" "Easyjet" "British A" ...
 $ links :'data.frame': 7176 obs. of 4 variables:
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 ..$ airB: int [1:7176] 2 38 1 7 8 10 14 15 17 18 ...
 ..$ line: int [1:7176] 1 1 1 1 1 1 1 1 1 1 ...
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 .. ..$ alpha3 : chr [1:40] "AUT" "BIH" "BEL" "BGR" ...
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 .. ..$ long : chr [1:40] "Austria" "Bosnia and Herzegovina" "Belgium" "Bulgaria
 .. ..$ region : chr [1:40] "Europe" "Europe" "Europe" "Europe" ...
 .. ..$ subregion: chr [1:40] "Western Europe" "Southern Europe" "Western Europe" "

```



3D layout of AirEu2013

Airports dendrogram, 3D layout

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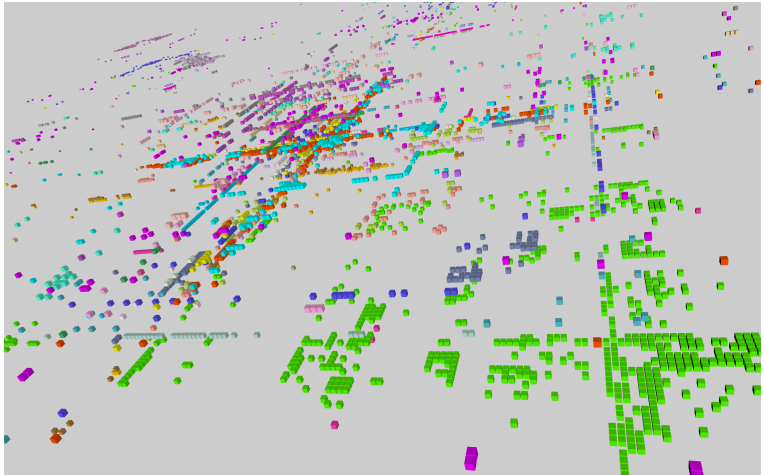
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The green links in the bottom right corner are provided by Ryan air – it is mostly linking airports that are not linked by other companies.

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The computational work reported in this presentation was performed in R using the packages `igraph`, `netsWeight`, and `MwNets`. The code and data are available at <https://github.com/bavla>.

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