Uncertainties in Hurricane Trajectories derived from an Empirical Orthogonal Function Decomposition of Best Track Data

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May 5, 2021

Abstract

A recently proposed hurricane track perturbation procedure is applied to a number of storms for evaluation. This procedure relies on the Best Track data to quantify the variability in the storm's forward speed during its lifespan, using an Empirical Orthogonal Function (EOF) procedure. The singular values are modified by two uncertain amplitudes which generate the alternative paths. The perturbed tracks envelop the Best Track data but their characteristics remain broadly similar to the Best Track which appear as the mean perturbation. This result holds even for storms that have complex trajectories including turns and loops.

1 Introduction

My research topic will evaluate a recently proposed procedure [Sochala et al., 2020] to perturb hurricane trajectories. The purpose of this research is to contribute to quantifying uncertainties in a storm surge forecast caused by the uncertainty in a storm's trajectory. A hurricane making landfall can cause catastrophic damage to thousands, if not millions of people. The most significant contributor to this damage is from the storm surge. Lives and money could be saved by creating more reliable forecasts by accounting for uncertainties in the hurricane's path.

The proposed procedure relies on the best track data to identify the variability of a hurricane's path. Once identified, this variability can be used to generate alternative plausible hurricane paths that can be used later to quantify the uncertainty in the storm surge. The research will rely on the Best Track data which provides observations of hurricane properties every six hours including center position, size, strength, etc. Then, we take the current path of the hurricane and its velocities, figure out the variability in the speed, and perturb it to see what kind of alternate trajectories the hurricane is likely to take. A model surrogate is created to show the change in the outputs of the model based on the inputs. Then, once the model has been established, statistical analysis is performed as well as exceedance probability maps to estimate which regions are most vulnerable to storm surge. The conclusion of the paper was that the biggest uncertainty is not in the strength of the hurricane, but its trajectory and where it will hit land. This procedure was illustrated in Sochala et al. [2020] for hurricane Gustav 2008; its suitability to other hurricanes remains unproven.

The goal of this research for MSC 411 is to apply this process by evaluating the perturbation procedure on many different hurricanes and to analyze the results. During previous work, I was able to store the data from several hurricanes and display their trajectories individually on a map using Python, with the help of Professor Iskandarani. My python program can read in all the hurricane best track data and place it into a tidy data structure so that we can repeat the proposed procedure on several hurricanes. This research will involve the application of techniques such as object oriented programming, linear algebra, and statistics. I took this course for 3 credits, requiring an estimated 9 hours per week of work. Professor Iskandarani and I agreed to meet on Tuesdays and Thursdays from 9:30 am to 11:00 am. This report presents the methodology in section 2, the results of applying to hurricanes in section 3, and section 4 presents a discussion of the results.

2 Methods

2.1 Calculating Hurricane Velocity

First, we needed to calculate the storm's velocities at each time given the position (λ, θ) of the storm's center at the time, t. The velocities are computed using:

$$u(t) = R\cos\theta \frac{\mathrm{d}\lambda}{\mathrm{d}t} \tag{1}$$

$$v(t) = R\frac{\mathrm{d}\theta}{\mathrm{d}t} \tag{2}$$

where $u(t)$ represents the zonal velocity (East to West) in m/s, and $v(t)$ represents the meridional velocity (North to South) in m/s. (λ, θ) are the longitude and latitude in radians, and R is the radius of Earth in meters. To calculate these velocities we need to approximate the time derivative. However, the derivative cannot be calculated because each velocity is a discrete position and you cannot take the derivative of each single point. To solve this we implement the finite difference formula to create the following expressions for the time derivatives:

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} \approx \frac{\lambda_{n+1} - \lambda_n}{\Delta t} \tag{3}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} \approx \frac{\theta_{n+1} - \theta_n}{\Delta t} \tag{4}$$

where $\Delta t = 6$ hours, where the derivatives are estimated at the half-way point between the two reporting times t_n and t_{n+1}. By using the following equations we are able to obtain the velocities:

$$u_{n+\frac{1}{2}} = R\cos\left(\frac{\theta_n + \theta_{n+1}}{2}\right)\frac{\lambda_{n+1} - \lambda_n}{\Delta t} \tag{5}$$

$$v_{n+\frac{1}{2}} = R\frac{\theta_{n+1} - \theta_n}{\Delta t} \tag{6}$$

$n = 0, 1, \ldots, N$ where N is the number of time snapshots reported in the Best Track report.

2.2 Calculating Trajectory from Hurricane Velocity

The previous calculations in equations 5 and 6 can be rearranged algebraically to compute the next position of the storm's center given its zonal and meridional velocities. We obtain the formula:

$$\lambda_{n+1} = \lambda_n + \frac{u_{n+\frac{1}{2}}\Delta t}{R\cos\frac{\theta_n + \theta_{n+1}}{2}} \tag{7}$$

$$\theta_{n+1} = \theta_n + \frac{v_{n+\frac{1}{2}}\Delta t}{R} \tag{8}$$

This step is trivial now, but later we will use these results to perturb the velocities from a single starting point, with a small variation for each perturbation at each time interval, resulting in the most likely variations from the Best Track data. The results of the new position calculation can be seen in figure 1. which illustrates the accuracy of the recomputed trajectory (seen in blue).

2.3 Calculating the Velocity Variability

The velocities from Section 2.2 are arranged into a matrix of the form:

$$\boldsymbol{V} = \begin{bmatrix} u_0 & u_1 & u_n & \cdots & u_N \\ v_0 & v_1 & v_n & \cdots & v_N \end{bmatrix} \tag{9}$$

In the python program, the matrix above is implemented as a 2-Dimensional array called Vel. Then we decompose the matrix which results in the form:

$$\boldsymbol{V} = \overline{\boldsymbol{V}} + \boldsymbol{\Sigma}\boldsymbol{V}' \tag{10}$$

where $\boldsymbol{\Sigma}$ is a diagonal scaling matrix filled with the standard deviations, $\overline{\boldsymbol{V}}$ is the time-mean of the matrix (along the rows), and $\boldsymbol{V}'$ is our unknown component, the velocity anomaly matrix. The matrix is named VelAnom in the program. Next we set up a singular value decomposition:

$$\boldsymbol{V} = \overline{\boldsymbol{V}} + \boldsymbol{\Sigma}\boldsymbol{U}\boldsymbol{S}\boldsymbol{W}^\top \tag{11}$$

where U is a 2 x 2 matrix of the left eigenvector, S is a 2 x 2 matrix containing the singular values, and W a 2 x N matrix containing the right eigenvector. The calculations proceed as follows:

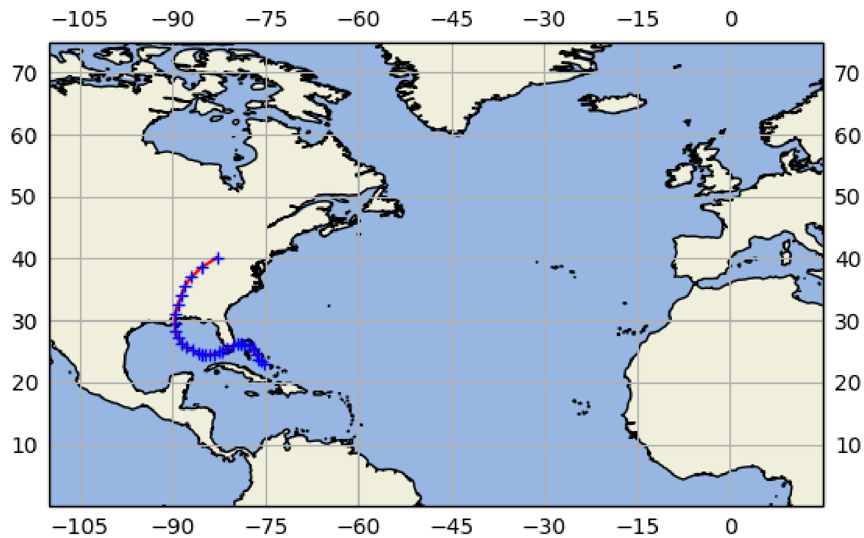


Figure 1: Best Track Data in red, Re-computed Trajectories from Algorithm in Section 2.2 in blue plus signs of Hurricane Katrina 2005

1. The matrix Vbar is obtained by replicating the column vector v (N+1) times. There is no explicit need to form it for the following calculations and the vector v will be sufficient. We compute the mean velocity:

$$\overline{\boldsymbol{v}} = \left[\begin{array}{c} \overline{u} \\ \overline{v} \end{array} \right] = \frac{1}{N+1} \sum_{n=0}^{N} \left[\begin{array}{c} u_n \\ v_n \end{array} \right], \tag{12}$$

   The matrix $\overline{\boldsymbol{V}}$ is obtained by replicating the column vector $\overline{\boldsymbol{v}}$ (N+1) times. There is no explicit need to form it for subsequent computations and the vector $\boldsymbol{v}$ will be sufficient.

2. Next we compute the velocity variance and the standard deviations:

$$\left[\begin{array}{c} \sigma_u^2 \\ \sigma_v^2 \end{array} \right] = \frac{1}{N} \sum_{n=0}^{N} \left[\begin{array}{c} (u_n - \overline{u})^2 \\ (v_n - \overline{v})^2 \end{array} \right], \quad \boldsymbol{\Sigma} = \left[\begin{array}{cc} \sigma_u & 0 \\ 0 & \sigma_v \end{array} \right] \tag{13}$$

   There is no reason for us to explicitly form the matrix $\boldsymbol{\Sigma}$ for the computations.

3. Then we form the normalized velocity anomaly matrix $\boldsymbol{V}'$

$$\boldsymbol{V}' = \left[\begin{array}{ccccc} \frac{u_0 - \overline{u}}{\sigma_u} & \frac{u_1 - \overline{u}}{\sigma_u} & \frac{u_n - \overline{u}}{\sigma_u} & \cdots & \frac{u_N - \overline{u}}{\sigma_u} \\ \frac{v_0 - \overline{v}}{\sigma_v} & \frac{v_1 - \overline{v}}{\sigma_v} & \frac{v_n - \overline{v}}{\sigma_v} & \cdots & \frac{v_N - \overline{v}}{\sigma_v} \end{array} \right] \tag{14}$$

4. Finally, we compute the Singular Value Decomposition of the matrix $\boldsymbol{V}'$ by invoking the function numpy.linalg.svd

2.4 Calculating the Perturbations

To perturb the track, we look at what the main modes of variability of the hurricane velocity values are. Then, we include an unknown (equation 8). The unknowns are the amplitudes of the modes, ξ_1 and ξ_2. We will use the following equation to modify the trajectory into 30 perturbed trajectories by changing the fluctuation in the velocity according to the following equation:

$$\boldsymbol{V}_{uq}(\xi_1, \xi_2) = \overline{\boldsymbol{V}} + \boldsymbol{\Sigma}\boldsymbol{U}(\boldsymbol{I}_2 + \alpha\boldsymbol{D})\boldsymbol{S}\boldsymbol{W}^\top \tag{15}$$

where $\boldsymbol{I}_2$ is the 2×2 identity matrix and $\boldsymbol{D}$ is a 2×2 diagonal matrix with entries ξ_1 and ξ_2 on the main diagonal. ξ_1 and ξ_2 are random variables with a uniform distribution between -1 and 1; setting $\xi_1 = \xi_2 = 0$ reproduces the Best Track trajectory. In Equation 9 from the original paper, the constant variable α is introduced, and the formula is restructured to include V_{bt} (of size 2 x N). This is the matrix which contains the Best Track data.

$$\boldsymbol{V}_{uq} = \boldsymbol{V}_{bt} + \alpha\boldsymbol{\Sigma}\boldsymbol{U}\boldsymbol{D}\boldsymbol{S}\boldsymbol{W}^\top \tag{16}$$

Matrix D is denoted by $a_{11} = 1 + \alpha * \xi_1$ and $a_{22} = 1 + \alpha * \xi_2$. α is a constant value that determines the size of the kick in the perturbations. The value of $1 + \alpha * \xi_1$ or ξ_2 is the magnitude of the variability in the perturbation. Note that if $1 + \alpha * \xi_1$ or $\xi_2 = 0$, then the Best track data will be produced because there is no variability. Finally, the uncertain hurricane track is calculated by integrating the velocities with respect to time using equations 7 and 8 in section 2.2, which are evaluated at the halfway point.

3 Results and Discussion

We reproduced Figure 5 from the original paper once the Best Track data was encapsulated by the Python program. The first plot shows the 30 velocity realizations of the storm's forward speed compared to the Best Track data (in red) for both the zonal (in green) and meridional (in orange) speeds. These perturbations were obtained while $\alpha = 0.15$, and ξ_1 and ξ_2 being uniformly distributed between the limits -1 and 1. The second plot shows the corresponding storm tracks for each of the 30 realizations (in blue), compared to the Best Track (in red). Then, we applied the procedure to numerous storms in the available data to generate the results to help determine the vailidity of this procedure. A summarized version of the documentation for the Python program is in the Appendix.

The procedure doesn't really generate radically new trajectories, but all of the perturbations seem to envelop the Best Track data. Although the procedure doesn't generate new tracks, it produces realizations that follow the BT no matter how the storm's track changes, which is visible in storms like Ivan, 2004, figure 3, that have a lot of turns. Twists and turns in the BT may be delayed or displaced, but qualitatively speaking their characters remain broadly the same. The best track data falls in the middle of the perturbations. It is also important to note the usage of the constant parameter α, which is the value that helps regulate the magnitude of how much our realizations deviate from the mean. For the figures produced, α equals 0.15. However, after producing the figures of many storms there were several characteristics identified in their tracks based on the velocity realizations. The basis functions produced by taking a singular value decomposition of the velocity fluctuation matrix are responsible for the changes to the best track data. These changes can be observed in the track and velocity perturbation figures to draw conclusions from their characteristics.

