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# Analysis of weighted temporal networks represented by time slices

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**Compstat 2023**

22-25 August 2023, Birkbeck, London



# Outline

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Current version of slides (August 23, 2023 at 10:58): [slides PDF](#)

<https://github.com/bavla/ibm3m/>



# World trade datasets

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We will discuss some approaches to **exploratory analysis** of weighted dense temporal networks of moderate size. For illustration, we use world trade data.

WTO and UN comtrade and some other institutions are collecting world trade data. The problem with the collected data is missing data and inconsistencies.

An early version of world trade flows was provided by NBER ([Feenstra et al.(2005)]; [download](#)).

The French research center CEPII is providing a consolidated version of the UN comtrade data as a database [BACI/CEPII](#) [[Gaulier and Zignago\(2010\)](#)].

An application of the network analysis to the BACI data was done already by de Benedictis et al.

[[De Benedictis et al.\(2014\)](#), [Hoang et al.\(2023\)](#)].



# BACI data

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Each BACI trade flow is described by 6 quantities:

- Year,
- Product category (HS 6-digit code),
- Exporter (ISO 3-digit country code),
- Importer (ISO 3-digit country code),
- Value of the trade flow (in thousands current USD),
- Quantity (in metric tons)

HS codes: [Harmonized Commodity Description and Coding Systems \(HS\)](#), [Classifications on economic statistics](#), [Harmonized System Codes \(HS Code 2017 - Current\)](#).

Currently, there are around 5300 different HS codes. They are hierarchically organized. The 97 2-digit codes are also called chapters. They are joined into 21 sections. This enables the user to study the data at different granularity levels. In this presentation, we will limit our attention to the top level – the values of total trade flows between countries in a year.



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The BACI data essentially determine a *weighted temporal multi-relational network*  $\mathcal{N} = (\mathcal{V}, (\mathcal{L}_h)_{h \in HS}, w, \mathcal{T})$  where  $\mathcal{V}$  is the set of world countries, relation  $\mathcal{L}_h$  is the set of links with the HS code  $h$ ,  $w : \mathcal{L} \rightarrow \mathbb{R}^+$  is the flow value (in thousands current USD), and  $\mathcal{T} = 1996 : 2021$  is the observed range of years.

In a temporal network, nodes  $v \in \mathcal{V}$  and links  $e \in \mathcal{L}$  are not necessarily present or active in all time points. Let  $T(v)$  be the *activity set* of time points for node  $v$  and  $T(e)$  the activity set of time points for link  $e \in \mathcal{L}$ . Besides the presence/absence of nodes and links also their properties can change through time – the weight  $w$  becomes a *temporal quantity* [Batagelj and Praprotnik(2016)]

$$w(t) = \begin{cases} w'(t) & t \in T(w) \\ \text{\#} & t \in \mathcal{T} \setminus T(w) \end{cases}$$

where  $T(w)$  is the *activity time set* of  $w$ ,  $w'(t)$  is the value of  $w$  in an instant  $t \in T(w)$ , and  $\text{\#}$  denotes the value *undefined*.



# Weighted temporal multi-relational network

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A *time slice* of  $\mathcal{N}$  in an instant  $t \in \mathcal{T}$  is called the network  $\mathcal{N}(t) = (\mathcal{V}(t), \mathcal{L}(t), w)$  where  $\mathcal{V}(t) = \{v \in \mathcal{V} : t \in T(v)\}$  and  $\mathcal{L}(t) = \{e \in \mathcal{L} : t \in T(e)\}$ .

They can be also considered weighted multiway network data on four sets of nodes Exporters, Importers, HS codes, Years, and the flow value as a weight.

We converted the BACI data into Pajek format at global, section, and chapter levels ([to Pajek](#)).

In this presentation, we limit our attention to time slices of the **globally aggregated network**.



# World trade 2019 graph

## spring embedder

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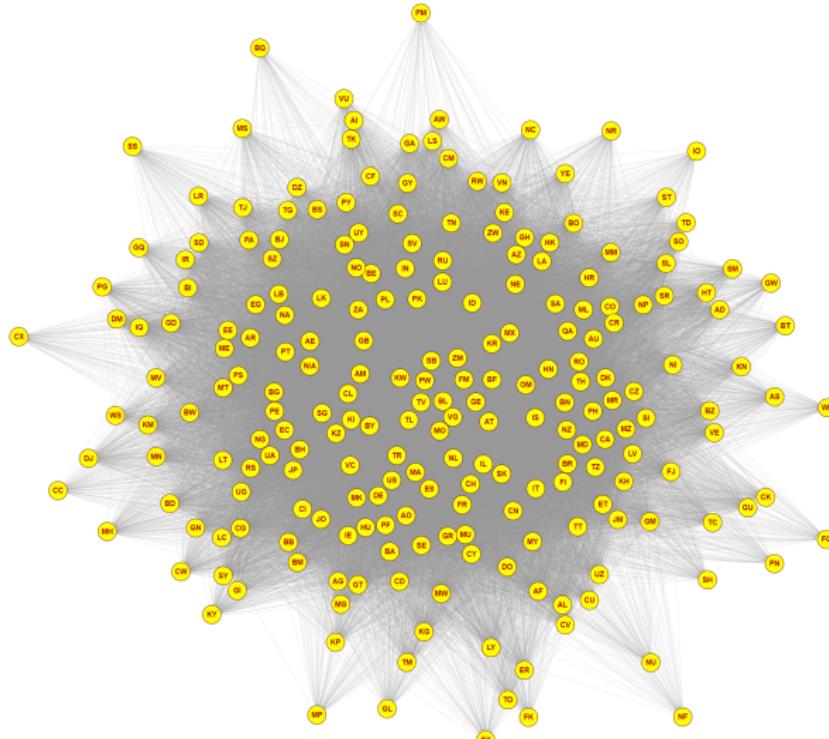
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# Binary matrix layout

