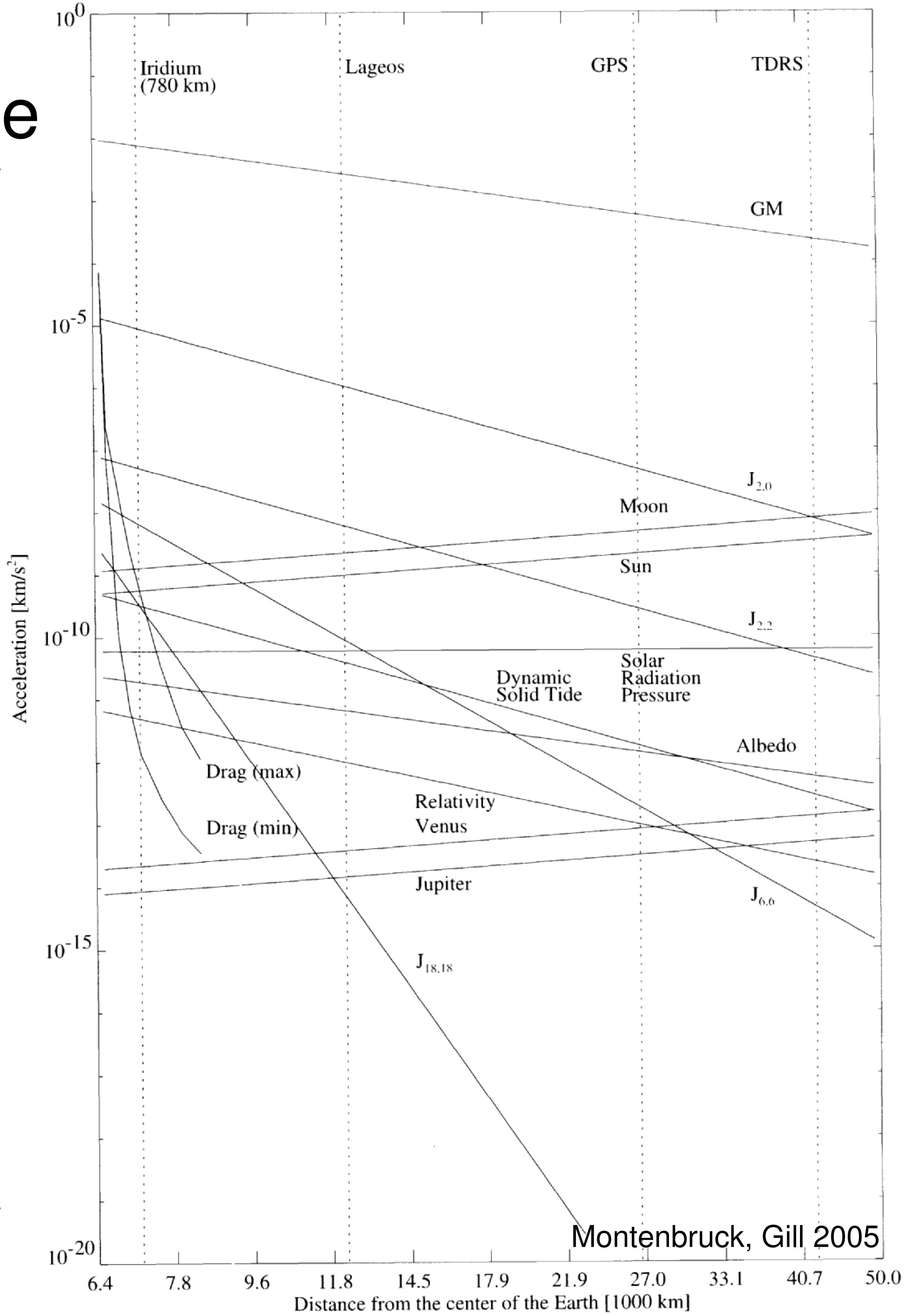
5 Orbit Perturbations

- Orders of magnitude
  - Orbit perturbations for GPS satellites
- Osculating orbital elements
- Decomposition of perturbing accelerations
  - TNW
  - RSW
- Geometrical considerations concerning perturbation
- Perturbation theory
  - General perturbation theory
  - Gauss perturbation equations
  - Lagrange perturbation equations

5.1 Orders of Magnitude

- Artificial Earth satellites are affected by a number of different perturbations.
- *Gravitational forces* such as from the potential of the Earth, attraction from Sun, Moon and planets, tides, relativistic effects.
- *Non-gravitational forces* such as air drag, radiation pressure, albedo, thermal emission.
- Because all these forces are small compared to the central force they are called *perturbing forces*. The orbits deviate from a Keplerian ellipse.



Montenbruck, Gill 2005

5.2 Orbit Perturbations for GPS Satellites

Perturbations for GPS satellites	Acceleration m/s^2	Orbit error after one day *)
Earth's attraction………………	0.59	330'000'000 m
Oblateness ……………………	$5 \cdot 10^{-5}$	24'000 m
Moon's attraction………………	$5 \cdot 10^{-6}$	2'000 m
Sun's attraction ……………….	$2 \cdot 10^{-6}$	900 m
Higher order terms in geopotential……………………	$3 \cdot 10^{-7}$	300 m
Direct radiation pressure	$9 \cdot 10^{-8}$	100 m
along solar panel axis	$5 \cdot 10^{-10}$	6 m
Neglect of Earth's shadow …..	-	10 – 20 m
Neglect of Moon's shadow	-	0.1 – 3 m
Solid Earth tides … ……………	$1 \cdot 10^{-9}$	0.3 m
General relativity ………………	$3 \cdot 10^{-10}$	0.3 m
Ocean tides ……………………	$1 \cdot 10^{-10}$	0.06 m

*) Remark: Fixed initial conditions (position and velocity)

5.2 Orbit Perturbations for GPS Satellites

Perturbations for GPS satellites	Acceleration m/s^2	Orbit error after one day
Direct radiation pressure	$9 \cdot 10^{-8}$	100 m
along solar panel axis	$5 \cdot 10^{-10}$	6 m
Solid Earth tides … ……………	$1 \cdot 10^{-9}$	0.3 m
Ocean tides ……………………	$1 \cdot 10^{-10}$	0.06 m
Venus (lower conjunction) ……	$2 \cdot 10^{-10}$	0.08 m
Jupiter (opposition) …………….	$2 \cdot 10^{-11}$	0.008 m
Mars (opposition) ………………	$1 \cdot 10^{-11}$	0.005 m
General relativity:		
Schwarzschild…………………	$3 \cdot 10^{-10}$	0.3 m
deSitter…………………………	$2 \cdot 10^{-11}$	0.02 m
Lense-Thirring ……………….	$1 \cdot 10^{-12}$	0.001 m

5.3 Osculating Orbital Elements

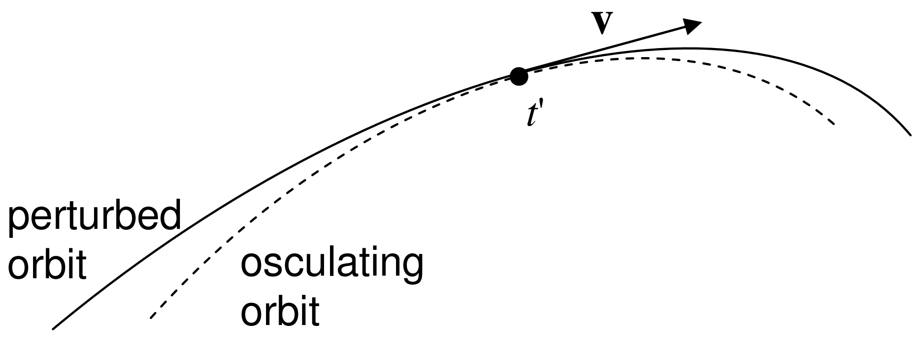
- Perturbing forces cause a deviation of the satellite orbit from a Keplerian ellipse. The orbit thus can no longer be represented by six constant orbital elements as in the case of the two body problem where Keplerian elements are integrals of motion.
- In general the satellite orbit has to be *integrated numerically*.
- Numerical integration in Cartesian coordinates provides *position* $\mathbf{x}(t)$ and *velocity* $\mathbf{v}(t)$ as functions of time.
- We already know that position and velocity can uniquely be converted into a set of six Keplerian elements (and vice versa)

$$\{\mathbf{x}(t), \mathbf{v}(t)\} \leftrightarrow \{a(t), e(t), i(t), \Omega(t), \omega(t), u_0(t)\}$$

- The six orbital elements are then in general not constant but *functions of time*.
- For each epoch t the elements represent the best fitting or *osculating ellipse*. The orbital elements are called *osculating orbital elements*.