The two most apparent characteristics in the realizations is that they diverge from the BT when the variability in the trajectory and velocities is relatively high, and that the realizations converge to the BT following a sudden turn in the trajectory. The high variability that results in diverging behaviour can be described as the storm taking many turns over a given time period. This characteristic can easily be seen in storms like Harvey, 2017, figure 6, which have a period of high variability. Alternatively, this can also be seen in storms that follow a straight path like Alex, 2010, figure 11, where the realizations are nearly identical to the BT. Looking at the velocity perturbation plots for those storms we can see that the realizations are much further from the BT in Harvey, 2017, than in Alex, 2010, which corroborates with the characteristic that a higher variability causes divergence from the BT. This result is expected because our calculations are entirely linear and do not account for atmospheric dynamics. The second characteristic that we observed was unexpected. In each trajectory plot it is apparent that the realizations diverge from the BT when there are gradual turns in the storm's forward speeds and trajectory relative to time, and the realizations quickly converge to the BT when there is a sudden turn in the track. The diverging behaviour in gradual turns is apparent in Igor, 2010, figure 5, Earl, 2010, figure 9, and Maria, 2017, figure 8. In Igor, 2010, the realizations are from the BT until there is a sudden turn in the track, and the realizations are corrected. We also observe the converging characteristic after a sudden turn in Wilma, 2005, figure 4, and Maria, 2017, figure 8, because they each have a period of high variability or a long turn (which cause the realizations to diverge) followed by an abrupt turn, and as long as there is a period of low variability following that turn the realizations quickly converge. Both properties that were discussed are visible in Karl, 2004, figure 10, because it shows in the first few days that a gradual turn causes realizations to diverge, and once the storm abruptly turns the realizations converge to the BT, and after another gradual turn the realizations diverge once again. Then, another sudden turn in Karl's track causes the realizations to converge once again. We expected the realizations to be less accurate following an abrupt turn, but our procedure does not give that result because it calculates each track using the existing BT data and does not predict entirely new tracks.

4 Conclusion

The purpose of this research was to generate perturbations to the velocity and use them to calculate new plausible tracks for any storm in the Best Track data, to test the validity of the procedure by analyzing the produced figures. The realizations are centered on the BT and entirely new tracks are not generated. This is an expected result because our procedure is entirely algebraic, and we do not account for other dynamics like atmospheric flow perturbations or the changes in the upper air flow. Additionally, the calculations rely on the availability of existing data, so producing entirely unique tracks is rare and only happens under specific circumstances, like when a storm has several gradual turns and high variability, with no abrupt turn at all, like in Maria, 2017, figure 8. The realizations diverge from the mean in these situations and can begin to produce unique turns before a sharp turn is made, which corrects the tracks to the BT. However, in the majority of the storms, all realizations were exaggerations of the BT data, and not unique paths. The most interesting result was that the realizations diverge from the mean in gradual turns but quickly converge after a sudden turn. When this happens, the basis functions which are used to recalculate the trajectory are formed by the result of the previous calculation. So a gradual turn in the track gives gradual changes to the realizations which accumulate with each calculation. When the storm suddenly turns, the realizations converge to the BT because the sudden change quickly returns the results to the mean. Overall, we can say that the realizations are relatively accurate for many different storms no matter the shape, however new tracks are not produced.

5 Appendix

The full documentation of the Python program is available in the .py file itself, this is a summarized version. The data is encapsulated in a dictionary and is separated into arrays depending on what storm is selected. These arrays are fed into matrices and a Singular Value Decomposition is performed using matrix notation. Then the resulting matrices of perturbed velocities are used to recompute the trajectory for each realization, and the final results are plotted to produce the figures.

References

Pierre Sochala, Chen Chen, Clint Dawson, and Mohamed Iskandarani. A polynomial chaos framework for probabilistic predictions of storm surge events. *Computational Geosciences*, 24:109–128, 2020. doi: 10.1007/s10596-019-09898-5.

