compute 1^{st} column of L

$a_{21} = l_{21}u_{11} \Rightarrow \boxed{l_{21} = \frac{a_{21}}{u_{11}}}$

$a_{31} = l_{31}u_{11} \Rightarrow l_{31} = \frac{a_{31}}{u_{11}}$

$l_{i1} = \frac{a_{i1}}{u_{11}}, \; i = 2, \dots n.$

Step-2 - compute 2^{nd} row of U

$a_{22} = l_{21}u_{12} + u_{22}$

$\boxed{u_{22} = a_{22} - l_{21}u_{12}}$

$a_{23} = l_{21}u_{13} + u_{23} \Rightarrow \boxed{u_{23} = a_{23} - l_{21}u_{13}}$

$\boxed{u_{2j} = a_{2j} - l_{21}u_{1j}}$

Doolittle's method $\boxed{l_{ii} = 1}$

Eg

$$A = \begin{bmatrix} 2 & 1 & -1 & 3 \\ -2 & 2 & 6 & -4 \\ 4 & 14 & 19 & 4 \\ 6 & 0 & -6 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & l_{43} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ 0 & u_{22} & u_{23} & u_{24} \\ 0 & 0 & u_{33} & u_{34} \\ 0 & 0 & 0 & u_{44} \end{bmatrix}$$

Step-1 compute 1^{st} row of U

$u_{1j} = a_{1j}$ i.e the 1^{st} row of U is the same as

$\boxed{u_{11} = 2}$ $\boxed{u_{12} = 1}$ $\boxed{u_{13} = -1}$ $\boxed{u_{14} = 3}$

compute the 1^{st} column of L

$l_{i1} = \frac{a_{i1}}{u_{11}}, \; i = 2, \dots n.$

$l_{21} = \frac{a_{21}}{u_{11}} = \frac{-2}{2} = -1 \Rightarrow l_{21} = -1$

$l_{31} = \frac{a_{31}}{u_{11}} = \frac{4}{2} = 2 \Rightarrow \boxed{l_{31} = 2}$

$$\left[\begin{array}{ccc|c|ccc} 0 & 1 & -\frac{7}{2} & -\frac{17}{2} & -1 & \frac{1}{2} & 0 \\ 0 & 3 & -11 & -27 & -3 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + m_{12}R_2 \\ R_3 \to R_3 + m_{32}R_2 \end{array} \quad \left[\begin{array}{ccc|c|ccc} 1 & 0 & \frac{11}{2} & \frac{35}{2} & 2 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{7}{2} & -\frac{17}{2} & -1 & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} & 0 & -\frac{3}{2} & 1 \end{array}\right]$$

$$R_3 \to -2R_3 \quad \left[\begin{array}{ccc|c|ccc} 1 & 0 & \frac{11}{2} & \frac{35}{2} & 2 & -\frac{1}{2} & 0 \\ 0 & 1 & -\frac{7}{2} & -\frac{17}{2} & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 3 & 0 & 3 & -2 \end{array}\right]$$

$1 + \frac{1}{2}$ $\quad 2 - \frac{1}{2}$ $\quad 2 - \frac{1}{2}$ $\quad 2 - \frac{1}{2}$

$$\begin{array}{l} R_1 \to R_1 + m_{13}R_3 \\ R_2 \to R_2 + m_{23}R_3 \end{array} \quad \left[\begin{array}{ccc|c|ccc} 1 & 0 & 0 & 1 & 2 & -17 & 11 \\ 0 & 1 & 0 & 2 & -1 & 11 & -7 \\ 0 & 0 & 1 & \underline{3} & 0 & 3 & -2 \end{array}\right]$$

A^{-1}

Matrix Decomposition Methods

$Ax = b_i, \quad i = 1, \ldots, 100{,}000$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ l_{21} & 1 & 0 & \cdots & 0 \\ l_{31} & l_{32} & 1 & \cdots & 0 \\ \vdots & & & & \\ l_{n1} & l_{n2} & l_{n3} & \cdots & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ 0 & u_{22} & \cdots & u_{2n} \\ 0 & 0 & u_{33} & u_{3n} \\ 0 & 0 & 0 \cdots & u_{nn} \end{bmatrix}$$

$u_{21} l_{21} + u_{22} = a_{22}$

Step 1 - comput 1st row of U

$a_{11} = u_{11} \Rightarrow \boxed{u_{11} = a_{11}}$

$a_{12} = u_{12} \Rightarrow \boxed{u_{12} = a_{12}}$

$$\boxed{u_{1j} = a_{1j}}$$

1st row of U is the same as first row of A.