5.3 Osculating Orbital Elements

- The Keplerian (elliptic) orbit computed using the osculating elements at epoch t' is defined for all epochs t. It is called *osculating orbit* at epoch t'.
- The actual (perturbed) and the osculating (unperturbed) orbit touch each other at epoch t', but usually have no common points neither before nor after t'.

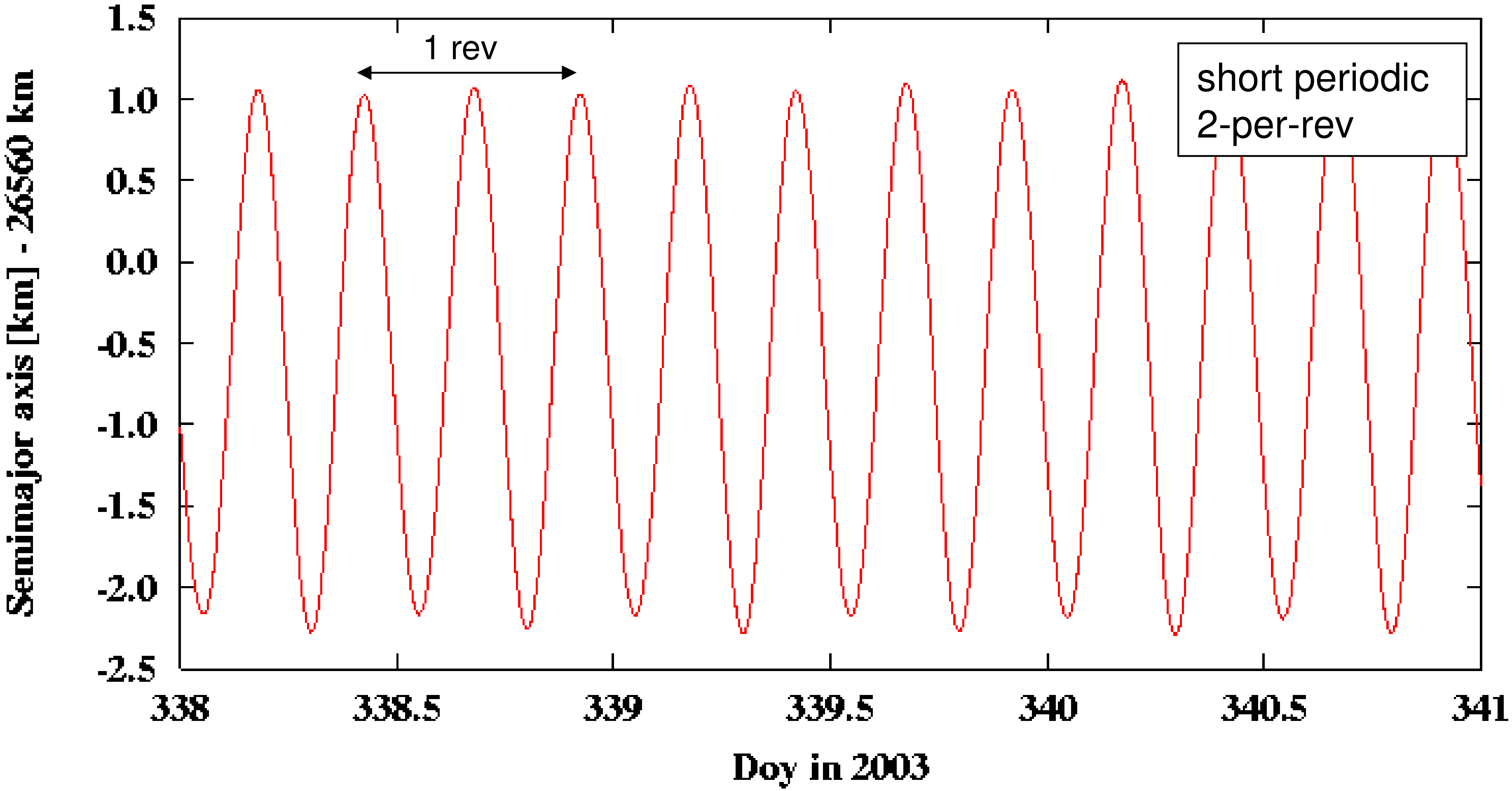


- At t' the position and velocity vectors of the two orbits are equal.
- The osculating orbit is nothing else than that orbit the satellite would follow if for all $t > t'$ all perturbations would be switched off.
- The actual orbit is the *envelope* of the osculating orbits.
- The satellite motion may be considered as Keplerian motion with time-dependent orbital elements.
- With osculating orbital elements the development of an orbit as a function of time under the influence of perturbations can be visualized.