## nodes ordered by wdeg

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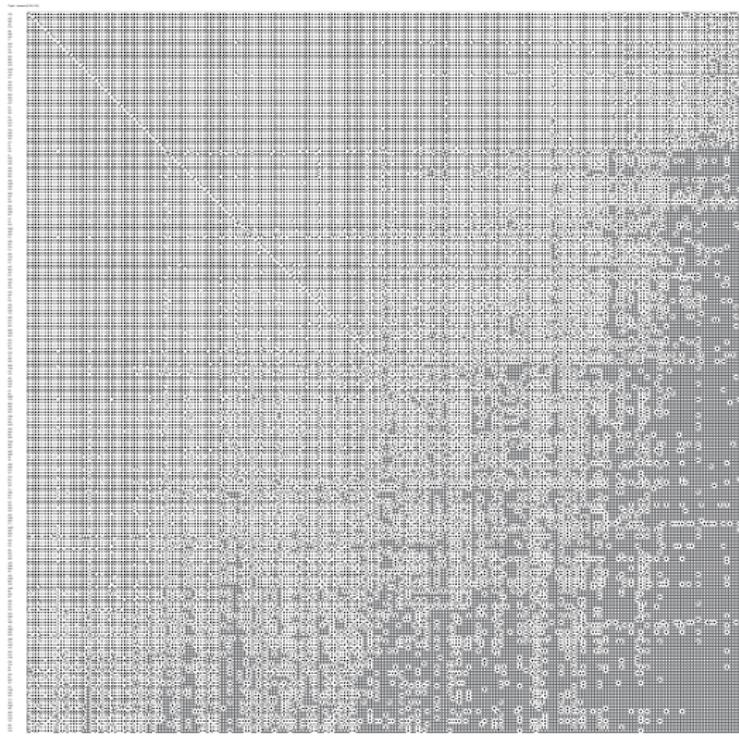
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# Comments

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- Larger,  $n > 20$ , dense graphs can't be presented readably with a graphical layout. For WT2019 the number of nodes is  $n = 226$  and the density  $\gamma = 0.6231$ . For dense graphs of moderate size (up to some hundreds of nodes) a better option is the matrix representation.
- What about weights? They can be represented by link thickness or levels of grey of matrix cells. The problem is a very large range and the distribution of weights – most weights give almost white cells. For WT2019  $w_{\min} = 0.001$  and  $w_{\max} = 430317776$ . Monotonic transformations such as  $w' = a \cdot w$ ,  $a > 0$  or  $w' = \sqrt{w}$  or  $w' = \log(w)$ , etc. In our case, we used  $w' = (1000 \cdot w)^{0.08}$ .
- A better ordering of rows can be obtained by network clustering [Batagelj et al.(2014)]. Additional reordering of subtrees can be made manually using Pajek by reordering nodes in the hierarchy.



# Pajek

## network graph and binary matrix layout

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```
File/Network/Read [WTtime.net]
Network/Temporal network/Generate in time/Interval [2019,2019] [OK]
File/Network/Change label [WT19]
Draw/Network
save the picture
Network/Create vector/Centrality/Weighted degree/All
Vector/Make permutation
Permutation/Mirror
Network/Create new network/Transform/Line values/Set all to 1
File/Network/Change label [WT19bin]
File/Network/Export as matrix/EPS/Using permutation [WT19bin.eps]
Network/Info/Line values
```

The extracted WT network for the year 2019 has 226 nodes and  $w_{\min} = 0.001$  and  $w_{\max} = 430317776$ . The range is of order  $10^{12}$ . To map it into the interval  $[1, 10]$  using the power law transformation  $w' = w^k$  we have to select  $k \approx 1/12 \approx 0.08$ .



# Pajek

## transforming weights

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We create two transformed networks, WT19t – transformed WT19, and WT19s – the network WT19 is first symmetrized by adding the weights of opposite arcs and afterward transformed.

