1- Tow degree of freedom (2 DOF)
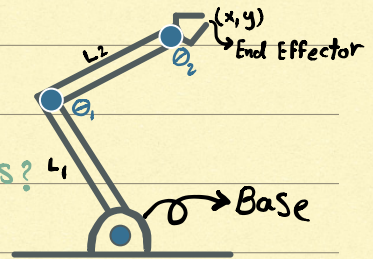
➡️ **Forword** :- to determain where the robot hand is ?

➡️ **Inverse** :- to calculate what each joint variable is ?
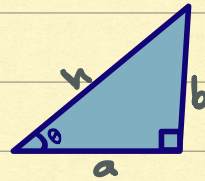

End Effector (x,y) — Base

Basic Rule

1-Pythagoras's Theorem : $a^2 + b^2 = h^2$

2- $\theta = \theta_1 + \theta_2$

3- قانون فك التربيع : $(x+y)^2 = x^2 + 2xy + y^2$

$\sin\theta = \dfrac{b}{h}$

$\cos\theta = \dfrac{a}{h}$

$\tan\theta = \dfrac{b}{a}$

1- Forword kinematics 2 DOF

$X , Y , \theta \leftarrow$ مجاهيل $\emptyset_1 , \emptyset_2 , L_1 , L_2 \leftarrow$ المعطيات

Transform joint coordinate to end effector coordinates $\leftarrow$ المطلوب

$L_1 = L_1 \cos\emptyset_1 \quad , \quad L_1 \sin\emptyset_1$

$L_2 = L_2 \cos(\emptyset_1 + \emptyset_2)$

$X = L_1 \cos\emptyset_1 + L_2 \cos(\emptyset_1 + \emptyset_2)$

$Y = L_1 \sin\emptyset_1 + L_2 \sin(\emptyset_1 + \emptyset_2)$

$\theta = \emptyset_1 + \emptyset_2$
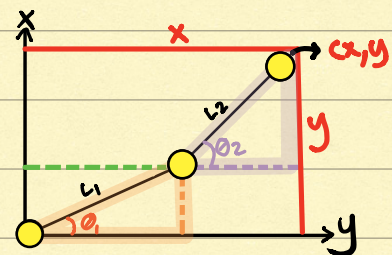
2- Inverse kinematics 2 DOF

$\emptyset_1 , \emptyset_2 \leftarrow$ مجاهيل $Y , X , \theta , L_1 , L_2 \leftarrow$ المعطيات

Transform end effector coordinates to joint coordinate $\leftarrow$ المطلوب

$J = L_2 \sin\emptyset_2$

$K = L_1 + L_2 \cos\emptyset_2$

$X^2 + Y^2 = J^2 + K^2$

Continue >>>
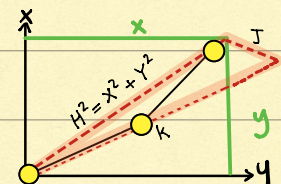
$$X = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$
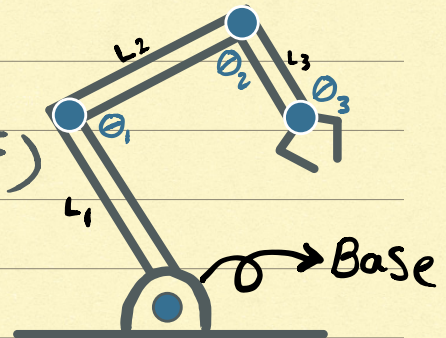$$= L_1 \cos\theta_1 + L_2 \cos\theta_1 \cos\theta_2 + L_2 \sin\theta_1 \sin\theta_2 \quad \rightarrow \text{details}$$

$$Y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$
$$= L_1 \sin\theta_1 + L_2 \sin\theta_1 \cos\theta_2 + L_2 \cos\theta_1 \sin\theta_2 \quad \rightarrow \text{details}$$

$$\cos\theta_2 = \frac{x^2 + y^2 - L_1^2 - L_2^2}{2 L_1 L_2}, \quad \theta_2 = \cos-\left(\frac{x^2 + y^2 - L_1^2 - L_2^2}{2 L_1 L_2}\right)$$

$$\emptyset_1 = \theta - \theta_2$$

2- Three degree of freedom (3 DOF)

1- Forword kinematics 3 DOF

المعطيات $\rightarrow \emptyset_1, \emptyset_2, \emptyset_3, L_1, L_2, L_3$ اجاهيل $\rightarrow X, Y, \theta$

$$X = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) + L_3 \cos(\theta_1 + \theta_2 + \theta_3)$$
$$Y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2) + L_3 \sin(\theta_1 + \theta_2 + \theta_3)$$
$$\theta = (\theta_1 + \theta_2 + \theta_3)$$

2- Inverse kinematics 3 DoF

المعطيات $\rightarrow X, Y, \theta$ اجاهيل $\rightarrow \emptyset_1, \emptyset_2, \emptyset_3, L_1, L_2, L_3$

$$k_1 = L_1 + L_2 \cos\theta_2 \qquad k_2 = L_2 \sin\theta_2$$

$$\sin \theta_2 = \sqrt{\pm(1 - \cos^2 \theta_2)}$$

$$\cos \theta_2 = \frac{x^2 + y^2 - L_1^2 - L_2^2}{2 L_1 L_2}$$

$$X_n = X - L_3 \cos \theta \qquad Y_n = Y - L_3 \sin \theta$$

Final equations are :-

$$\emptyset_1 = \text{atan} 2 \left((K_1 y_n - K_2 x_n), (K_1 x_n - K_2 y_n) \right)$$

$$\emptyset_2 = \text{atan} 2 \left(\sin \theta_2, \cos \emptyset_2 \right)$$

$$\emptyset_3 = \theta - (\emptyset_1 + \emptyset_2)$$

Done

Bayan Baqa

Task 4

Robotics Track