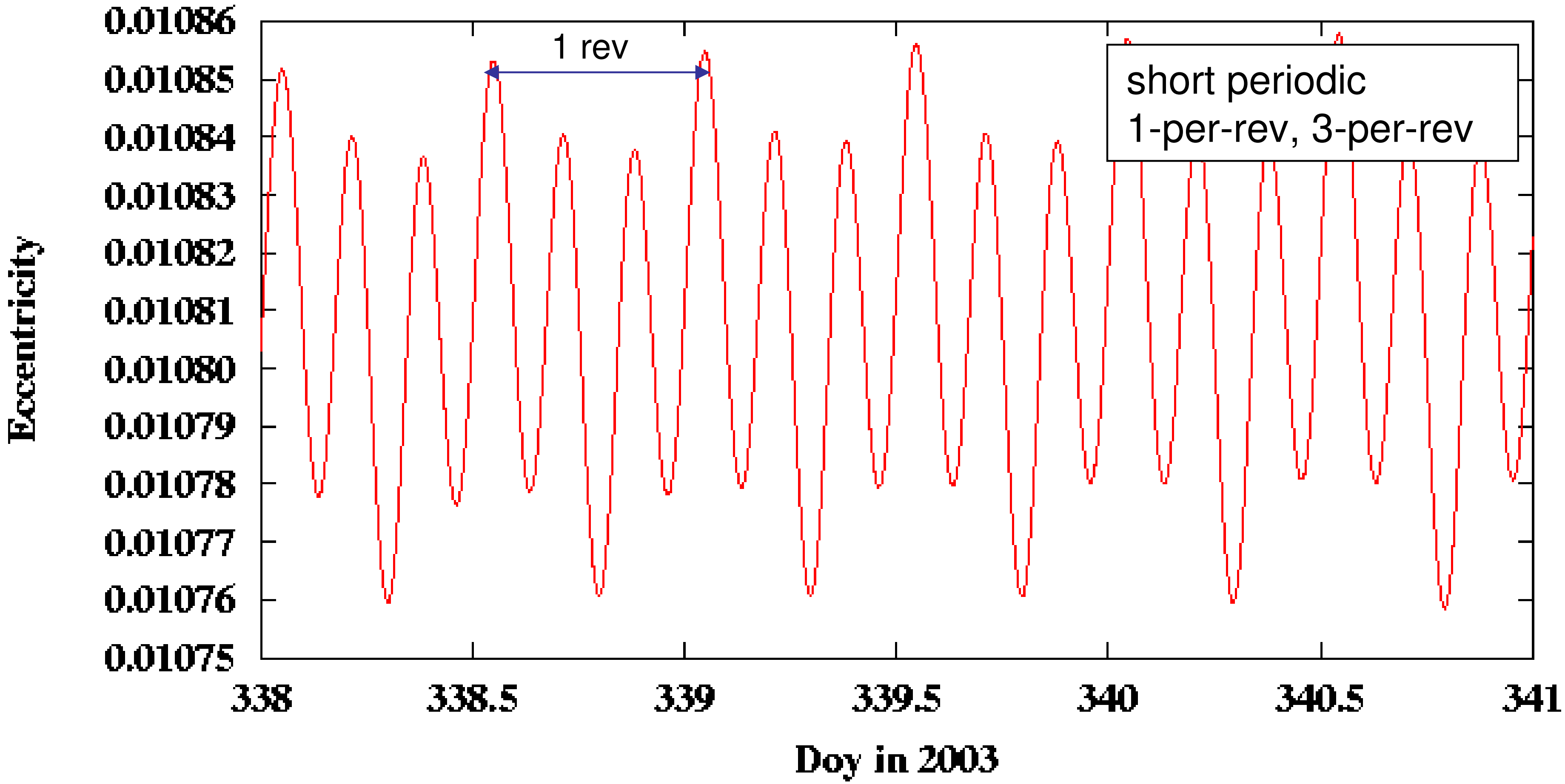
5.3 Osculating Orbital Elements

- For small perturbations the variations of the osculating elements around mean elements are small.
- The osculating elements, plotted as a function of time, visualize the effect of the perturbations.
- In the following we will see the osculating elements of GPS satellite PRN 25 over a time interval of 3 days, i.e., six revolutions.
- The perturbations are primarily caused by the oblateness of the Earth.
- We find perturbations with different characteristics:
  - *Short periodic* perturbations with typical periods of the revolution period U and integer fractions thereof $U/2$, $U/3$, $U/4$, ...
  - *Long periodic* perturbations with typical periods of years to decades.
  - *Secular* (i.e. monotonically advancing) perturbations.