```
select the network WT19
Network/Create new network/Transform/Line values/Multiply by [1000]
Network/Create new network/Transform/Line values/Power [0.08]
File/Network/Change label [WT19t]
select the network "Multiply by [1000]"
Network/Create new network/Transform/Arcs -> Edges/All [1]
Network/Create new network/Transform/Line values/Power [0.08]
File/Network/Change label [WT19s]
```



# Weighted matrix

order determined by clustering + manual editing

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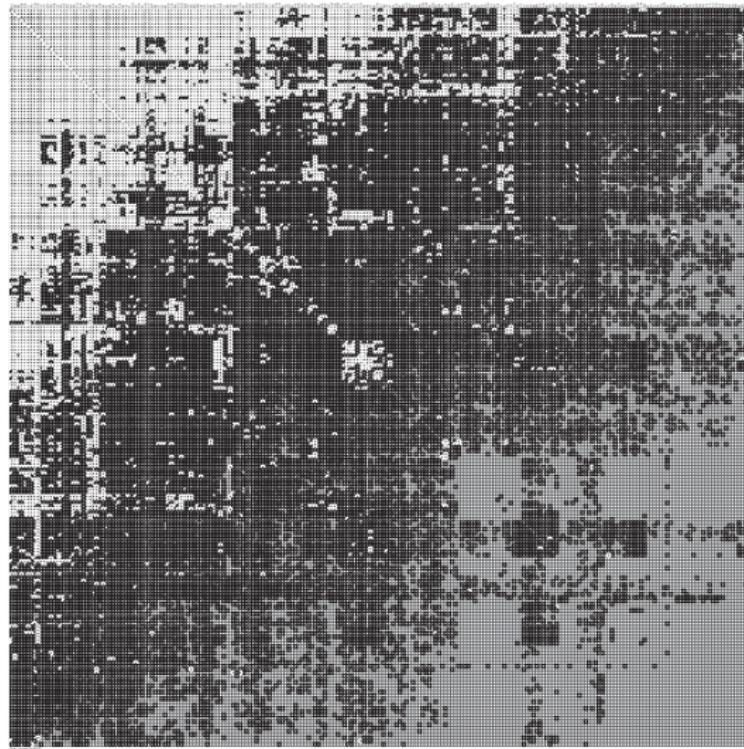
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- center-periphery
- blocks

LC Lucia	CR Costa Rica	AF Afghanistan	AI Anguilla
VC S Vincent Grenad	DO Dominican R	UZ Uzbekistan	TC Turks Caicos
AG Antigua Barbuda	EC Ecuador	KG Kyrgyzstan	MS Montserrat
BB Barbados	GT Guatemala	MN Mongolia	AS American Samoa
BM Bermuda	UY Uruguay	TJ Tajikistan	GU Guam
GY Guyana	CO Colombia	TM Turkmenistan	TK Tokelau
GD Grenada	PE Peru		KP DPR Korea
BZ Belize	PA Panama		KY Cayman Is
SR Suriname	TT Trinidad Tobago		KN S Kitts Nevis
JM Jamaica	SV El Salvador		VG Br Virgin Is
BS Bahamas	HN Honduras		DM Dominica
	NI Nicaragua		CW Curaçao
	BO Bolivia		AW Aruba



# Pajek

## network clustering

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```
Select the network WT19s
Cluster/Create complete cluster [226] [OK]
Operations/Network+cluster/Dissimilarity/Network based/d5
[1] [OK] [dendro.eps]
Select the transformed WT19
File/Network/Export as matrix/Options/Labels up-right
File/Network/Export as matrix/EPS/Using permutation [WT19mat.eps]
```

Double-click on the clustering in the Hierarchy register. Drill into the structure of the clustering. Select a subtree node and using Edit/Push Up or Edit/Push Down change the order of subtrees.  
When done

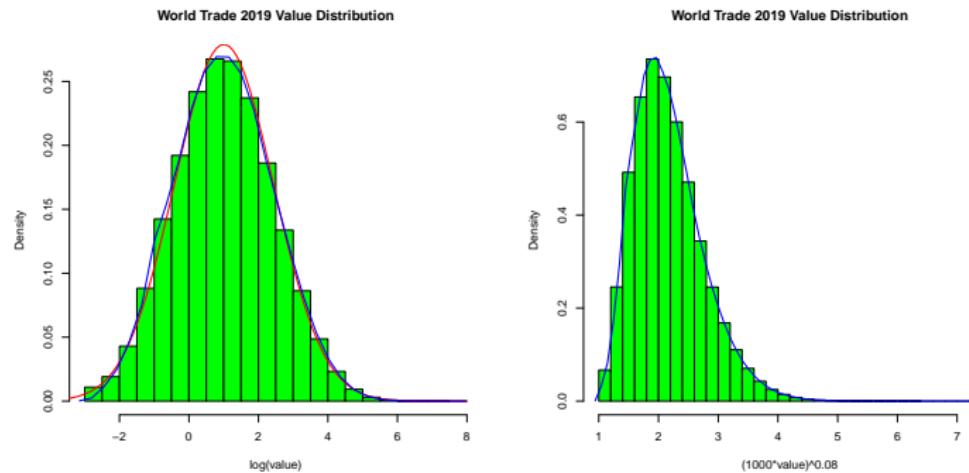
```
Hierarchy/Make permutation
File/Network/Export as matrix/EPS/Using permutation [WT19mat.eps]
```

# World trade 2019 all flows value distribution

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# World trade 2019 all flows value distribution