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$l_{41} = \frac{a_{41}}{u_{11}} = \frac{6}{2} = 3 \Rightarrow \boxed{l_{41} = 3}$

step-2 compute 2^{nd} row of u

$a_{22} = l_{21}u_{12} + u_{22} \Rightarrow u_{22} = a_{22} - l_{21}u_{12}$

$= 2 - (-1)(1) = 3$

$\boxed{u_{22} = 3}$

$a_{23} = l_{21}u_{13} + u_{23} \Rightarrow u_{23} = a_{23} - l_{21}u_{13}$

$= 6 - (-1)(-1)$

$6 - 1 = 5$

$\boxed{u_{23} = 5}$

$\boxed{u_{11} = 2}$, $\boxed{u_{12} = 1}$, $\boxed{u_{13} = -1}$, $\boxed{u_{14} = 3}$

compute the 2^{nd} column of L

$a_{24} = l_{21}u_{14} - u_{24}$

$\Rightarrow u_{24} = a_{24} - l_{21}u_{14}$

$= -4 - (-1)(3)$ $\boxed{u_{24} = -1}$

compute the 2^{nd} column of L

$a_{32} = l_{31}u_{12} + l_{32}u_{22}$

$\Rightarrow l_{32} = \frac{a_{32} - l_{31}u_{12}}{u_{22}}$

$= \frac{14 - 2(1)}{3}$ $\boxed{l_{32} = 4}$

$a_{42} = l_{41}u_{12} + l_{42}u_{22}$

$\Rightarrow l_{42} = \frac{a_{42} - l_{41}u_{12}}{u_{22}}$

step-3 compute the 3^{rd} row of u

$a_{33} = l_{31}u_{13} + l_{32}u_{23} + u_{33}$

$\Rightarrow u_{33} = a_{33} - (l_{31}u_{13} + l_{32}u_{23})$

$= 19 - (2)(-1) + 4 \times 5)$

$\boxed{u_{33} = 1}$

$a_{11} = l_{11} \Rightarrow \boxed{l_{11} = a_{11}}$

$a_{21} = l_{21} \Rightarrow \boxed{l_{21} = a_{21}}$

$\boxed{l_{i1} = a_{i1}}$

compute 1^{st} row of U

$a_{12} = l_{11}u_{12} \Rightarrow u_{12} = \frac{a_{12}}{l_{11}}$

$a_{13} = l_{11}u_{13} \Rightarrow u_{13} = \frac{a_{13}}{l_{11}}$

$\boxed{u_{1j} = \frac{a_{1j}}{l_{11}}}$ $\quad j = 2 - - - n.$

step-2 $\quad$ 2^{nd} column of L

$a_{i2} = l_{i1}u_{12} + l_{i2}$

$\boxed{l_{i2} = a_{i2} - l_{i1}u_{12}}$ $\quad i = 2 - - - n.$

- 2^{nd} row of U

$a_{2j} = l_{21}u_{1j} + l_{22}u_{2j}$, $j = 3, - - - n.$

$u_{2j} = \frac{a_{2j} - l_{21}u_{1j}}{l_{22}}$

step-k $\quad$ compute the k^{th} column of L

~~$a_{kj} = l_{k1}u_{1j} + l_{k2}u_{2j}$~~

$a_{ik} = l_{i1}u_{1k} + l_{i2}u_{2k} + l_{i3}u_{3k} + - - - l_{ik}u_{kk}, j = k+1 \cdots n$