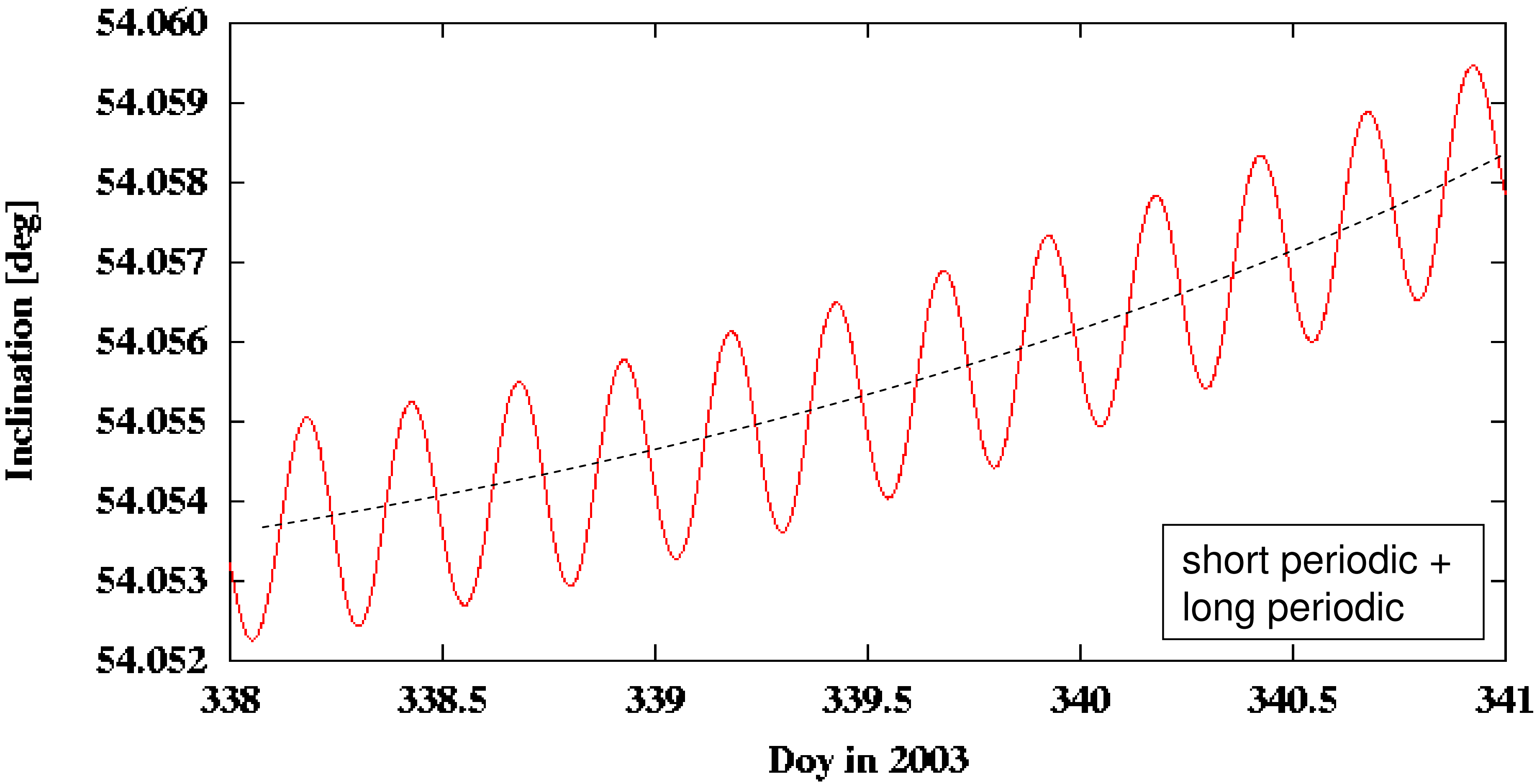
5.3 Osculating Orbital Elements



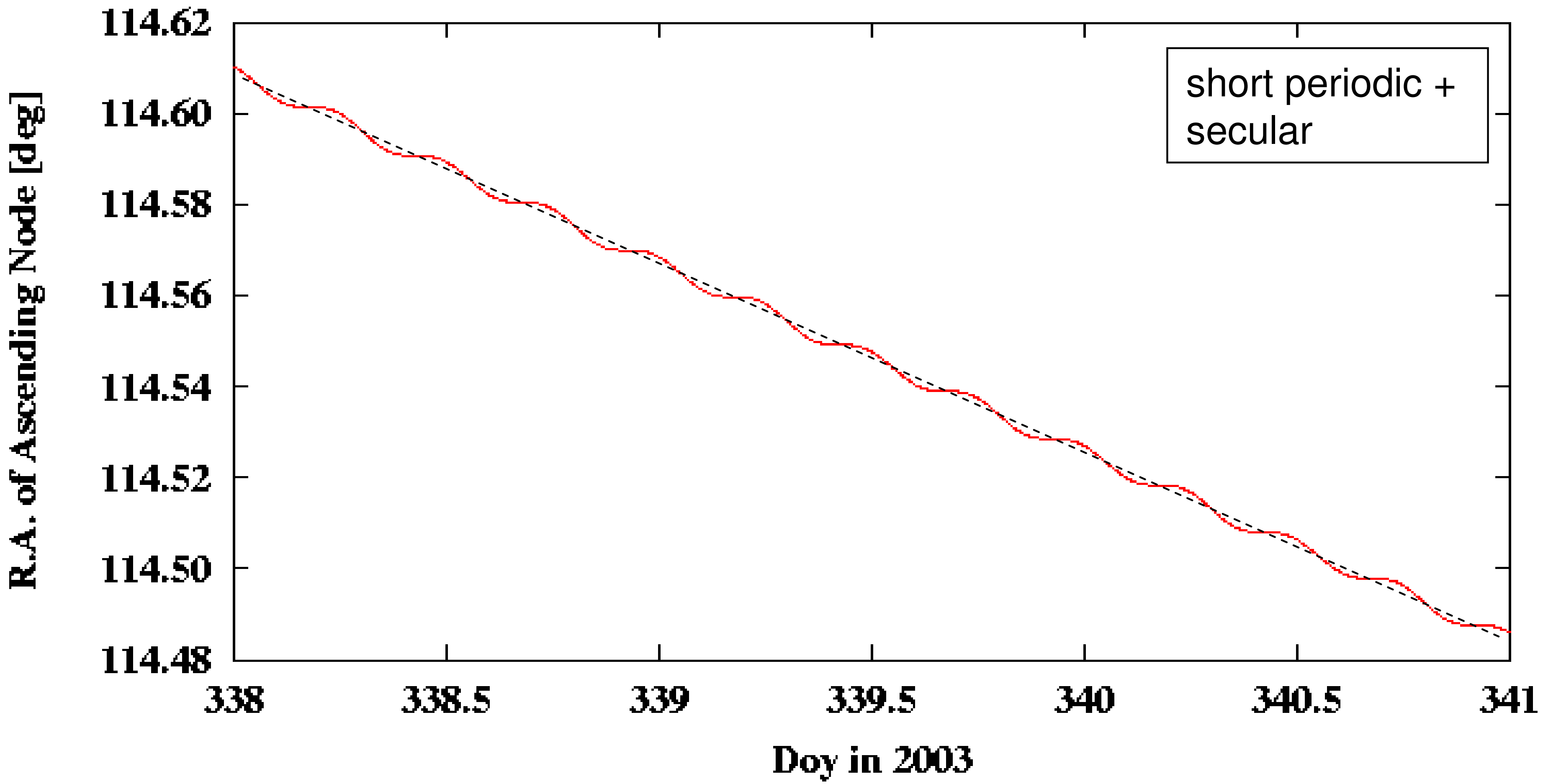
5.3 Osculating Orbital Elements



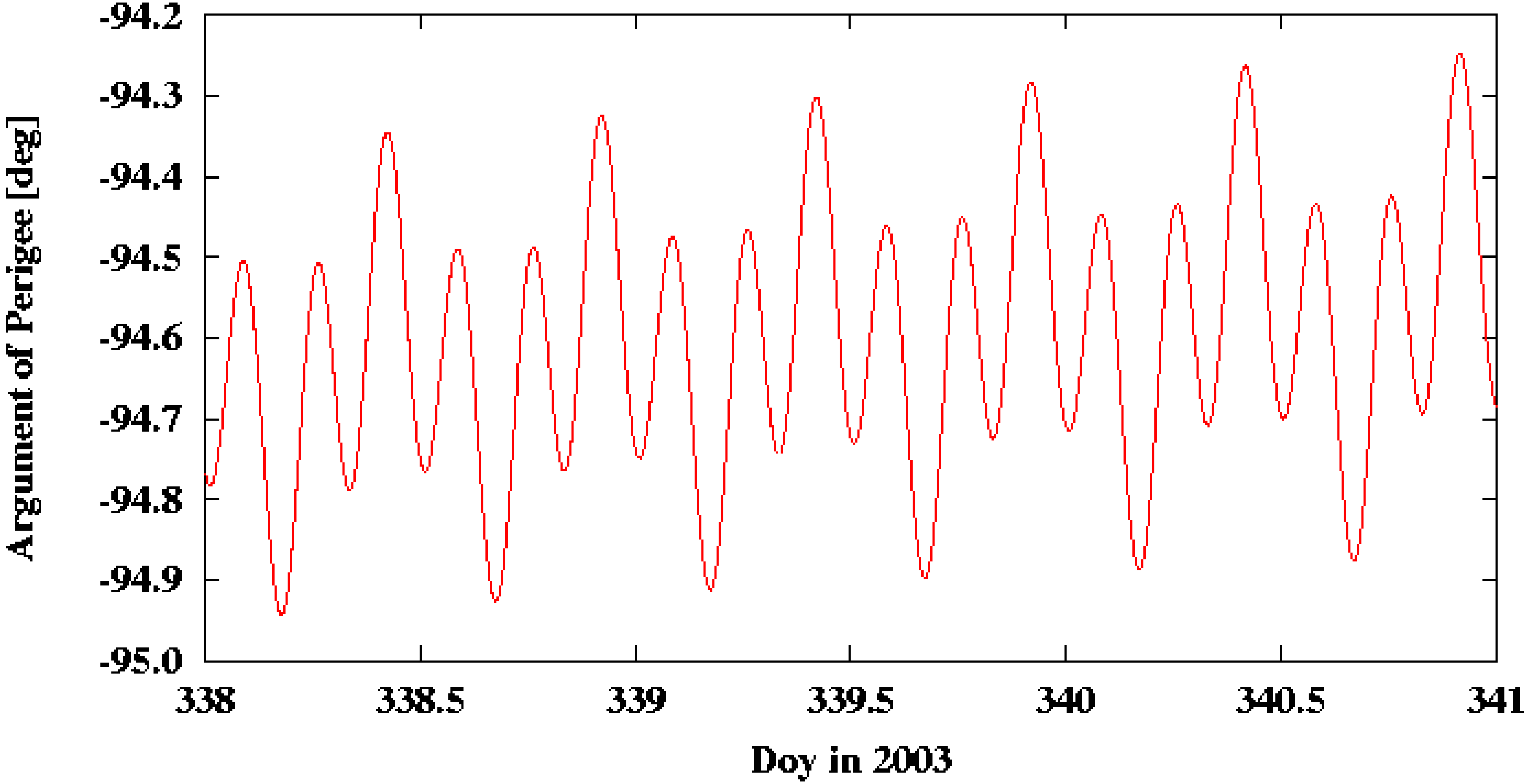
5.3 Osculating Orbital Elements



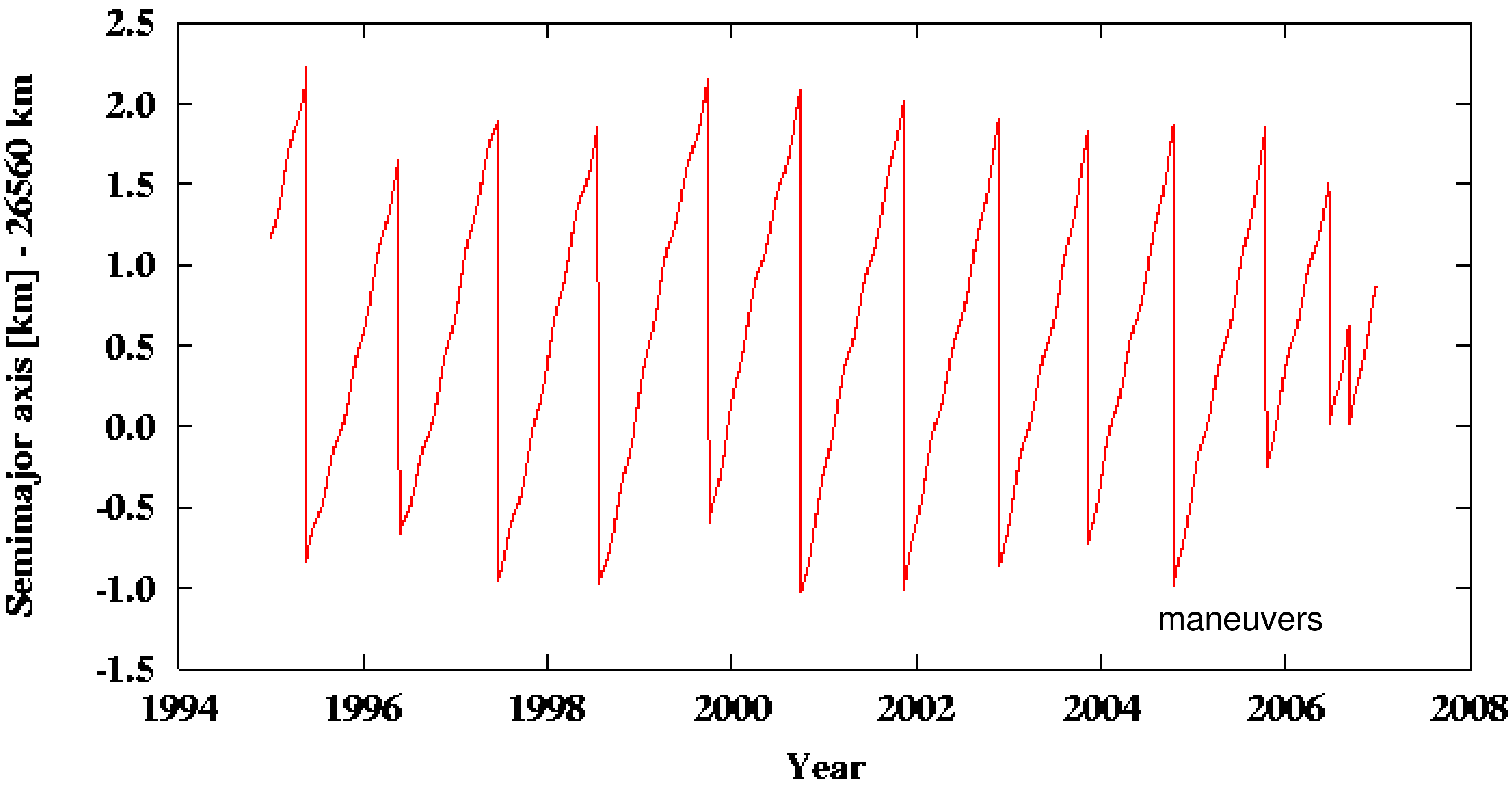
5.3 Osculating Orbital Elements



5.3 Osculating Orbital Elements

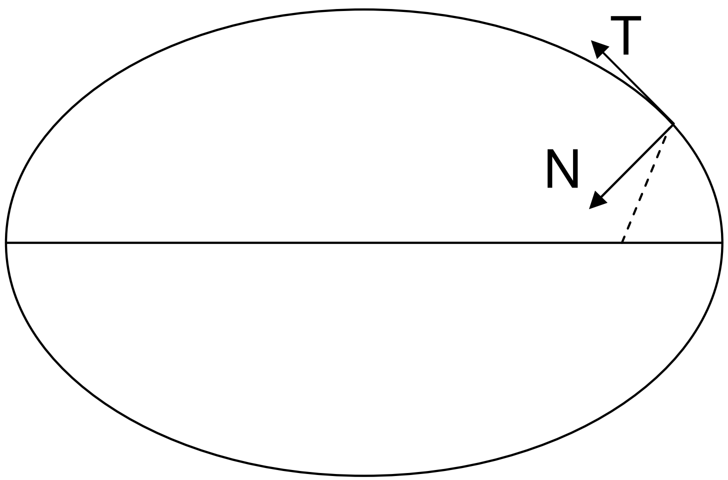


5.3 Osculating Orbital Elements



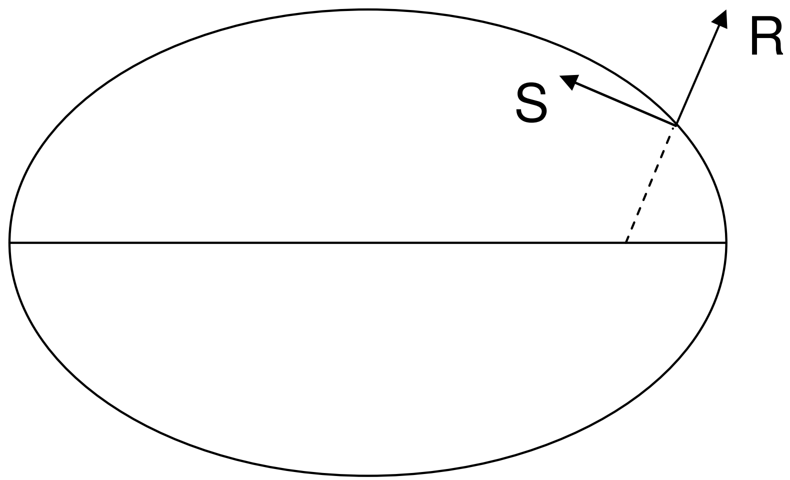
5.4 Decomposition of Perturbing Acceleration

- In the following we will consider only *elliptical orbits*.
- We will decompose an arbitrary perturbing acceleration, acting on a point along the orbit, into an orbit oriented coordinate frame.
- We consider the two decompositions called TNW und RSW.
- The axes of the *TNW system* are oriented in the following way:
  - *T-axis*: Parallel to the instantaneous velocity vector (tangential to the orbit).
  - *N-axis*: Normal to the T-axis, in the orbital plane, pointing to the inside of the ellipse.
  - *W-axis*: Perpendicular to the two other axes and perpendicular to the orbital plane. Parallel to the orbital angular momentum vector.



5.4 Decomposition of Perturbing Acceleration

- The axes of the *RSW system* are oriented in the following way:
  - *R-axis*: Parallel to the instantaneous position vector, pointing radially outward from the focal point.
  - *S-axis*: Normal to the R-axis, in the orbital plane, pointing into the direction of motion.
  - *W-axis*: Perpendicular to the other two axes and perpendicular to the orbital plane. Parallel to the orbital angular momentum vector.
- Both systems are right-handed.
- The W-axis is the same for both decompositions.
- The S-axis coincides with the T-axis only for circular orbits. For elliptical orbits they coincide only in perigee and in apogee.



5.5 Perturbation Theory

- The motion on a conic section as a solution of the two-body problem is called *unperturbed motion* in celestial mechanics.
- A *perturbed motion* deviates from an unperturbed orbit due to perturbing acceleration that act in addition to the two-body accelerations.
- The notion „perturbation“ implies that the perturbing acceleration is *small* with respect to the two-body acceleration.
- Including perturbations the *equation of motion* may be written in the following way ($\mu = GM$)

$$\ddot{\mathbf{r}} = -\mu \frac{\mathbf{r}}{r^3} + \mathbf{f}_1(\mathbf{r}, \dot{\mathbf{r}}, t)$$

- The first term is called two-body- or *Keplerterm*, the second term is called perturbation term and may depend on position and velocity of the satellite as well as explicitly on time.

5.5.1 General Perturbation Theory