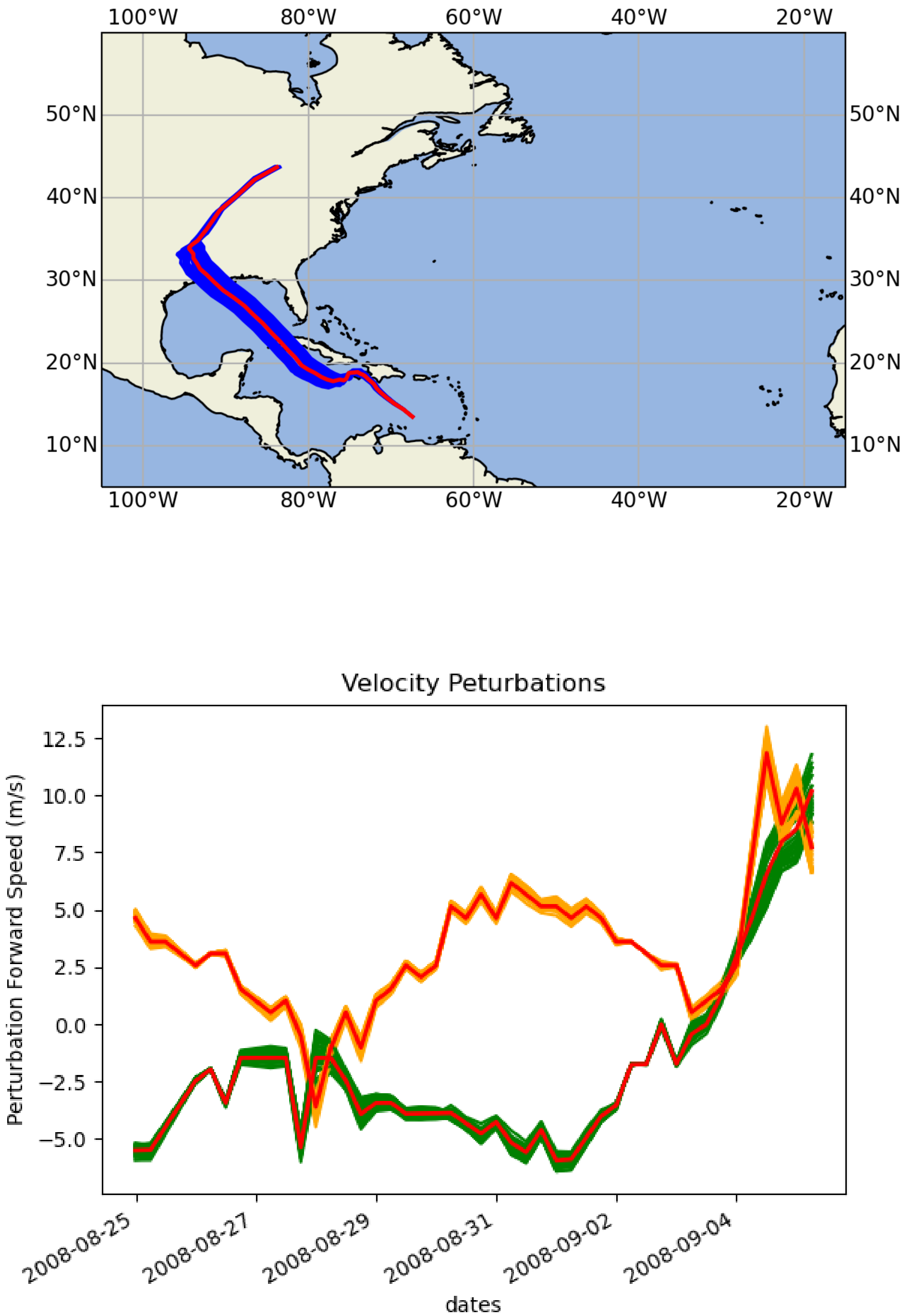


Figure 2: Gustav, 2008

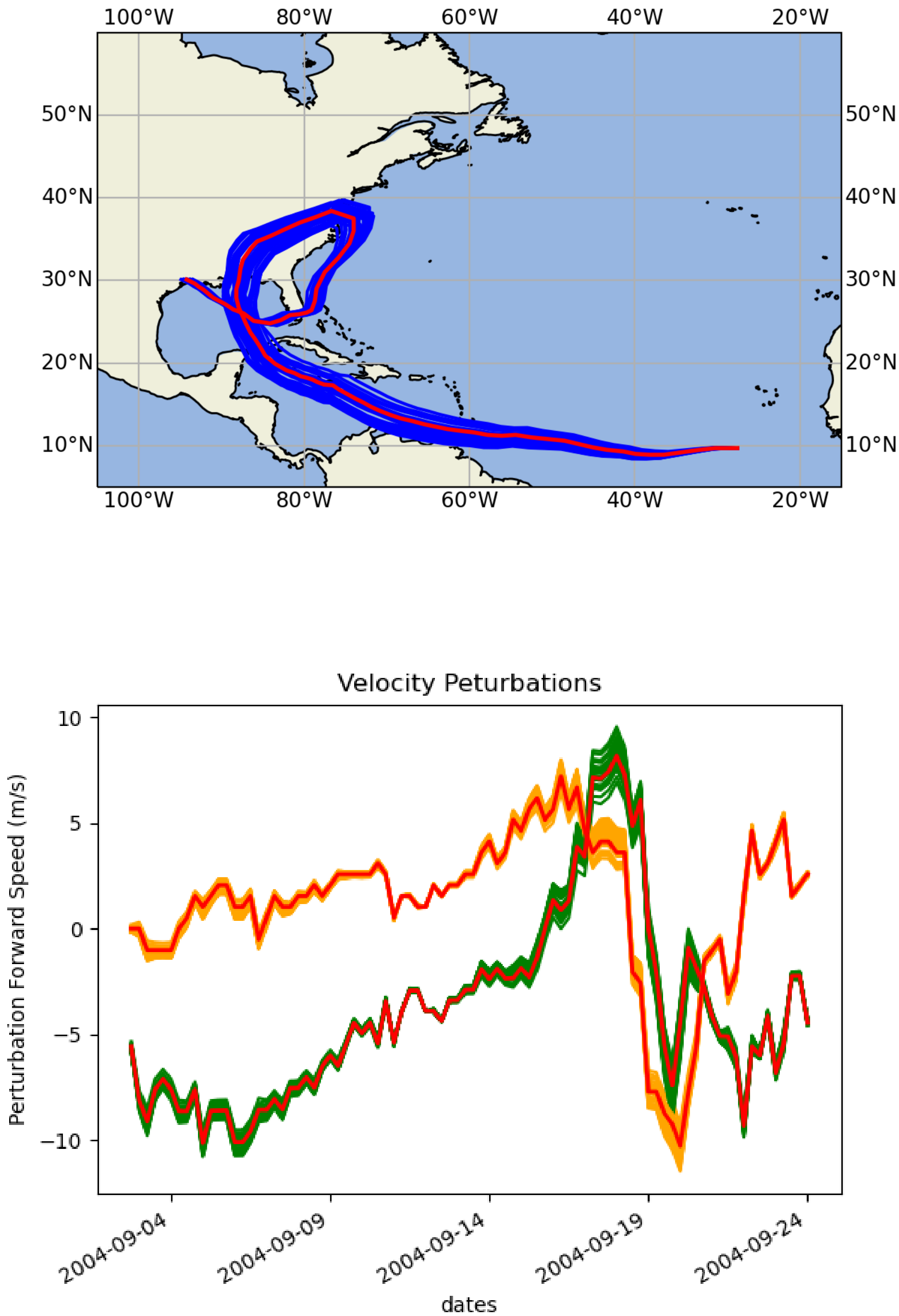


Figure 3: Ivan, 2004