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```
> source("https://raw.githubusercontent.com/bavla/Rnet/master/R/Pajek.R")
> C <- read.csv("country_codes_V202301.csv")
> B <- read.csv(unzip("../BACI_HS92_V202301.zip",
+   files="BACI_HS92_Y2019_V202301.csv"))
> str(B)
'data.frame': 10823621 obs. of 6 variables:
...
$ v: num 1.596 1.114 0.484 6.153 2.399 ...
...
> library(ggplot2)
> hist(log10(B$v), col="green", border="black", prob=TRUE,
+   xlab="log(value)", main="World Trade 2019 Value Distribution")
> m <- mean(log10(B$v)); s <- sd(log10(B$v))
> m
[1] 1.005583
> s
[1] 1.429857
> curve(dnorm(x,m,s), from=-4, to=8, lwd=2, col="red", xaxt="n", yaxt="n",
+   add=TRUE)
> lines(density(log10(B$v), n=32), lwd=2, col="blue")
> hist((1000*B$v)**0.08, col="green", border="black", prob=TRUE,
+   xlab="(1000*value)^0.08", main="World Trade 2019 Value Distribution")
> lines(density((1000*B$v)**0.08, n=64), lwd=2, col="blue")
```

# World trade 2019 aggregated flows distribution

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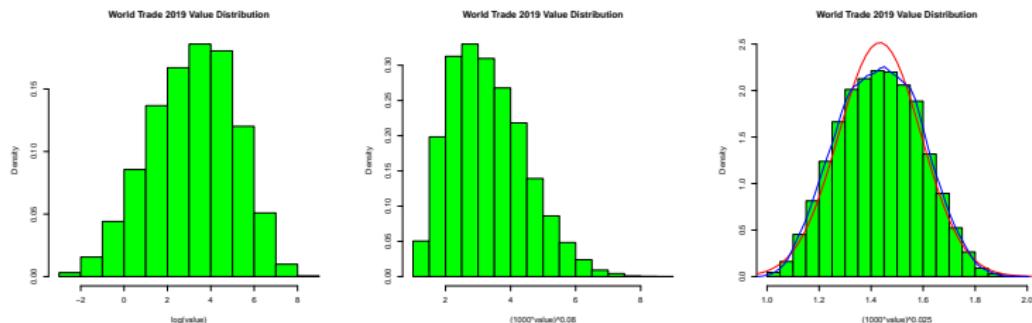
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# World trade 2019 aggregated flows distribution

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```
> library(ggplot2)
> P <- net2matrix("WT2019.net"); V <- P[P>0]
> hist(log10(V),col="green",border="black",prob=TRUE,
+   xlab="log(value)",main="World Trade 2019 Value Distribution")
> hist((1000*V)**0.08,col="green",border="black",prob=TRUE,
+   xlab="(1000*value)^0.08",main="World Trade 2019 Value Distribution")
> a <- 0.025; T <- (1000*V)**a
> hist(T,col="green",border="black",prob=TRUE,ylim=c(0,2.5),
+   xlab="(1000*value)^0.025",main="World Trade 2019 Value Distribution")
> m <- mean(T); s <- sd(T)
> m
[1] 1.433429
> s
[1] 0.1585575
> curve(dnorm(x,m,s),from=0.6,to=2.6,lwd=2,col="red",xaxt="n",yaxt="n",
+   add=TRUE)
> lines(density(T,n=64),lwd=2,col="blue")
```



# Corrected Salton index

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*Corrected Salton index* of the link  $(u, v) \in \mathcal{L}$

$$S[u, v] = \frac{w(u, .) \bullet w(v, .) + (w[u, u] - w[u, v]) \cdot (w[v, v] - w[v, u])}{\sqrt{w[u, .]^2 \cdot w[v, .]^2}}$$

where  $\mathbf{x} \bullet \mathbf{y} = \sum_i x_i \cdot y_i$  and  $\mathbf{x}^2 = \mathbf{x} \bullet \mathbf{x}$ .