- In general perturbation theory the solution of the perturbed equation of motion is searched in the form of an *analytical* solution.
- The Keplerian orbital elements are integrals of motion (i.e., constant in time) for the unperturbed motion

$$\ddot{\mathbf{r}} = -\mu \frac{\mathbf{r}}{r^3} = \mathbf{f}_0$$

- The equation of motion for the perturbed orbit may be written as

$$\ddot{\mathbf{r}} = -\mu \frac{\mathbf{r}}{r^3} + \mathbf{f}_1(\mathbf{r}, \dot{\mathbf{r}}, t) = \mathbf{f}_0 + \mathbf{f}_1$$

  where $\mathbf{f}_1$ includes all perturbing accelerations.
- Let us denote one of the Keplerian elements with $I(t)$:

$$I_k(t) \in \left\{ a(t), e(t), i(t), \Omega(t), \omega(t), T_p(t) \right\} \qquad k = 1, \ldots, 6$$

5.5.1 General Perturbation Theory

- According to the definition of the osculating elements $I_k(t)$ may be written as a function of position and velocity of the perturbed body's trajectory at time t:

$$I_k(t) = I_k\left(\mathbf{r}(t), \dot{\mathbf{r}}(t)\right)$$

no explicit time depencency

- $I_k(t)$ does not explicitly depend on time but only implicitly through the time dependence of position and velocity.
- We obtain a *differential equation* for I_k by computing the total derivative of the above equation with respect to time using the chain rule.

$$\frac{d}{dt} I_k = \sum_{l=1}^{3} \left\{ \frac{\partial I_k}{\partial r_l} \dot{r}_l + \frac{\partial I_k}{\partial \dot{r}_l} \ddot{r}_l \right\} = \nabla_r I_k \cdot \dot{\mathbf{r}} + \nabla_v I_k \cdot \ddot{\mathbf{r}}$$

no partial derivative w.r.t time

- Here $\nabla_r I$ designates the gradient of the scalar function I w.r.t spatial coordinates while $\nabla_v I$ designates the gradient w.r.t velocity coordinates.

5.5.1 General Perturbation Theory

- The equation above may be rewritten by replacing $\ddot{\mathbf{r}}$ by the perturbed equation of motion

$$\frac{d}{dt}I_k = \nabla_r I_k \cdot \dot{\mathbf{r}} + \nabla_v I_k \cdot \ddot{\mathbf{r}} = \nabla_r I_k \cdot \dot{\mathbf{r}} + \nabla_v I_k \cdot (\mathbf{f}_0 + \mathbf{f}_1)$$

- Because I_k is a constant of integration of the unperturbed problem we know

$$\nabla_r I_k \cdot \dot{\mathbf{r}} + \nabla_v I_k \cdot \mathbf{f}_0 = 0$$

- With this we obtain

coupling through dependency on $\mathbf{r}$ and $\mathbf{v}$

$$\frac{d}{dt}I_k = \nabla_v I_k \cdot \mathbf{f}_1$$

- This is a coupled system of six nonlinear differential equations of first order.
- It is equivalent to the original equation of motion (coupled system of three nonlinear differential equations of second order).
- The above differential equation is probably the most general representation of the *Gaussian perturbation equations*.

