The answer is a median of $nums[i]$.

Proof. Suppose that

$$M = \max\left\{nums[i] \;\middle|\; 1 \le i \le n\right\} \text{ and } m = \min\left\{nums[i] \;\middle|\; 1 \le i \le n\right\}.$$

We first prove that the answer lies in $[m, M]$. Let t be a number that is less than m. Then there is $\varepsilon > 0$ such that $m = t + \varepsilon$. Hence

$$|m - nums[i]| = |t + \varepsilon - nums[i]| \le |t - nums[i]| + |\varepsilon - nums[i]|.$$

Thus, $|m - nums[i]| < |t - nums[i]|$. So that t is not the answer. Similarly, if $t > M$, then we have $|M - nums[i]| < |t - nums[i]|$. Therefore, the answer is lies in $[m, M]$.
Next, notice that for any $t \in [m, M]$,

$$|t - m| + |t - M| = m - t + t - M = m - M,$$

which is remain constant. Hence, we can remove all element of $nums$ that are equal to m and M since that will not change the answer.
Now we obtained a new array $nums'$ from $nums$. By the above discussion, we have that the answer is lies in the minimum and the maximum of $nums'$.
Continue this process, until the number of element in the new array is less or equal to 2, say this array is $final\_nums$.

- If this number is 1, then the answer is $final\_nums[1]$, since $final\_nums[1]$ is the number that minimize

$$|x - final\_nums[1]| = |final\_nums[1] - final\_nums[1]| = 0$$

- If this number is 2, then by the above discussion, for any number x lies in $final\_nums[1]$ and $final\_nums[2]$ can be the answer since

$$|x - final\_nums[1]| = |x - final\_nums[2]|.$$

Finally, since for every step, we remove the minimum and maxium from $nums$, hence, the median can be a answer.