# World trade 2019 Salton

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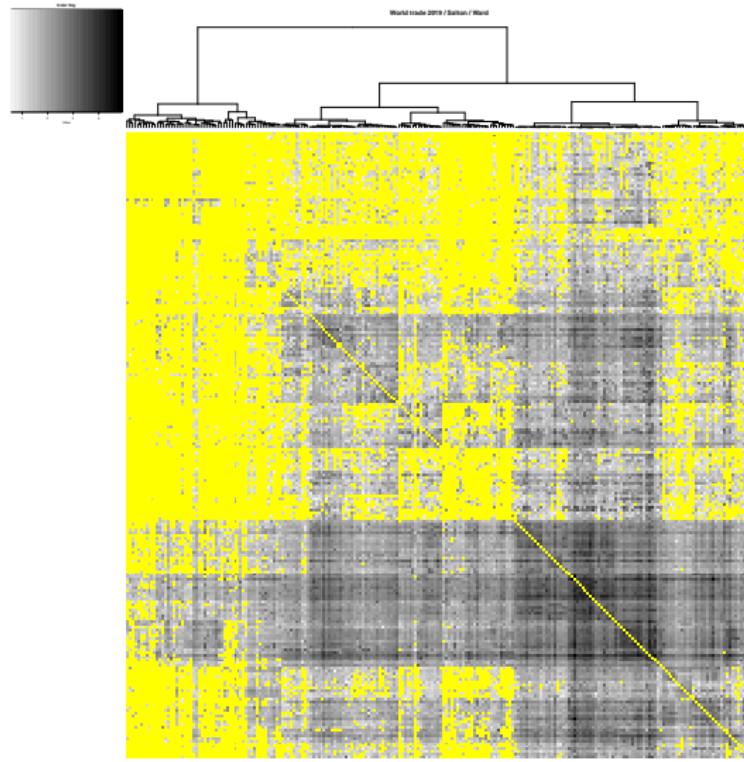
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```
> source("https://raw.githubusercontent.com/bavla/Rnet/master/R/Pajek.R")
> library(gplots)
> Salton <- function(W,p=1){
+   S <- W; diag(S) <- 1; n = nrow(S)
+   for(u in 1:(n-1)) for(v in (u+1):n) S[v,u] <- S[u,v] <-
+     (as.vector(W[u,]*%*%W[v,])+(W[u,u]-W[u,v])*(W[v,v]-W[v,u]))/
+     sqrt(as.vector(W[u,]*%*%W[u,])*as.vector(W[v,]*%*%W[v,]))
+   return(S)
+ }

> Z <- P <- net2matrix("WT2019.net")
> n <- nrow(P); diag(P) <- 0
> for(u in 1:n) for(v in 1:n) Z[u,v] <- P[u,v]**0.08
> t <- hclust(1-as.dist(Salton(Z)),method="ward.D")
> Z[P == 0] <- NA
> pdf(file="WT19salt.pdf",width=30,height=30)
> heatmap.2(Z,Rowv=as.dendrogram(t),Colv="Rowv",
+ dendrogram="column",scale="none",revC=TRUE,
+ col=colorpanel(30,low="grey95",high="black"),na.color="yellow",
+ trace="none", density.info="none", keyszie=0.8, symkey=FALSE,
+ main=paste("World trade ",2019," / Salton / Ward",sep=""))
> dev.off()
```

# Normalizations – activity or Balassa index

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In networks with weights with a large range usually a few strong nodes prevail. To diminish or neutralize the influence of size on results different normalizations were proposed and used [Batagelj and Mrvar(2003), Matveeva et al.(2023)].

Let  $T = \sum_{e \in \mathcal{L}} w(e)$  and for  $(u, v) \in \mathcal{L}$  (Balassa index)

$$A(u, v) = \frac{w(u, v) \cdot T}{\text{woutdeg}(u) \cdot \text{windeg}(v)}$$

then the *activity normalization*  $w'$

$$w'(u, v) = \log_2 A(u, v)$$

Corrected Euclidean distance

$$D[u, v] = \sqrt{(w[u, v] - w[v, u])^2 + (w[u, u] - w[v, v])^2 + \sum_{t: t \neq u, t \neq v} (w[u, t] - w[v, t])^2}$$



# World trade 2019 activity

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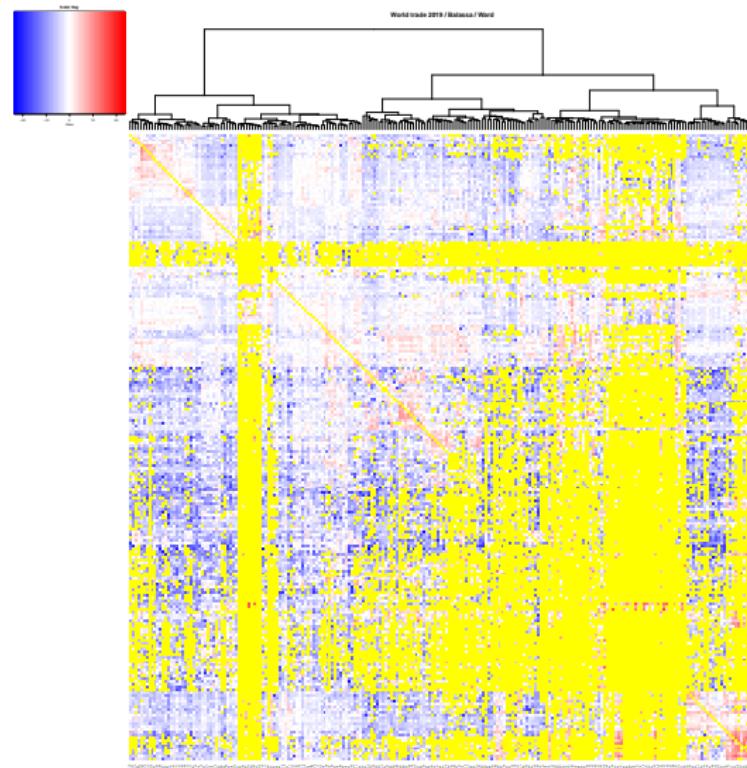
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# Activity or Balassa index

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```
> source("https://raw.githubusercontent.com/bavla/Rnet/master/R/Pajek.R")
> library(gplots)
> CorEu <- function(W,p=1){
+   D <- W; diag(D) <- 0; n = nrow(D)
+   for(u in 1:(n-1)) for(v in (u+1):n) D[v,u] <- D[u,v] <-
+     sqrt(sum((W[u,]-W[v,])**2) -
+       (W[u,u]-W[v,u])**2 - (W[u,v]-W[v,v])**2 +
+       p*((W[u,u]-W[v,v])**2 + (W[u,v]-W[v,u])**2))
+   return(D)
+ }
> P <- net2matrix("WT2019.net"); diag(P) <- 0
> D <- rowSums(P); T <- sum(D); n <- nrow(P)
> for(u in 1:n) for(v in 1:n) P[u,v] <- P[u,v]*T/D[u]/D[v]
> Z <- log2(P); Z[Z == -Inf] <- 0;
> t <- hclust(as.dist(CorEu(Z)),method="ward.D")
> Z[Z == 0] <- NA
> pdf(file="WT19ba.pdf",width=30,height=30)
> heatmap.2(Z,Rowv=as.dendrogram(t),Colv="Rowv",dendrogram="column",
+   scale="none",revC=TRUE,col=bluered(100),na.color="yellow",
+   trace="none", density.info="none", keyszie=0.8, symkey=FALSE,
+   main=paste("World trade ",2019," / Balassa / Ward",sep=""))
> dev.off()
```