5.5.2 Gaussian Perturbation Equations

- As an example we derive the perturbation equation for the semimajor axis a. For this we start with the energy equation of celestial mechanics (vis-viva equation)

$$\dot{\mathbf{r}}^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

- We apply the gradient ∇_v to both sides and obtain

$$\nabla_v(\dot{\mathbf{r}}^2) = 2\dot{\mathbf{r}} = \mu\nabla_v\left(\frac{2}{r}\right) - \mu\nabla_v\left(\frac{1}{a}\right) = \frac{\mu}{a^2}\nabla_v a$$

- Using the perturbation equation for $I=a$ we obtain thus

$$\dot{a} = \frac{2a^2}{\mu}\dot{\mathbf{r}} \cdot \mathbf{f}_1$$

5.5.2 Gaussian Perturbation Equations

- With

$$\mathbf{f}_1 = \begin{pmatrix} T \\ N \\ W \end{pmatrix}$$

representation of perturbing acceleration vector in TNW orbit frame

$$\dot{\mathbf{r}} = \begin{pmatrix} \mathrm{v} \\ 0 \\ 0 \end{pmatrix}$$

representation of velocity n vector in TNW orbit frame

we get

$$\dot{a} = \frac{2a^2 \mathrm{v}}{\mu} \cdot T$$

unit: acceleration

→ semimajor axis only changes by acceleration in tangential direction

- With

$$\mathbf{f}_1 = \begin{pmatrix} R \\ S \\ W \end{pmatrix} \qquad \dot{\mathbf{r}} = \begin{pmatrix} \dot{r} \\ r\dot{\nu} \\ 0 \end{pmatrix} = \sqrt{\frac{\mu}{p}} \begin{pmatrix} e \sin\nu \\ \frac{p}{r} \\ 0 \end{pmatrix}$$

we get

$$\dot{a} = \sqrt{\frac{p}{\mu}} \frac{2a}{1-e^2} \left(e \sin\nu \cdot R + \frac{p}{r} \cdot S \right)$$

5.5.2 Gaussian Perturbation Equations

- The *Gaussian perturbation equations* are

$$\dot{a} = \sqrt{\frac{p}{\mu}} \frac{2a}{1-e^2} \left[e \sin\nu \cdot R + \frac{p}{r} \cdot S \right]$$

$$\dot{e} = \sqrt{\frac{p}{\mu}} \left[\sin\nu \cdot R + (\cos\nu + \cos E) \cdot S \right]$$

$$\dot{i} = \frac{r \cos u}{n a^2 \sqrt{1-e^2}} \cdot W$$

→ orientation of orbital plane can only be changed by an out-of-plane acceleration

$$\dot{\Omega} = \frac{r \sin u}{n a^2 \sqrt{1-e^2} \sin i} \cdot W$$

$$\dot{\omega} = \frac{1}{e} \sqrt{\frac{p}{\mu}} \left[-\cos\nu \cdot R + \left(1 + \frac{r}{p}\right) \sin\nu \cdot S \right] - \dot{\Omega} \cos i$$

$$\dot{T}_p = -\frac{1-e^2}{n^2 a e} \left[\left(\cos\nu - 2e \frac{r}{p} \right) \cdot R - \left(1 + \frac{r}{p}\right) \sin\nu \cdot S \right] - \frac{3}{2a} (t - T_p) \dot{a}$$

5.5.2 Gaussian Perturbation Equations

- Frequently the mean anomaly at a reference epoch t_0 is used instead of the perigee passing time T_p

$$M_0 = n(t_0 - T_p)$$

- The corresponding perturbation equation reads as

$$\dot{M}_0 = \frac{1-e^2}{nae}\left[\left(\cos\nu - 2e\frac{r}{p}\right)\cdot R - \left(1+\frac{r}{p}\right)\sin\nu\cdot S\right] - \frac{3n}{2a}(t-t_0)\dot{a}$$

5.5.2 Gaussian Perturbation Equations

- The Gaussian perturbation equations may alternatively be written in TNW

$$\dot{a} = \frac{2a^2}{\mu} \mathrm{v} \cdot T$$

$$\dot{e} = \frac{1}{\mathrm{v}}\left[-\frac{r}{a} \sin\nu \cdot N + 2(\cos\nu + e) \cdot T \right]$$

$$\dot{i} = \frac{r\cos u}{na^2\sqrt{1-e^2}} \cdot W$$

$$\dot{\Omega} = \frac{r\sin u}{na^2\sqrt{1-e^2}\sin i} \cdot W$$

$$\dot{\omega} = \frac{1}{\mathrm{v}e}\left\{ \left[\frac{r}{p}\cos\nu + e\left(1+\frac{r}{p}\right)\right] \cdot N + 2\sin\nu \cdot T \right\} - \dot{\Omega}\cos i$$

$$\dot{T}_p = -\frac{\sqrt{1-e^2}}{n\mathrm{v}e}\left[-\frac{r}{p}\cos\nu \cdot N - 2\left(1+e^2\frac{r}{p}\right)\sin\nu \cdot T \right] - \frac{3}{2a}(t-T_p)\dot{a}$$

5.5.3 Lagrangian Perturbation Equations

- An alternative representation of the perturbation equations can be obtained if the perturbing acceleration $\mathbf{f}_1$ can be written as the gradient of a scalar *potentials* $\tilde{R}$

$$\mathbf{f}_1 = \nabla_r \tilde{R}$$

- With the abbreviation

$$L = \sqrt{\mu a (1 - e^2)}$$

we obtain the so-called Lagrangian perturbation equations:

5.5.3 Lagrangian Perturbation Equations

- The *Lagrangian perturbation equations*:

Derivative along the satellite orbit

$$\dot{a} = -\frac{2a^2}{\mu} \cdot \frac{\partial \tilde{R}}{\partial T_p}$$

$$\dot{e} = -\frac{L^2}{\mu^2 e} \cdot \frac{\partial \tilde{R}}{\partial T_p} - \frac{1-e^2}{eL} \cdot \frac{\partial \tilde{R}}{\partial \omega}$$

$$\dot{i} = \frac{1}{L \sin i} \left(\cos i \cdot \frac{\partial \tilde{R}}{\partial \omega} - \frac{\partial \tilde{R}}{\partial \Omega} \right)$$

$$\dot{\Omega} = \frac{1}{L \sin i} \cdot \frac{\partial \tilde{R}}{\partial i}$$

$$\dot{\omega} = \frac{1-e^2}{eL} \cdot \frac{\partial \tilde{R}}{\partial e} - \frac{e \cos i}{(1-e^2) \sin i} \cdot \frac{\partial \tilde{R}}{\partial i}$$

$$\dot{T}_p = \frac{2a^2}{\mu} \cdot \frac{\partial \tilde{R}}{\partial a} + \frac{L^2}{\mu^2 e} \cdot \frac{\partial \tilde{R}}{\partial e}$$