$\Rightarrow l_{ik} = a_{ik} - \left(\sum_{j=1}^{k-1} l_{ij}u_{jk} \right)$

k^{th} row of U

$a_{kj} = l_{k1}u_{1j} + l_{k2}u_{2j} + - - - + l_{kk}u_{kj}$

$u_{kj} = \frac{a_{kj} - \left(\sum_{i=1}^{k-1} l_{ki}u_{ij} \right)}{l_{kk}}$

$a_{34} = l_{31}u_{14} + l_{32}u_{24} + u_{34}$

$u_{34} = a_{34} - (l_{31}u_{14} + l_{32}u_{24})$

$= 4 - (2\times3) + 4(-1)$ $\boxed{u_{23} = 5}$

$\boxed{u_{34} = 2}$ $\boxed{u_{11} = 2}$ $\boxed{u_{12} = 1}$ $\boxed{u_{13} = -1}$ $\boxed{u_{14} = 3}$

$a_{43} = l_{41}u_{13} + l_{42}u_{23} + l_{43}u_{33}$

$l_{43} = \dfrac{a_{43} - (l_{41}u_{13} + l_{42}u_{23})}{u_{33}}$

$= \dfrac{-6 - (3\times(-1) + (-1\times5))}{u_{33}}$

$\boxed{l_{43} = 2}$

$a_{44} = l_{41}u_{14} + l_{42}u_{24} + l_{43}u_{34} + u_{44}$

$u_{44} = 12 - (3\times3 + (-1)(-1) + (2\times2))$

$\boxed{u_{33} = 1}$ $\boxed{u_{44} = -2}$

$$A = \begin{bmatrix} 2 & 1 & -1 & 3 \\ -2 & 2 & 6 & -4 \\ 4 & 14 & 19 & 4 \\ 6 & 0 & -6 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 \\ 3 & -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 & 3 \\ 0 & 3 & 5 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

Crout Reduction

$u_{ii} = 1$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & & a_{n3} \end{bmatrix} \begin{bmatrix} l_{11} & 0 & 0 & \cdots & 0 \\ l_{21} & l_{22} & 0 & \cdots & 0 \\ \vdots & \vdots & & & \\ l_{n1} & l_{n2} & \cdots & & l_{nn} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & 1 & u_{23} & \cdots & u_{2n} \\ \vdots & & & & \\ 0 & 0 & \cdots & & 1 \end{bmatrix}$$

Step-1 1^{st} column of L

1^{st} column of L is the same as 1^{st} column of A.

$j=4$ $\quad u_{24} = \dfrac{a_{24} - l_{21}u_{14}}{l_{22}}$

$= \dfrac{-2 - 3 \times \frac{1}{3}}{9} = -\frac{1}{3}$

$\boxed{u_{24} = -1/3}$

Step-3 $\quad$ 3rd column of L

$a_{i3} = l_{i1}u_{13} + l_{i2}u_{23} + l_{i3}$

$l_{i3} = a_{i3} - (l_{i1}u_{13} + l_{i2}u_{23})$

$i=3$ $\quad l_{33} = a_{33} - (l_{31}u_{13} + l_{32}u_{23})$

$= 18 - (3 \times \frac{1}{3} + (-3)(-\frac{1}{3}))$

$\boxed{l_{33} = 16}$

$i=4$ $\quad l_{43} = a_{43} - (l_{41}u_{13} + l_{42}u_{23})$

$= 10 - (3 \times 1/3 + (-3)(-1/3))$

$\boxed{l_{43} = 8}$

$a_{34} = l_{31}u_{14} + l_{32}u_{24} + l_{33}u_{34}$

$u_{34} = \dfrac{a_{34} - (l_{31}u_{14} + l_{32}u_{24})}{l_{33}} = \dfrac{10 - (3 \times \frac{1}{3} + (-3)(-\frac{1}{3}))}{16} = \frac{1}{2}$