# Skeletons

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To get insight into the structure of a large network we can reduce it to its skeleton by removing less important links and/or nodes [Batagelj (2011)].

- Most often the spanning tree, link cut, or node cut are used.
- In the closest  $k$ -neighbor skeleton for each node only the largest  $k$  incident links are preserved. Invariant for monotonic transformations.
- The Pathfinder algorithm was proposed in the 1980s by Schvaneveldt [Schvaneveldt et al.(1988), Schvaneveldt(1990), Batagelj et al.(2014)]. It removes from the network with a dissimilarity weight all links that do not satisfy the triangle inequality – if a shorter path exists that connects the link's end-nodes then the link is removed.
- Cores are a very efficient tool to determine the most cohesive (active) subnetworks [2]. The subset of nodes  $\mathbf{C} \subseteq \mathcal{V}$  induces a  $P_s$  core at level  $t$  if for all  $v \in \mathbf{C}$  it holds  $w\deg_{\mathbf{C}}(v) \geq t$ , and  $\mathbf{C}$  is the maximum such subset.



# $P_s$ cores

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	Rank	Id	1995	Value	Id	2019	Value
	1	US	257693269.0000		MX	658761422.0000	
	2	CA	257693269.0000		US	658761422.0000	
	3	GB	234199078.8000		CN	658761422.0000	
	4	JP	234199078.8000		CA	658761422.0000	
	5	IT	234199078.8000		JP	520093954.0000	
	6	DE	234199078.8000		KR	504015412.0000	
	7	BE	234199078.8000		DE	504015412.0000	
	8	FR	234199078.8000		GB	500566393.0000	
	9	NL	234199078.8000		FR	500566393.0000	
	10	N/A	162498504.7000		NL	500566393.0000	
	11	KR	162498504.7000		IT	468723319.0000	
	12	HK	162498504.7000		BE	452244774.0000	
	13	CN	162498504.7000		HK	441557136.3000	
	14	ES	143785957.0000		N/A	435748876.4000	
	15	CH	136849606.8000		ES	403209053.3000	
	16	SG	136849606.8000		CH	398697522.1000	
	17	MX	135153628.7000		VN	398697522.1000	
	18	MY	126873428.8500		SG	395561634.0000	
	19	TH	104745321.6000		IN	395561634.0000	
	20	AT	94004220.3200		AU	395561634.0000	
	21	SE	93440360.4800		MY	383134139.7000	
	22	AU	87125103.4300		RU	383134139.7000	
	23	ID	79999822.3200		PL	359390045.2000	
	24	BR	70865384.5000		TH	343993122.8000	
	25	DK	68975678.4900		AE	310082847.8000	
	26	IE	68975678.4900		ID	310082847.8000	
	27	NO	68022683.4700		BR	310082847.8000	
	28	SA	67949085.4900		SA	310082847.8000	
	29	RU	64736216.2870		CZ	274596220.6000	
	30	FI	61497981.5330		AT	274174974.0100	