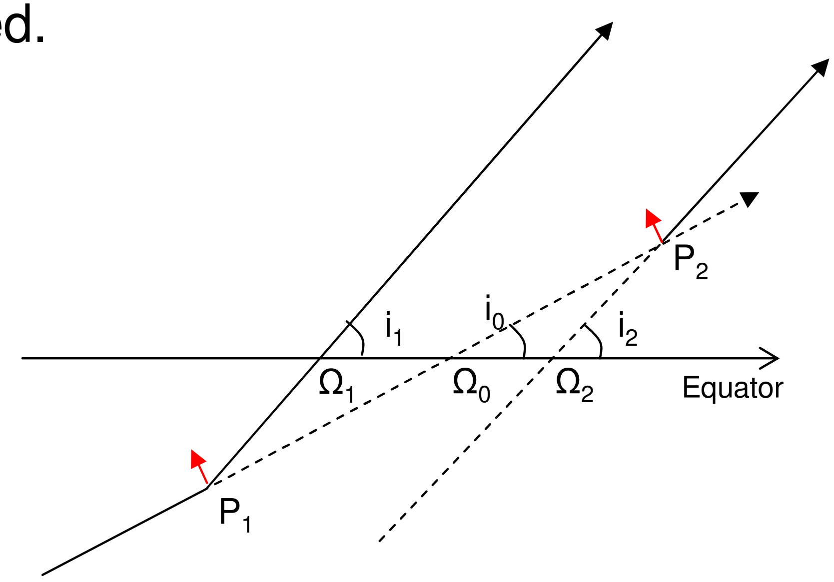
$$L \equiv \sqrt{\mu a (1-e^2)}$$

5.6 Geometrical Considerations of Perturbations

- After writing down the perturbation equations we will study the impact of perturbing accelerations on orbital elements separately for the directions T, N and W using *geometrical considerations*.
- These considerations stem from Moulton (1914) and were formulated in a similar way already by Isaac Newton in his Principia (1687).
- We study short specific impulses (*instantaneous velocity changes*) in the three directions in specific points along the orbit and analyze their impact on the orbital elements.
- The same considerations are valid also for continuous perturbing accelerations acting in a particular direction since a continuous sequence of infinitesimal specific impulses can be considered as an *acceleration*.

5.6.1 Perturbations by the W-Component

- The W-component of a perturbation is orthogonal to the orbital plane and affects *only the orbital plane*, i.e., the elements i and Ω but has no influence on the elements a, e, T_0.
- The change in $d\omega = -d\Omega \cos i$ is only due to the displacement of the node.
- We consider a pulse in positive W-direction in the point P_1 close to the ascending node. The satellite changes its direction of motion on the celestial sphere.
- *Orbital inclination and node* have changed.
  We have
  $$i_1 > i_0, \quad \Omega_1 < \Omega_0$$
  i.e. an increase of the inclination and a *recession of the node*.
- If a pulse with equal magnitude is executed in point P_2, the inclination increases too, but the node precesses in positive direction.



5.6.1 Perturbations by the W-Component

- The corresponding figure in the *descending node* shows that a pulse in negative W-direction leads to a decrease of the inclination. A pulse before the passage through the node leads to a prograde motion of the node, a pulse after the passage through the node to a retrograde motion of the node.
- A pulse acting exactly *in the node* only changes the inclination of the orbit.
- A pulse in the point with *highest or lowest elevation* above the equator only changes the location of the node and not the inclination.
- In summary: *a perturbing acceleration* in positive W-direction
  - causes a *prograde motion of the node* in the first and the second quadrant (above the equatorial plane) and a *retrograde motion of the node* in the third and the fourth quadrant of the orbit (below the equatorial plane).
  - causes an *increase of the orbital inclination* in the first and the fourth quadrant (close to the ascending node) and a *decrease of the orbital inclination* in the second and third quadrant (close to the descending node).

5.6.2 Action of T-Component on Semimajor Axis

- We use the *vis-viva* or energy equation of the two-body problem

$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$

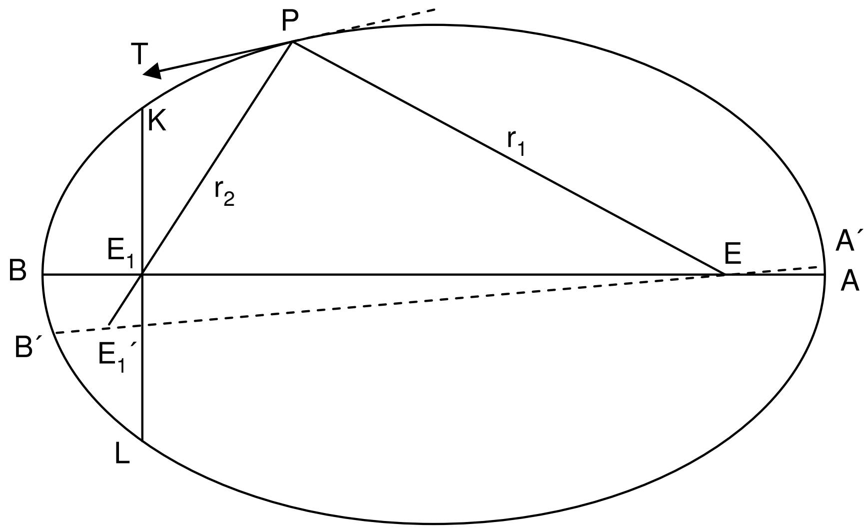
- A positive pulse in T-direction increases v^2 but does not cause an instantaneous change of the distance r, therefore $1/a$ has to become smaller or a larger

$$v\Delta v \propto \Delta(v^2) \propto -\Delta\left(\frac{1}{a}\right) = \frac{\Delta a}{a^2} \qquad \rightarrow \qquad \Delta a \propto a^2 v \Delta v$$

- A positive perturbing acceleration in T-direction increases the semimajor axis independently on the position along the orbit.
- For given Δv the change in a largest when v is maximal, i.e., in the *perigee*.
- The change in a is larger for larger a.

5.6.3 Action of T-Component on Line of Apsides

- The *line of apsides* of an ellipse is the line connecting perigee and apogee.
- The straight lines from a point P on the circumference of an ellipse to the two focal points E and E_1 enclose same angles with the tangent to the ellipse at point P.
- An impulse in positive tangential direction causes an increase of the velocity but no instantaneous change of the direction of motion.
- The position of focal point E does not change and thus the distance r_1 remains unchanged.
- Because of $r_2 = 2a - r_1$ the increase in the semimaior axis causes an increase of the value of r_2 while the direction of r_2 remains unchanged.
- The second focal point thus moves from E_1 to $E_1{}'$ and we obtain an advancement of the line of apsides.



5.6.3 Action of T-Component on Line of Apsides

- We observe that a perturbing acceleration in positive T-direction causes an *prograde motion* of the line of apsides in the first half of the revolution and a *retrograde motion* during the second half of the revolution.
- The effect is the same (with different sign) for points on the ellipse that are symmetrical to the major axis.
- In points K and L the motion perpendicular to the major axis of the second focal point is maximal and the line of apsides exhibits a *maximum rate of rotation*.
- The point of maximum rotation of the line of apsides is displaced from K resp. L towards the perigee because the semimajor axis changes faster if the satellite is close to the perigee.
- The rotation of the line of apsides vanishes for a pulse acting either in the *perigee* or in the *apogee*.

5.6.4 Action of T-Component on Eccentricity

- A pulse acting in tangential direction in the perigee does not change the distance $q=a(1-e)$.
- Therefore the eccentricity is *increased* if the semimajor axis increases:

$$\Delta e = \frac{1-e}{a}\Delta a$$

- If the pulse acts in the apogee the distance $Q=a(1+e)$ remains unchanged. As a consequence the eccentricity is *reduced* if the semimajor axis is increased:

$$\Delta e = -\frac{1+e}{a}\Delta a$$

- Between perigee and apogee there is therefore a point in which a pulse in tangential direction does not change the eccentricity.