$\boxed{u_{34} = \frac{1}{2}}$

$l_{11} = 9$, $l_{22} =$

Step-4: $a_{44} = l_{41}u_{14} + l_{42}u_{24} + l_{43}u_{34} + l_{44}$

$l_{44} = a_{44} - (l_{41}u_{14} + l_{42}u_{24} + l_{43}u_{34})$

$= 10 - (3 \times \frac{1}{3} + (-3)(-\frac{1}{3}) + 8\,\frac{1}{2})$ $\quad \boxed{l_{44} = 4}$

$$\begin{bmatrix} 9 & 3 & 3 & 3 \\ 3 & 10 & -2 & -2 \\ 3 & -2 & 18 & 10 \\ 3 & -2 & 10 & 10 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 & 0 \\ 3 & 9 & 0 & 0 \\ 3 & -3 & 16 & 0 \\ 3 & -3 & 8 & 4 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 24 \\ 17 \\ 45 \\ 29 \end{bmatrix}$$

$Ux = y$ $\qquad LY = b$

Example solve (p.d. f.f matrix) by using Crouts method

$$\begin{bmatrix} 9 & 3 & 3 & 3 \\ 3 & 10 & -2 & -2 \\ 3 & -2 & 12 & 10 \\ 3 & -2 & 10 & 10 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ l_{31} & l_{32} & l_{33} & 0 \\ l_{41} & l_{42} & l_{43} & l_{44} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} & u_{14} \\ 0 & 1 & u_{23} & u_{24} \\ 0 & 0 & 1 & u_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step-1 $\Rightarrow$ 1st column of L

$a_{i1} = l_{i1} \Rightarrow l_{i1} = a_{i1}$, $i = 1, \dots n$.

$\boxed{l_{11} = 9}$, $\boxed{l_{21} = 3}$, $\boxed{a_{31} = 3}$, $\boxed{l_{41} = 3}$

* 1st row of U

$a_{1j} = l_{11} u_{1j}$, $j = 2, 3, 4$

$$u_{1j} = \frac{a_{1j}}{l_{11}} = \boxed{u_{12} = \frac{1}{3}}, \boxed{u_{13} = \frac{1}{3}}, \boxed{u_{14} = \frac{1}{3}}$$

Step-2 2nd column of L

$a_{i2} = l_{i1} u_{12} + l_{i2} u_{22}$ $i = 2, 3, 4$

$\Rightarrow l_{i2} = a_{i2} - l_{i1} u_{12}$

$i=2$ $l_{22} = a_{22} - l_{21} u_{12} = 10 - 3 \times \frac{1}{3} = 9$

$\boxed{l_{22} = 9}$

$i=3$ $l_{32} = a_{32} - l_{31} u_{12} = -2 - 3 \times \frac{1}{3} = -3$

$\boxed{l_{32} = -3}$

$i=4 =$ $l_{42} = a_{42} - l_{41} u_{12} = -2 - 3 \times \frac{1}{3} = -3$

$\boxed{l_{42} = -3}$

* 2nd row of U

$a_{2j} = l_{21} u_{1j} + l_{22} u_{2j}$

$$u_{2j} = \frac{a_{2j} - l_{21} u_{1j}}{l_{22}} \quad j = 3, 4$$

$$j=3 \quad u_{23} = \frac{a_{23} - l_{21} u_{13}}{l_{22}}$$

$$u_{23} = -2 - 3 \times \frac{1}{3} = -\frac{1}{3} \quad \boxed{u_{23} = -\frac{1}{3}}$$

18.5/20 V. good!

ARBA MINCH UNIVERSITY
COLLEGE OF NATURAL SCIENCES
DEPARTMENT OF MATHEMATICS

Course Title: Numerical Methods	Test 2
Name: Birut Gutema	**ID number:** 015/09

1. Solve the system $\boldsymbol{Ax} = b$ using the Doolittle decomposition where **(7 points)**

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 0 \\ 3 & 5 & 2 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

2. Find the solution of the system of equations **(7 points)**

$$x_1 - 3x_2 + 7x_3 = 17$$
$$3x_1 - 6x_2 + 2x_3 = 23$$
$$-4x_1 + x_2 - x_3 = -8$$

correct to one decimal place using Gauss Siedel method with an initial guess of $\boldsymbol{x}^{(0)} = [0.9, -3.1, 0.9]^T$.