## WT19t closest neighbor skeleton

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## Matrix layout

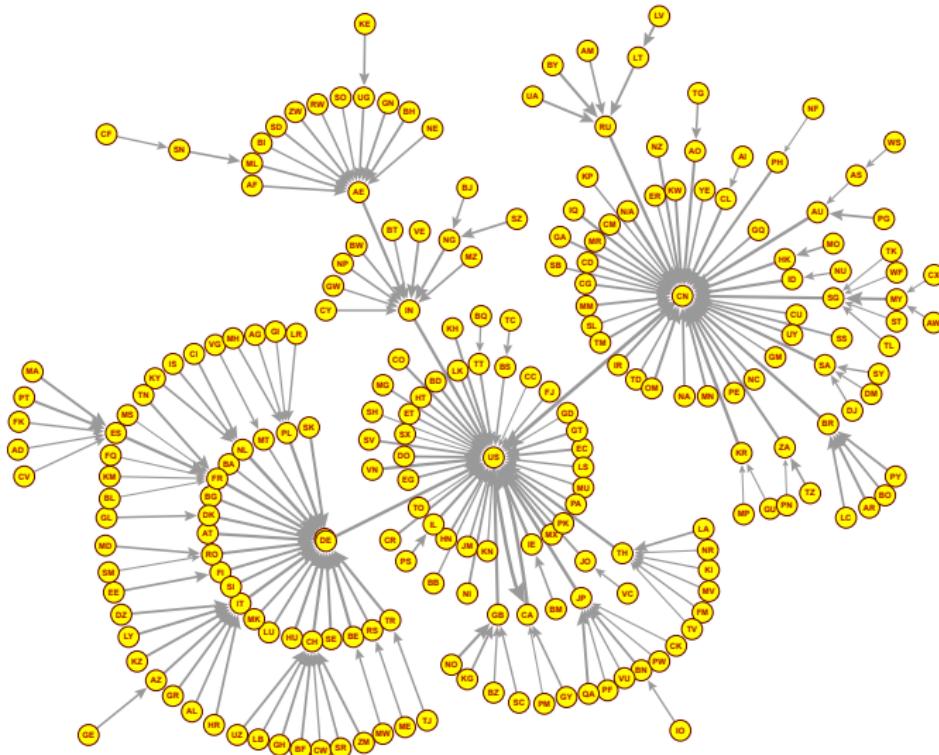
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# World trade closest neighbor skeletons

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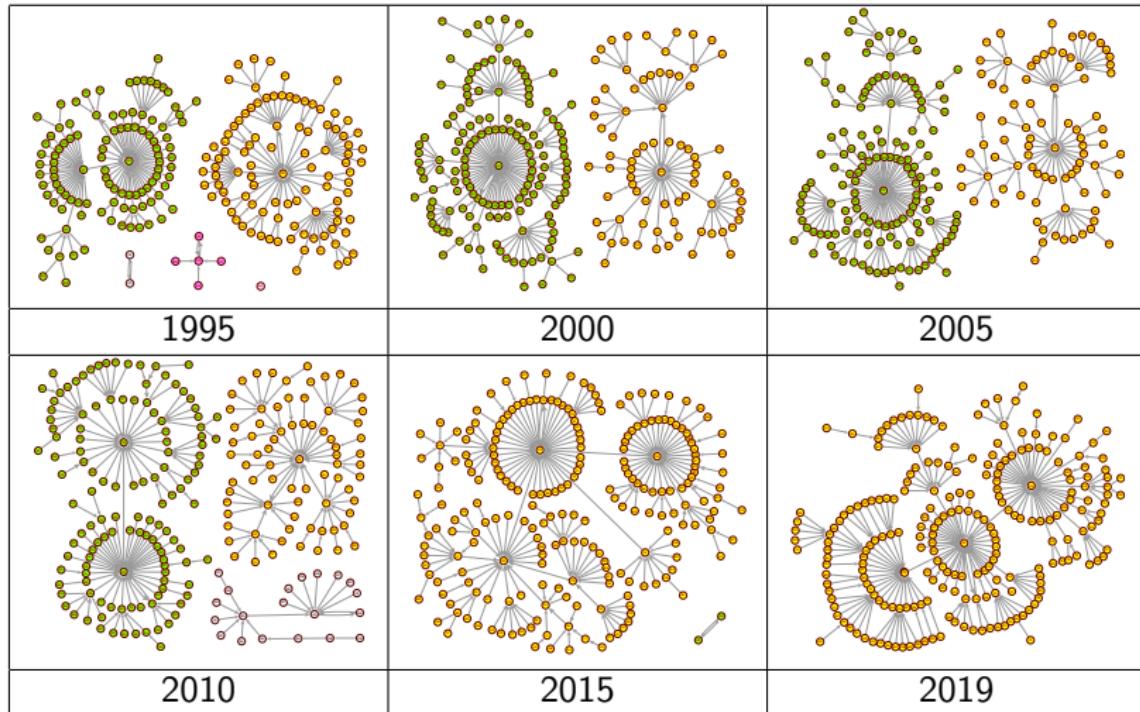
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## WT19s Pathfinder skeleton