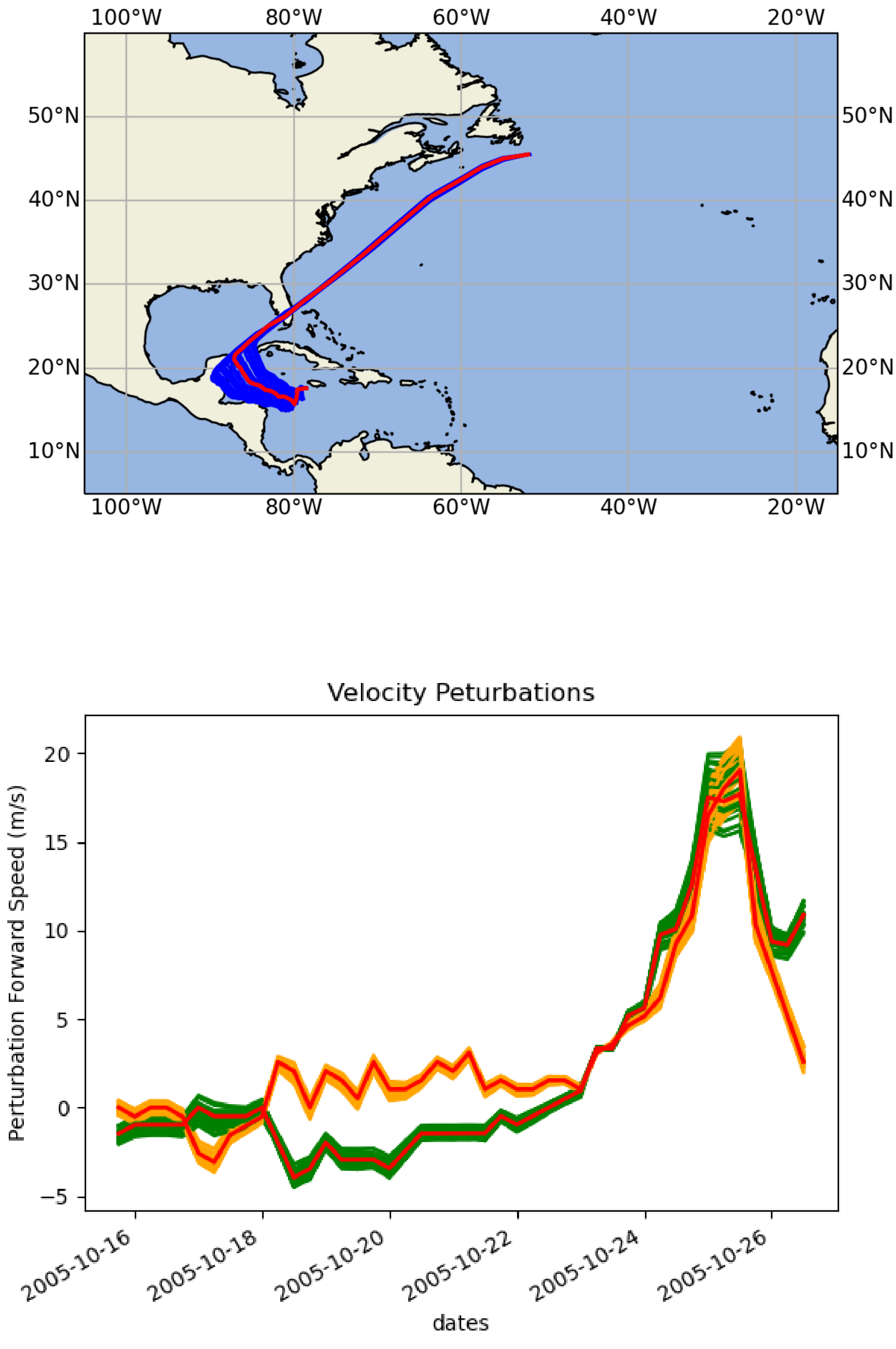


Figure 4: Wilma, 2005

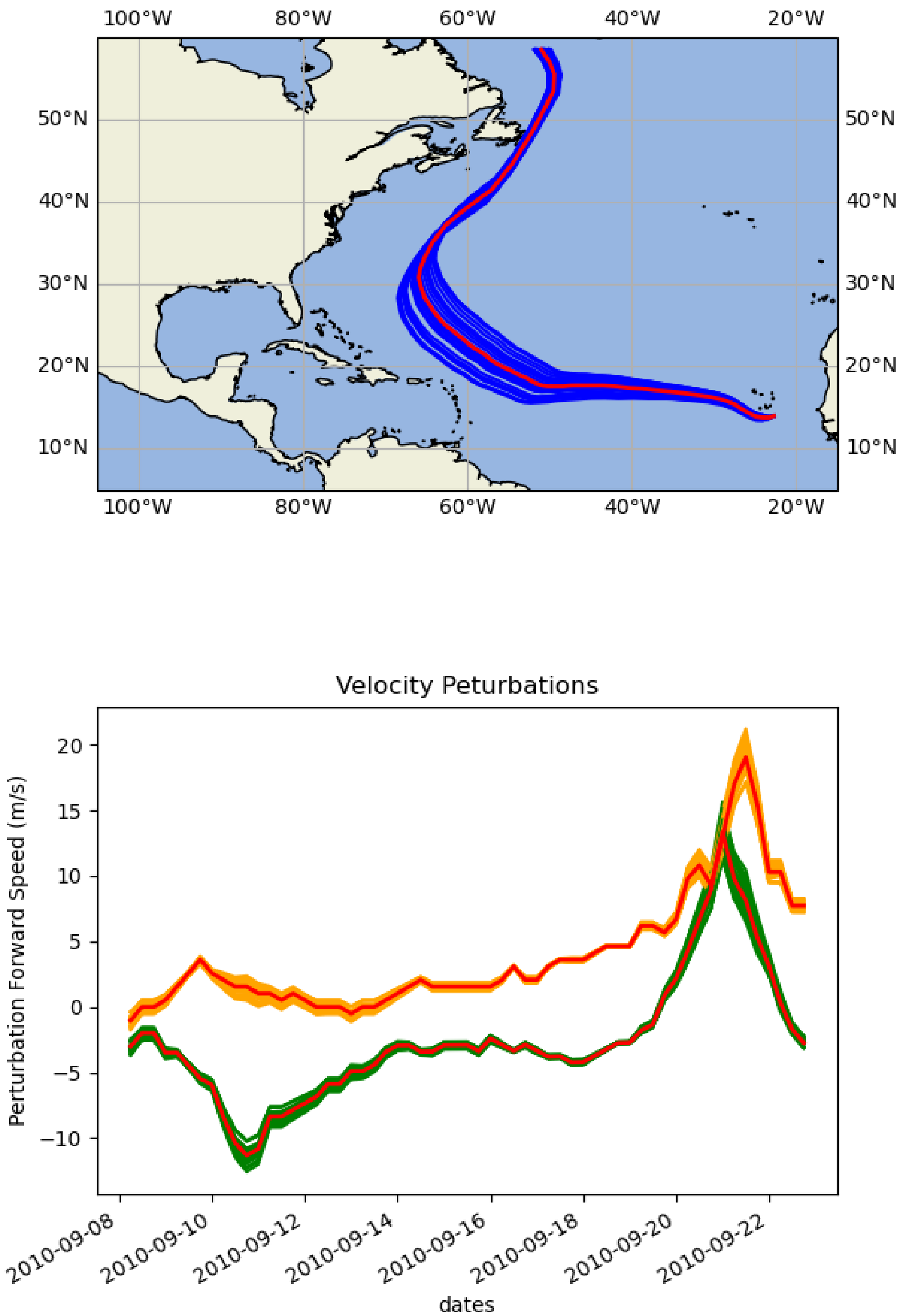
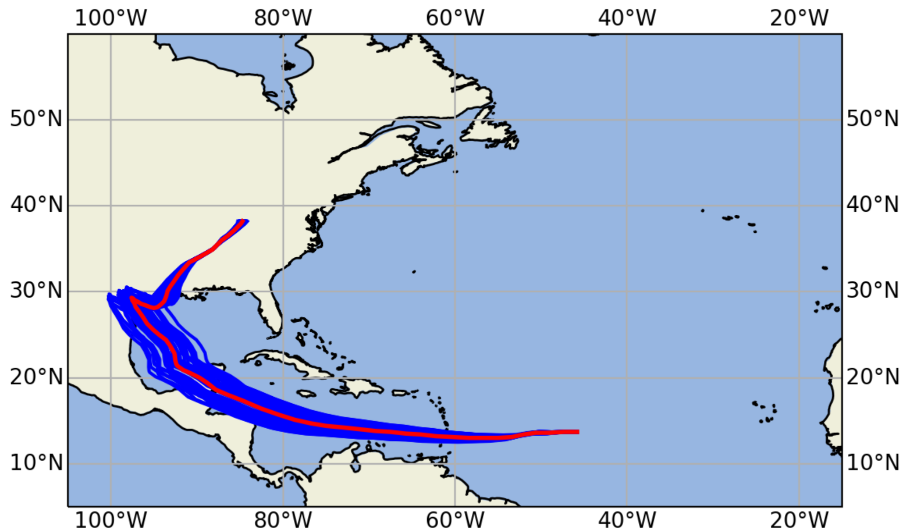


