HW 6

Saturday, March 24, 2018 3:32 PM

1.
000
011
110
101

2.

a) The minimum distance is 2
b) This code can't correct any errors, however it can detect up to 1 error.
c) $G = [\boldsymbol{I} \quad \mathbf{1}]\ (k\ X\ k+1)$
d) $H = [\mathbf{1}](1\ X\ k+1)$
e) $(p + (1-p))^n = \sum_{i}^{n} \binom{n}{i} p^i (1-p)^{n-i}$

$$\left(-p + (1-p)\right)^n = \sum_{i}^{n} \binom{n}{i} (-p)^i (1-p)^{n-i}$$

$$\left(p + (1-p)\right)^n + \left(-p + (1-p)\right)^n = 1 + (1-2p)^n = 2 \sum_{i \in even}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

$$\sum_{i \in even}^{n} \binom{n}{i} p^i (1-p)^{n-i} = \frac{1 + (1-2p)^n}{2}$$

3.

a) $G = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

Row 1 new = row 2 old
Row 2 New = sum of old rows
Row 3 new = row 3 old

b) $H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4.)

a) $GH^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
b) O-output, I = input
O1=I1,
O2=xor(I2,I3)
O3=xor(I1,xor(I2,I3))
O4=I2
O5=xor(I1,I3)
O6=xor(I1,I2)

c) P1 =xor(xor(O1,O2),xor(O4,O5))
P2 =xor(xor(O1,O3),xor(O5,O6))
P3 =xor(O2,xor(O5,O6))

d) $G^{\perp} = range(H) = \begin{cases} 000000 \\ 110110 \\ 101011 \\ 011101 \\ 010011 \\ 100101 \\ 111000 \\ 001110 \end{cases}$

e) a

000000	101011	011101	110110	011010	110001	000111	101100
100000	001011	111101	010110	111010	010001	100111	001100
010000	111011	001101	100110	001010	100001	010111	111100
001000	100011	010101	111110	010010	111001	001111	100100
000100	101111	011001	110010	011110	110101	000011	101000
000010	101001	011111	110100	011000	110011	000101	101110
000001	101010	011100	110111	011011	110000	000110	101101
100010	001001	111111	010100	111000	010011	100101	001110

f)

Syndrome	Bits to flip
000	000000
110	100000
101	010000
010	001000
100	000100
111	000010
011	000001
001	100010

g) A

i	0	1	2	3	4	5	6
A_i	1	0	0	4	3	0	0

$A(z) = 1 + 4z^3 + 3z^4$

h) 110110

i) 111001->110001->101

j) $B(z) = q^{-k}\big(1 + (q-1)z\big)^n A\left(\frac{1-z}{1+(q-1)z}\right)$

$$B(z) = \frac{1}{8}(1+z)^6\left(1 + 4\left(\frac{1-z}{1+z}\right)^3 + 3\left(\frac{1-z}{1+z}\right)^4\right)$$

$$= \frac{1}{8}(1+z)^6 + \frac{1}{2}\big((1-z)(1+z)\big)^3 + \frac{3}{8}(1-z)^4(1+z)^2$$

$B(z) = 1 + 4z^3 + 3z^4$

k) $P_u(E) = 4p^3(1-p)^3 + 3p^4(1-p)^2 = 3.91E-6$

l) $P_d(E) = 1 - (1-p)^n - P_u(E) = .0585$

m) $\alpha_0 = 1, \alpha_1 = 6, \alpha_2 = 1,$

$$P(E) = 1 - \sum_{j=0}^{n} \alpha_j p^j (1-p)^j = 1 - 0.99^6 - 6 \cdot 0.01 \cdot 0.99^5 - 0.01^2 \cdot 0.99^4 = 0.001364$$

n) $P(E) = \sum_{j=dmin}^{n} A_j \sum_{l=0}^{\tau} P_l^j = .001163$

o) $P(f) = 1 - \sum_{j=0}^{floor\left(\frac{dmin-1}{2}\right)} \binom{n}{j} p^j (1-p)^{n-j} - P(E) = .000288$

p) $H' = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$

$G = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$

q) $G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$