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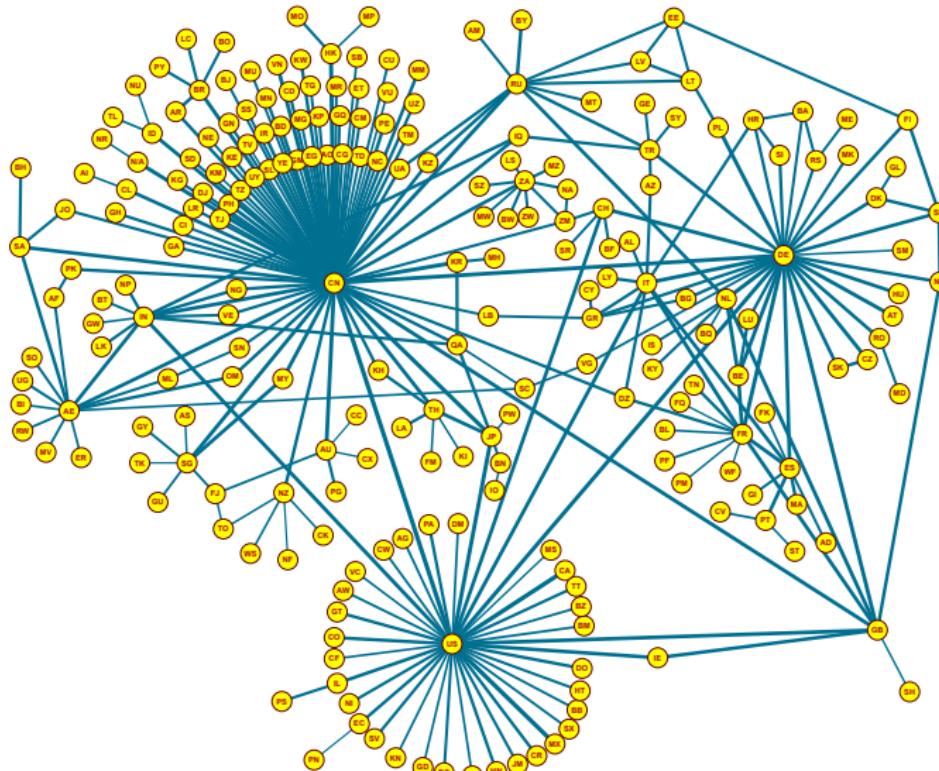
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# World trade Pathfinder skeletons

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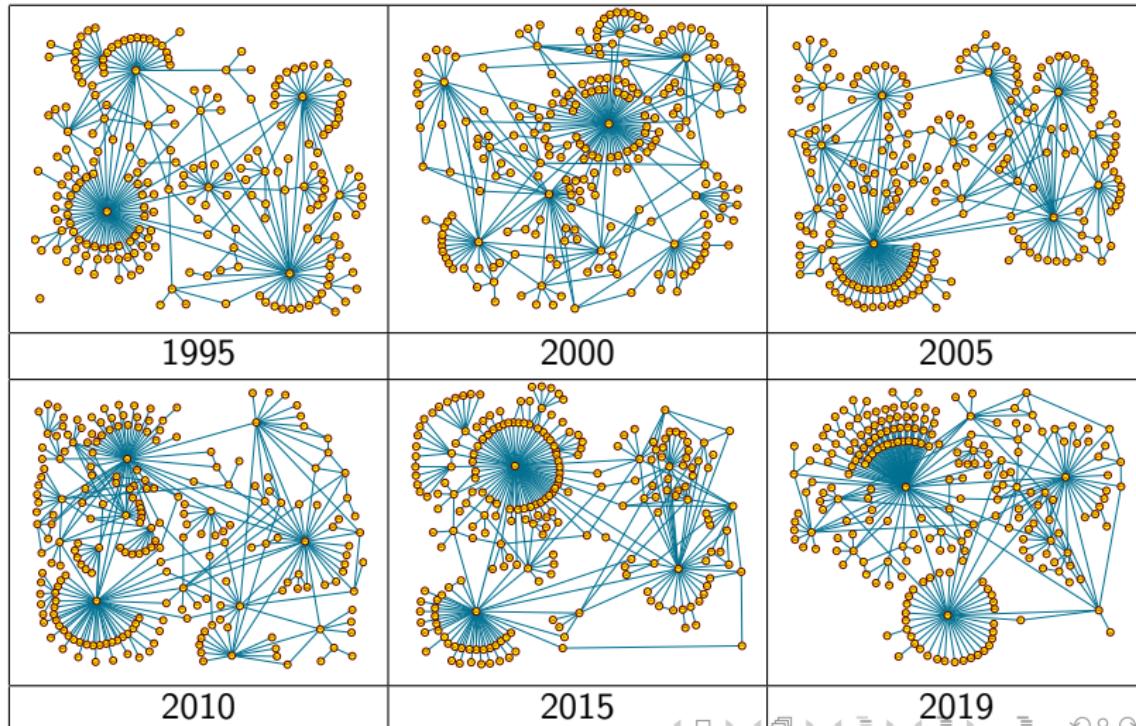
Distributions

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# Pajek skeletons

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```
select the network WT19t
Network/Create new network/Transform/Remove/All arcs except/
    k with highest [1]
Draw network
Kamada-Kawai, manual edit, export picture in SVG
select the network WT19s
Network/Create new network/Transform/Line values/Power [-1]
Network/Create new network/Transform/Reduction/Pathfinder [10] [OK]
select the network WT19s as the second network
Networks/Cross-intersection/Second
Draw network
Kamada-Kawai, manual edit, export picture in SVG
```



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- BACI is a gold mine
- margin labels in large matrices are not very handy – SVG/JS support with tooltips.
-



# Acknowledgments

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The computational work reported in this presentation was performed using R and Pajek. The code and data are available at

This work is supported in part by the Slovenian Research Agency (research program P1-0294 and research projects J5-2557, J1-2481, and J5-4596), and prepared within the framework of the COST action CA21163 (HiTEc).



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