

HW 5

Monday, March 19, 2018 7:30 PM

2.1

A) There are 10 elements in this group

B)

	F1	F2	F3	F4	F5	R0	R1	R2	R3	R4
F1	R0	R3	R1	R4	R2	F1	F3	F5	F2	F4
F2	R2	R0	R3	R1	R4	F2	F4	F1	F3	F5
F3	R4	R2	R0	R3	R1	F3	F5	F2	F4	F1
F4	R1	R4	R2	R0	R3	F4	F1	F3	F5	F2
F5	R3	R1	R4	R2	R0	F5	F2	F4	F1	F3
R0	F1	F2	F3	F4	F5	R0	R1	R2	R3	R4
R1	F4	F5	F1	F2	F3	R1	R2	R3	R4	R0
R2	F2	F3	F4	F5	F1	R2	R3	R4	R0	R1
R3	F5	F1	F2	F3	F4	R3	R4	R0	R1	R2
R4	F3	F4	F5	F1	F2	R4	R0	R1	R2	R3

C) No this isn't abelian. It's not Symmetric

D) Subgroup with 5 elements is the group composing of only the rotation operation A subgroup must contain R0 and anything that is it's own inverse(any flip). I.E. {R0,F4}

E) No 4 is not a prime factorization of 10 which is the cardinality of the original group

2.2

Using isomorphism you can rearrange the table such that the first row and column is the identity i.e.

	I	X	Y
I	I	X	Y
X	X		
Y	Y		

Once sorted in this fashion we know that each row and column can't have duplicates. This forces the (x,x) combination to be Y otherwise there'd be 2 X in the 2nd row and column. It also forces the (y,y) combination to be x for the same reason. This leaves only the 2 identities to be placed in the remaining 2 spots. There is no rearranging that can be performed besides Isomorphic transforms (for example rearranging the rows and columns)

2.6

A) $\{4, 8, 12, 0\} \in H$

B) $S_0 = \{0, 4, 8, 12\}, S_1 = \{1, 5, 9, 13\}, S_2 = \{2, 6, 10, 14\}, S_3 = \{3, 7, 11, 15\}$

C)

+	S0	S1	S2	S3
S0	S0	S1	S2	S3
S1	S1	S2	S3	S0
S2	S2	S3	S0	S1
S3	S3	S0	S1	S2

D) This is isomorphic with the $\langle \mathbb{Z}_4, + \rangle$ group

2.18)

Test for multiplication distributes over addition $a(b+c) = ab+ac$

Counter example

$$2(2 + 3) = 2 \cdot 2 + 2 \cdot 3 = 3 + 1 = 0$$

$$2 \cdot (2 + 3) = 2 \cdot 1 = 2 \neq 0$$

This is not a field

2.28)

Let $G = [v_1 \ v_2 \ \dots \ v_n]$

$$G \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix} = v$$

If G is full rank than

$$\rightarrow \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix} = G^{-1}v$$

Otherwise you will need to perform a psudo inverse $(G^T G)^{-1} G^T$