3. Perform two iterations of Newton's Method in approximating the solution of the non-linear system

$$x^2 + y^2 - 2 = 0$$
$$x^3 - y = 0$$

an initial guess $\boldsymbol{x}^0 = [0.5, 0]^T$. **(6 points)**

X_0, X_1

$$\begin{bmatrix} 9 & 0 & 0 & 0 \\ 3 & 9 & 0 & 0 \\ 3 & -3 & 16 & 0 \\ 3 & -3 & 8 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 24 \\ 17 \\ 45 \\ 29 \end{bmatrix} \qquad y = \begin{bmatrix} 8/3 \\ 1 \\ 5/2 \\ 1 \end{bmatrix}$$

$$9y_1 = 24 \Rightarrow \boxed{y_1 = 8/3}$$

$$3y_1 + 9y_2 = 17 \qquad 8 + 9y_2 = 17 \Rightarrow \boxed{y_2 = 1}$$

$$\begin{bmatrix} 1 & 1/3 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & -\frac{1}{3} & -1/3 \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 8/3 \\ 1 \\ 5/2 \\ 1 \end{bmatrix} \qquad \boxed{x_4 = 1}$$

$$x_3 = \frac{x_4}{2} = \frac{5}{2}$$

$$x_3 = \frac{5}{2} - \frac{1}{2} = 2 \Rightarrow \boxed{x_3 = 2}$$

$$x_2 - \frac{1}{3}x_3 - \frac{1}{3}x_4 = 1$$

$$x_2 - \frac{2}{3} - \frac{1}{3} = 1 \Rightarrow \boxed{x_2 = 2}$$

$$x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 + \frac{1}{3}x_4 = 8/3$$

$$x_1 + 2/3 + 2/3 + 1/3 = 8/3$$

$$x_1 = 8/3 - 5/3 = 1$$

$$x = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} \quad /\!/$$

7

1) $\begin{bmatrix} 1 & 2 & 1 \\ -2 & -3 & 0 \\ 3 & 5 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$

step 1) a) 1st row of U

$U_{11} = 1, \quad U_{12} = 2, \quad U_{13} = 1$

b) 1st column of l

$l_{21} U_{11} = -2$, $\quad l_{31} U_{11} = 3$

$l_{21} = \frac{-2}{1} = -2$ $\quad l_{31} = \frac{3}{1} = 3$

step 2) a) 2nd row of U

$l_{21} U_{12} + U_{22} = -3$ $\quad l_{21} U_{13} + U_{23} = 0$

$U_{22} = -3 - (-2 \times 2)$ $\quad U_{23} = -(-2 \times 1)$

$U_{22} = 1$ $\quad U_{23} = 2$

b) 2nd column of l

$l_{31} U_{12} + l_{32} U_{22} = 5$

$3 \times 2 + l_{32} = 5$

$l_{32} = -1$

step 3) a) 3rd row of U

$l_{31} U_{13} + l_{32} U_{23} + U_{33} = 2$

$U_{33} = 2 - (3 \times 1) - (-1 \times 2)$

$U_{33} = 2 - 1 = 1$

$L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$Ax = b$

$L\underbrace{Ux}_{y} = b, \quad Ux = y$

$Ly = b$

$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$y_1 = 1$

$-2y_1 + y_2 = 0$

$-2 + y_2 = 0$

$y_2 = 2$

$3y_1 + (-y_2) + y_3 = 1$

$3y_1 - y_2 + y_3 = 1$

$3 - 2 + y_3 = 1$

$y_3 = 0$

$$y = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$\therefore Ux = y$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$x_3 = \underline{0}$