Figure 5: Igor, 2010



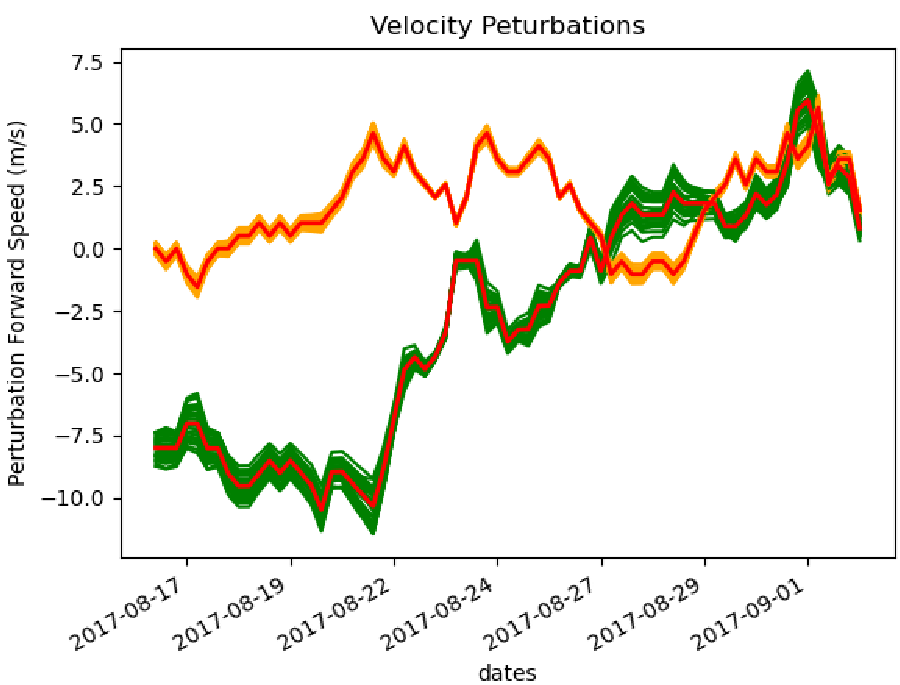


Figure 6: Harvey, 2017

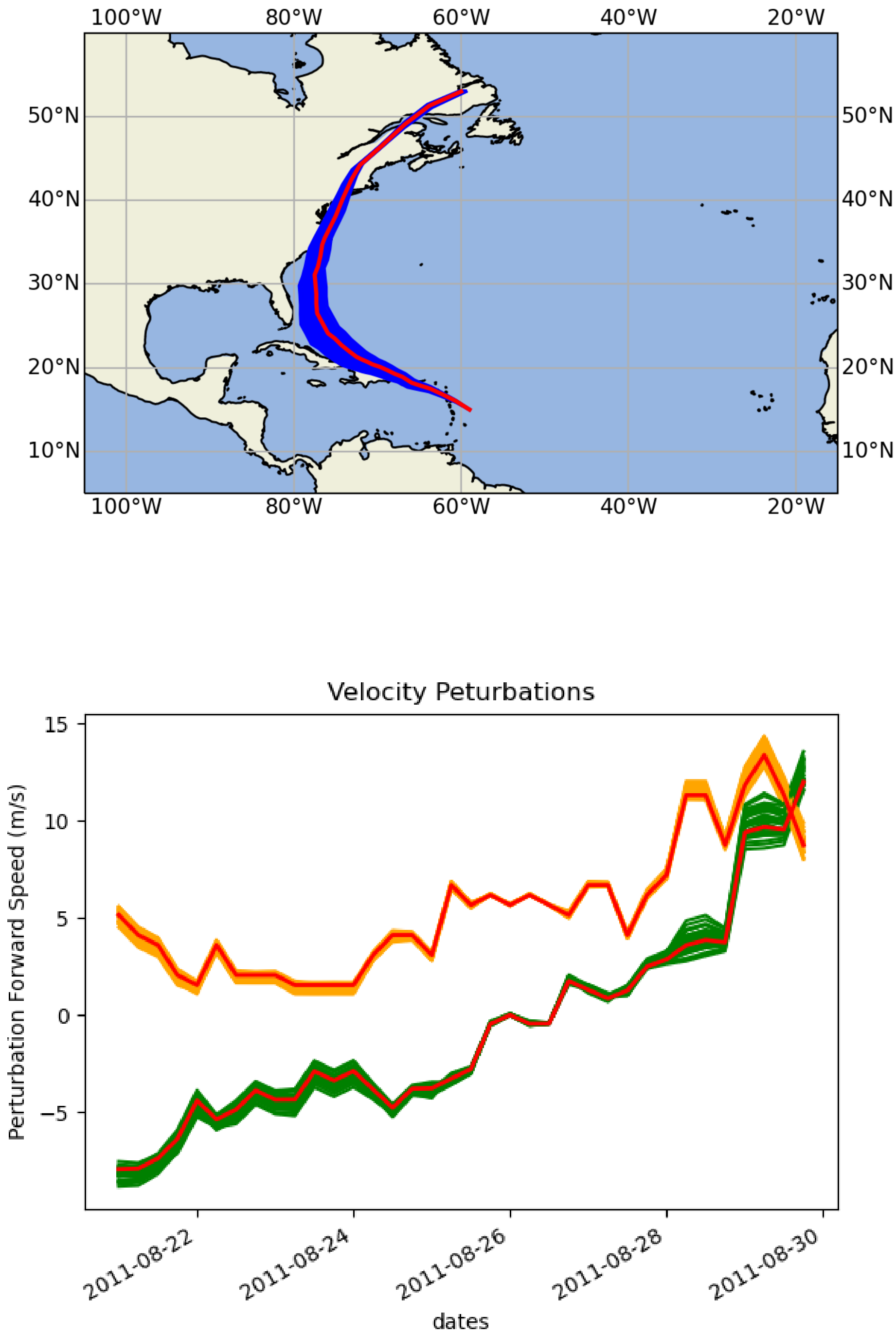


