Institute for Astronomical and Physical Geodesy
Technische Universität München

Winter Semester 2014/2015

ESPACE: Orbit Mechanics, Exercise 1
Numerical Integration of Satellite Orbits

For numerical integration MATLAB provides the functions **ode23**, **ode45** and **ode113** (see **help**). As input parameters the integration time steps, the initial conditions (position, velocity) of the integration **y_0** and the program **yprime,** which contains the differential equation are required.

Compute numerically the trajectory of the GOCE satellite for 3 revolutions. Use the following Kepler elements for the GOCE satellite:

Satellit	**a [km]**	**e**	**i [°]**	**Ω [°]**	**ω [°]**	**τ[sec]**
GOCE	6629	0.004	96.6	257.7	144.2	0

Compare the numerically integrated trajectories for the un-disturbed and for the disturbed (due to flattening of the Earth) cases against each other as well as against the analytical derived trajectory (from the Kepler elements). In particular the following sub tasks have to be solved:

1. Analytically compute the undisturbed trajectory (positions & velocities) for 3 revolutions from the Kepler elements (program should be available from previous exercise in orbit mechanics).

2. Write a program **yprime**, providing the differential equation for the un-disturbed Kepler problem. Compute the trajectory (positions & velocities) for 3 revolutions using two different MATLAB integrators and two different step sizes.

3. Compare the analytical and numerical derived results in terms of plots for each component.

4. Take into account the Earth's flattening and update your program **yprime** accordingly. Apply the formulas given below. Plot the differences between the un-disturbed and the disturbed case. The disturbed equation of motion is given by:

$$\ddot{\mathbf{r}} = -\frac{GM}{r^3}\mathbf{r}\left[1-\frac{3}{2}J_2\frac{a_e^2}{r^2}\begin{pmatrix}5\left(\frac{z}{r}\right)^2-1\\ 5\left(\frac{z}{r}\right)^2-1\\ 5\left(\frac{z}{r}\right)^2-3\end{pmatrix}\right] \qquad with:\ \mathbf{r}=\begin{pmatrix}x\\ y\\ z\end{pmatrix}$$

with $J_2 = -C_{20} = 0.00108263$; GM = 398600.5 km^3/s^2; a_e = 6378 km

5. Develop your own simple integrator for the undisturbed case and investigate the impact of the step size on your result. Compare results with the analytical solution computed above.

Due date for delivery of written report: 15. December 2014