$x_2 + 2x_3 = 2$

$x_2 = \underline{2}$

$x_1 + 2x_2 + x_3 = 1$

$x_1 = 1 - 4$

$x_1 = \underline{-3}$

② 7

$-4x_1 + x_2 - x_3 = -8$

$3x_1 - 6x_2 + 2x_3 = 23$

$x_1 - 3x_2 + 7x_3 = 17$

$$x^0 = \begin{bmatrix} 0.9 \\ -3.1 \\ 0.9 \end{bmatrix}$$

$$x_1 = \frac{-8 - x_2 + x_3}{-4} = \frac{8 + x_2 - x_3}{4}$$

$$x_2 = \frac{23 - 3x_1 - 2x_3}{-6} = \frac{-23 + 3x_1 + 2x_3}{6}$$

$$x_3 = \frac{17 - x_1 + 3x_2}{7}$$

$\frac{1}{2} \times 10^{1}$

0.5×10^{5}

$$x_1^1 = \frac{8 + (-3.1) - 0.9}{4} = \underline{1}$$

$$x_2^1 = \frac{-23 + 3(1) + 2(0.9)}{6} = -3.03$$

$$x_3^1 = \frac{17 - 1 + 3(-3.03)}{7} = 0.98$$

$$\boxed{\frac{[x_1^1 - x^0]}{x_1^1} < \varepsilon} = \frac{|1 - 0.9|}{1} = 0.1$$

$\frac{|0.98 - 0.9|}{0.98}$ 0.082

$\cancel{|-3.03 + 3.1|} = \cancel{0.14}$ $\frac{|-3.03 + 3.1|}{}$ $= 0.023 < \varepsilon$

$x_1^2 = \frac{8+(-3.05)-0.98}{4} = 0.99$

$x_2^2 = \frac{-23+3(0.99)+2(0.98)}{6} = -3.01$

$x_3^2 = \frac{17-0.99+3(-3.01)}{7} = 0.99$

$$\frac{|x_1^2 - x_1^1|}{x_1^2} = \frac{|0.99-1|}{|0.99|} < \varepsilon$$

$x_1^3 = \frac{8+(-3.01)-0.99}{4} = 1$

$x_2^3 = \frac{-23+3(1)+2(0.99)}{6} = -3.00$

$x_3^3 = \frac{17-1+3(-3)}{7} = 1$

4.5

(3) $x^2+y^2-2=0$ $\quad x^0 = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$

$x^3 - y = 0$

$f_1(x,y) = x^2+y^2-2=0$

$f_2(x,y) = x^3-y=0$

$J(\bar{x})\bar{h} = -f(\bar{x})$

$$J(\bar{x}) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x & 2y \\ 3x^2 & -1 \end{bmatrix} \quad J(\bar{x}^0) = \begin{bmatrix} 1 & 0 \\ 0.75 & -1 \end{bmatrix}$$

$$f(\bar{x}^0) = \begin{bmatrix} -1.75 \\ 0.125 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 0 \\ 0.75 & -1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} 1.75 \\ -0.125 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} 1.75 \\ -1.438 \end{bmatrix}$$

$$-h_2 = -1.488 \qquad h_1 = \underline{1.75} \qquad h = \begin{bmatrix} 1.75 \\ 1.488 \end{bmatrix}$$

$$h_2 = \underline{1.438}$$

$$x^1 = x^0 + h \qquad \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} + \begin{bmatrix} 1.75 \\ 1.488 \end{bmatrix} = \begin{bmatrix} 2.25 \\ 1.438 \end{bmatrix}$$

Step②

$$J(x) = \begin{bmatrix} 2x & 2y \\ 3x^2 & -1 \end{bmatrix}, \quad J(x^1) = \begin{bmatrix} 4.5 & 2.876 \\ 15.188 & -1 \end{bmatrix}$$

$$F(x^1) = \begin{bmatrix} 5.13 \\ 1.536 \end{bmatrix}$$

-3.375

$$\begin{bmatrix} 4.5 & 2.876 \\ 15.188 & -1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} -5.13 \\ -1.536 \end{bmatrix}$$

$$\begin{bmatrix} 4.5 & 2.876 \\ 0 & -10.707 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} -5.13 \\ 15.778 \end{bmatrix}$$

$$-10.707 h_2 = 15.778 \qquad 4.5h_1 + 2.876h_2 = -5.13$$

$$h_2 = -\underline{1.474} \qquad h_1 = \frac{-5.13 + 4.239}{4.5}$$

$$h_1 = 0.198$$

$$h = \begin{bmatrix} 0.198 \\ -1.474 \end{bmatrix}$$

$$x^2 = x^1 + h = \begin{bmatrix} 2.25 \\ 1.438 \end{bmatrix} + \begin{bmatrix} 0.198 \\ -1.474 \end{bmatrix} = \begin{bmatrix} 2.448 \\ -0.036 \end{bmatrix}$$

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