Figure 7: Irene, 2011

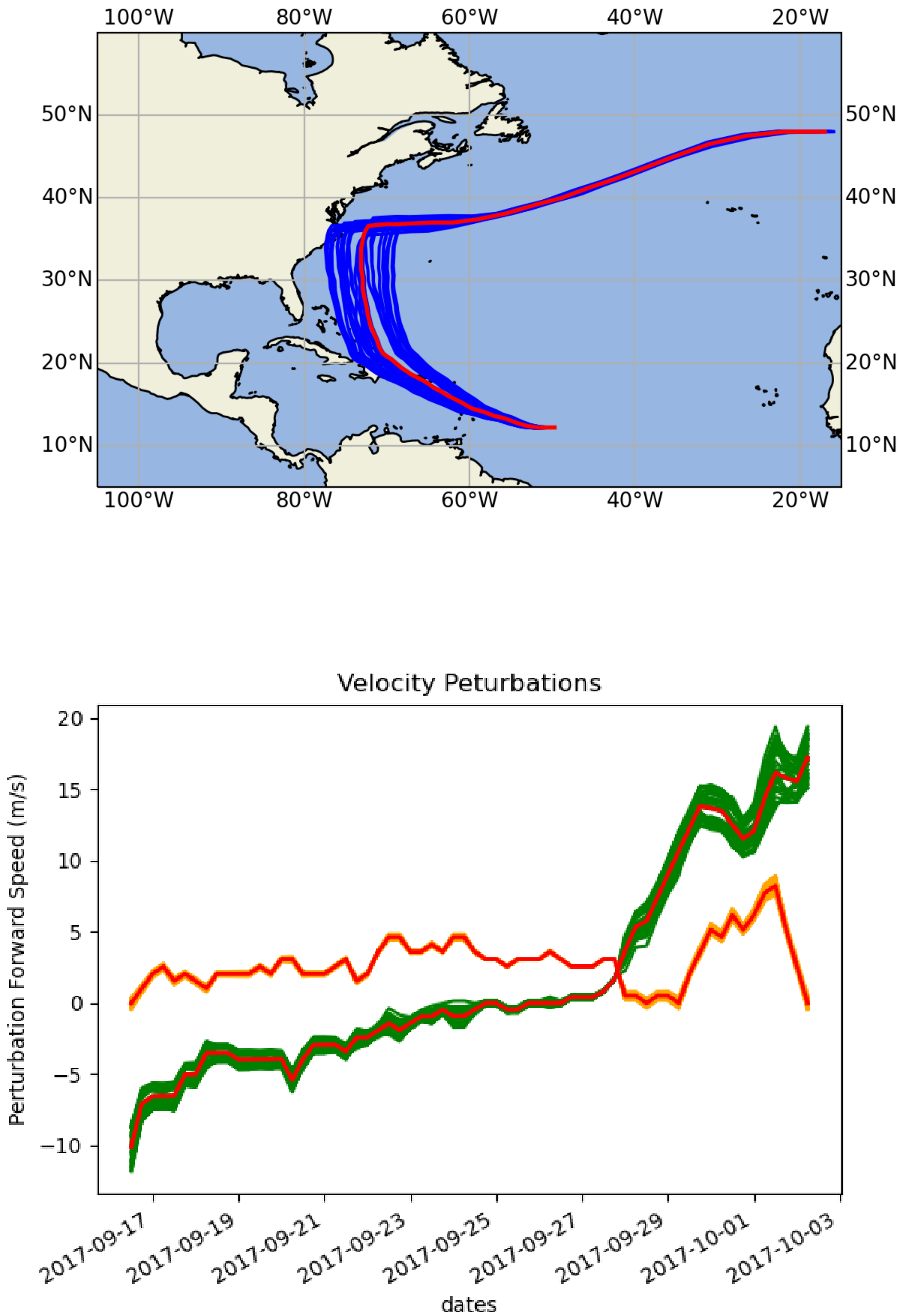


Figure 8: Maria, 2017

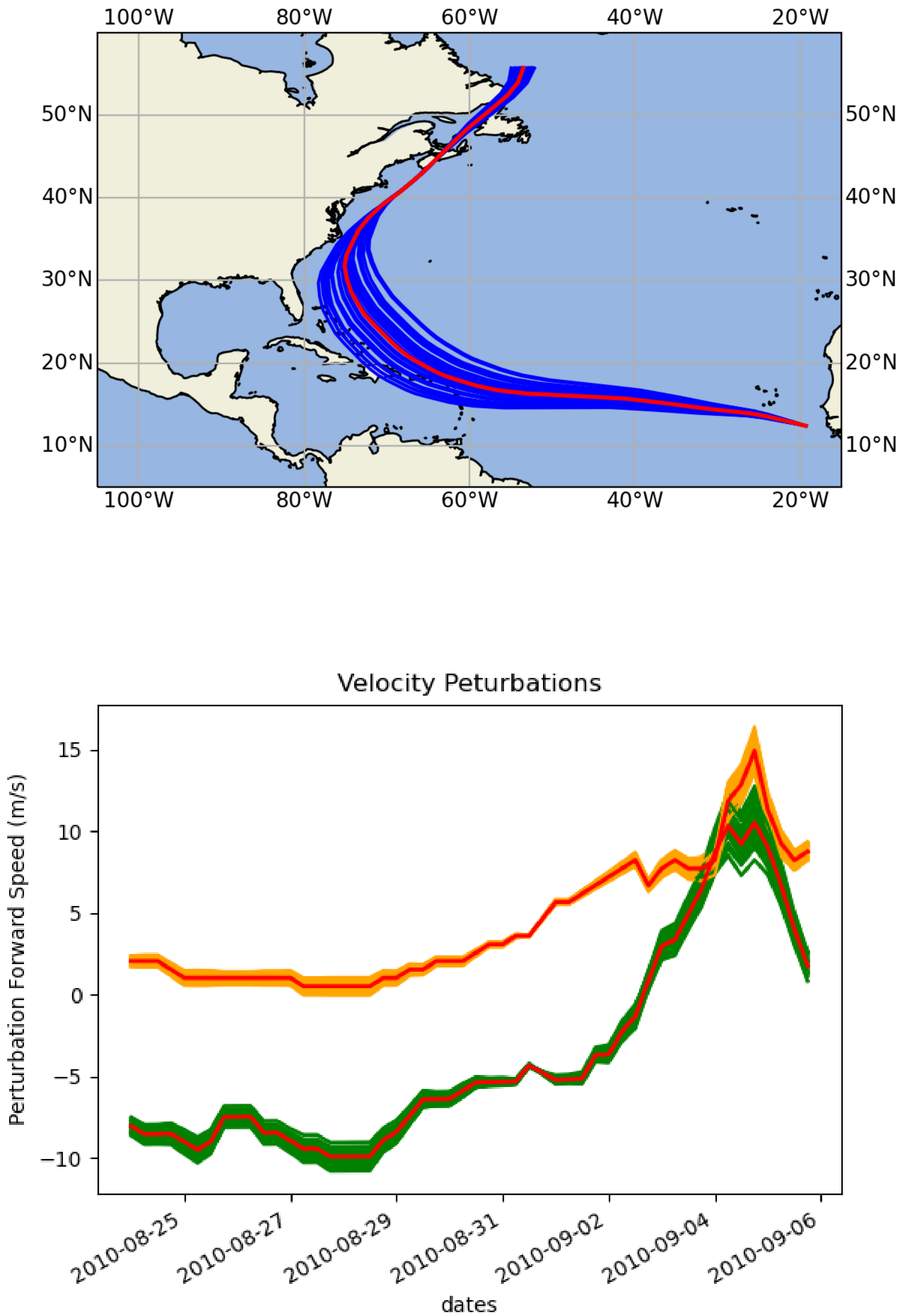


Figure 9: Earl, 2010