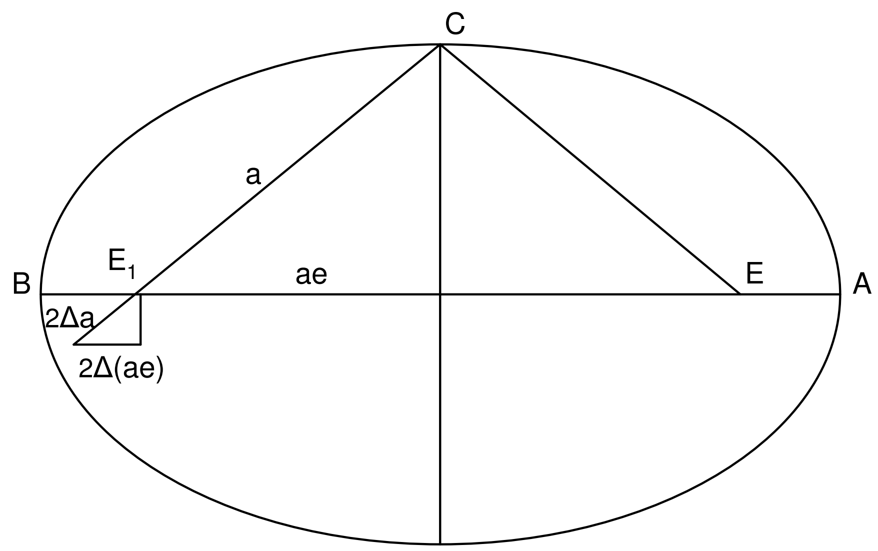
5.6.4 Action of T-Component on Eccentricity

- This point is the endpoint of the minor axis (point C).
- The figure shows that from the two similar triangles we get the following relation:

$$\frac{ae}{a} = \frac{2\Delta(ae)}{2\Delta a}$$

- Rearranged $\qquad e\Delta a = \Delta(ae) = e\Delta a + a\Delta e$

or $\qquad \Delta e = 0.$

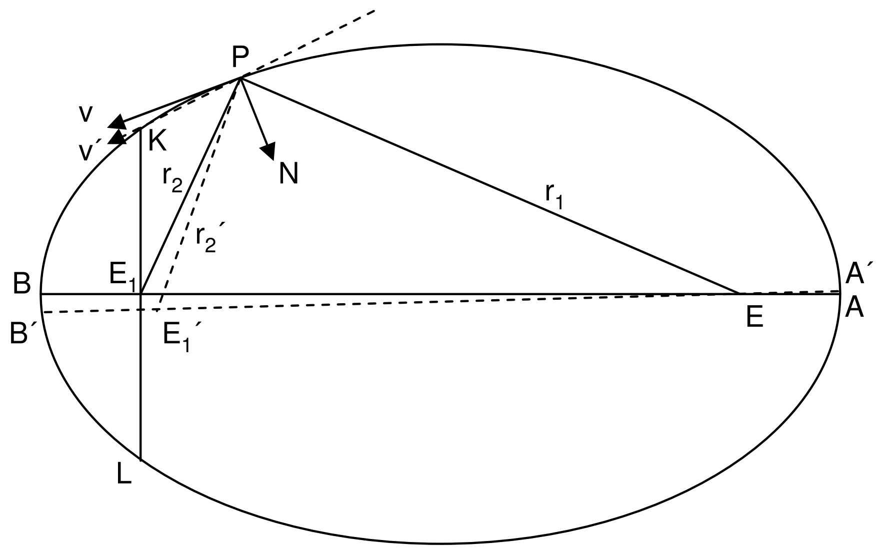


5.6.5 Action of N-Component on Semimajor Axis

- According to the energy equation the *semimajor axis* depends on the absolute value and not on the direction of the velocity.
- The semimajor axis remains unaffected by a pulse acting in normal direction to the velocity vector because neither the absolute value of the velocity nor the distance to the focal point is instantaneously changed by such a perturbation.
- The semimajor axis *is not changed* by the normal component of a perturbing acceleration.

5.6.6 Action of N-Component on Line of Apsides

- A pulse in N-direction in point P changes the direction of the tangential.
- This changes the direction of r_2 to r_2' because the direction to the focal point E remains unchanged and because the angles between the tangential and the direction to the two focal points are equal.
- The length of r_2 and r_2' remain the same because the semimajor axis does not change.
- A perturbing acceleration in positive N-direction therefore leads to an *prograde motion* of the line of apsides in the arc LAK.
- In the arc KBL the line of apsides performs a *retrograde motion*.
- A pulse in the N-direction in the points K and L leaves the orientation of the line of apsides unchanged.



5.6.6 Action of N-Component on Line of Apsides

- In the following we compare the rate of rotation of the line of apsides for a pulse of equal magnitude in N-direction in the perigee and in the apogee.
- If the pulse acts in one of the two points the second focal point $E_1{'}$ moves along the line KL.
- The change in orientation of the tangential by a given normal velocity change is inversely proportional to the velocity.
- According to the law of the areas the velocity in the perigee and in the apogee is inversely proportional to the distance from the focal point.
- Therefore

$$\Delta v / v_A : \Delta v / v_B = v_B : v_A = a(1-e) : a(1+e)$$

- The displacement of the line of apsides is proportional to the displacement of the second focal point E_1 along the line KL.
- This displacement in turn is proportional to change in direction of the velocity vector ($\Delta v/v$ for small angles) multiplied with the distance to E_1.

5.6.6 Action of N-Component on Line of Apsides

- The angle of rotation R of the line of apsides is proportional to this displacement d of E_1.
- Together we get

$$R_A : R_B = d_A : d_B = a(1+e)\Delta v / v_A : a(1-e)\Delta v / v_B$$
$$= a(1+e)v_B : a(1-e)v_A = 1:1$$

- We find that equal normal pulses in the perigee and the apogee cause a displacement of the line of apsides that is *equal in magnitude and opposite in sign*.
- The velocity in the apogee is smaller than in the perigee by a factor of $(1-e)/(1+e)$. A continuous acceleration has thus more time to act in the vicinity of the apogee.
- The average displacement of the line of apsides caused by a constant normal acceleration is therefore *retrograde*.

5.6.7 Action of N-Component on Eccentricity

- The eccentricity is given by

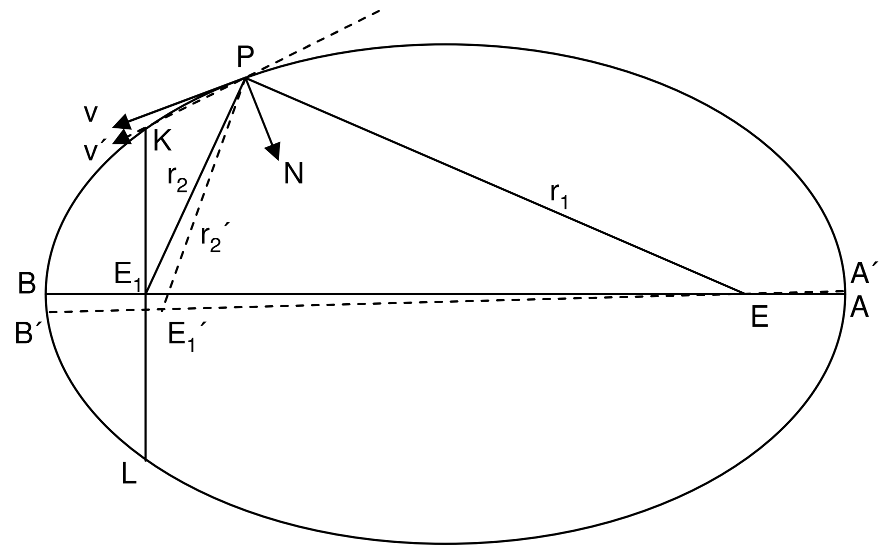
$$e = \frac{EE_1}{2a}$$

- After the action of a normal component of a pulse the eccentricity is

$$e' = \frac{EE_1'}{2a}$$

- Since the semimajor axis a does not change the figure tells that a positive normal perturbation *reduces* the eccentricity in the first half revolution and *increases* the eccentricity in the second half revolution because $EE1$ gets smaller in the first and larger in the second case.

- A normal pulse in the perigee or in the apogee leaves the eccentricity *unchanged*.



5.6.8 Summary of Perturbation Effects

Component	T	N	W
Node	0	0	Advance above, recession below the equatorial plane
Inclination	0	0	Increase in the first and fourth, decrease in the second and third quadrant.
Semimajor axis	Increases steadily	0	0
Line of apsides	Prograde in interval ACB, retrograde in interval BDA	Prograde in interval LAK, retrograde in interval KBL	$-d\Omega \cos i$
Eccentricity	Increase in interval DAC, decrease in interval CBD	Decrease in interval ACB, increase in interval BDA.	0

All information for positive perturbing accelerations

5.6.8 Summary of Perturbation Effects

- We find with the geometrical consideration what we already got for the perturbation equations:
  - The perturbation equations of the elements that describe shape, size, and time dependence of the orbit (a, e, T_p) depend only on the perturbing acceleration components R and S.
  - The perturbation equations of the elements that describe the orientation of the orbital plane in space (i, Ω), contain only the perturbing acceleration component W.