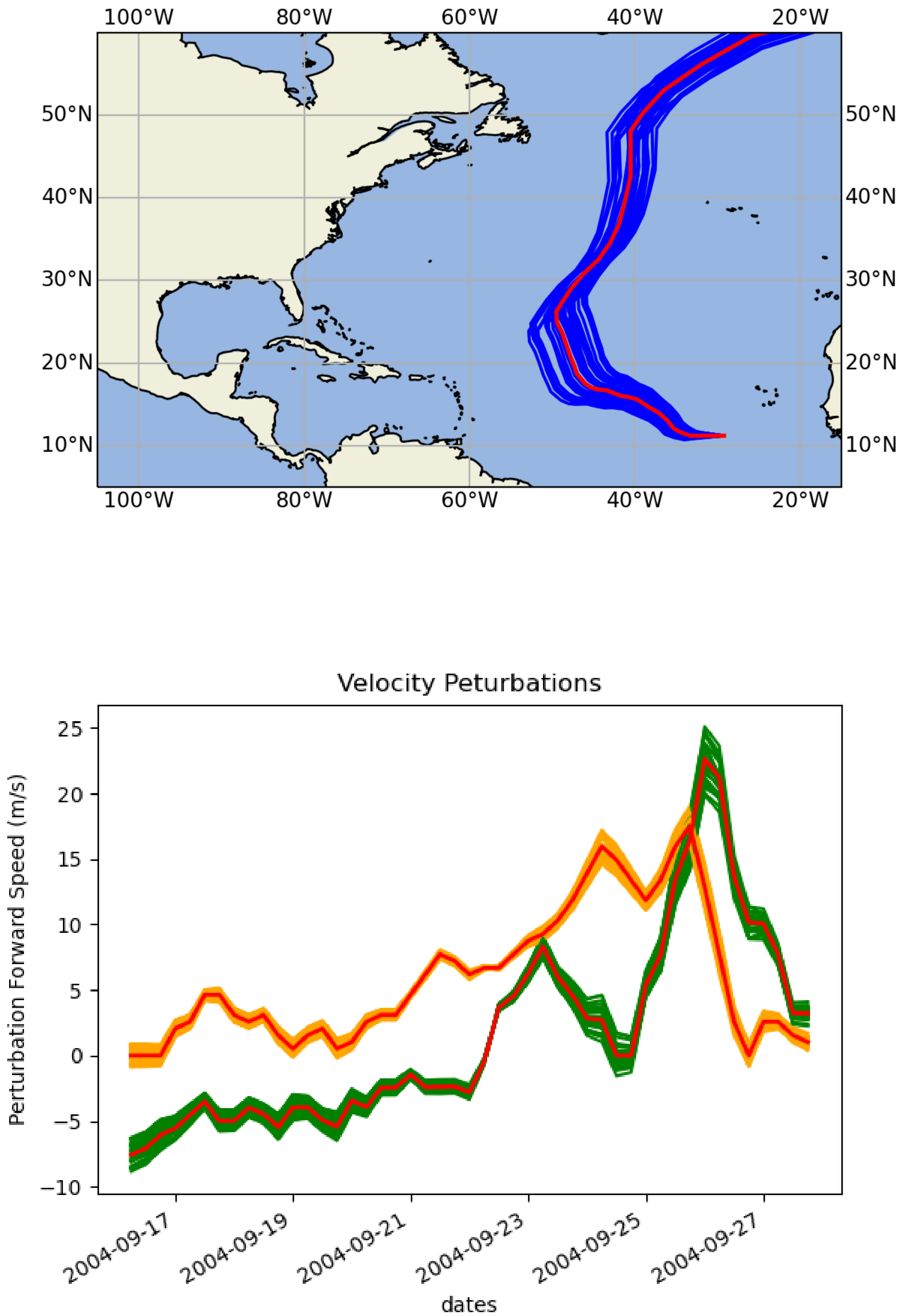


Figure 10: Karl, 2004

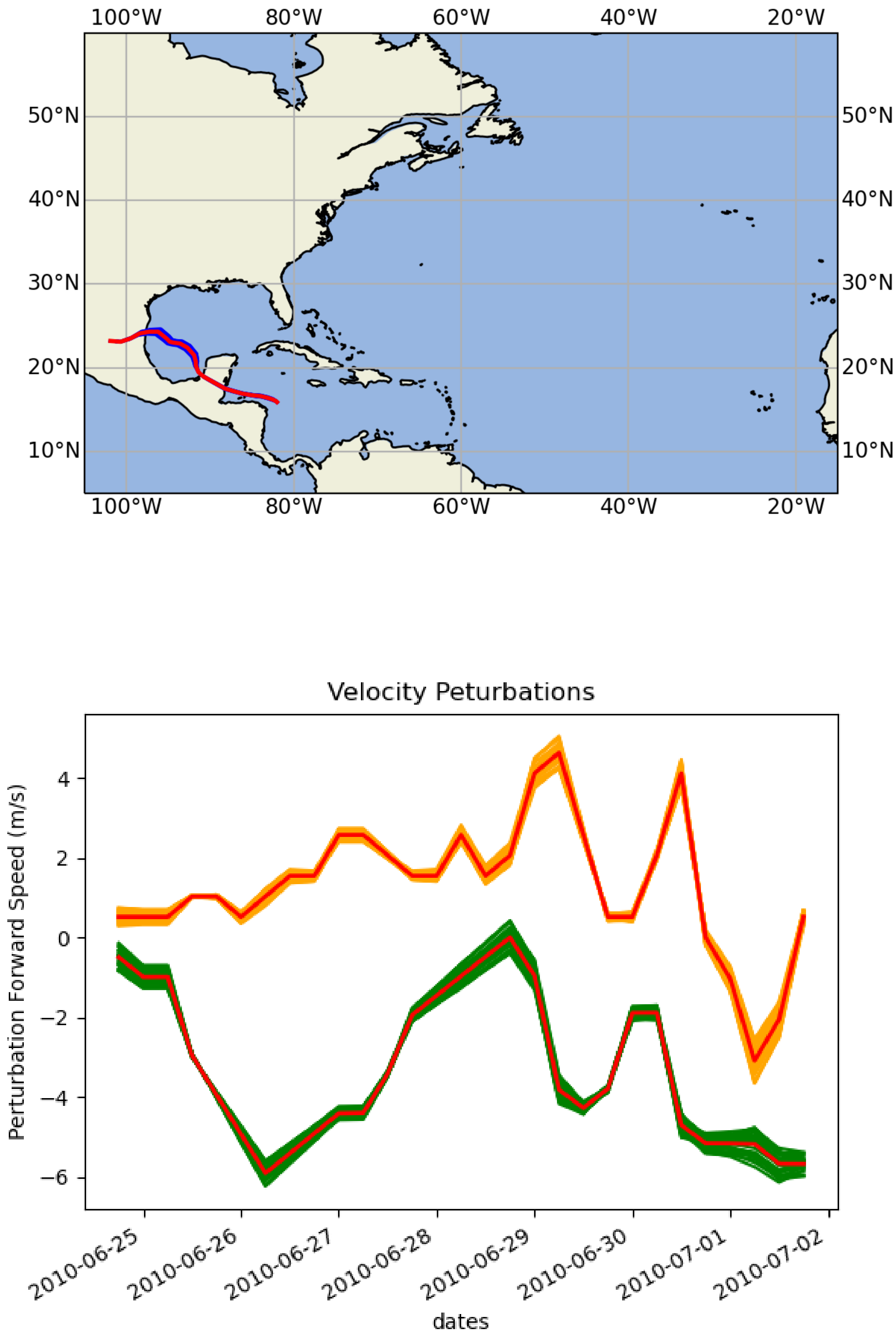


Figure 11: Alex, 2010